بسم الله الرحمن الرحيم

الحلول المختارة لطلاب الهندسة والعمارة

تحليل دوائر (1)

إعداد

د/ محمد يوسف رهاوي

رحمه الله

و سلطان محمد العاكي

الاستاذ المساعد بقسم الهندسة الكهربية والحاسبات

بجامعة ام القرى

الحمد لله وحده والصلاة والسلام على من لا نبي بعده سيدنا محمد وعلى آله وصحبه ومن تبعهم بإحسان،

وبعد، فهذه مجموعة من المسائل المحلولة في مادة تحليل دوائر (١) لطلبة الهندسة والعمارة إختارها بعضها الدكتور/ محمد يوسف برماوي رحمه الله حين درّس هذه المادة في الفصل الأول للعام ١٤٠٨هـ، وهي مأخوذة من كتاب:

Basic Electrical Engineering

للمؤلفين: A. Fitzgerald, D. Higginbotham & A. Grabel

الطبعة الخامسة ١٩٨٢م

وحرصاً على ألا ينقطع عمله رحمه الله فقد رجوت الدكتور/ مستشير أن يأذن لي في إخراج هذا البعض لأبنائنا الطلاب من نسخته الخاصة فاستجاب مشكوراً وزودني بما حضره للطلاب حين درّسهم هذه المادة في الفصل الأول للعام ١٤١٠هـ. ثم زدت الآن بعضاً آخر هو حصيلة ما أعطيتهم إياه آنذاك لتخرج هذه المجموعة في إخراجها الثاني.

هذا وقد قمت بتبويبها وترتيبها واستدراك ما فيها من الهفوات لتسهل وتتم المراجعة فيها، واكتفيت عند ذكر المسألة بذكر رقمها والصفحة التي وردت فيها في الكتاب المذكور بعاليه والمقرر لمادة تحليل دوائر (١) لطلاب الهندسة والعمارة الإسلامية بجامعة أم القرى،

والله أسأل أن يوفقه وينفع بهذا العمل كل من طالعه وأن يرحم كاتبه الأول برحمته التي وسعت كل شيء إنه سميع قريب مجيب.

سالم العمام
١٤١٤/٤/٢هـ

الفهرس

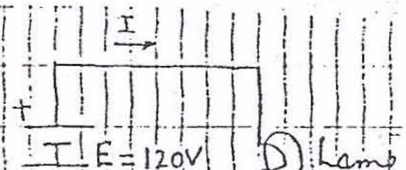
الصفحة

الصفحة

الصفحة

$\boxed{\frac{1-1}{28}}$

(a) $I = \frac{P}{E} = \frac{100}{120} = 0.833\,A$



(b) $Q = It = 0.833\,(60 \times 60) = 3{,}000\,C$

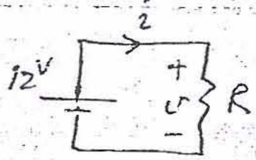
(c) $W = P_l t = 100\,(365 \times 10) = 365{,}000\ Wh = 365\ kWh$

$\therefore$ Cost = W x cost of energy = 365 x 6 = 2190 cents = 21.9 dollars

$\boxed{\frac{1-2}{28}}$

a) $120 = P = iV = 12i$

$\therefore$ $i = 10$ Amp, the load current



b) $60\ Ah = iT = 10A.T$

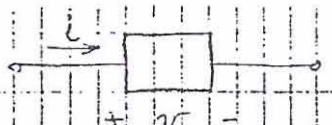
$\therefore$ T = 6 hours, the duration of supply

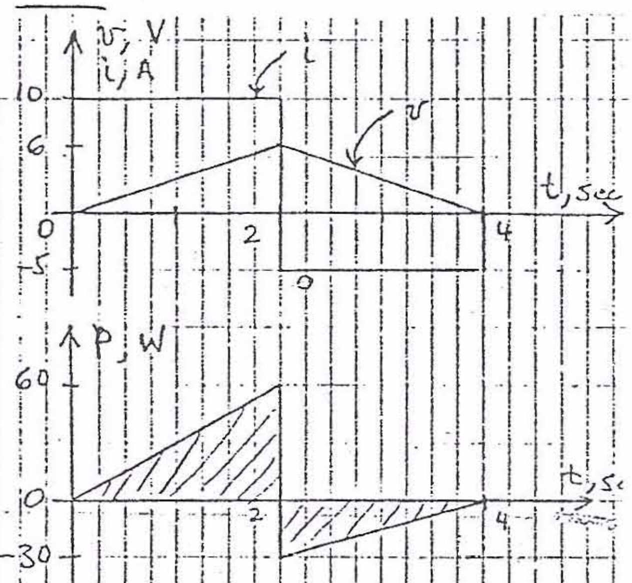
c) $q = iT = 10A * 6h = 60\ Ah = 60\,\frac{C}{s} * 3600\,s = 216000\,C$

$\therefore$ q, the total charge = 216 KC

d) $R = \frac{v}{i} = \frac{12V}{10A} = 1.2\,\Omega$, the load resistance.

$\boxed{\frac{1-3}{28}}$





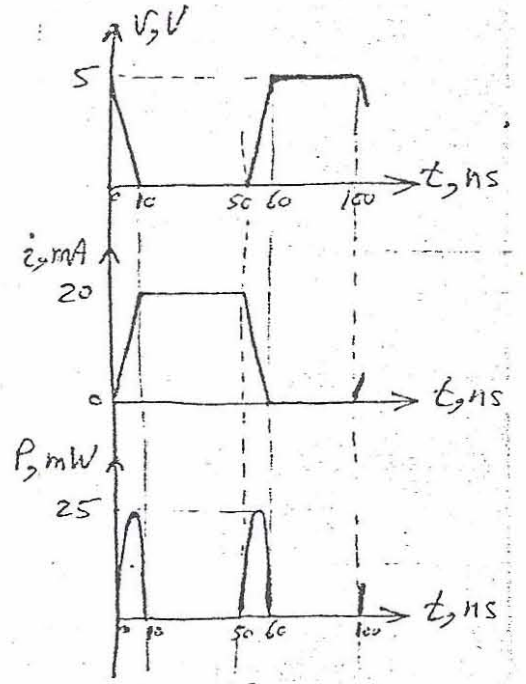
(a) $p = iv$

(b) Average power is $\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p\,dt$; $t_1 = 0$ sec ; $t_2 = 4$ sec

= (area under curve divided by the time)

$\therefore$ $P = \frac{1}{4-0}\left(\frac{1}{2} \times 2 \times 60 - \frac{1}{2} \times 2 \times 30\right) = \frac{30}{4} = 7.5\ W$

$\boxed{\frac{1-4}{29}}$ a) In $t \in [0, 10]$ ns;



$v(t) = -.5t + 5$, Volts &

$i(t) = 2t$, mA

$\therefore p(t) = 2t\,(5 - .5t) =$

$= 10t - t^2$, mW

This is a parabolic shape with maxima of 25 mW at $t = 5$ ns

Note: $P(0) = P(10) = 0$

In $t \in [10, 50] \cup [60, 100]$ ns;

$v(t) * i(t) = 0 \quad \therefore p(t) = 0$

In $t \in [50, 60]$ ns; the shape of $p(t)$ is just similar to that in $t \in [0, 10]$ ns

The sketch is therefore as shown above.

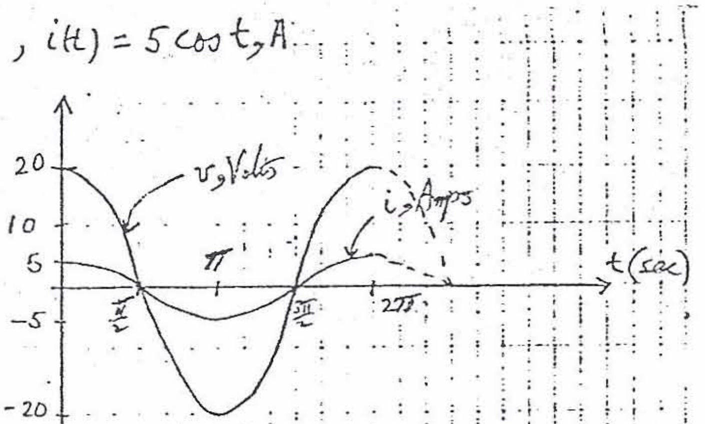
b) $P_{av} = \frac{1}{100} * 2 * \frac{2}{3} * 10 * 25 = 3.\bar{3}$ mW, average power during a period

c) $V_{av} = \frac{1}{100} * \frac{60+40}{2} * 5 = 2.5$ Volts, average voltage during a period.

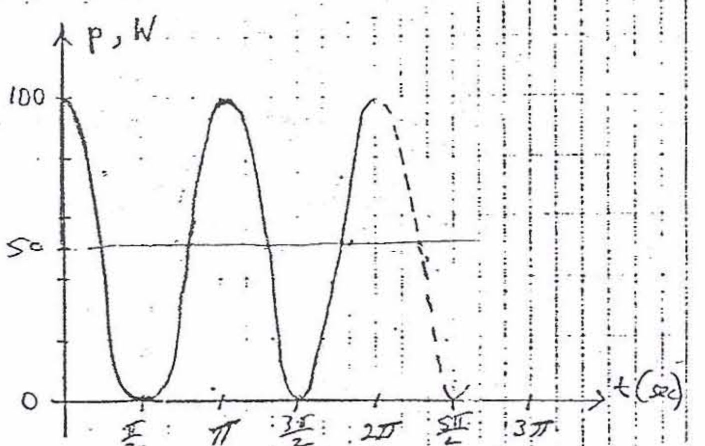
$i_{av} = \frac{1}{100} * \frac{60+40}{2} * 20 = 10$ mA, average current during a period.

d) $v_{av} \cdot i_{av} = 2.5 * 10 = 25$ mW $> 3.\bar{3}$ mW $= P_{av}$. The reason is due v in $t \in [10, 50]$ ns being zero hence no pow[illegible] its average gives fals

$\boxed{\frac{1-5}{29}}$ $v(t) = 20\cos t$, V , $i(t) = 5\cos t$, A



Notice that the voltage across a Resistor is in phase with the current throught it.



ⓐ $p = vi$

$= (20\cos t)(5\cos t)$

$= 100\cos^2 t$

$= \frac{100}{2}(1+\cos 2t)$

$= 50\,(1+\cos 2t)$ W

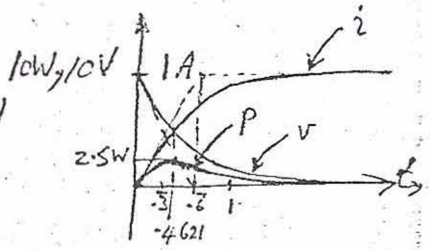
ⓑ $P_{av} = \frac{1}{t_2 - t_1}\int_{t_1}^{t} p\,dt$

$= \frac{1}{2\pi}\int_0^{2\pi}\frac{100}{2}(1+\cos 2t)\,dt = \frac{50}{2\pi}\left[t + \cancel{\frac{\sin 2t}{2}}^{\,0}\right]_0^{2\pi} = \frac{50}{2\pi}\times 2\pi = 50$ W

$\therefore \boxed{P_{av} = 50\text{ W}}$

$\boxed{\frac{1-9}{30}}$ a) $v(t) = 10e^{-1.5t}, V$ as shown

$i(t) = 1 - e^{-1.5t}, A$ as shown



$\therefore\ p(t) = v(t).i(t) = 10e^{-1.5t}\cdot(1-e^{-1.5t}), W$

$p(t)$ is max. at $t = t_1$

$\therefore\ p'(t_1) = 10(-1.5e^{-1.5t_1} + 3e^{-3t_1}) = 0$

$\therefore\ e^{1.5t_1} = 2 \quad \therefore t_1 = \frac{\ln 2}{1.5} = .4621$, sec

$\therefore\ p(t_1) = P_{max} = 10 * \frac{1}{2}\cdot(1 - \frac{1}{2}) = 5 * \frac{1}{2} = 2.5\,W$

$\therefore$ Max. power occurs at $t = .4621$ sec with value of 2.5 W

Note that $p(0) = p(\infty) = 0$ W & $p(t)$ is as shown.

$W\{\text{in } t \in [0, .\bar{3}]\} = \int_0^{.\bar{3}} p(t)dt = 10\int_0^{.\bar{3}}(e^{-1.5t} - e^{-3t})dt = 10\left[\frac{e^{-1.5t}}{-1.5} - \frac{e^{-3t}}{-3}\right.$

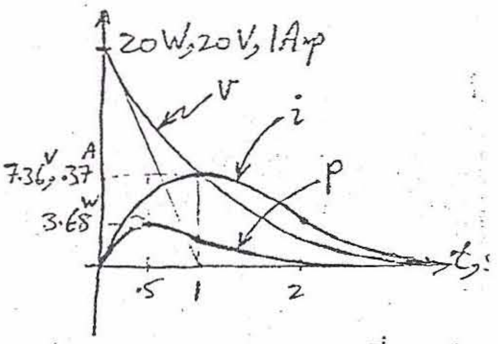
$= -\frac{10}{3}\left[2(e^{-1/2} - 1) - (e^{-1} - 1)\right] = -\frac{10}{3}\left(\frac{2}{\sqrt{e}} - 2 - \frac{1}{e} + 1\right) = \frac{10(e - 2\sqrt{e} + 1)}{3e}$

$\therefore\ W\{\text{in } t \in [0, .\bar{3}] \text{ sec}\} = \frac{10(e - 2\sqrt{e} + 1)}{3e} = 0.5161$ J

$W\{\text{in } t \in [0, 1] \text{ sec}\} = \int_0^1 p(t)dt = -\frac{10}{3}\left[2(e^{-1.5} - 1) - (e^{-3} - 1)\right] = 2.0118$ J

b) $v(t) = 20e^{-t}, V$ as shown

$i(t) = te^{-t}, A$ as shown



max i is at t_1: $e^{-t_1}(1 - t_1) = 0 \Rightarrow t_1 = 1$ sec.

$i_{max} = i(t_1) = 1e^{-1} = .36788$ Amp

$i(0) = i(\infty) = 0$

$\therefore\ p(t) = 20te^{-2t}, W$

$p(t)$ is max at $t = t_2$: $e^{-2t_2}(1 - 2t_2) = 0 \quad \therefore t_2 = .5$ sec & $P(t_2) = 20 * \frac{1}{2} * e^{-1} = 3.678$

$\therefore$ Max. power occurs at $t = 0.5$ sec with value of 3.6788 W.

Note that $P(0) = P(\infty) = 0W$ giving the shape of $p(t)$ shown above.

$W\{\text{in } t \in [0, .\bar{3}] \text{ sec}\} = \int_0^{.\bar{3}} 20te^{-2t}dt = -10te^{-2t} - 5e^{-2t}\Big|_0^{.\bar{3}} = 5e^{-2t}(-1 - 2t)\Big|_0^{.\bar{3}} = 5e^{-2t}(1 + 2t)\Big|_{.\bar{3}}^{0}$

$= 5e^{0}(1 + 0) - 5e^{-\frac{2}{3}}(1 + \frac{2}{3}) = 5 - \frac{25}{3}e^{-2/3} = 0.72152$ J

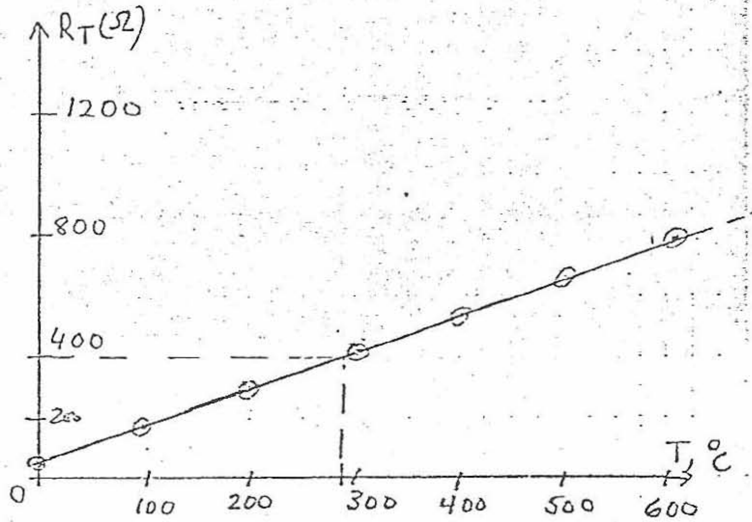
$W\{\text{in } t \in [0, 1] \text{ sec}\} = 5e^{-2t}(1 + 2t)\Big|_1^{0} = 5 - 15e^{-2} = 2.96997$ J

$\boxed{\frac{1-12}{30}}$ $R_T = R_o(1 + 5.42 \times 10^{-2} T - 4.17 \times 10^{-6} T^2)\ \Omega$

R_o = resistance @ $T = 0^\circ C$

(a) $R_o = 25\ \Omega$, $0^\circ C \leq T \leq 600^\circ C$

T (°C)	R_T (Ω)
0	25
100	159.5
200	291.8
300	422.1
400	550.3
500	676.4
600	800.5



(b) $V = 6\,V$, $I = 15\,mA$

$$R_T = \frac{V}{I} = \frac{6}{15 \times 10^{-3}} = 400\ \Omega$$

From the R_T curve ; for $R_T = 400\ \Omega \rightarrow \boxed{T \simeq 285^\circ C}$

To get correct answer; solve for T the above quadratic

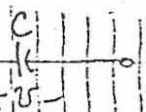
$$\therefore\ 400 = 25\left(1 + 5.42 \times 10^{-2} T - 4.17 \times 10^{-6} T^2\right)$$

$$4.17 \times 10^{-6} T^2 - 5.42 \times 10^{-2} T + 15 = 0$$

$$\frac{5.42 \times 10^{-2} \pm \sqrt{(5.42 \times 10^{-2})^2 - 4 \times 4.17 \times 10^{-6} \times 15}}{2 \times 4.17 \times 10^{-6}} = 12{,}714.7^\circ C \text{ OR } 282.9^\circ C$$

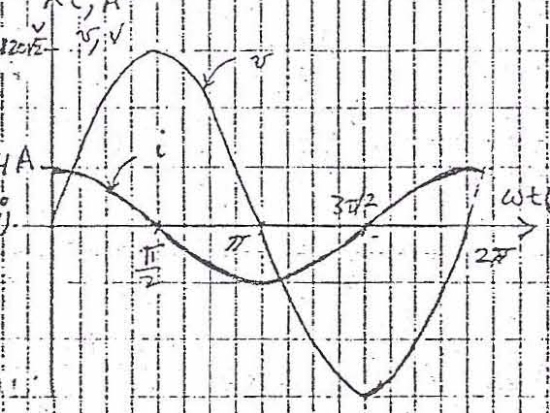
(out of range $[0, 600^\circ C]$) OR (within range

1-13 / 30 $C = 100\,\mu F$, $v = 120\sqrt{2} \sin 377t$ V



(a) $i = C\dfrac{dv}{dt} = 100\times10^{-6} \dfrac{d}{dt}(120\sqrt{2} \sin 377t) = 10^{-4}(120\sqrt{2}(377 \cos 377t))$

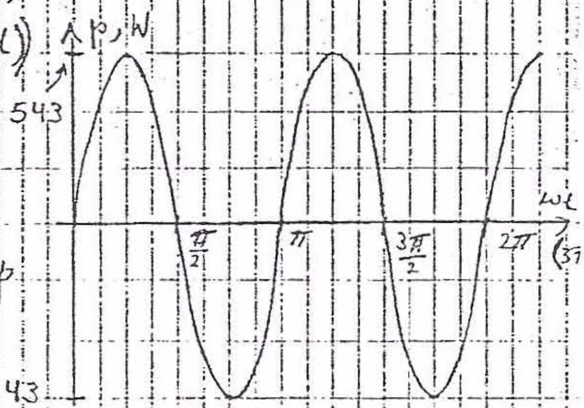
$\therefore \boxed{i = 6.398 \cos 377t \text{ A}}$



(b) Notice that in a Capacitor: the current leads the voltage by (90°).

(c) $p = vi = (120\sqrt{2})(6.398)(\sin 377t \cos 377t)$
$= 1086\left(\frac{1}{2} \sin(2\times377t)\right)$

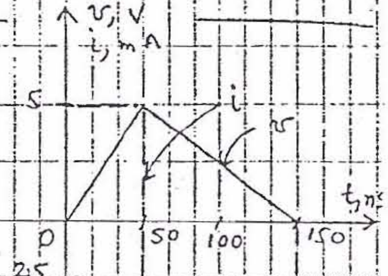
$\therefore \boxed{p = 543 \sin 754t \text{ W}}$



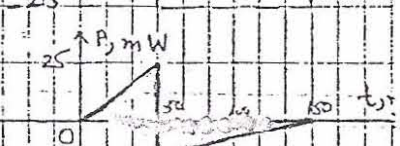
Notice that the frequency of p is twice that of v and i.

1-15 / 31 $C = 50\,pF$

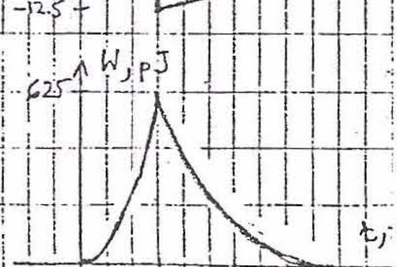
$$v(t) = \begin{cases} 10^8 t \text{ V} & 0 \le t \le 50\text{ns} \\ 7.5 - 5\times10^7 t \text{ V} & 50 \le t \le 150\text{ns} \\ 0 \text{ V} & t > 150\text{ns} \end{cases}$$



(a) $i = C\dfrac{dv}{dt} = 50\times10^{-12} \times \begin{cases} 10^8 \\ -5\times10^7 \\ 0 \end{cases} = \begin{cases} 5 \text{ mA} & 0 \le t \le 50\text{ns} \\ -2.5 \text{ mA}; & 50 \le t \le 150\text{ns} \\ 0 \text{ mA} & t > 150\text{ns} \end{cases}$

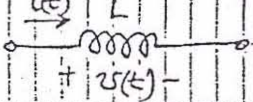


(b) $p(t) = vi = \begin{cases} 5\times10^5 t; \text{ W} & 0 \le t \le 50\text{ns} \\ -18.75\times10^{-3} + 12.5\times10^4 t; \text{ W}; & 50 \le t \le 150\text{ns} \\ 0, \text{ Watt} & t > 150\text{ns} \end{cases}$



$W = \dfrac{Cv^2}{2} = 25\times10^{-12} \times \begin{cases} 10^{16} t^2, \text{ Joul} & 0 \le t \le 50\text{ns} \\ (7.5 - 5\times10^7 t)^2, \text{ Joul} & 50 \le t \le 150\text{ns} \\ 0, \text{ Joul.} & t > 150\text{ns} \end{cases}$

$\boxed{\frac{1-16}{31}}$ $i(t) = 0.1414 \sin 377t$, A, $L = 10$ mH

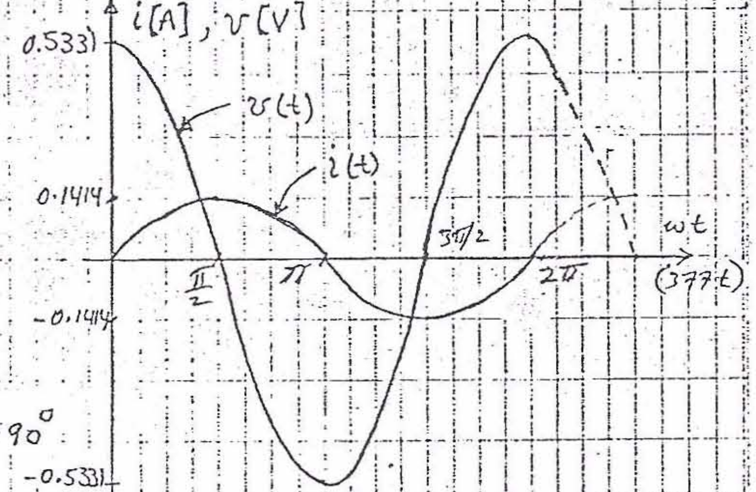


(a) $v(t) = L\frac{di}{dt} = 10^{-2}\frac{d}{dt}(0.1414 \sin 377t) = 0.5331 \cos 377t$ V

(b) Waveforms;

Note that in Inductance,

the voltage leads the current by $\frac{\pi}{2} \equiv 90^\circ$



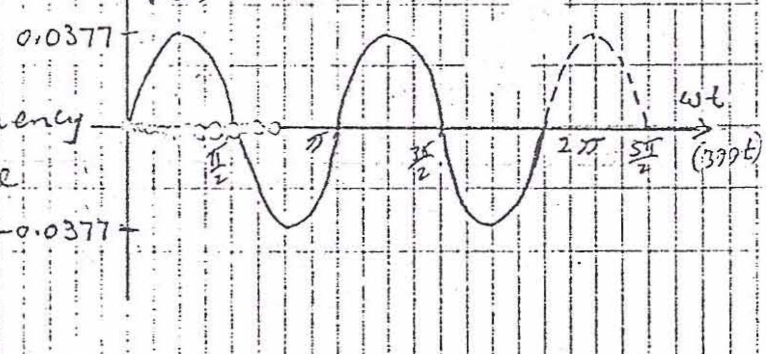
(c) $p = v\,i = (0.5331 \cos 377t)(0.1414 \sin 377t)$

$= 0.0754 \sin 377t \cos 377t$

$= 0.0754 \left(\frac{1}{2} \sin 2 \times 377t\right)$, W

$\therefore p = 0.0377 \sin 754t$, Watt

Note that the frequency of the power is twice that of current or voltage.



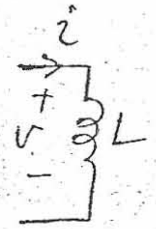
$\boxed{\frac{1-17}{31}}$ a) $v = L\,di/dt$

$\because\ i(t) = 1 - e^{-10^6 t}$, mA

& $L = 10$ mh

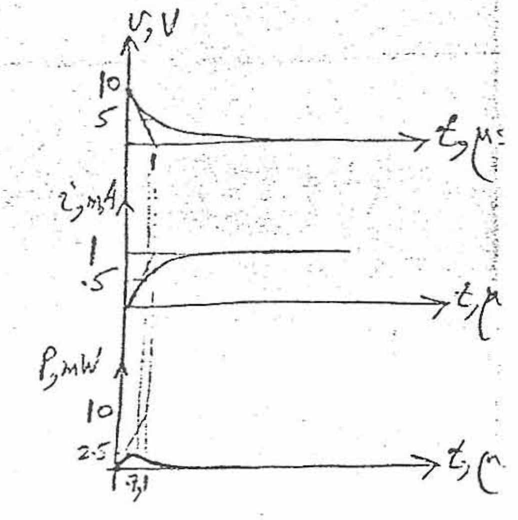
$\therefore\ v = 10m * 10^6 * e^{-10^6 t}$, mV

$= 10e^{-10^6 t}$, V



$\therefore\ v(t) = 10e^{-10^6 t}$, V

b) Sketches are as shown



c) $P(t) = i(t) . v(t) =$
$= 10e^{-10^6 t} \cdot (1 - e^{-10^6 t})$, mW

& Sketch of $p(t)$ is shown.

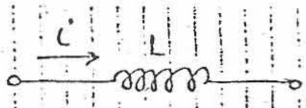
Note that $P(0) = P(\infty) = 0$
and that max. power is at $t = t_1$ where $P'(t_1) = 0$

$\therefore\ 10\left[e^{-10^6 t_1}(-10^6) - e^{-2*10^6 t_1} * (-2*10^6)\right] = 0 \qquad \therefore\ 10^6 e^{-10^6 t_1} = 2*10^6 e^{-2*10^6 t_1}$

$\therefore\ 1 = 2e^{-10^6 t_1} \qquad \therefore\ t_1 = 10^{-6} \ln 2$ sec $= 0.693$ μsec

$\therefore\ P(t)$ is max at $t = .693$ μs and $P(t_1) = P_{max} = \frac{10}{2}(1 - \frac{1}{2}) = 2.5$, mW

$\boxed{\frac{1-18}{31}}$ $L = 5.00\,\mu H = 5\times10^{-4} H$



$$i(t) = \begin{cases} 10^4 t \ , A & 0 \le t \le 2\,\mu s \\ 2.5\times10^{-2} - 2.5\times10^{3} t, A & 2 < t \le 10\,\mu s \\ 0 \ , A & t > 10\,\mu s \end{cases}$$

(a) $$v = L\frac{di}{dt} = 5\times10^{-4} \times \begin{cases} 10^4 \\ -2.5\times10^3 \\ 0 \end{cases} = \begin{cases} 5 \quad V & 0 \le t \le 2\,\mu s \\ -1.25\ V & 2 < t \le 10\,\mu s \\ 0 \quad V & t > 10\,\mu s \end{cases}$$

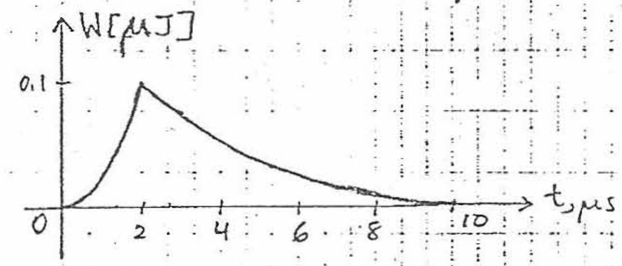
(b) Energy, $W = \frac{1}{2} L i^2$

$$\therefore W = \frac{5\times10^{-4}}{2} \times \begin{cases} 10^8 t^2 & , J & 0 \le t \le 2\,\mu s \\ (2.5\times10^{-2} - 2.5\times10^3 t)^2 & , J & 2 < t \le 10\,\mu s \\ 0 & , J & t > 10\,\mu s \end{cases}$$

@ $t = 0$, $W = 0\ J$

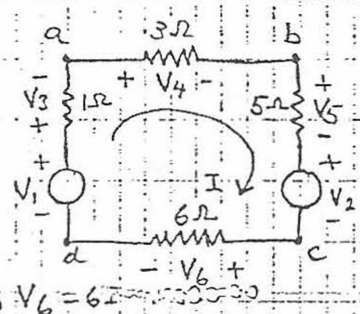
@ $t = 2\,\mu s$, $W = 0.1\,\mu J$

@ $t \ge 10\,\mu s$, $W = 0\ J$



$\boxed{\frac{1-21}{31}}$ $V_1 = 45\ V$, $V_2 = 15\ V$

(a) Same current I flows in the ckt.



KVL $\longrightarrow$ $V_1 - V_3 - V_4 - V_5 - V_2 - V_6 = 0$

Ohm's Law $\longrightarrow$ $V_3 = 1I$, $V_4 = 3I$, $V_5 = 5I$, $V_6 = 6I$

$\therefore 45 - I - 3I - 5I - 15 - 6I = 0 \Rightarrow \boxed{I = \frac{30}{15} = 2\ A}$

(b) $V_{ac} = V_4 + V_5 + V_2 = 2\times3 + 2\times5 + 15 = 31\ V$

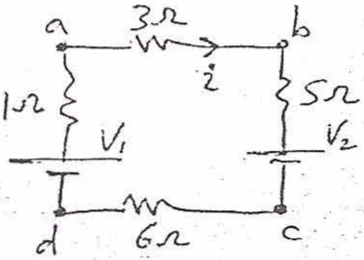
(c) $P_1 = V_1 I = 45\times2 = 90\ W$

(d) $P_{bc} = I^2R + V_2 I = 5(2)^2 + 15(2) = 50\ W$

Notice Source V_2 absorbs power $\Rightarrow V_1$ charges battery V_2.

$\boxed{\frac{1-22}{32}}$ $V_{ac} = 12V = V_2 + 8i$ (1)

$V_{bd} = 21V = V_2 + 11i$ (2)



(2) − (1) $\Rightarrow$

$21 - 12 = 3i \quad \therefore i = \frac{21-12}{3} = 3A$

into (1):

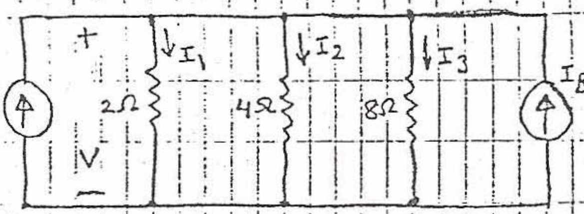
$\therefore 12 = V_2 + 8 * 3 \Rightarrow V_2 = 12 - 24 = -12$ Volts

but $i = \frac{V_1 - V_2}{1+3+5+6} = \frac{V_1 + 12}{15} = 3A$

$\therefore V_1 = 3 * 15 - 12 = 45 - 12 = 33$ Volts

$\therefore V_1 = 33V$ & $V_2 = -12V$

$\frac{1-23}{32}$ $I_A = 3\,A$, $I_B = 5\,A$



(a) KCL @ upper Node

$I_A - I_1 - I_2 - I_3 + I_B = 0$

from Ohm's law

$I_1 = \frac{V}{2}$, $I_2 = \frac{V}{4}$, $I_3 = \frac{V}{8}$

$\therefore\ 3 - \frac{V}{2} - \frac{V}{4} - \frac{V}{8} + 5 = 0 \Rightarrow \boxed{V = \frac{64}{7} = 9.14\ V}$

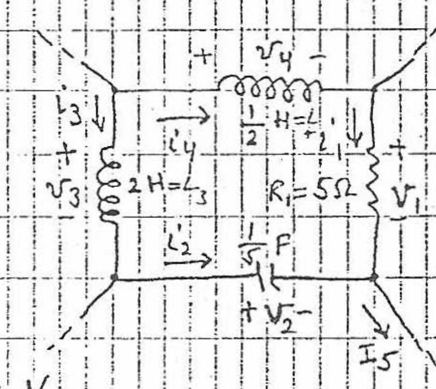
(b) $\therefore\ I_1 = \frac{9.14}{2} = 4.57\,A$, $I_2 = \frac{9.14}{4} = 2.285\,A$ & $I_3 = \frac{9.14}{8} = 1.142\,A$

(c) $P_B = I_B V = 5 \times 9.14 = 45.7\ W$

$\frac{1-24}{33}$ $i_1 = 2\,A$, $i_3(t) = 10e^{-t}\,A$

$i_4(t) = 5\cos 2t\ A$

$v_1, v_2, v_3, v_4, i_2, i_5 = ?$



$v_1 = i_1 R_1 = 2 \times 5 = 10V$

$V_3 = L_3 \frac{di_3}{dt} = 2\frac{d}{dt}(10e^{-t}) = -20e^{-t}\ V$

$v_4 = L_4 \frac{di_4}{dt} = \frac{1}{2}\frac{d}{dt}(5\cos 2t) = -5\sin 2t\ V$

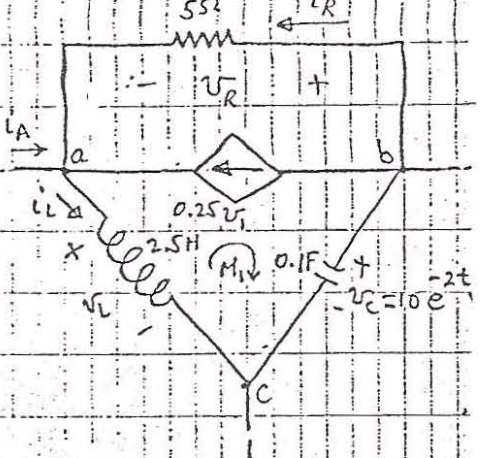
$\therefore$ KVL $\longrightarrow$ $v_3 - v_4 - v_1 + v_2 = 0$ $\therefore v_2 = 10 + 20e^{-t} - 5\sin 2t, V$

$\therefore\ i_2 = \frac{1}{5}\frac{dv_2}{dt} = \frac{1}{5}\frac{d}{dt}(10 + 20e^{-t} - 5\sin 2t) = -4e^{-t} - 2\cos 2t, A$

KCL $\rightarrow$ $i_5 = i_1 + i_2 = 2 - 4e^{-t} - 2\cos 2t, A$

$\boxed{\dfrac{1-25}{33}}$ $i_L = 2e^{-2t}$ A

i_A , $v_{ab} = ?$



$v_L = 2.5\,\frac{d}{dt}(2e^{-2t}) = -10\,e^{-2t}$ V

KVL around Mesh $M_1 \longrightarrow v_L - v_{ab} - v_c = 0$

$-v_{ab} = v_R = v_c - v_L = 10e^{-2t} + 10e^{-2t} = 20e^{-2t}$ V

$\therefore V_{ab} = -20e^{-2t}$, V.

$i_R = \frac{v_R}{5} = 4e^{-2t}$ A

KCL @ Node a : $\therefore i_A = i_L - 0.25v_1 - i_R$.

1st $\boxed{\text{Assume } V_1 = V_L}$

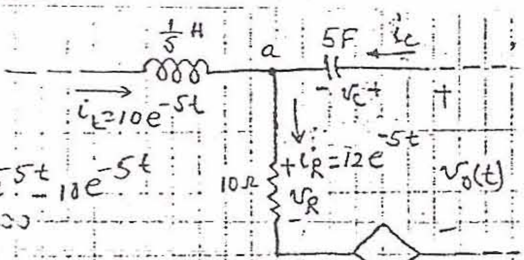
$\therefore i_A = 2e^{-2t} - 0.25(-10e^{-2t}) - 4e^{-2t} = 0.5e^{-2t}$ A

2nd $\boxed{\text{Assume } v_1 = v_c}$

$\therefore i_A = 2e^{-2t} - 0.25(10e^{-2t}) - 4e^{-2t} = -4.5e^{-2t}$ A

$\boxed{\dfrac{1-26}{34}}$ $v_o(t) = ?$

KCL @ Node a



$i_c = i_R - i_L = 12e^{-5t} - 10e^{-5t}$

$\therefore i_c$ [illegible] e^{-5t} A

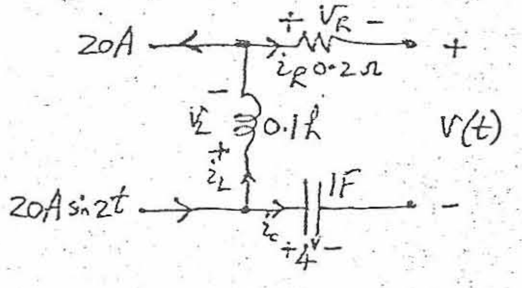
KVL $\longrightarrow v_o = v_c + v_R - 5i_L = v_c + 10(12e^{-5t}) - 5(10e^{-5t})$

$v_c = \frac{1}{C}\int i_c\,dt = \frac{1}{5}\int 2e^{-5t}\,dt = \frac{-2}{25}e^{-5t} + K = -0.08e^{-5t} + K$

$\therefore v_o = K - 0.08e^{-5t} + 120e^{-5t} - 50e^{-5t} = 69.92e^{-5t} + K$, V

(K could be found from capacitor initial charge).

$\boxed{\frac{1-28}{34}}$ $i_c = C\dfrac{dV_c}{dt} = 1*0 = 0\,A$



$\therefore\ i_L = 20\sin 2t - i_c$

$= 20\sin 2t,\ A$

$\therefore\ V_L = L\dfrac{di_L}{dt} =$

$= 0.1 * 20 * 2\cos 2t$

$= 4\cos 2t,\ V$

& $i_R = i_L - 20 = 20\sin 2t - 20 = 20(\sin 2t - 1),\ A$

$\therefore\ V_R = R\cdot i_R = 0.2 * 20(\sin 2t - 1) = 4(\sin 2t - 1),\ V$

$\therefore\ v(t) = 4 - V_L - V_R = 4 - 4\cos 2t - 4(\sin 2t - 1) = 4(2 - \sin 2t - \cos 2t)$

& $P_R = \dfrac{1}{T}\displaystyle\int_0^T i_R^2 R\,dt = \dfrac{\omega}{2\pi}\int_0^{2\pi/\omega} i_R^2 R\,dt$ (with $\omega = 2$) $=$

$= \dfrac{2}{2\pi}\displaystyle\int_0^{\pi} 20^2(\sin 2t - 1)^2 * 0.2\,dt = \dfrac{80}{\pi}\int_0^{\pi}(\sin^2 2t - 2\sin 2t + 1)\,dt =$

$= \dfrac{80}{\pi}\displaystyle\int_0^{\pi}\left(\frac{1-\cos 4t}{2} - 2\sin 2t + 1\right)dt = \frac{80}{\pi}\left[\frac{3t}{2} - \frac{\sin 4t}{8} + \cos 2t\right]_0^{\pi} =$

$= \dfrac{80}{\pi}\left[\dfrac{3\pi}{2} - 0 - 0 + 0 + 1 - 1\right] = 120\ W$

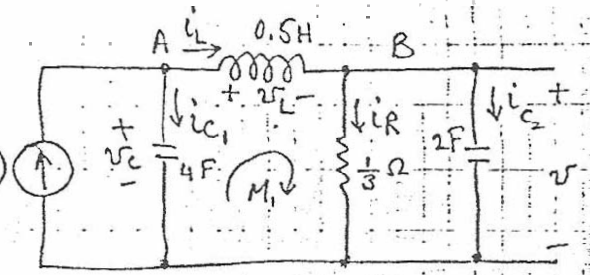
OR $P_R =$ power due to $20\sin 2t, A$ + power due to $-20\,A$ =

$= \left(\dfrac{20}{\sqrt{2}}\right)^2 * 0.2 + (-20)^2 * 0.2 = \left(\dfrac{400}{2} + 400\right) * 0.2 = 600 * .2 = 120W$

($\therefore$ OK

$\frac{1-29}{35}$

$v(t) = 5e^{-t}$ V., $i(t) = ?$



$i_{c_2} = 2 \frac{d}{dt}(5e^{-t}) = -10e^{-t}$ A

$i_R = \frac{v}{1/3} = 15e^{-t}$ A

KCL @ Node B $i_L = i_R + i_{c_2} = 15e^{-t} - 10e^{-t} = 5e^{-t}$ A

KVL around Mesh M_1 $\therefore v_c = v_L + v$

$v_L = 0.5 \frac{di_L}{dt} = 0.5(-5e^{-t}) = -2.5e^{-t}$ V

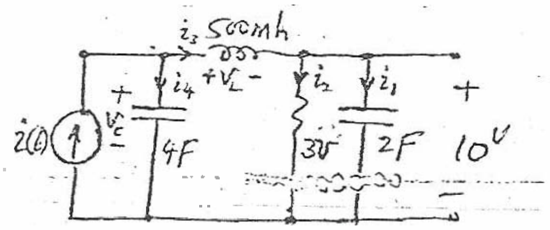
$\therefore v_c = -2.5e^{-t} + 5e^{-t} = 2.5e^{-t}$ V

$\therefore i_{c_1} = 4\frac{dv_c}{dt} = 4\frac{d}{dt}(2.5e^{-t}) = -10e^{-t}$ A

KCL @ Node A $i(t) = i_{c_1} + i_L = -10e^{-t} + 5e^{-t}$

$\therefore$ $\boxed{i(t) = -5e^{-t} \text{ A}}$

$\frac{1-30}{35}$ Assuming notations shown:



$\therefore$ $i_1 = c\frac{d}{dt}(10V) = 2 * 0 = 0$ Ap

& $i_2 = 3Ʊ * 10V = 30$

$\therefore$ KCL: $i_3 = i_1 + i_2 = 30$ A

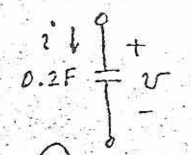
$\therefore$ $V_L = L\frac{di_3}{dt} = .5 * \frac{d\,30}{dt} = 0$ Volts

$\therefore$ KVL: $V_c = V_L + 10V = 0 + 10 = 10$ V.

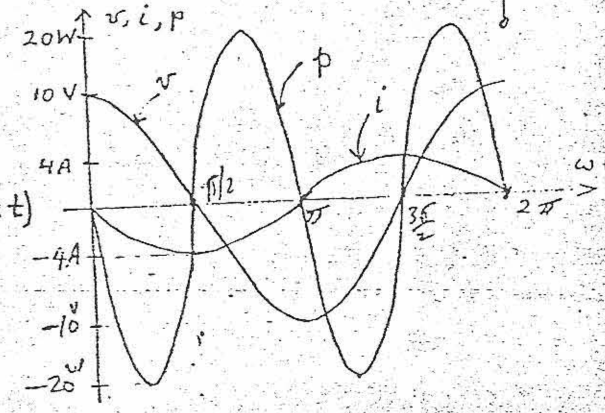
$\therefore$ $i_4 = c\frac{dV_c}{dt} = 4\frac{d10^V}{dt} = 4 * 0 = 0$A

$\therefore$ $i(t) = i_3 + i_4 = 30 + 0 = 30$ A, the source current.

$v = 10\cos 2t$ V, Find i, p & sketch v, i & p.



ⓐ $i = C\dfrac{dv}{dt} = 0.2(-20\sin 2t) = -4\sin 2t$, A

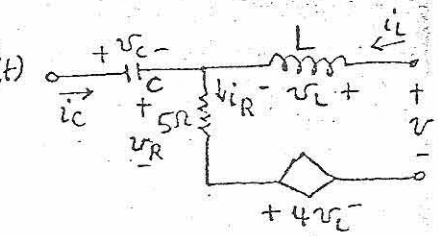


ⓑ Sketches of v & i

ⓒ $p = vi = (10\cos 2t)(-4\sin 2t)$

$\therefore p = -20\sin 4t$

$i_L = 5\sin 2t$ A, $v_C = 10e^{-5t}$ V, Find $v(t)$



$v(t) = v_L + v_R + 4v_L$ [KVL]

$v_L = L\dfrac{di_L}{dt} = 10L\cos 2t$ V

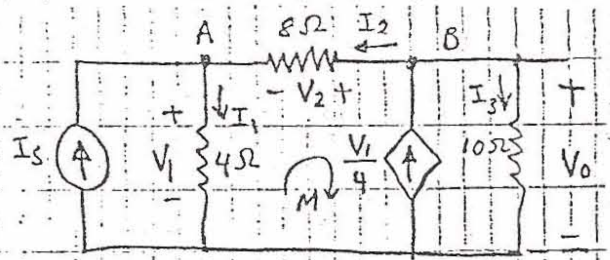
$i_R = i_L + i_C$ (KCL)

$i_C = C\dfrac{dv_C}{dt} = -50C\, e^{-5t}$ A

$\therefore i_R = 5\sin 2t - 50Ce^{-5t}$ A

$v_R = 5i_R = 25(\sin 2t - 10Ce^{-5t})$ V

$\therefore v(t) = 10L\cos 2t + 25(\sin 2t - 10Ce^{-5t}) + 40L\cos 2t$
$= 25[2L\cos 2t + \sin 2t - 10Ce^{-5t}]$ V



$\boxed{\frac{1-33}{35}}$ $V_0 = 5V$, $I_s = ?$

$I_3 = \frac{V_0}{10} = 0.5\ A$

KCL @ Node B $\quad I_2 = \frac{V_1}{4} - I_3 = 0.25V_1 - 0.5$

KVL around Mesh M $\quad V_1 = V_0 - V_2 = 5 - \overbrace{8(0.25V_1 - 0.5)}^{\text{Ohm's Law}}$

$\therefore V_1 = \frac{9}{3} = 3V$

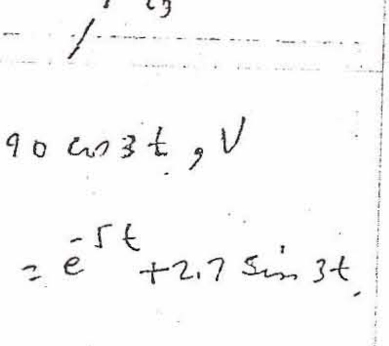
$\therefore I_2 = 0.25 \times 3 - 0.5 = 0.25\ A$

$\therefore I_1 = \frac{V_1}{4} = \frac{3}{4} = 0.75\ A$

$\therefore$ KCL @ Node A

$I_s = I_1 - I_2 = 0.75 - 0.25 = 0.5\ A$

$P_{4\Omega} = I_{4\Omega}^2 * 4 = \left(\frac{3}{4}\right)^2 * 4 = \frac{9}{4} = 2.25$ Watt



$i_1 = 5\ A$, $i_3 = 10 \sin 3t\ A$, $i_4 = 2e^{-5t}\ A$
V_1, V_2, V_3, V_4, i_2 & $i_5 = ?$

$v_1 = 2i_1 = 10\ V$

$v_3 = 3\frac{di_3}{dt} = 3(30 \cos 3t) = 90 \cos 3t\ \ V$

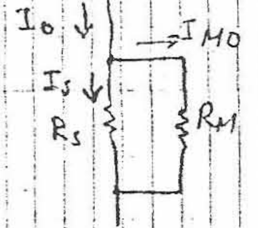
$v_4 = 2\frac{di_4}{dt} = 2(-10e^{-5t}) = -20e^{-5t}\ V$

KVL $\quad v_2 = v_1 + v_4 - v_3 = 10 - 20e^{-5t} - 90\cos 3t$, V

$i_2 = C\frac{dv_2}{dt} = 0.01\left(+100e^{-5t} + 270 \sin 3t\right) = e^{-5t} + 2.7 \sin 3t$

KCL $\quad \therefore i_5 = i_1 - i_4 = 5 - 2e^{-5t}$, A.

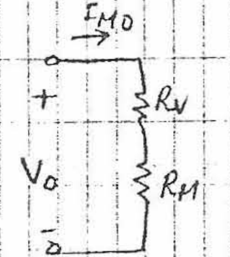
2-1 / 78 $I_{MO} = 1\,mA$, $R_M = 25\,\Omega$



ⓐ for ammeter, $R_S = ?$ if $I_O = 5A$

$$R_S = \frac{R_M I_{MO}}{I_O - I_{MO}} = \frac{25 \times 10^{-3}}{5 - 10^{-3}} \simeq 0.005\,\Omega$$

ⓑ for Voltmeter, $R_V = ?$ if $V_O = 50V$

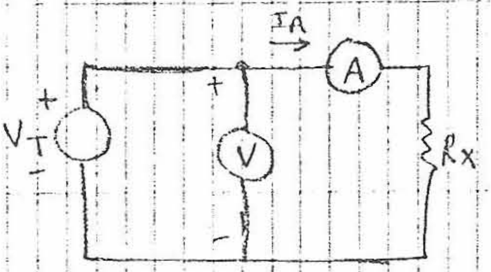


$$R_V = \frac{V_O}{I_{MO}} - R_M = \frac{50}{10^{-3}} - 25 = 50{,}000 - 25$$
$$\simeq 50\,K\Omega$$

2-5 / 79 R_V = Voltmeter Resistance = $10\,K\Omega$

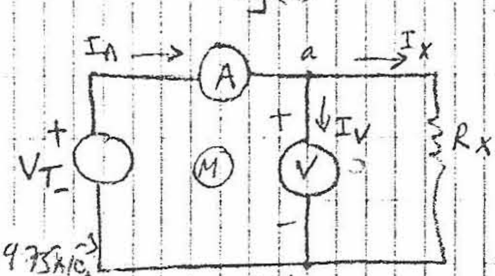
R_A = Ammeter " $= 1\,\Omega$

$V_T = 10V$, $R_X = 2000\,\Omega = 2k\Omega$



Fig(a)

ⓐ Fig(a): $V = V_T = 10V$



Fig(b)

KVL $\longrightarrow V = I_A R_A + I_A R_X$

$$\therefore I_A = \frac{V_T}{R_A + R_X} = \frac{10}{2001} = 4.9975 \times 10^{-3}\,A$$
$$= 4.9975\,mA$$

Fig(b)

$I_X R_X = V = I_V R_V \quad \therefore \frac{I_X}{I_V} = \frac{R_V}{R_X} = \frac{10}{2} = 5$ ①

KCL @ ⓐ $\longrightarrow I_A = I_V + I_X \overset{①}{=} I_V + 5I_V = 6I_V \quad \therefore I_V = \frac{I_A}{6}$ ②

KVL for Mesh M $\quad V_T = I_A R_A + I_V R_V \overset{②}{=} 1 I_A + 10^4 \left(\frac{I_A}{6}\right) = 1667.\bar{6}\, I_A$

$$\therefore I_A = \frac{10}{1667.667} = 0.0059964\,A = 5.9964\,mA$$

$$V = I_V R_V = \frac{I_A}{6}(10^4) = 9.994\,V$$

(b) Measured $R_x = \frac{V}{I_A}$

Fig (a) : $R_x(\text{measured}) = \frac{10V}{4.9975mA} = 2.001 K\Omega \therefore \%e = \left|\frac{2-2.001}{2}\right| \times 100 = .05\%$

Fig (b) : $R_x(\text{measured}) = \frac{9.994V}{5.9964mA} = 1.\bar{6} K\Omega \therefore \%e = \left|\frac{2-1.\bar{6}}{2}\right| \times 100 = 16.\bar{6}\%$

Note that the measured value of R_X from Fig (a) is very close from the real value (2000Ω) because I_A is the current in R_X and the voltage drop in the Ammeter of $R_A = 1\Omega$ is negligibly small compared to voltage drop in $R_X = 2000\Omega$ and hence the reading of V is almost the voltage of $R_X \Rightarrow \frac{V}{I_A} \simeq R_X$ (accurate).

Note that $\frac{V}{I_A} = R_A + R_X \approx R_X$ when $R_X \ggg R_A$.

In the connection of Fig (b) the reading of V is exactly the voltage of R_X, however, the current $I_V = 0.2 I_X$, i.e., I_V is comparable to I_X and $I_A \neq I_X$.

$\therefore \frac{V}{I_A} \neq R_X$

Note that $\frac{V}{I_A} = R_{eq}$, where R_{eq} = equivalent resis. of $R_V // R_X$ and this is equal to R_X only when $R_X \lll R_V$.

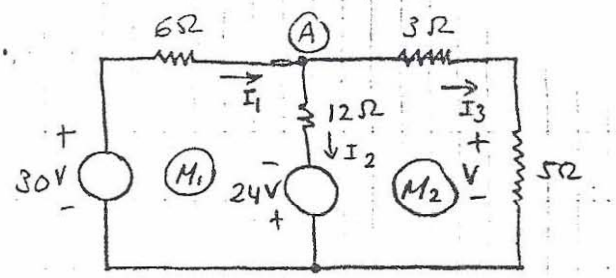
On the other hand if R_X was small comparable to R_A, then the connection of Fig (a) will result in large drop in R_A comparable to the drop of R_X, i.e., $V \neq I_A R_X$ and $\frac{V}{I_A} \neq R_X$.

Conclusion

Connection of Fig (a) is accurate for $R_X \ggg R_A$

and " " Fig (b) " " " $R_X \lll R_V$

$\boxed{\frac{2-12}{81}}$ V = ? by direct application.



KCL @ node A

$$I_1 = I_2 + I_3 \quad (1)$$

KVL for Mesh M_1 $\quad 30 + 24 = 6I_1 + 12I_2 \Rightarrow I_1 = 9 - 2I_2 \quad (2)$

from (1) & (2) $\quad I_2 + I_3 = 9 - 2I_2 \Rightarrow I_2 = 3 - \frac{I_3}{3} \quad (3)$

KVL for Mesh M_2 $\quad 12I_2 = 3I_3 + 5I_3 + 24 \quad (4)$

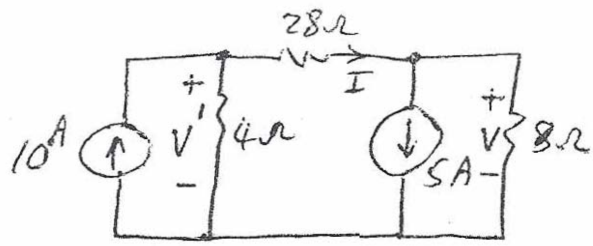
using (3) in (4) $\rightarrow$ $\boxed{I_3 = 1A}$ & $\boxed{V = 5I_3 = 5V}$

$\boxed{\frac{2-13}{81}}$

$V = 8 * (I - 5) \quad (1)$

$V' = V + 28I \quad (2)$

$V' = 4 * (10 - I) \quad (3)$



(2) into (3)

$\therefore V + 28I = 40 - 4I$

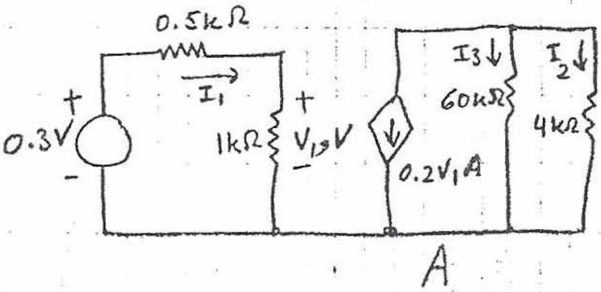
$\therefore V = 40 - 32I$

into (1)

$\therefore 40 - 32I = 8I - 40$

$\therefore 40I = 80 \qquad \therefore I = 2$ Amp.

2-14 / 81



(a) $I_1 = \dfrac{0.3\ V}{(1+0.5)\ K\Omega} = 0.2\ mA$

$\therefore V_1 = (0.2\ mA)(1K\Omega) = 0.2\ V$

$\therefore 0.2\ V_1 = 0.04\ A = 40.0\ mA$

$4 I_2 = 60 I_3 \quad \therefore \quad I_3 = \dfrac{I_2}{15}$ (1) $\quad I_2$ & I_3 in mA

@ node A $\quad I_3 = -(0.2 V_1 + I_2)$ (2)

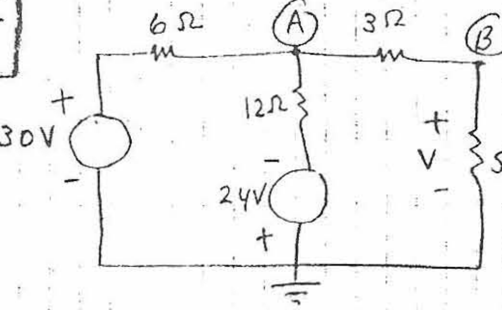
from (1) & (2) $\quad \dfrac{I_2}{15} = -40.0 - I_2 \Rightarrow \boxed{I_2 = \dfrac{-600.0}{16} = -37.5\ mA}$

(b) $P_{in} = 0.3 I_1 = .06\ mW = 60\ \mu W$

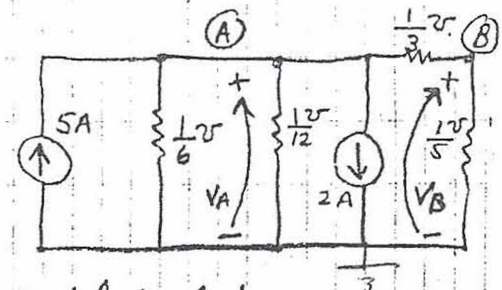
(c) $P_{out} = I_2^2 (4k) = (-37.5 \times 10^{-3})^2 (4 \times 10^3) = 5.625 W$

It is an amplifier

2-17 / 82



Convert → Redraw



$V = ?$ using Nodal Analysis

@ (A) $\quad \left(\frac{1}{6}+\frac{1}{12}+\frac{1}{3}\right)V_A - \frac{1}{3}V_B = 5-2$

$$\Rightarrow \quad 7V_A - 4V_B = 36 \quad (1)$$

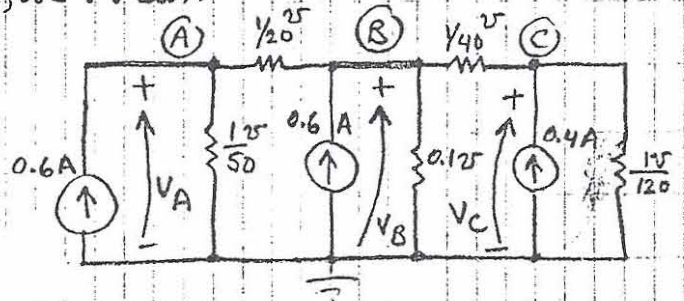
@ (B) $\quad -\frac{1}{3}V_A + \left(\frac{1}{3}+\frac{1}{5}\right)V_B = 0$

$$\Rightarrow \quad V_A = \frac{8}{5}V_B \quad (2)$$

(2) in (1) $\Rightarrow \; 7\left(\frac{8}{5}V_B\right) - 4V_B = 36$

$$\boxed{V = V_B = \frac{180}{36} = 5\text{ V}}$$

2-20 / 82 Converting and Redrawing, we obtain



(a) To find V_B :

@ (A) $\quad \left(\frac{1}{50}+\frac{1}{20}\right)V_A - \frac{V_B}{20} - 0\,V_C = 0.6$

$$\Rightarrow \quad 0.07V_A - 0.05V_B - 0V_C = 0.6 \quad (1)$$

@ (B) $\quad -\frac{V_A}{20} + \left(\frac{1}{20}+0.1+\frac{1}{40}\right)V_B - \frac{V_C}{40} = 0.6$

$$\Rightarrow \quad -0.05V_A + (0.175)V_B - 0.025V_C = 0.6 \quad (2)$$

@ (C) $\quad -0\,V_A - \frac{V_B}{40} + \left(\frac{1}{40}+\frac{1}{120}\right)V_C = 0.4$

$$\Rightarrow \quad -0\,V_A - 0.025V_B + 0.033\bar{3}V_C = 0.4 \quad (3)$$

$$V_B = \frac{\begin{vmatrix} 0.07 & 0.6 & 0 \\ -0.05 & 0.6 & -0.025 \\ 0 & 0.4 & 0.033\bar{3} \end{vmatrix}}{\begin{vmatrix} 0.07 & -0.05 & 0 \\ -0.05 & 0.175 & -0.025 \\ 0 & -0.025 & 0.033\bar{3} \end{vmatrix}} = 11.0\bar{2}\text{ Volts}$$

(b) To find I_3 :

KVL $\qquad V_B = 10I_3 + 6$

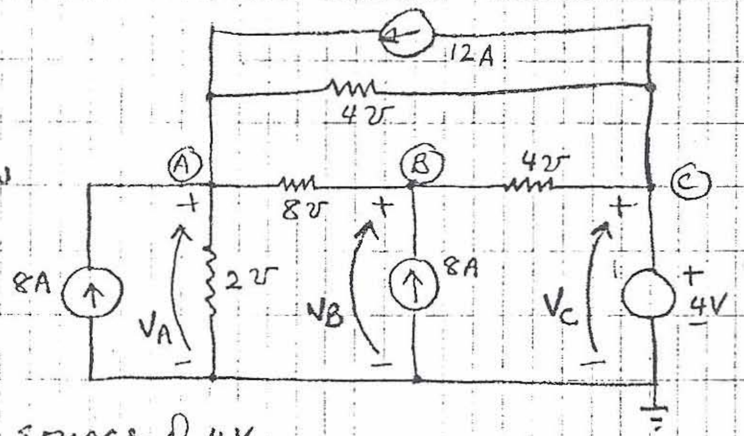
$$\therefore \quad \boxed{I_3 = \frac{11.022\bar{2}-6}{10} = 0.5022\bar{2}\text{ A}}$$

2-21

82

$V_B = ?$

Convert & Redraw



Notice that voltage source of 4V has no series resistance ⇒ can't be converted to current source.

However, $V_C = 4V$ (known) ⇒ need only 2 eqs. for V_A & V_B.

@ node (A)

$$(2+8+4)V_A - 8V_B - 4V_C = 8 + 12$$

OR

$$7V_A - 4V_B = 18 \quad (1)$$

@ node (B)

$$-8V_A + (8+4)V_B - 4V_C = 8$$

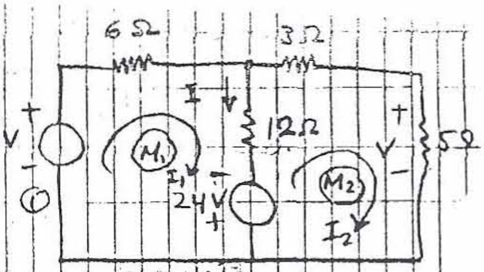
OR

$$-2V_A + 3V_B = 6 \quad (2)$$

from (2) $V_A = \frac{3}{2}V_B - 3 \quad (3)$

(3) in (1) $7\left(\frac{3}{2}V_B - 3\right) - 4V_B = 18 \Rightarrow \boxed{V_B = 6V}$

2-24 / 83 Find I using mesh analysis.



M_1 $\quad (6+12)I_1 - 12 I_2 = 30+24 = 54 \quad ①$

M_2 $\quad -12 I_1 + (12+3+5) I_2 = -24$

OR $\quad -3I_1 + 5I_2 = -6 \quad ②$

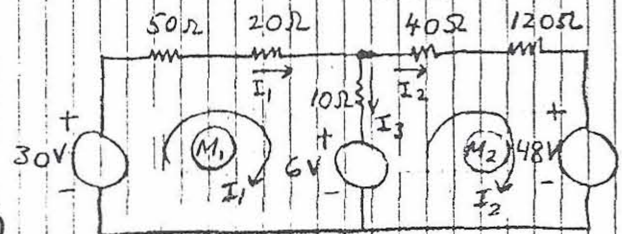
$① \longrightarrow \quad I_1 = 3 + \frac{2}{3} I_2$

Substitute in ② $\longrightarrow \quad -3\left(3+\frac{2}{3}I_2\right) + 5I_2 = -6$

Solving for $I_2 \Rightarrow \boxed{I_2 = 1\,A}$ $\quad \therefore \boxed{I_1 = 3 + \frac{2}{3} = 3.667\,A}$

$\therefore \boxed{I = I_1 - I_2 = 2.667\,A}$

2-25 / 83 Convert and redraw:



M_1 $\quad (50+20+10)I_1 - 10I_2 = 30-6$

$\therefore \quad 80I_1 - 10I_2 = 24 \quad ①$

M_2 $\quad -10I_1 + (10+40+120)I_2 = 6-48 = -42 \quad ②$

from ① : $I_2 = 8I_1 - 2.4$

substitute in ② : $-10I_1 + 170(8I_1 - 2.4) = -42$

$\Rightarrow I_1 = 0.27\bar{1}\,A \quad \& \quad I_2 = -0.23\bar{1}\,A$

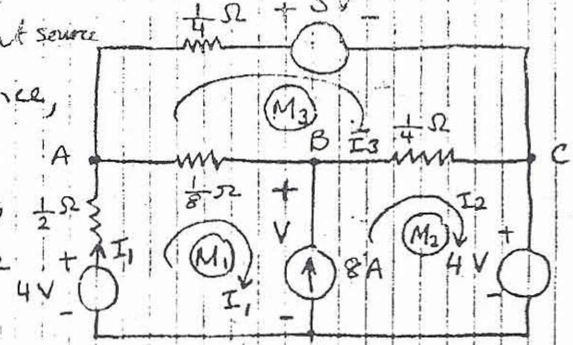
$\therefore \quad I_1 = 0.27\bar{1}\,A \quad , \quad I_2 = -0.23\bar{1}\,A$

& $I_3 = I_1 - I_2 = 0.50\bar{2}\,A$

2-28 / 84

Convert the current source with parallel conductance only and redraw the Ckt.

Notice that the 8 A-current source has no parallel conductance, so we can't convert it. However, a voltage exists across this current source say V as shown. This voltage is considered as a source in the Mesh Analysis.



(M1) $\left(\frac{1}{2}+\frac{1}{8}\right)I_1 - \frac{1}{8}I_3 = 4 - V \Rightarrow 5I_1 - I_3 = 32 - 8V$ (1)

(M2) $\frac{1}{4}I_2 - \frac{1}{4}I_3 = V - 4 \Rightarrow -2I_2 + 2I_3 = 32 - 8V$ (2)

from (1) & (2): $5I_1 - I_3 = -2I_2 + 2I_3 \Rightarrow 5I_1 + 2I_2 - 3I_3 = 0$ (3)

(M3) $-\frac{1}{8}I_1 - \frac{1}{4}I_2 + \left(\frac{1}{8}+\frac{1}{4}+\frac{1}{4}\right)I_3 = -3 \Rightarrow -I_1 - 2I_2 + 5I_3 = -24$ (4)

But $8 = I_2 - I_1 \Rightarrow -I_1 + I_2 + 0I_3 = 8$ (5)

Solve eqs. (3), (4) & (5) simultaneously:

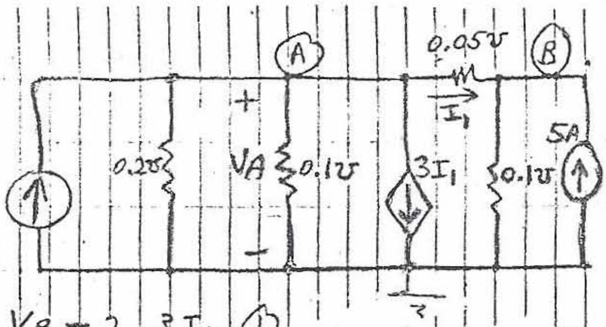
$$I_1 = \frac{\begin{vmatrix} 0 & 2 & -3 \\ -24 & -2 & 5 \\ 8 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 5 & 2 & -3 \\ -1 & -2 & 5 \\ -1 & 1 & 0 \end{vmatrix}} = \frac{(0)(0-5)-(-24)(0+3)+(8)(10-6)}{5(0-5)-(-1)(0+3)+(-1)(10-6)}$$

$$= \frac{72+32}{-25+3-4} = \frac{104}{-26} = -4\,A$$

$\therefore$ $\boxed{I_1 = -4\,A}$, is the current in the $\frac{1}{2}\Omega$ resistor.

2-30
84

Convert all voltage sources and redraw.



Node Ⓐ

$$(0.2+0.1+0.05)V_A - 0.05\,V_B = 2-3I_1 \quad (1)$$

Node Ⓑ

$$-0.05\,V_A + (0.05+0.1)V_B = 5 \quad (2)$$

Constraint

$$I_1 = 0.05(V_A - V_B) \quad (3)$$

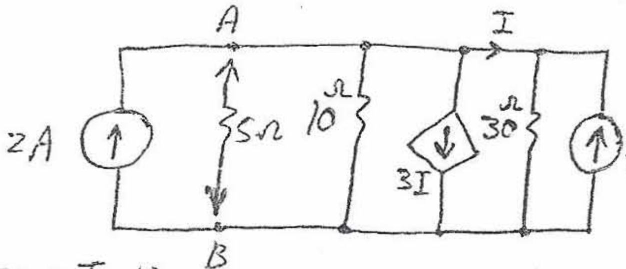
substitute (3) in (1) $\Rightarrow$ $V_B = -10 + 2.5\,V_A$ (4)

substitute (4) in (2) $\Rightarrow$ $\boxed{V_A = 20\,V}$, the voltage across 5Ω

from (4) $V_B = 40\,V$, from (3) $I_1 = -1A$

2-30
84

Remove the 5Ω and convert to current sources

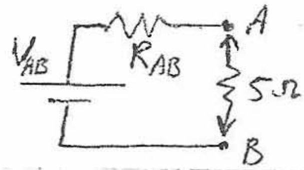


$\therefore V_{AB} = (2+\frac{5}{3}-3I)*(10//30)$

$= \frac{11-9I}{3} * \frac{300}{40} = 27.5 - 22.5\,I$ (1)

Constraint Eq:

$I = \frac{V_{AB}}{30} - \frac{5}{3} = \frac{V_{AB}-50}{30}$ (2)



(2) into (1) $\Rightarrow$ $V_{AB} = 27.5 - 22.5\left(\frac{V_{AB}-50}{30}\right)$ $\therefore V_{AB}\left(1+\frac{22.5}{30}\right) = 27.5 + \frac{22.5*5}{3}$

$\therefore V_{AB}\left(1+\frac{3}{4}\right) = 27.5 + 37.5 = 65$

$\therefore V_{AB} = 65 * \frac{4}{7} = \frac{260}{7}$ Volts $= 37.143$ Volts

To find I_{AB} short AB, $\therefore$ from (2) $I = \frac{0-50}{30} = \frac{-5}{3}A$

$\therefore I_{AB} = 2 + \frac{5}{3} - 3I - 0/(10//30) = \frac{6+5-3*(-5)}{3} = \frac{26}{3}A = 8.\bar{6}A$

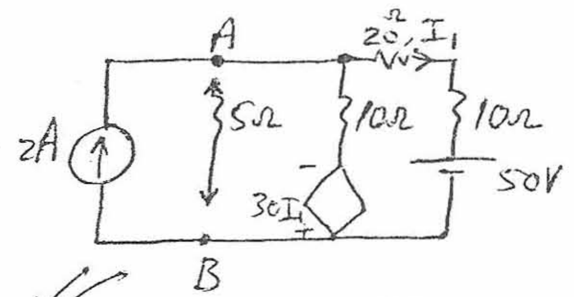
$\therefore R_{AB} = V_{AB}/I_{AB} = \frac{260}{7} * \frac{3}{26} = \frac{30}{7}\,\Omega = 4.2857\,\Omega$

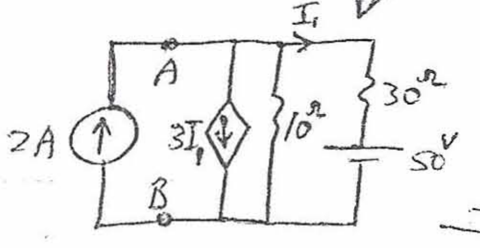
$\therefore V_{5\Omega} = V_{AB} * \left(\frac{5}{5+R_{AB}}\right) = \frac{260}{7} * \left(\frac{5*7}{5*7+30}\right) = \frac{260*5*7}{7*65} = 20$ Volts.

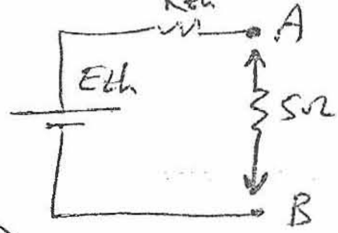
\# For max. power replace 5Ω by $\frac{30}{7}\Omega$, hence getting max. power of $\left(\frac{13}{3}\right)^2 \cdot \frac{30}{7} = \frac{1690}{21}W = 80.476W$

2-30 / 84

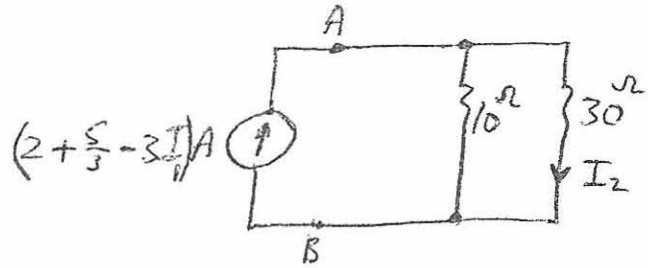
Solution by Thevenin Theorem:
Take away the 5Ω resistor & denote the terminals by AB.
V_{AB} when the 5Ω is out $= E_{th}$
& R_{AB} " " " " " $= R_{th} = \frac{E_{th}}{I_{th}}$
where I_{th} is the short circuit current through AB.

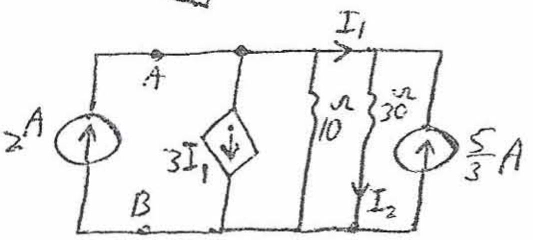






Using simplification:
$I_2 = I_1 + \frac{5}{3}$, A





$$\therefore\ I_2 = \left(2 + \frac{5}{3} - 3I_1\right) * \frac{10}{10+30} = I_1 + \frac{5}{3}$$

$$\therefore\ \frac{11}{3} - 3I_1 = 4I_1 + \frac{20}{3} \Rightarrow 7I_1 = \frac{11-20}{3} = \frac{-9}{3} = -3$$

$$\therefore\ I_1 = \frac{-3}{7}\ A = -.4286\ A$$

$$\therefore\ I_2 = \frac{-3}{7} + \frac{5}{3} = \frac{-9+35}{21} = \frac{26}{21} = 1.2381\ Amp$$

$$\therefore\ V_{AB} = E_{th} = 30\,I_2 = \frac{260}{7} = 37.1429\,V$$

Now recalculate with AB shorted together, hence $I_2 = 0$ & $I_1 = -\frac{5}{3}A$

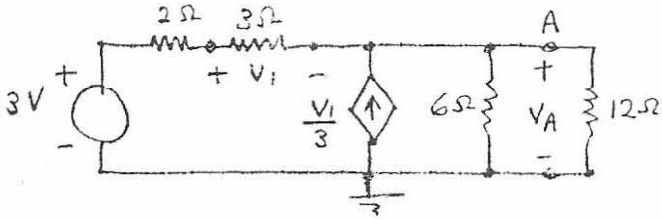
$$\therefore\ I_{th} = 2 + \frac{5}{3} - 3I_1 = \frac{11}{3} + 5 = \frac{11+15}{3} = \frac{26}{3} = 8.\bar{6}\ A$$

$$\therefore\ R_{th} = E_{th}/I_{th} = (260/7)/\left(\frac{26}{3}\right) = \frac{260*3}{7*26} = \frac{30}{7} = 4.2857\,\Omega$$

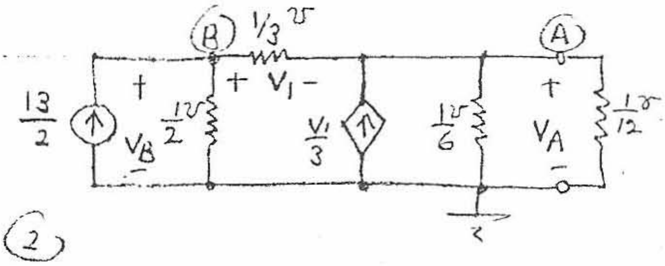
$$\therefore\ i_{5\Omega} = E_{th}/(R_{th}+5) = \frac{260/7}{30/7+5} = \frac{260}{65} = 4\ Amp\ \&\ V_{5\Omega} = 20\ Volts.$$

& max. power is when 5Ω is replaced by $\frac{30}{7}\,\Omega$ hence giving $\left(\frac{26}{6}\right)^2 \cdot \frac{30}{7} = 80.4762$ W

2-31 / 85 Convert voltage source to current source.



Node (B)



$\left(\frac{1}{2}+\frac{1}{3}\right)V_B - \frac{1}{3}V_A = \frac{13}{2}$ (1)

Node (A)

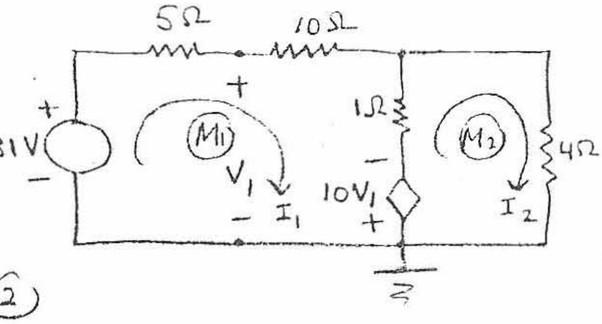
$-\frac{1}{3}V_B + \left(\frac{1}{3}+\frac{1}{6}+\frac{1}{12}\right)V_A = \frac{V_1}{3}$ (2)

Constraint Eq. $V_1 = V_B - V_A$ (3)

(3) in (2) $-\frac{1}{3}V_B + \frac{7}{12}V_A = \frac{V_B - V_A}{3} \Rightarrow V_B = \frac{11}{8}V_A$ (4)

(4) in (1) $\frac{5}{6}\times\frac{11}{8}V_A - \frac{1}{3}V_A = \frac{13}{2} \Rightarrow \boxed{V_A = 8\text{ V}}$, voltage across 12Ω

2-34 / 86 (a) Power dissipated, $P_d = I_2^2(4)$ W



(M1) $(5+10+1)I_1 - (1)I_2 = 31 + 10V_1$ (1)

(M2) $-(1)I_1 + (1+4)I_2 = -10V_1$ (2)

Constraint Eq.: $V_1 = 31 - 5I_1$ (3)

(3) in (1) : $\rightarrow 16I_1 - I_2 = 31 + 10(31 - 5I_1) \Rightarrow I_2 = 66I_1 - 341$ (4)

(4) & (3) in (2) $\rightarrow -I_1 + 5(66I_1 - 341) = -10(31 - 5I_1) \Rightarrow \boxed{I_1 = 5\text{ A}}$ (5)

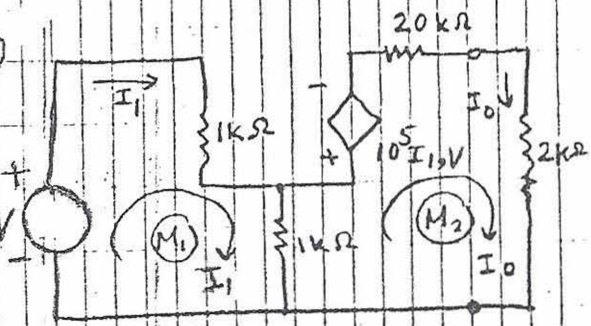
from (4) $\boxed{I_2 = -11\text{ A}}$ $\therefore \boxed{P_d = (-11)^2(4) = 484.0\text{W}}$

(b) Power supplied, $P_{in} = 31\,I_1 = 155$ W

$\frac{2-35}{86}$

(a) (M_1) $(10^3+10^3)I_1 - 10^3 I_o = 10^{-3}$ (1)

(M_2) $-10^3 I_1 + (1+20+2)10^3 I_o = -10^5 I_1$ (2)



(2) $\Longrightarrow$ $99I_1 + 23I_o = 0$ $\Longrightarrow$ $I_o = -\frac{99}{23}I_1$

into (1) $\Longrightarrow$ $(2+\frac{99}{23})\times10^3 I_1 = 10^{-3}$ $\Rightarrow$ $\boxed{I_1 = 0.159\,\mu A}$

$\therefore$ $\boxed{I_o = -0.683\,\mu A}$

(b) Power input by the source, in $= 10^{-3} I_1 = 0.159\times10^{-9}$ W

(c) Power output, $P_{out} = I^2(2\times10^3) = (-0.683\times10^{-6})^2\times2\times10^3$

$P_{out} = 0.932\times10^{-9}$ W

(d) Power in the dependent source, $= (10^5 I_1)\, I_o$, into it

$\therefore$ $P_d = (10^5\times0.159\times10^{-6})(-0.683\times10^{-6}) = -10.83\times10^{-9}$ W

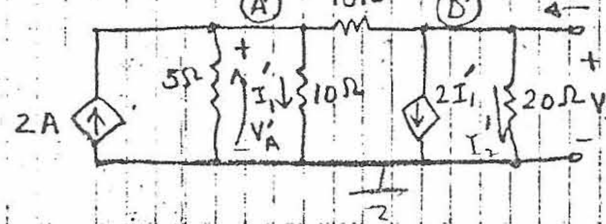
$\therefore$ power source is -10.83 nW

OR power out of dependent source is 10.83 nW

$\therefore$ Dependent supplies power of 10.83 n

2-38 / 88

1st suppress the current source to see the effect of the independent voltage source.
Converting the voltage source, redraw the ckt.



Node (A')

$$\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}\right)V_A' - \frac{1}{10}V_B' = 2 \qquad (1)$$

Node (B')
$$-\frac{1}{10}V_A' + \left(\frac{1}{10}+\frac{1}{20}\right)V_B' = -2I_1' \qquad (2)$$

Constraint
$$I_1' = \frac{V_A'}{10} \qquad (3)$$

(3) in (2) $\longrightarrow -0.1V_A' + 0.15V_B' = -0.2V_A'$

$$\Rightarrow V_A' = -1.5V_B' \qquad (4)$$

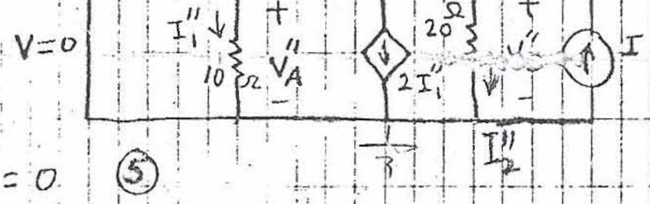
(4) in (1) $\longrightarrow -0.4\times1.5V_B' - 0.1V_B' = 2 \Rightarrow V_B' = -2.857V = V_2'$

$\therefore V_A' = 4.286V$

(3) $\longrightarrow$

$\therefore I_1' = 0.4286\ A \qquad \& \ I_2' = \frac{V_2'}{20} = -0.14286\ A$

2nd suppress the voltage source to see the effect of the current source.
Redraw the ckt.



Node (A'')
$$\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}\right)V_A'' - \frac{1}{10}V_B'' = 0 \qquad (5)$$

Node (B'')
$$-\frac{1}{10}V_A'' + \left(\frac{1}{10}+\frac{1}{20}\right)V_B'' = I - 2I_1'' \qquad (6)$$

Constraint
$$I_1'' = \frac{V_A''}{10} \qquad (7)$$

(7) in (6) $\longrightarrow -0.1V_A'' + 0.15V_B'' = I - 0.2V_A'' \Rightarrow V_A'' = 10I - 1.5V_B''$ (

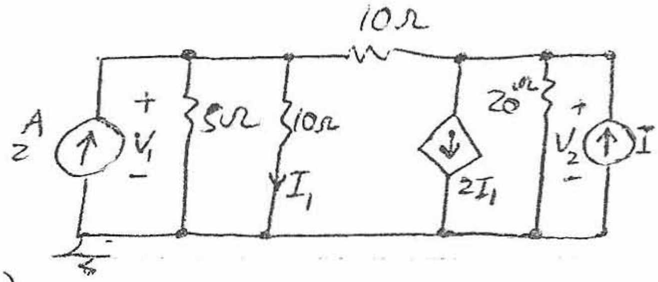
⑧ in ⑤ $\rightarrow$ $0.4(10I - 1.5V_B'') - 0.1V_B'' = 0 \Rightarrow V_B'' = 5.714I$ $V = V_2''$

$\therefore I_2'' = \frac{V_2''}{20} = 0.2857I$ A & $I_2 = I_2' + I_2''$ $\rightarrow$ ⑨

Since $V_2 = I_2(20)$ $\therefore$ For $V_2 = 0$ $\Rightarrow I_2 = 0$

$\therefore$ from ⑨ $\rightarrow$ $-0.14286 + 0.2857I = 0$ $\boxed{I = 0.5 \text{ A}}$

2-38 / 88 Solution by Nodal analysis:
Convert the 10V source;
denote the terminal voltage
of the new source by V_1.



$\therefore V_1*\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}\right)-V_2\left(\frac{1}{10}\right)=2$ (1)

& $V_1\left(-\frac{1}{10}\right)+V_2\left(\frac{1}{10}+\frac{1}{20}\right)=I-2I_1$ (2)

with constraint Eq: $I_1 = V_1/10$ (3)

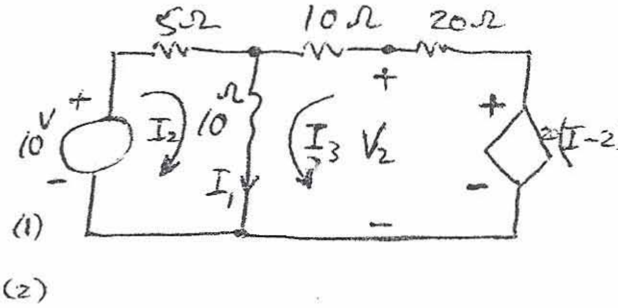
$\because V_2 = 0$

$\therefore$ from (1) : $\frac{4}{10}V_1 = 2$ $\therefore V_1 = \frac{20}{4} = 5V$ into (3)

$\therefore I_1 = 5/10 = 0.5$ Amp into (2)

$\therefore 5\left(-\frac{1}{10}\right) = I - 1$ $\therefore I = 1 - \frac{1}{2} = 0.5$ Amp.

Solution by Mesh analysis:
convert the combined current sources into voltage one.



$\therefore (10+10+20)I_3 + 10I_2 = 20(I-2I_1)$ (1)

& $10I_3 + (5+10)I_2 = 10$ (2)

with constraint equation: $I_1 = I_2 + I_3$ (3)

$\because V_2 = 0$

$\therefore 10I_3 + 10(I_2+I_3) = 0$ $\Rightarrow$ $I_2 + 2I_3 = 0$ $\Rightarrow I_2 = -2I_3$ (4)

(4) into (2) $\Rightarrow$ $10I_3 + 15(-2I_3) = 10$ $\Rightarrow$ $-20I_3 = 10$ $\therefore I_3 = -.5A$

into (4) $\Rightarrow$ $I_2 = 1$ Amp

into (3) $\Rightarrow$ $I_1 = 0.5$ Amp

into (1) $\Rightarrow$ $40(-.5) + 10(1) = 20(I-1) \Rightarrow$ $-10 = 20I - 20$

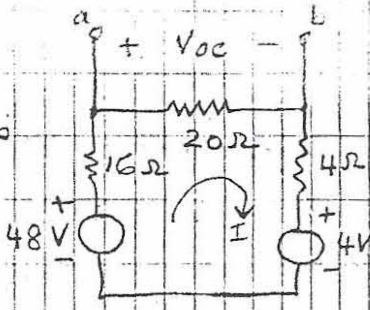
$\therefore I = (20-10)/20 = 0.5$ Amp

Solution by simplification:

$\because V_2 = 0$ $\therefore$ $I_2 = \frac{10}{5+10\|10} = \frac{10}{5+5} = 1$ Amp $\therefore I_1 = 0.5A = -I_3 = 2I_1 - I$ $\therefore I = .5A$

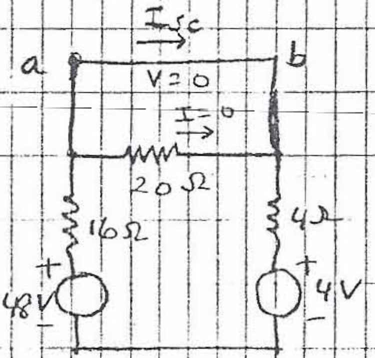
2-39
88

Convert the current source,
and open the ckt @ the terminals ab



ⓐ $I = \frac{48-4}{16+20+4} = \frac{44}{40} = 1.1\ A$

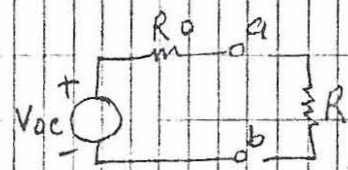
$\therefore V_{oc} = 20\,I = 22\ V$



Now, short circuit the terminals ab.

Since voltage across 20 Ω resistor = 0,
∴ I in the resistor = 0.
∴ Isc exists in the outer loop only.

From KVL, $I_{sc} = \frac{48-4}{16+4} = \frac{44}{20} = 2.2\ A$



$\therefore R_o = \frac{V_{oc}}{I_{sc}} = \frac{22}{2.2} = 10\ \Omega$

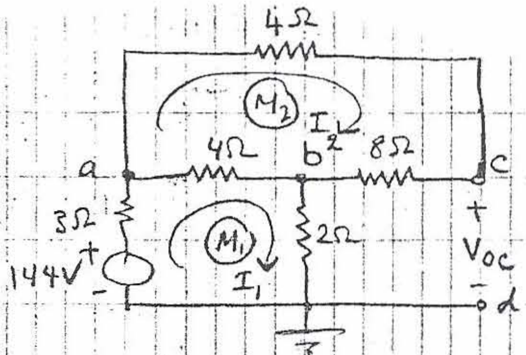
ⓑ For max Power delivery to Resistance, R, between ab:

∴ R must equal $R_o = 10\ \Omega$.

ⓒ $P_{R_{max}} = \left(\frac{V_{oc}}{R_o+R_o}\right)^2 R_o = \left(\frac{22}{20}\right)^2 * 10 =$

$= (1.1)^2 * 10 = 1.21 \times 10 = 12.1\ W$

$\boxed{\frac{2-40}{88}}$ 1^{st} convert the current source, and open ckt between c d.



(M_1) $(3+4+2)I_1 - 4I_2 = 144$ (1)

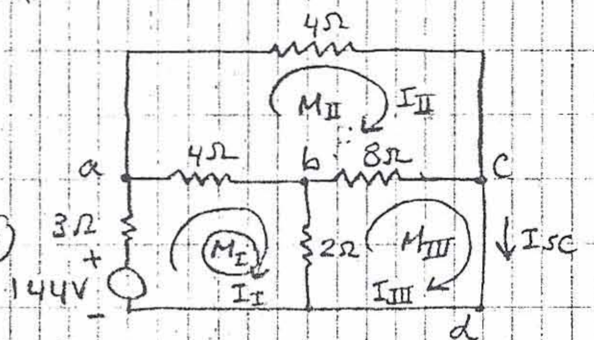
(M_2) $-4I_1 + (4+4+8)I_2 = 0$ (2)

from (2) $I_1 = 4I_2$ (3)

(3) in (1) $9(4I_2) - 4I_2 = 144$ $\Rightarrow \boxed{I_2 = 4.5\,A} \Rightarrow \boxed{I_1 = 18\,A}$

KVL $\longrightarrow$ $V_{oc} = 8I_2 + 2I_1 = 36 + 36 = 72\,V$

2^{nd} Short circuit terminals c d:



(M_I) $(3+4+2)I_I - 4I_{II} - 2I_{III} = 144$ (4)

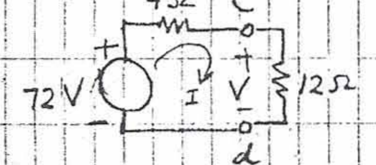
(M_{II}) $-\cancelto{1}{4}I_I + \cancelto{4}{(4+4+8)}I_{II} - \cancelto{2}{8}I_{III} = 0$ (5)

(M_{III}) $-\cancelto{1}{2}I_I - \cancelto{4}{8}I_{II} + \cancelto{5}{(2+8)}I_{III} = 0$ (6)

$$I_{sc} = I_{III} = \frac{\begin{vmatrix} 9 & -4 & 144 \\ -1 & 4 & 0 \\ -1 & -4 & 0 \end{vmatrix}}{\begin{vmatrix} 9 & -4 & -2 \\ -1 & 4 & -2 \\ -1 & -4 & 5 \end{vmatrix}} = \frac{144(4+4)}{9(20-8)-(-1)(-20-8)+(-1)(8+8)}$$

$$= \frac{1152}{64} = 18\,A$$

$\therefore \boxed{I_{SC} = 18\,A}$ $\Rightarrow R_O = \frac{V_{oc}}{I_{sc}} = \frac{72}{18} = 4\,\Omega$

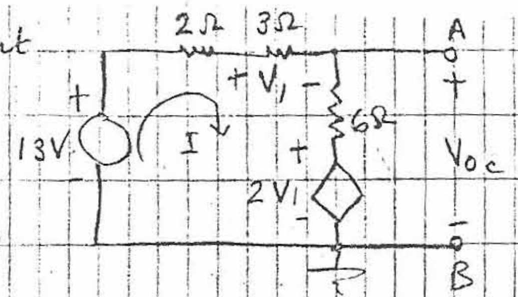


$I = \frac{72}{4+12} = 4.5\,A$

$\therefore V = 12 \times 4.5 = 54\,V$, the voltage across $12\,\Omega$ resistor.

2-42
89

(a) 1st Convert the controlled current source & Opent ckt @ AB.



KVL → $(2+3+6)I = 13 - 2V_1$ (1)

Constraint $V_1 = 3I$ (2)

(2) in (1) $11I = 13 - 2\times3I \Rightarrow I = \frac{13}{17} = 0.765\,A$

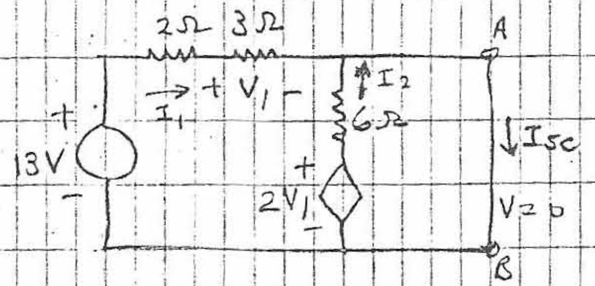
$V_{oc} = 6I + 2V_1 = (6+6)I = 12\times0.765 = 9.176\ V.$

2nd short ckt AB.

KCL:

$I_{sc} = I_1 + I_2$ (3)

KVL (Right hand side Loop)



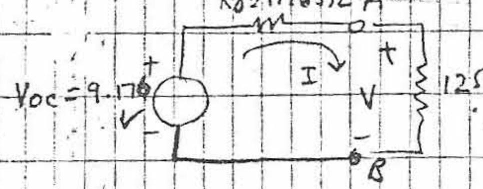
$2V_1 = 6I_2$ (4)

Constraint $V_1 = 3I_1$ (5)

KVL (outer loop) $(2+3)I_1 = 13 \rightarrow I_1 = \frac{13}{5} = 2.6\,A$ (6)

(5) → $V_1 = 3\times2.6 = 7.8\ V$

(4) → $I_2 = \frac{V_1}{3} = \frac{7.8}{3} = 2.6\,A.$

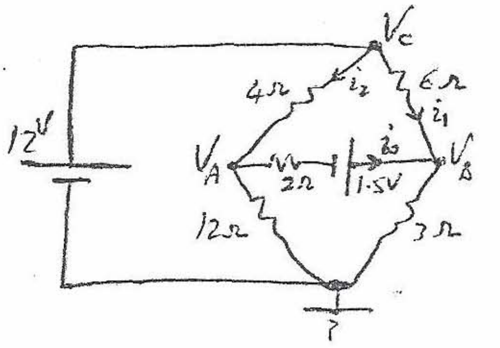


(3) → $I_{sc} = 2.6 + 2.6 = 5.2\ A$

$\therefore R_o = \frac{V_{oc}}{I_{sc}} = \frac{9.176}{5.2} = 1.765\ \Omega$

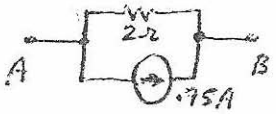
(b) $V = 12I = 12\times\frac{9.176}{12+1.765} = 8\ V$, the voltage across 12 Ω resistor.

2-45 / 89 Assume the notations shown:



a) Nodal Analysis:

∴ Converting the voltage source between AB to current source, we'll get



∴ The equations are:

$V_c = 12$ Volts (1)

$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{12}\right)V_A - \frac{1}{2}V_B - \frac{1}{4}V_c = -.75$ (2)

$-\frac{1}{2}V_A + \left(\frac{1}{6}+\frac{1}{2}+\frac{1}{3}\right)V_B - \frac{1}{6}V_c = .75$ (3)

(1) into (2): $\frac{10V_A}{12} - \frac{6V_B}{12} - \frac{36}{12} = -\frac{9}{12} \Rightarrow 10V_A - 6V_B = 27$

(1) into (2): $-\frac{6V_A}{12} + \frac{12V_B}{12} - \frac{24}{12} = \frac{9}{12} \Rightarrow -2V_A + 4V_B = 11$

∴ $V_A = \frac{27*4+6*11}{10*4-6*2} = \frac{108+66}{40-12} = \frac{174}{28}$ V & $V_B = \frac{10*11+2*27}{10*4-6*2} = \frac{110+54}{28} = \frac{164}{28}$ V

∴ $i_0 = \frac{V_A - V_B + 1.5}{2} = \frac{174-164+42}{56} = \frac{52}{56} = \frac{13}{14}$ Amp $= 0.92857$ A

b) Mesh Analysis:

∵ Sources are already converted.

∴ Equations are:

loop1: $12V - 6\Omega - 3\Omega$: $(6+3)i_1 + 0\,i_2 + 3\,i_0 = 12$

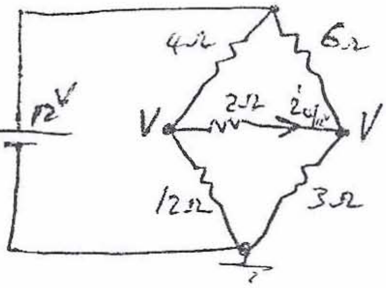
loop2: $12V - 4\Omega - 12\Omega$: $0\,i_1 + (4+12)i_2 - 12\,i_0 = 12$

loop3: $1.5V - 3\Omega - 12\Omega - 2\Omega$: $3\,i_1 - 12\,i_2 + (3+12+2)\,i_0 = 1.5$

$$\therefore\ i_0 = \frac{\begin{vmatrix} 9 & 0 & 12 \\ 0 & 16 & 12 \\ 3 & -12 & 1.5 \end{vmatrix}}{\begin{vmatrix} 9 & 0 & 3 \\ 0 & 16 & -12 \\ 3 & -12 & 17 \end{vmatrix}} = \frac{9*16*1.5+0+0-12*16*3-0+9*12*12}{9*16*17+0+0-3*16*3-0-9*12*12} = \frac{936}{1008} = \frac{13}{14}\text{A} = .92857\text{A}$$

c) Superposition Method

$\therefore\ i_o = i_o\big|_{12V} + i_o\big|_{1.5V}$



$i_o\big|_{12V}$: circuit is as shown

$$\therefore\ i_o\big|_{12V} = \frac{12-V}{4} - \frac{V}{12} = 3 - \frac{V}{3} \quad (4)$$

$$\& \ V - 2\,i_o\big|_{12V} = V' \ (5) \ \& \ \frac{12-V'}{6} - \frac{V'}{3} = -i_o\big|_{12V} \quad (6)$$

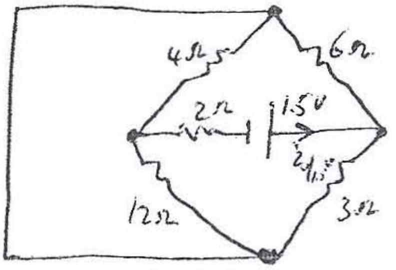
$$(4) \Rightarrow V = 9 - 3\,i_o\big|_{12V} \quad (7)$$

$$\& \ (6) \Rightarrow 2 - \frac{V'}{2} = -i_o\big|_{12V} \Rightarrow \quad V' = 4 + 2\,i_o\big|_{12V} \quad (8)$$

(7) & (8) into (5):

$$\therefore\ 9 - 3i_o\big|_{12V} - 2\,i_o\big|_{12V} = 4 + 2i_o\big|_{12V} \quad \Rightarrow \therefore\ i_o\big|_{12V} = \frac{9-4}{3+2+2} = \frac{5}{7}\ \text{Amp}$$

$i_o\big|_{1.5V}$: Circuit is as shown



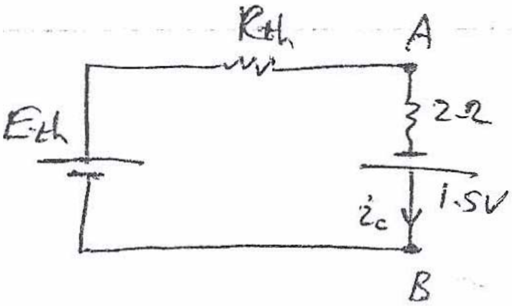
$$\therefore\ i_o\big|_{1.5V} = \frac{1.5}{2 + 6/\!/3 + 4/\!/12} =$$

$$= \frac{1.5}{2 + \frac{6\times3}{6+3} + \frac{4\times12}{4+12}} = \frac{1.5}{2+2+3} = \frac{1.5}{7}\text{A}$$

$$\therefore\ i_o = \frac{1.5}{7} + \frac{5}{7} = \frac{6.5}{7} = \frac{13}{14}\ \text{Amp} = 0.92857\ \text{A}$$

d) Thevenin Theorem:

Take the branch between AB away.



$$\therefore\ E_{th} = V_{AB} =$$

$$= \frac{12}{4+12} \times 12 - \frac{12}{6+3} \times 3$$

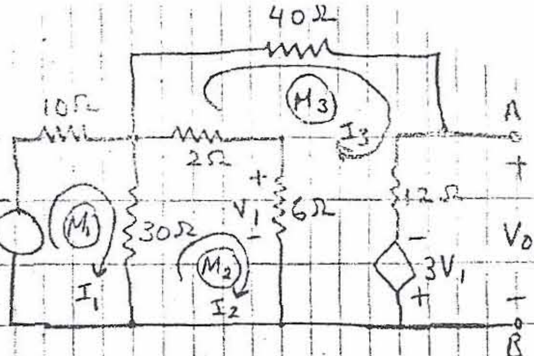
$$= \frac{144}{16} - \frac{36}{9} = 9 - 4 = 5\text{V}$$

$$\& \ R_{th} = 4/\!/12 + 6/\!/3 = 3 + 2 = 5\Omega$$

$$\therefore\ i_c = \frac{E_{th} + 1.5}{R_{th} + 2} = \frac{5 + 1.5}{5+2} = \frac{6.5}{7} = \frac{13}{14}\ \text{Amp} = 0.92857\text{A}$$

2-46 / 89

(a) 1st open ckt AB



(M_1) $(10+30)I_1 - 30I_2 = 0.55$ (1)

(M_2) $-30I_1 + (30+2+6)I_2 - (2+6)I_3 = 0$ (2)

(M_3) $-(2+6)I_2 + (2+40+12+6)I_3 = 3V_1$ (3)

Constraint $V_1 = 6(I_2 - I_3)$ (4)

(4) in (3) $I_3 = \dfrac{I_2}{3}$ (5)

(5) in (2) $I_1 = \dfrac{106}{90} I_2$ (6)

(6) in (1) $I_2 = 32.143\ mA$

from (5) $I_3 = 10.714\ mA$

$$V_{oc} = 12I_3 - 3V_1 = 12 \times 10.714 \times 10^{-3} - 3 \times 6 \times 10^{-3}(32.143 - 10.714)$$

$$\boxed{V_{oc} = -0.257\ V}$$

2nd Short ckt AB ⇒ Points A & B are at the same potential

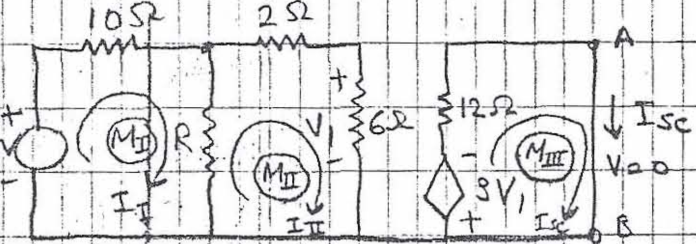
⇒ 40Ω & 30Ω in parallel, $R = \dfrac{30 \times 40}{30+40} = 17.143\Omega$

(M_1): $(10 + 17.143)I_I - 17.143 I_{II} = 0.55$ (7)

(M_2): $-17.143 I_I + (17.143 + 2 + 6) I_{II} = 0$ (8)

(M_3): $I_{sc} = \dfrac{-3V_1}{12}$ (9)

Constraint $V_1 = 6I_{II}$ (10)



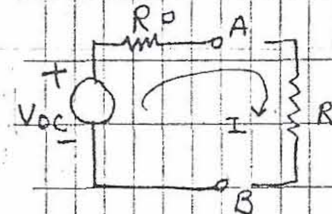
(10) in (9) $I_{sc} = -1.5 I_{II}$ (11)

from (8) $I_I = 1.467 I_{II}$ (12)

(12) in (7) $I_{II} = 24.265\ mA$

from (11) ∴ $I_{sc} = -36.397\ mA$

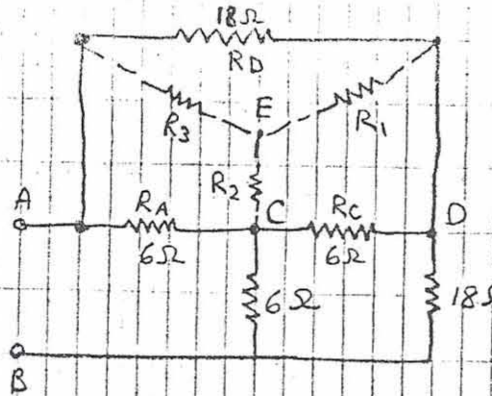
$$\therefore R_o = \frac{V_{oc}}{I_{sc}} = \frac{-0.257}{-36.397 \times 10^{-3}} = 7.065\ \Omega$$



(b) For max. power delivery, $R = R_o = 7.065\Omega$

$$P_{max} = I^2 R_o = \left(\frac{V_{oc}}{2R_o}\right)^2 R_o = \frac{(-0.257)^2}{4 \times 7.065} = 2.340\ mW$$

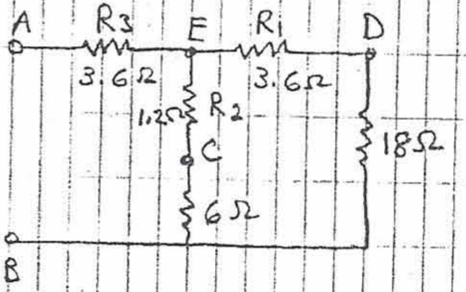
2-47 / 89 Convert Δ_{ACD} into Y and redraw.



$$R_1 = \frac{6\times18}{6+6+18} = \frac{108}{30} = 3.6\,\Omega$$

$$R_2 = \frac{6\times6}{30} = 1.2\,\Omega$$

$$R_3 = \frac{18\times6}{30} = 3.6\,\Omega$$



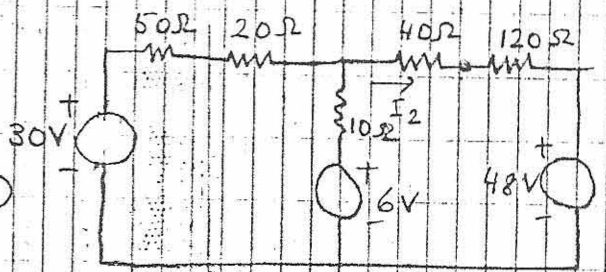
$$R_{EDB} = 3.6 + 18 = 21.6\,\Omega$$

$$R_{ECB} = 1.2 + 6 = 7.2\,\Omega$$

$$R_{EB} = R_{ECB} \,||\, R_{EDB} = \frac{7.2\times21.6}{7.2+21.6} = 5.4\,\Omega$$

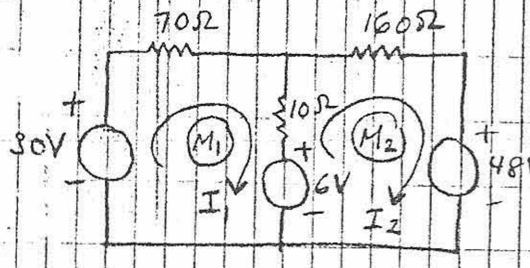
$$R_{AB} = R_3 + R_{EB} = 3.6 + 5.4 = 9.0\,\Omega.$$

2-48 / 89 Convert current sources



(M1) $(70+10)I_1 - 10I_2 = 30 - 6$ (1)

(M2) $-10I_1 + (10+160)I_2 = 6 - 48$ (2)



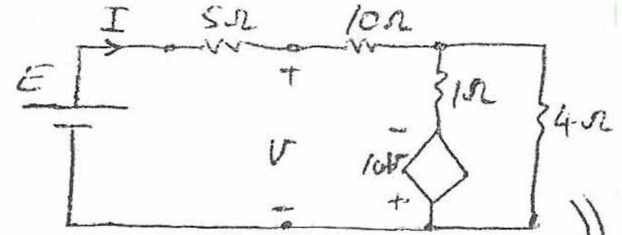
from (2) $I_1 = 17I_2 + 4.2$ (3)

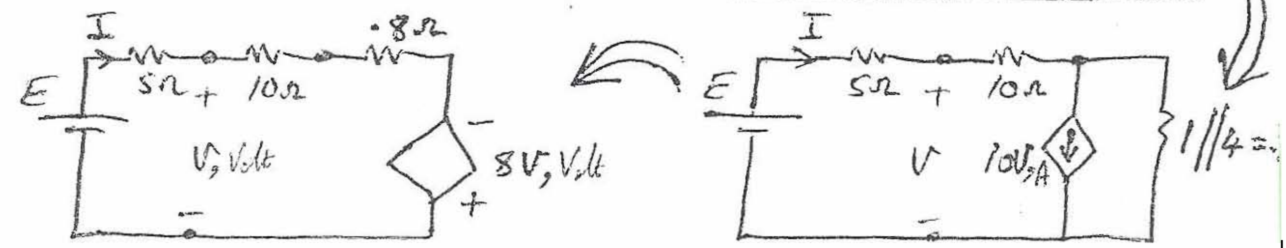
(3) in (1) $I_2 = -0.231\ A$

OR, you can convert the two voltage sources to current ones, then reduce them to a single current source, the convert to voltage source then solve for I_2.

2-49 / 89

Assume a supply current, I. Using circuit simplification as shown:





$\therefore \quad V = E - 5I \qquad (1)$

$\& \quad I = \dfrac{E+8V}{5+10+.8} = \dfrac{E+8V}{15.8} \qquad (2)$

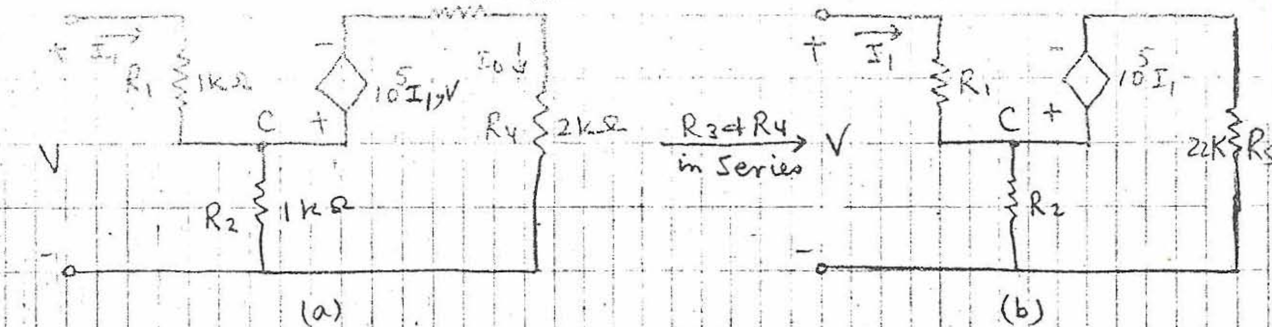
(1) into (2) $\quad \therefore \quad 15.8\,I = E + 8\,(E - 5I) = 9E - 40I$

$\therefore \; 15.8\,I + 40\,I = 9E \qquad \therefore \; I = \dfrac{9E}{55.8}$

$\because$ The equivalent resistor, R, will take same current I

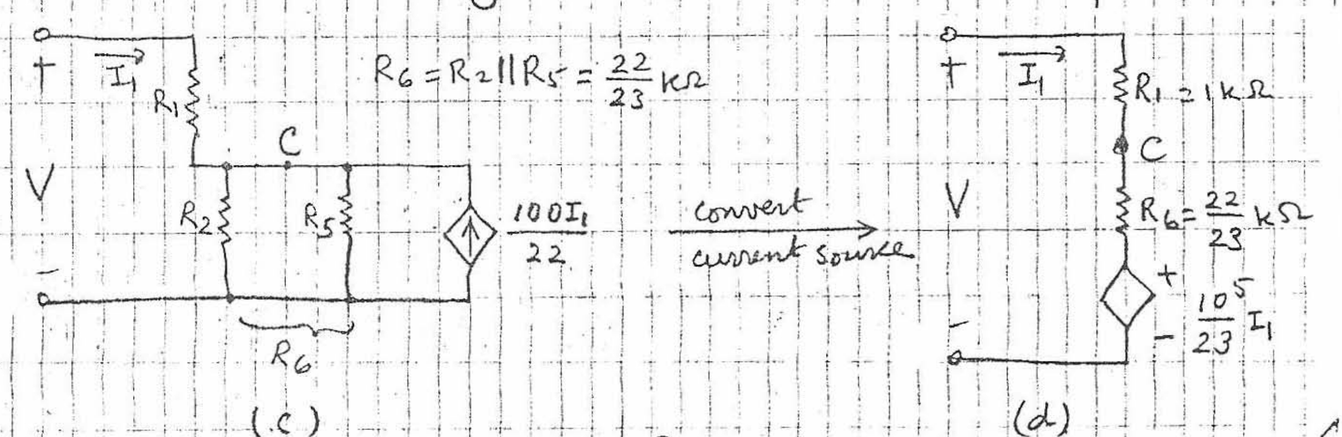
$\therefore \; IR = E \quad \therefore \; R = \dfrac{E}{I} = \dfrac{55.8}{9} = 6.2\ \Omega$, resistance seen by E.

2-50 / 89



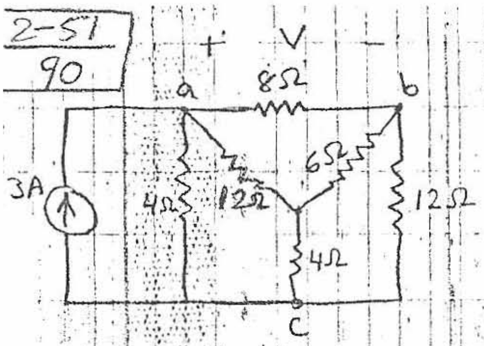
convert controlled voltage source

$R_6 = R_2 \| R_5 = \frac{22}{23}\ k\Omega$

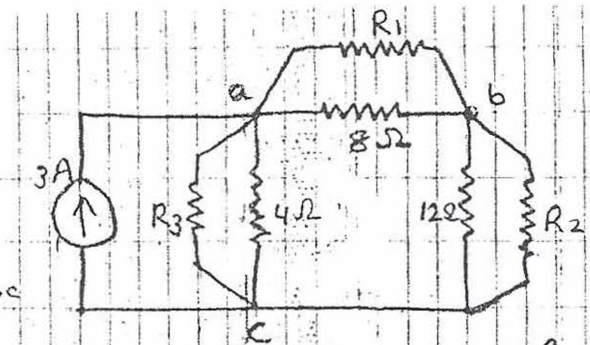


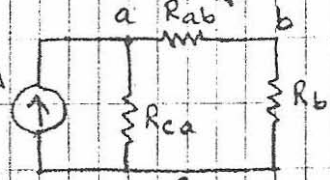
From Fig (d): $\quad V = (R_1 + R_6)\,I_1 + \dfrac{10^5 I_1}{23} \Rightarrow \boxed{R_{in} = \dfrac{V}{I_1} = 6.304\ k\Omega}$

2-57
90



convert
$Y_{abc} \rightarrow \Delta_{abc}$



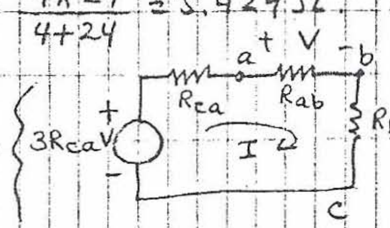


$$R_1 = \frac{(6\times12)+(12\times4)+(4\times6)}{4} = \frac{144}{4} = 36\,\Omega$$

$$R_2 = \frac{144}{12} = 12\,\Omega \quad , \quad R_3 = \frac{144}{6} = 24\,\Omega$$

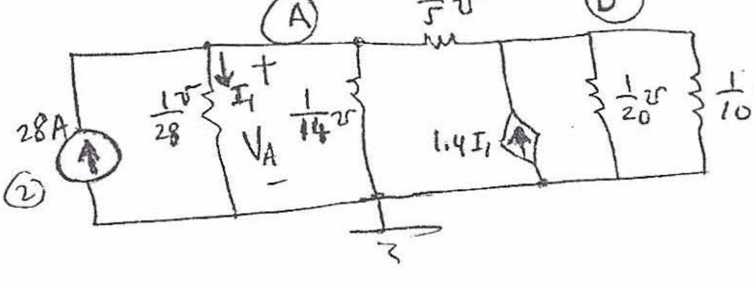
$$R_{ab} = \frac{8\times36}{8+36} = 6.\overline{54}\,\Omega \;,\; R_{bc} = \frac{12\times12}{12+12} = 6\,\Omega \;,\; R_{ca} = \frac{4\times24}{4+24} = 3.429\,\Omega$$

Convert current source ∴ $V = \underbrace{\frac{3R_{ca}}{R_{ca}+R_{ab}+R_{bc}}}_{I} \times R_{ab}$



$$\therefore \boxed{V = \frac{3\times3.429}{15.974}\times6.\overline{54} = 4.215\,V}$$

To find V_A by Node Voltage Method:



$$\left(\frac{1}{28}+\frac{1}{14}+\frac{1}{5}\right)V_A - \frac{1}{5}V_B = 28 \quad (1)$$

$$-\frac{1}{5}V_A + \left(\frac{1}{5}+\frac{1}{20}+\frac{1}{10}\right)V_B = 1.4I_1 \quad (2)$$

Constraint Eq.: $I_1 = \frac{V_A}{28}$ (3)

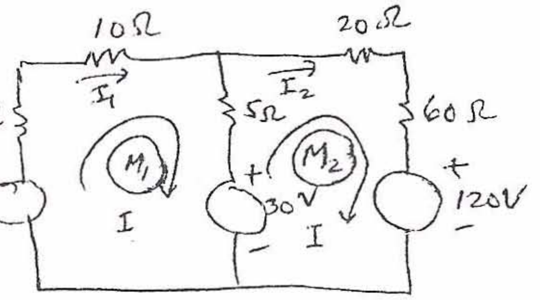
(3) in (2) $\rightarrow \left(\frac{1.4}{28}+\frac{1}{5}\right)V_A = 0.35\,V_B \Rightarrow V_B = 0.7143\,V_A$ (4)

(4) in (1) $\rightarrow (0.307 - 0.2\times0.7143)V_A = 28$

$\therefore V_A = 170\ 435$ Volts.

To find I_1 & I_2 by Mesh Analysis:

(M1) $(25+10+5)I_1 - 5I_2 = 75-30$ (1)



(M2) $-5I_1 + (5+20+60)I_2 = 30-120$ (2)

from (1) $I_2 = 8I_1 - 9$

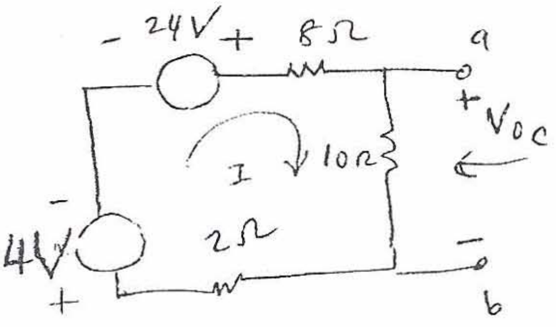
in (2)

$$-I_1 + 17(8I_1 - 9) = -18 \Rightarrow I_1 = \frac{153-18}{136-1} = 1\text{ A}$$

in (1) $\Rightarrow I_2 = 8 - 9 = -1\text{ A}$

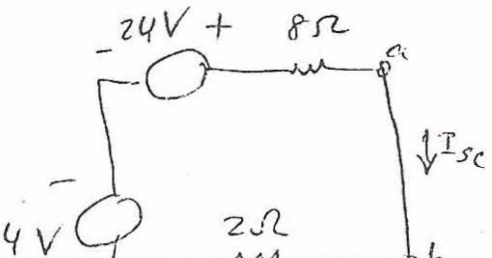
(a) To find Thevenin Equivalent at ab:

$$I = \frac{24-4}{8+10+2} = 1\text{ A}$$



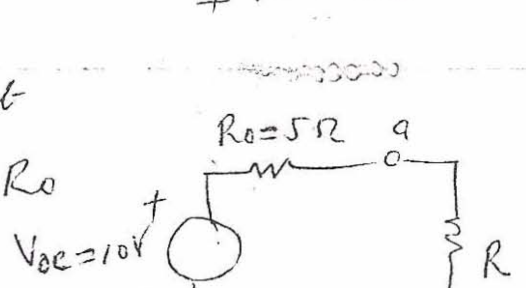
$$V_{oc} = 10I = 10\text{ V}$$

$$I_{sc} = \frac{24-4}{8+2} = 2\text{ A}$$



$$\therefore R_o = \frac{V_{oc}}{I_{sc}} = 5\ \Omega$$

(b) For R to receive the greatest Power, it has to equal to R_o



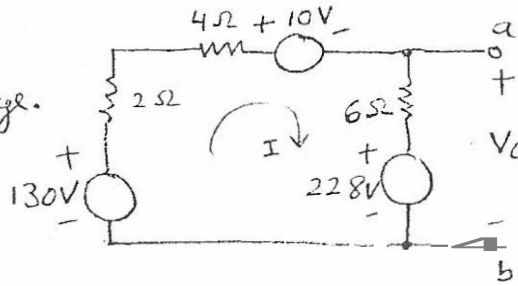
$\Rightarrow R = R_o = 5\Omega$.

(c) $P_{max} = I^2 R_o = \left(\frac{10}{2R_o}\right)^2 R_o = 5\text{ W}$

ⓐ Thevenin Equivalent at ab:

Open ckt @ ab — A.

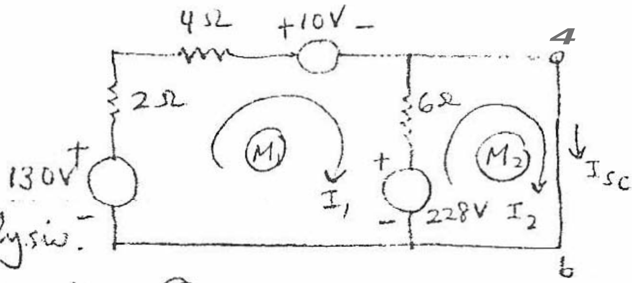
convert current source to voltage.



KVL

$$I(2+4+6) = 130-10-228$$

$$\therefore I = \frac{-108}{12} = -9\ A$$



$$\therefore V_{oc} = 6I + 228 = 174\ V$$

Now, find I_{sc} using Mesh analysis.

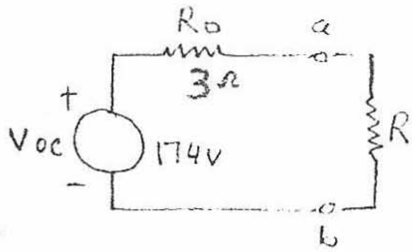
(M1) $(2+4+6)I_1 - 6I_2 = 130-228-10$ ①

(M2) $-6I_1 + 6I_2 = 228 \Rightarrow I_1 = I_2 - 38$ ②

in ①

$$12(I_2-38) - 6I_2 = -108 \Rightarrow I_{sc} = I_2 = 58\ A$$

$$R_o = \frac{V_{oc}}{I_{sc}} = \frac{174}{58.0} = 3\Omega.$$



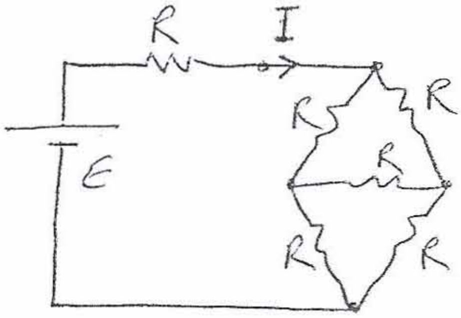
ⓑ R of max. power transfer:

For Max. Power, $R = R_o = 3\Omega$

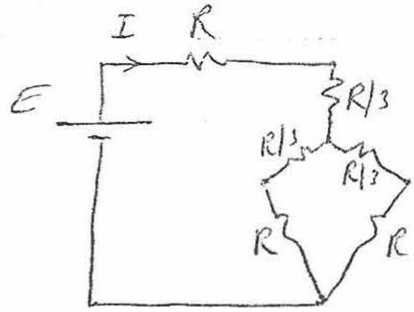
ⓒ Maximum output power:

$$P_{max} = \left(\frac{V_{oc}}{2R_o}\right)^2 (R_o) = \frac{(174)^2}{4 \times 3.0} = 2.523\ kW$$

To find I of the network shown by simplification we change one of the Δ's (upper or lower) to Y. All values of resistors in Y are equal



$= \frac{R \cdot R}{R+R+R} = \frac{R}{3}$



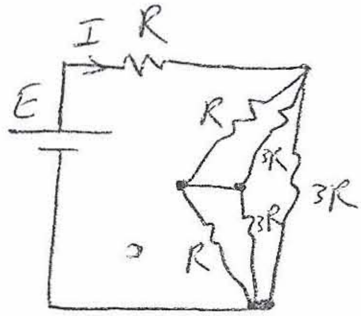
$$\therefore\ I = \frac{E}{R + \frac{R}{3} + \left(R+\frac{R}{3}\right) \| \left(R+\frac{R}{3}\right)}$$

$$= \frac{E}{\frac{4R}{3} + \frac{1}{2} * \frac{4R}{3}}$$

$$= \frac{3E}{4R+2R} = \frac{3E}{6R} = \frac{E}{2R}$$

OR Change one of the Y's (right or left) to Δ. All values of resistors in Δ are equal $= \frac{R \cdot R + R \cdot R + R \cdot R}{R}$

$= \frac{3R^2}{R} = 3R$



$$\therefore\ I = \frac{E}{R + \left[(R \| 3R) + (R \| 3R)\right] \| 3R}$$

$$= \frac{E}{R + \left[\frac{R * 3R}{R+3R} * 2\right] \| 3R} = \frac{E}{R + \left(\frac{6R^2}{4R}\right) \| 3R} =$$

$$= \frac{E}{R + 1.5R \| 3R} = \frac{E}{R + \frac{1.5R * 3R}{1.5R+3R}} = \frac{E}{R + \frac{4.5R}{4.5R}} = \frac{E}{2R}$$

B-1
908

(a) $2.24\angle 26.4^\circ = 2.24(\cos 26.4^\circ + j\sin 26.4^\circ) = 2.01 + j1.00$

$10e^{j30^\circ} = 10(\cos 30^\circ + j\sin 30^\circ) = 8.66 + j5.00$

$15\angle -72^\circ = 15(\cos 72^\circ - j\sin 72^\circ) = 4.64 - j14.27$

$20e^{-j126.9^\circ} = 20(\cos 126.9^\circ - j\sin 126.9^\circ) = -(12 + j16)$

$0.3\angle 140^\circ = 0.3(\cos 140^\circ + j\sin 140^\circ) = -0.230 + j0.193$

(b) $3 - j4 = \sqrt{(3)^2+(4)^2}\ \angle \tan^{-1}\frac{-4}{3} = 5\angle -53.1^\circ = 5e^{-j53.1^\circ}$

$-15 + j5 = 15.81\angle 180^\circ - 18.43^\circ = 15.81\angle 161.57^\circ = 15.81e^{j161.57^\circ}$

$10 + j20 = 22.36\angle 63.43^\circ = 22.36e^{j63.43^\circ}$

$-10 - j24 = 26\angle 180^\circ + 67.38^\circ = 26\angle 247.38^\circ = 26e^{j247.38^\circ}$

$0.25 + j0.433 = 0.5\angle 60^\circ = 0.5e^{j60^\circ}$

B-2
908

$A = 8 - j6 = 10\angle -36.9^\circ = 10e^{-j36.9^\circ}$

$B = 20 + j10 = 22.36\angle 26.57^\circ = 22.36e^{j26.57^\circ}$

$C = 0.866 - j0.5 = 1\angle -30^\circ = 1e^{-j30^\circ}$

(a) $A - B + C = (8 - j6) - (20 + j10) + (0.866 - j0.5) = -11.134 - j16.5$

(b) $BC - A = (22.36\angle 26.57^\circ)(1\angle -30^\circ) - A = 22.36\angle -3.44^\circ - A$

$= (22.32 - j1.34) - (8 - j6) = 14.32 + j4.66$

(c) $(ABC)^2 = \left[(10e^{-j36.9^\circ})(22.36e^{j26.57^\circ})(1e^{-j30^\circ})\right]^2$

$= (223.6\angle -40.31^\circ)^2 = 49997.8\angle -80.61^\circ$

(d) $AC/B - BC/A = \dfrac{(10\angle -36.9^\circ)(1\angle -30^\circ)}{22.36\angle 26.57^\circ} - \dfrac{(22.36\angle 26.57^\circ)(1\angle -30^\circ)}{10\angle -36.9^\circ}$

$= 0.45\angle -93.44^\circ - 2.236\angle 33.43^\circ$

$= [-0.45(\cos 86.56^\circ + j\sin 86.56^\circ)] - [1.87 + j1.23]$

$= (-0.03 - 1.87) - j(0.45 + 1.23) = -(1.9 + j1.68)$

43

B-3
908

$A = 10e^{-j45^\circ} = 7.07 - j7.07 = 10\angle{-45^\circ}$

$B = -4 + j2 = 4.47\angle{180 - 26.57^\circ} = 4.47\angle{153.43^\circ} = 4.47e^{j153.43^\circ}$

$C = 12\angle{30^\circ} = 12e^{j30} = 10.39 + j6$

(a) $AB - C/B = (10\angle{-45^\circ} \times 4.47\angle{153.43^\circ}) - \dfrac{12\angle{30^\circ}}{4.47\angle{153.43^\circ}} =$

$= 44.7\angle{108.43^\circ} - 2.68\angle{-123.43^\circ}$

$= 44.7(-\cos 71.57^\circ + j\sin 71.57^\circ) - 2.68(-\cos 56.57^\circ - j\sin 56.57^\circ)$

$= -12.66 + j44.67$

(b) $C + A/(B - C) = (10.39 + j6) + \dfrac{10\angle{-45^\circ}}{-14.39 - j4}$

$= (10.39 + j6) - \dfrac{10\angle{-45^\circ}}{14.94\angle{15.53^\circ}} = (10.39 + j6) - 0.67e^{-j60.53^\circ}$

$= 10.06 + j6.58$

(c) $BC = 53.67\angle{183.43^\circ} = 53.64(-\cos 3.43^\circ - j\sin 3.43^\circ)$

$= -53.57 - j3.22$

$AB = 44.7\angle{108.43^\circ} = -14.14 + j42.43$

$A + BC = (7.07 - j7.07) + (-53.57 - j3.22) = -46.50 - j10.29$

$= -47.62\angle{12.47^\circ}$

$C - AB = (10.39 + j6) - (-14.14 + j42.43) = 24.53 - j36.43$

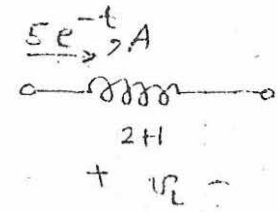
$= 43.92\angle{-56.04^\circ}$

$\therefore \dfrac{A + BC}{C - AB} = \dfrac{-47.62\angle{12.47^\circ}}{43.92\angle{-56.04^\circ}} = -1.08\angle{68.51^\circ} = 1.08\angle{248.51^\circ}$

(d) $A - B = 11.07 - j9.07 = 14.31\angle{-39.33^\circ}$

$\therefore \sqrt{C(A - B)} = \sqrt{171.75\angle{-9.33^\circ}} = 13.11\angle{-4.66^\circ}$

3-1 / 145



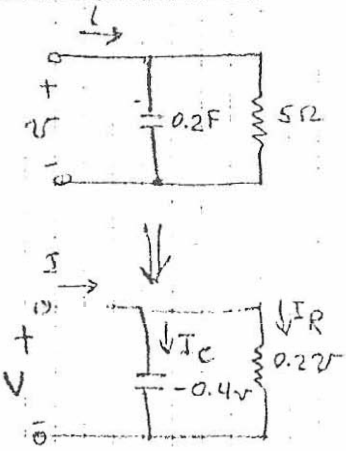
$s = -1/\text{sec}$

(a) $Z_L = sL = -2\ \Omega$

(b) $Y_L = \frac{1}{Z_L} = \frac{1}{sL} = \frac{1}{-2} = -0.5\ \mho$

(c) $v_L = L\frac{di}{dt} = -10e^{-t}$, V

3-3 / 145



$v = 10e^{-2t}$, V , $V = 10$ volt

$\therefore s = -2/\text{sec}$

$G = \frac{1}{R} = 0.2\ \mho$, $Y_C = sC = -0.4\ \mho$

(a) $I_R = GV = 0.2 \times 10 = 2\ A \Rightarrow i_R = 2e^{-2t}$, A

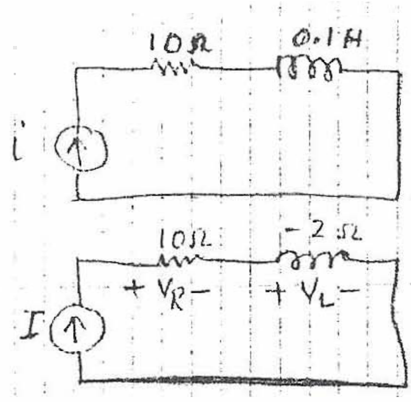
$I_C = Y_C V = -0.4 \times 10 = -4\ A \Rightarrow i_C = -4e^{-2t}$, A

(b) $I = I_R + I_C = -2 \Rightarrow i = -2e^{2t}$, A

(c) $Y = Y_C + G = 0.2 - 0.4 = -0.2\ \mho$

3-4 / 145

$i = 3e^{-20t}$, A , $I = 3\ A$, $s = -20/s$



$Z_L = sL = -2\ \Omega$

(a) $V_R = IR = 30\ V$, $v_R = 30e^{-20t}$ V

& $V_L = Z_L I = -6\ V$, $v_L = -6e^{\dots}$ V

(b) $V = V_R + V_L = 30 - 6 = 24\ V$, $v = 24e^{-20t}$, V

(c) $Z = R + sL = 10 - 2 = 8\ \Omega$

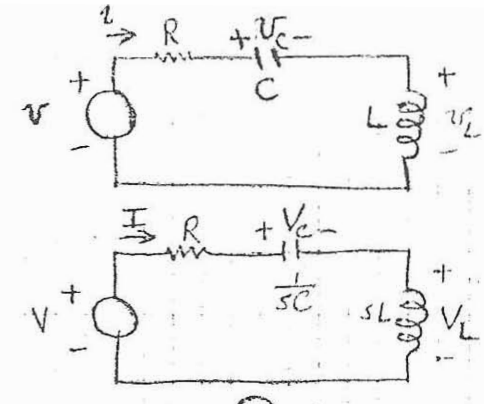
3-5 / 145 $i = I e^{st}$ A

$Z_C = \frac{1}{sC}\ \Omega,\ Z_L = sL\ \Omega$

(a) ∴ Network is as shown in Fig. (a).

(b) KVL → $V = (R + \frac{1}{sC} + sL)\, I$

$$\therefore v(t) = Ve^{st} = (R + sL + \frac{1}{sC}) I e^{st}, V$$



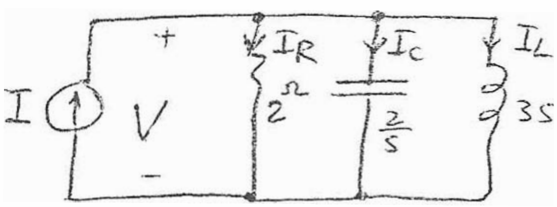
(a)

3-6 / 14-6 $R = 2\Omega$ = Transformed impedence

$C = .5F$ ∴ " " $= \frac{1}{.5s} = \frac{2}{s}, \Omega$

$L = 3H$ ∴ " " $= 3s, \Omega$

∴ Network becomes as shown:



$\therefore i_L(t) = 12 e^{-t/2}$, Amp

$\therefore s = -\frac{1}{2}$ rad/sec

& $I_L = 12$ Amp

$\therefore V = 12 * 3s = 36 * -\frac{1}{2} = -18$ Volts

$\therefore v(t) = -18\, e^{-t/2}$, volts

& $I_c = \frac{V}{2/s} = \frac{-18}{2/(-\frac{1}{2})} = \frac{-18 * (-\frac{1}{2})}{2} = \frac{9}{2}$ Amp

& $I_R = \frac{V}{R} = \frac{-18}{2} = -9$ Amp

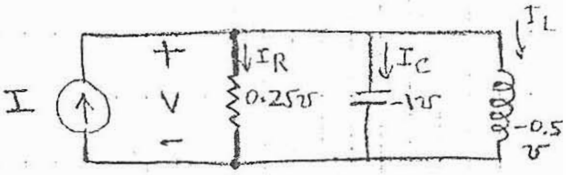
$\therefore I = I_R + I_c + I_L = -9 + \frac{9}{2} + 12 = \frac{-18+9+24}{2} = \frac{15}{2} = 7.5A$

$\therefore i(t) = 7.5\, e^{-t/2}$, Amp.

$\boxed{\frac{3-7}{146}}$ $R = 4\Omega$, $C = 1F$, $L = 2H$, $i_R = 3e^{-t}$ A

$s = -1$ /sec

$Y_C = sC = -1 \mho$, $Y_L = \frac{1}{sL} = -0.5 \mho$



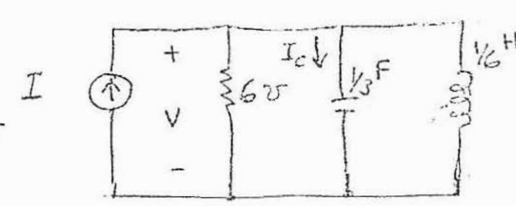
$V = R\, I_R = 4 \times 3 = 12$ V

$I_C = Y_C V = -12$ A , $I_L = Y_L V = -6$ A

KCL $\rightarrow I = I_R + I_C + I_L = 3 - 12 - 6 = -15$ A $\Rightarrow \boxed{i = -15 e^{-t} \text{ A}}$

$\boxed{\frac{3-10}{146}}$ $i(t) = 12 e^{-3t}$ A $\rightarrow s = -3$ /sec & $I = 12$ A

$G = 6\mho$, $Y_C = sC = -1\mho$, $Y_L = \frac{1}{sL} = -2\mho$



KCL $\rightarrow I = 12 = V(G + Y_C + Y_L) = 3 V$

$\therefore V = 4$ volts

$\therefore I_C = Y_C V = -4$ A $\Rightarrow i_c(t) = -4 e^{-3t}$ A

$\boxed{\frac{3-13}{147}}$ $i(t) = 10^{-3} \cos(3.14 \times 10^5 t - 30^\circ) = 10^{-3} \cos(3.14 \times 10^5 t - \frac{\pi}{6})$, A

(a) $\omega = 2\pi f = 3.14 \times 10^5 \Rightarrow f \simeq 50$ kHz

(b) $i \rightarrow I_{max}$ when $3.14 \times 10^5 t_1 - \frac{\pi}{6} = 0 \Rightarrow \boxed{t_1 = 1.67 \mu\text{sec}}$

(c) $i \rightarrow I_{min}$ when $3.14 \times 10^5 t_2 - \frac{\pi}{6} = \pi \Rightarrow \boxed{t_2 = 11.67 \mu\text{sec}}$

(d) Time between max. & min $= \frac{T}{2} = t_2 - t_1 = 10.0\ \mu$sec

$\therefore$ Time between two consecutive maxima = T

$\therefore \boxed{T = 2 \times 10 = 20\ \mu\text{sec}}$

3-14
147

$f = 60\,Hz$, $V_m = 169V$, $v = 102V$ @ $t=0$

$\omega = 2\pi f = 376.99$ rad/sec.

(a) Assume, $v = V_m \cos(\omega t + \theta) = 169\cos(376.99t + \theta)$

@ $t=0$, $v = 102 = 169\cos\theta \Rightarrow \theta = 52.88^\circ = 0.92$ rad.

$\therefore v = 169\cos(376.99t + 52.88^\circ)$, V

(b) $v \to 0$ @ t_1: $376.99t_1 + 0.92 = \frac{\pi}{2} \Rightarrow \boxed{t_1 = 1.72 \times 10^{-3} sec}$

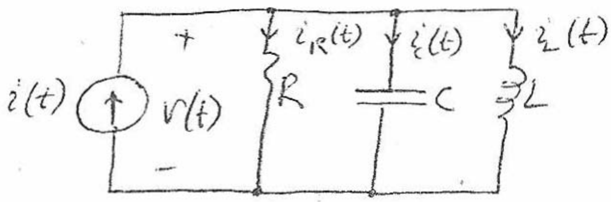
(c) Notice the period $T = \frac{1}{f} = \frac{1}{60}$ sec $= 1.\bar{6} \times 10^{-2}$ sec $= 16.\bar{6}$ msec

@ $t = \frac{1}{30}$ sec $= 2T$ from the time origin, the voltage will have the same value as it has at the origin, i.e., at $t=0 \Rightarrow v(2T) = v(0) = 102V$.

$\boxed{\frac{3-17}{147}}$ $R = \frac{1}{6}\,\Omega$ = impedence

$C = \frac{1}{3}F$ & $L = \frac{1}{6}h$

$i(t) = 12\cos(3t + 45^\circ)$, A

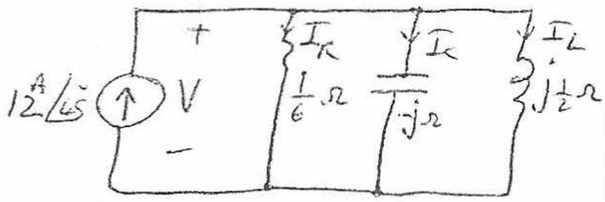


$\therefore\ s = j\omega = j3$ rad/sec

& $I = 12\,A\ \angle 45^\circ$

impedence of $C = \frac{1}{\frac{1}{3}\times j3} = \frac{1}{j} = -j\Omega$

" " $L = \frac{1}{6}\times j3 = j.5\Omega$



Transformed network is as shown.

$$\therefore\ I_c = 12\angle 45^\circ * \frac{(\frac{1}{6}\,//\,j\frac{1}{2})}{(\frac{1}{6}\,//\,j\frac{1}{2}) + (-j)} = 12\angle 45^\circ * \frac{\frac{j/12}{1/6 + j/2}}{\frac{j/12}{1/6+j/2} - j} =$$

$$= 12\angle 45^\circ * \frac{j/12}{j/12 - j(\frac{1}{6} + j/2)} = 12\angle 45^\circ * \frac{j}{j - 2j + 6} = \frac{12\angle 45^\circ \times \angle 90^\circ}{6 - j}$$

$$= \frac{12\angle 135^\circ}{\sqrt{37}\angle -9.46^\circ} = 1.9728\angle 144.46^\circ, \text{Ap}$$

$\therefore\ i_c(t) = 1.9728\ \cos(3t + 144.46^\circ)$, Amp

OR: Total impedence $= \frac{1}{6}\,//\,{-j}\,//\,j.5 = 1/\left[\frac{1}{1/6} + \frac{1}{-j} + \frac{1}{j.5}\right] = 1/\left[6 + j - j2\right] = \frac{1}{6-j} = Z, \Omega$

$$\therefore\ V = IZ = 12\angle 45^\circ * \frac{1}{6-j} = \frac{12\angle 45^\circ}{\sqrt{37}\angle -9.46^\circ} = 1.9728\angle 54.46^\circ, V$$

$$\therefore\ I_c = \frac{V}{-j} = \frac{1.9728\angle 54.46^\circ}{1\angle -90^\circ} = 1.9728\angle 144.46^\circ, \text{Ap}$$

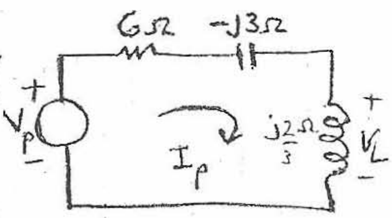
$\therefore\ i_c(t) = 1.9728\ \cos(3t + 144.46^\circ)$, Amp.

3-18 / 147

$R = 6\,\Omega,\ C = \frac{1}{6}\,F,\ L = \frac{1}{3}\,H\ ;\ v_L(t) = ?$

(a) $v(t) = 10\cos(2t - 30^\circ) \equiv \underbrace{10e^{-j30^\circ}}_{V_P}, V\ \ \&\ \ s = j2 = j\omega$

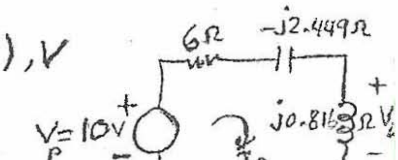
$Z_C = \frac{1}{j\omega C} = \frac{-j}{2/6} = -j3\,\Omega\ \ \&\ \ Z_L = j\omega L = j2/3 = j\frac{2}{3}\,\Omega$



$I_p = \frac{V_p}{Z} = \frac{10\angle -30^\circ}{6 - j3 + j\frac{2}{3}} = \frac{10\angle -30^\circ}{6 - j2.333} = \frac{10\angle -30^\circ}{6.4377\angle -21.25^\circ} = 1.553\angle -8.75^\circ, A$

$V_{L_p} = I_p Z_L = (1.553\angle -8.75^\circ)(0.667\angle 90^\circ) = 1.0356\angle 81.25^\circ, V$

$\therefore\ V_{L_p} \equiv v_L(t) = 1.0356\cos(2t + 81.25^\circ), V$



(b) $v(t) = 10\cos\sqrt{6}\,t \equiv 10e^{j0^\circ}\,Volt = V_p\ \ \&\ \ s = j\sqrt{6}$

$\therefore\ I_p = \frac{V_p}{Z} = \frac{10\angle 0^\circ}{6 + j(0.816 - 2.449)} = \frac{10}{6.218\angle -15.23^\circ} = 1.61\angle 15.23^\circ, A$

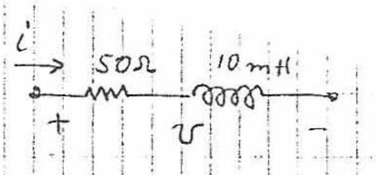
$V_{L_p} = I_p Z_L = (1.61\angle 15.23^\circ)(0.816\angle 90^\circ) = 1.313\angle 105.23^\circ\ V$

$\therefore\ V_{L_p} \equiv v_L(t) = 1.313\cos(\sqrt{6}\,t + 105.23^\circ), Volt.$

3-19 / 147

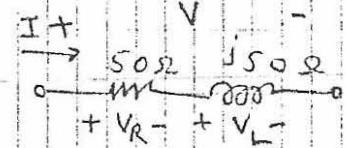
$i = 10^{-2}\cos 5\times10^3 t,\ A \ \Rightarrow\ I = \frac{10^{-2}}{\sqrt{2}}\angle 0^\circ\ A$

$\omega = 5\times10^3\ rad/sec$



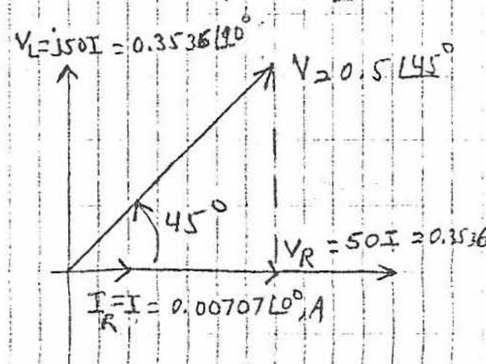
(a) $Z_L = j\omega L = j\,5\times10^3\times10\times10^{-3} = j50\ \Omega$

$\therefore\ Z = 50 + j50 = 70.71\angle 45^\circ,\ \Omega$



(b) $V = Z I = 70.71\angle 45^\circ \times \frac{10^{-2}}{\sqrt{2}} = 0.5\angle 45^\circ\ V$

$\therefore\ v(t) = 0.7071\cos(5\times10^3 t + 45^\circ)\ V$



(c) $V_R = I R = 50\times\frac{10^{-2}}{\sqrt{2}}\angle 0^\circ = 0.3536\ V$

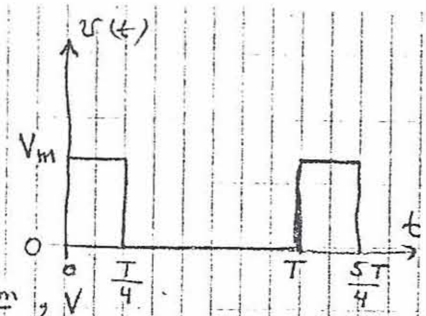
$V_L = I Z_L = (50\angle 90^\circ)\left(\frac{10^{-2}}{\sqrt{2}}\right) = 0.3536\angle 90^\circ\ V$

$V = V_R + V_L$

3-21 / 148

$v = V_m \qquad 0 \le t \le \frac{T}{4}$

$v = 0 \qquad \frac{T}{4} < t \le T$



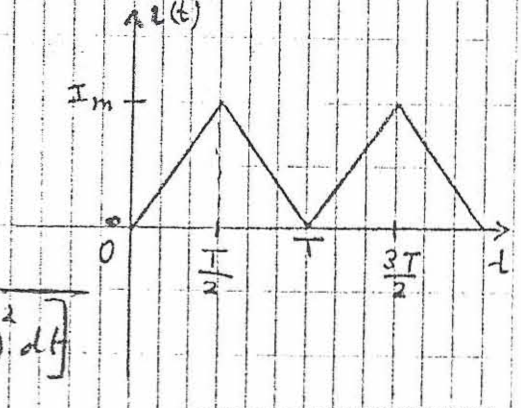
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T/4} V_m^2\,dt} = V_m\sqrt{\frac{1}{T}[t]_0^{T/4}} = \frac{V_m}{2}, V$$

$$V_{av} = \frac{1}{T}\int_0^{T/4} V_m\,dt = \frac{V_m}{T}[t]_0^{T/4} = \frac{V_m}{4}, V$$

3-24 / 148

$i(t) = \frac{I_m}{T/2}\, t \qquad 0 \le t \le \frac{T}{2}$

& $i(t) = -\frac{I_m}{T/2}\, t + 2I_m \qquad \frac{T}{2} < t \le T$



$$I_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{T/2}\left(\frac{I_m}{T/2}t\right)^2 dt + \int_{T/2}^{T}\left(\frac{-I_m t}{T/2} + 2I_m\right)^2 dt\right]}$$

$$= \sqrt{\frac{1}{T}\left\{\left(\frac{I_m^2}{T^2/4}\right)\left[\frac{t^3}{3}\right]_0^{T/2} + \left(\frac{I_m^2}{T^2/4}\right)\left[\frac{t^3}{3}\right]_{T/2}^{T} - \left(\frac{4I_m^2}{T/2}\right)\left[\frac{t^2}{2}\right]_{T/2}^{T} + (4I_m^2)[t]_{T/2}^{T}\right\}}$$

$$= 2I_m\sqrt{\frac{1}{T}\left\{\frac{1}{T^2}\left[\frac{T^3/8}{3}\right] + \frac{1}{T^2}\left[\frac{T^3 - T^3/8}{3}\right] - \frac{2}{T}\left[\frac{T^2 - T^2/4}{2}\right] + \left[T - \frac{T}{2}\right]\right\}}$$

$$= 2I_m\sqrt{\frac{1}{12}} \implies \boxed{I_{rms} = \frac{I_m}{\sqrt{3}}}$$

$$I_{av} = \frac{1}{T}\left\{\int_0^{T/2}\left(\frac{I_m t}{T/2}\right)dt + \int_{T/2}^{T}\left(-\frac{I_m t}{T/2} + 2I_m\right)dt\right\}$$

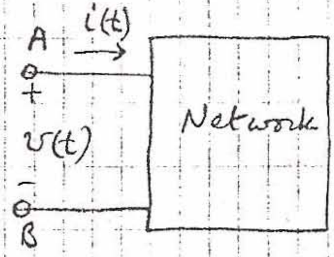
$$= \frac{2I_m}{T}\left\{\frac{1}{T}\left[\frac{t^2}{2}\right]_0^{T/2} - \frac{1}{T}\left[\frac{t^2}{2}\right]_{T/2}^{T} + [t]_{T/2}^{T}\right\} = 2I_m\left\{\frac{1}{4}\right\}$$

$$\therefore \boxed{I_{av} = \frac{I_m}{2}}$$

3-25 / 148

$v(t) = 100\sqrt{2}\cos 10^3 t$, V

$i(t) = 10\sqrt{2}\cos(10^3 t + 60^\circ)$, A



(a) $V = 100\angle 0^\circ$, V & $I = 10\angle 60^\circ$, A

$\therefore P = VI\cos\theta = 100 \times 10 \cos 60 = 500$ W

& $Q = VI\sin\theta = 100 \times 10 \sin 60 = 866$ var

& $VA = 100 \times 10 = 1000$ VA = 1KVA

(b) $Z = \dfrac{V}{I} = \dfrac{100\angle 0}{10\angle 60} = 10\angle -60 = 5 - j8.66$, Ω

(c) X is −ve ⇒ capacitive $\therefore -jX_c = -j8.66$ Ω

$\therefore X_c = \dfrac{1}{\omega C} = 8.66$, $\omega = 10^3$ rad/s

$\therefore C = \dfrac{1}{8.66 \times 10^3} = 115.47\,\mu F$

3-27 / 149

$v(t) = 220\sqrt{2}\cos(377t - 15^\circ)$ V ; $i(t) = 10\cos(377t - 60^\circ)$A

$\therefore V = 220\angle -15^\circ$ V & $I = \dfrac{10}{\sqrt{2}}\angle -60^\circ$ A & $\omega = 377$ rad/s

(a) $P = VI\cos(-15^\circ - (-60^\circ)) = \dfrac{220 \times 10}{\sqrt{2}}\cos(45^\circ) = 1100\,W = 1.1$ KW

$Q = VI\sin 45^\circ = 1100$ var $= 1.1$ KVar

$VA = VI = \dfrac{220 \times 10}{\sqrt{2}} = 1555.6$ VA $= 1.5$ KVA

(b) $Z = R + jX = \dfrac{V}{I} = \dfrac{220\angle -15^\circ}{(10/\sqrt{2})\angle -60^\circ} = 31.11\angle 45^\circ = 22 + j22$ Ω

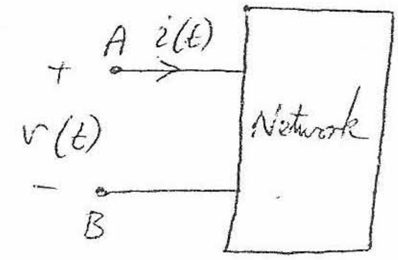
(c) Since X is +ve ⇒ It is inductive ckt. $\therefore jX_L = j22$

$\therefore X = X_L = \omega L = 377L = 22 \Rightarrow L = 58.4$ mH

$\frac{3-28}{149}$ $v(t) = 4\cos 3t + 3\sin 3t$, V

&

$i(t) = 3\cos 3t - 4\sin 3t$, A



$\because \sin 3t = \cos(3t - 90^\circ)$

$\therefore v(t) = 4\cos 3t + 3\cos(3t - 90^\circ) = (4 + 3\angle{-90^\circ})\cos 3t$, V

$\therefore V_{rms} = \frac{4+3\angle{-90^\circ}}{\sqrt{2}} = \frac{4-j3}{\sqrt{2}} = \frac{5}{\sqrt{2}}\angle{-36.87^\circ} = 3.5355\angle{-36.87^\circ}$, V

& $i(t) = 3\cos 3t - 4\cos(3t - 90^\circ) = (3 - 4\angle{-90^\circ})\cos 3t$, A

$\therefore I_{rms} = \frac{3-4\angle{-90^\circ}}{\sqrt{2}} = \frac{3+j4}{\sqrt{2}} = \frac{5\angle{53.13^\circ}}{\sqrt{2}} = 3.5355\angle{53.13^\circ}$, A

a) $\therefore P_{av} = I_{rms} \cdot V_{rms} \cdot \cos \angle_{I_{rms}}^{V_{rms}} = 3.5355 * 3.5355 * \cos(-36.87^\circ - 53.13^\circ)$

$= 12.5\cos(-90^\circ) = 0$ Watt.

& $Q = I_{rms} \cdot V_{rms} \cdot \sin \angle_{I_{rms}}^{V_{rms}} = 12.5\sin(-90^\circ) = -12.5$ Vars $= 12.5$ Cap. Vars

& $S = I_{rms} \cdot V_{rms} = 12.5$ VA.

b) $Z_{AB} = \frac{V_{rms}}{I_{rms}} = \frac{3.5355\angle{-36.87^\circ}}{3.5355\angle{53.13^\circ}} = 1\angle{-90^\circ}, \Omega = -j\Omega = R - jX_c$

$\therefore R = 0\ \Omega$, the equivelant resistance between AB

& $X_c = 1\Omega$, " " capacitive reactance between AB.

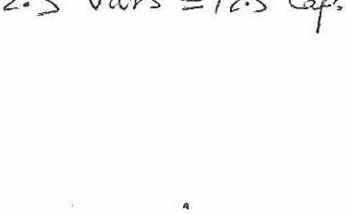
c) $\because \omega = 3$ rad/sec & $X_c = \frac{1}{\omega C}$

$\therefore C = \frac{1}{\omega X_c} = \frac{1}{3*1} = \frac{1}{3}$ F, the capacitance realizing X_c at 3 rad/sec.

\# $I_{av} = \frac{1}{4a} \cdot \left[I*a + \frac{I*2a}{2} + 0\right] = \frac{2Ia}{4a} = I/2$

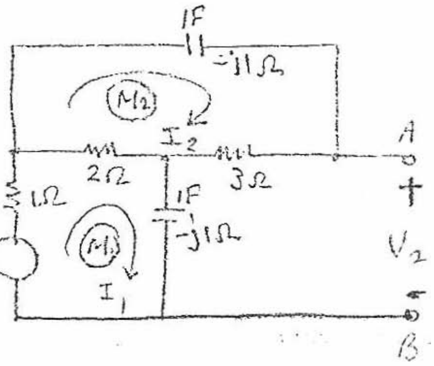
& $I_{rms} = \sqrt{\frac{1}{4a} \cdot \left[I^2 a + \frac{I^2 * 2a}{3} + 0\right]} = \sqrt{\frac{3I^2a + 2I^2a}{12a}} = \sqrt{\frac{5}{12}} \cdot I$

$= .6455 I$



$\boxed{\dfrac{3-29}{149}}$ $v = 16\sqrt{2}\cos t$ V

$\therefore \omega = 1$ rad/sec



(a) (M_1) $(3-j)I_1 - 2I_2 = 16\angle 0^\circ$ (1)

(M_2) $-2I_1 + (5-j)I_2 = 0$ (2)

(2) $\rightarrow$ $I_1 = (2.5 - j0.5)I_2$ (3)

(3) in (1) $[(3-j)(2.5-j0.5) - 2]I_2 = 16$

$$\therefore I_2 = \frac{16}{5-j4} = \frac{16}{6.4\angle -38.7^\circ} = 2.5\angle 38.7^\circ = 1.95 + j1.56 \text{ A}$$

$$I_1 = (2.55\angle -11.31^\circ)(2.5\angle 38.7^\circ) = 6.37\angle 27.3^\circ = 5.66 + j2.93 \text{ A}$$

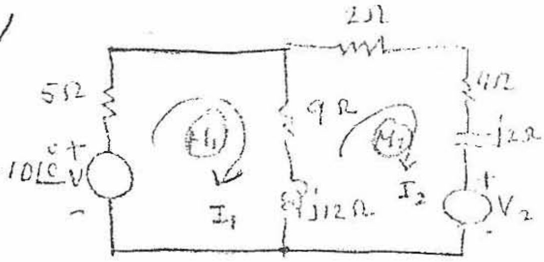
$$V_2 = 3I_2 - jI_1 = 8.78 - j0.98 = 8.83\angle -6.34^\circ \text{ V}$$

$$v_2 = 8.83\sqrt{2}\cos(t - 6.34^\circ) \text{ V} = 12.5\cos(t - 6.34^\circ), \text{ V}$$

3-30 / 150

$V_2 = (4-j2)(2\angle{-30^\circ}) = 8.94\angle{-56.57^\circ}\,V$

$= 4.93 - j7.46\,V$



(M1) $(14+j12)I_1 - (9+j12)I_2 = 10\angle 0^\circ$ (1)

(M2) $-(9+j12)I_1 + (15+j10)I_2 = -8.94\angle{-56.57^\circ}$ (2)

(1) $\rightarrow$ $I_1 = \dfrac{10+(9+j12)I_2}{14+j12}$ (3)

(3) in (2). $\dfrac{-(9+j12)\times 10}{14+j12} + \left[\dfrac{-(9+j12)(9+j12)}{14+j12} + (15+j10)\right] I_2 =$

$= -8.94\angle{-56.57^\circ}$

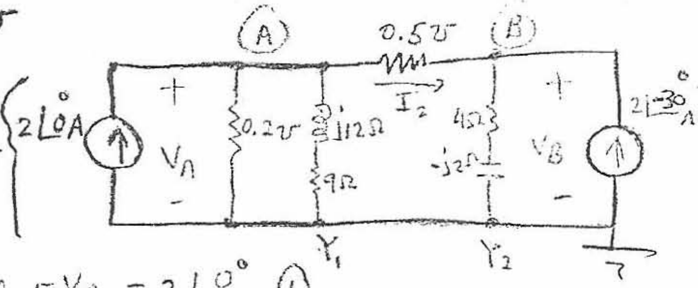
$\Rightarrow I_2 = .196 + j.948 = 0.968\angle 78.3^\circ, A$

(3) $\rightarrow$ $I_1 = .400 + j.434 = 0.590\angle 47.3^\circ$, A

3-31 / 150

$Y_1 = \dfrac{1}{9+j12} = 0.07\angle{-53.1^\circ}\,℧$

$Y_2 = \dfrac{1}{4-j2} = 0.22\angle{+26.57^\circ}\,℧$



(A) $(0.2 + 0.07\angle{-53.1^\circ} + 0.5)V_A - 0.5V_B = 2\angle 0^\circ$ (1)

(B) $-0.5\ V_A + (0.5 + 0.22\angle{+26.57^\circ})V_B = 2\angle{-30^\circ}$ (2)

(1) $\rightarrow$ $V_A = \dfrac{2+0.5V_B}{0.74\angle{-4.12^\circ}}$ (3)

(3) in (2): $-0.5\left(\dfrac{2+0.5\,V_B}{0.74\angle{-4.12^\circ}}\right) + (0.7+j0.1)V_B = 2\angle{-30^\circ}$

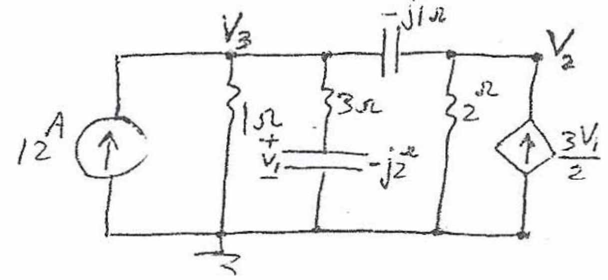
$\Rightarrow V_B = 7.61 - j4.07 = 8.63\angle{-28.12^\circ}$, Volts

$\therefore V_A = 8.00 - j2.17 = 8.29\angle{-15.18^\circ}$, Volts

$\therefore I_2 = \dfrac{V_A - V_B}{2} = 0.196 + j.948 = 0.968\angle 78.32^\circ$, Amp.

3-33 / 150

Convert to phasor voltages and impedences using current sources as shown.



$$\therefore \left(\frac{1}{1}+\frac{1}{3-j2}+\frac{1}{-j}\right)V_3-\frac{1}{-j}V_2=12 \qquad (1)$$

$$\& \quad -\frac{1}{-j}V_3+\left(\frac{1}{-j}+\frac{1}{2}\right)V_2=\frac{3V_1}{2} \qquad (2)$$

Constraint Eq. $\quad V_1=\frac{V_3}{3-j2}*(-j2) \qquad (3)$

(3) into (2) $\Rightarrow \quad \frac{V_3}{j}+\left(\frac{j-2}{2j}\right)V_2=\frac{3}{2}\left(\frac{-j2}{3-j2}\right)V_3$

$$\therefore V_2=V_3\left[\frac{-j3}{3-j2}-\frac{1}{j}\right]\Big/\left(\frac{j-2}{2j}\right)=\frac{-j3+j(3-j2)}{3-j2}\cdot\frac{2j}{j-2}\cdot V_3=\frac{4j\,V_3}{-4+j7} \qquad (4)$$

(4) into (1) $\Rightarrow \left[\frac{3-j2+1+j(3-j2)}{3-j2}-j\cdot\frac{4j}{-4+j7}\right]V_3=12$

$$\therefore \left(\frac{6+j}{3-j2}+\frac{4}{-4+j7}\right)V_3=12 \Rightarrow \frac{(6+j)(-4+j7)+4(3-j2)}{(3-j2)(-4+j7)}\cdot V_3=12$$

$$\therefore \frac{-31+38j+12-j8}{(3-j2)(-4+j7)}\cdot V_3=12 \Rightarrow V_3=\frac{12*(3-j2)*(-4+j7)}{-19+j30}$$

into (4) $\Rightarrow \quad V_2=\frac{4j}{(-4+j7)}\cdot\frac{12*(3-j2)(-4+j7)}{-19+j30}=\frac{48j(3-j2)}{-19+j30}=$

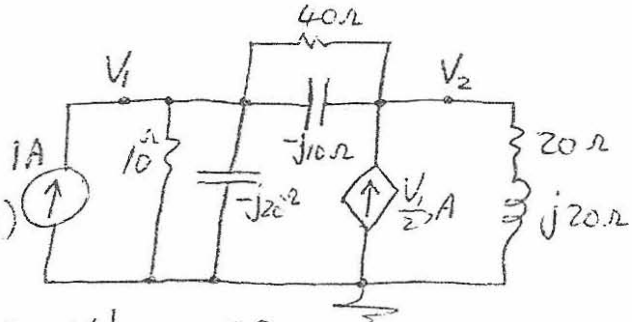
$$=\frac{48(2+j3)(-19-j30)}{(-19)^2-(j30)^2}=\frac{48(52-j117)}{361+900}=\frac{48*13(4-j9)}{12\ldots}=$$

$$\therefore V_2=\frac{48}{97}\cdot\sqrt{97}\angle-\tan^{-1}\left(\frac{9}{4}\right)=\frac{48}{\sqrt{97}}\angle-\tan^{-1}(2.25)=4.8737\angle-66.04^{\circ}, V$$

$$\therefore v_2(t)=4.8737\sqrt{2}\cos(2t-66.04^{\circ})=6.8924\cos(2t-66.04^{\circ}), V$$

3-39 / 153

Solution by Nodal Analysis:
Sources are already converted.



$$\therefore \left(\frac{1}{10}+\frac{1}{-j20}+\frac{1}{40}+\frac{1}{-j10}\right)V_1-\left(\frac{1}{40}+\frac{1}{-j10}\right)V_2=1 \quad (1)$$

$$\& \; -\left(\frac{1}{40}+\frac{1}{-j10}\right)V_1+\left(\frac{1}{40}+\frac{1}{-j10}+\frac{1}{20+j20}\right)V_2=+V_1/2 \quad (2)$$

$$(2)\Rightarrow \left(-\frac{1}{2}-\frac{1}{40}-j\frac{1}{10}\right)V_1+\left(\frac{1}{40}+j\frac{1}{10}+\frac{20-j20}{20^2+20^2}\right)V_2=0$$

$$\therefore \frac{21+j4}{40}\cdot V_1=\frac{20+j80+20-j20}{800}V_2=\frac{40+j60}{800}V_2=\frac{2+j3}{40}V_2$$

$$\therefore V_1=\left(\frac{2+j3}{21+j4}\right)V_2=\left(\frac{3.6056\angle +56.31^\circ}{21.3776\angle +10.75^\circ}\right)V_2=\left(0.16866\angle +45.53^\circ\right)V_2 \quad (3)$$

$$(3) \text{ into } (1)\Rightarrow \left[\left(\frac{j4-2+j-4}{j40}\right)\left(.16866\angle 45.53^\circ\right)-\frac{j-4}{j40}\right]V_2=1$$

$$\therefore \left[(j5-6)(.16866\angle 45.53^\circ)-j+4\right]V_2=j40$$

$$\therefore \left((7.8102\angle 140.19^\circ)(.16866\angle 45.53^\circ)+4-j\right)V_2=j40$$

$$\therefore \left(1.3173\angle 185.72^\circ+4-j\right)V_2=j48$$

$$\therefore \left(-1.3107-j.1313+4-j\right)V_2=j40$$

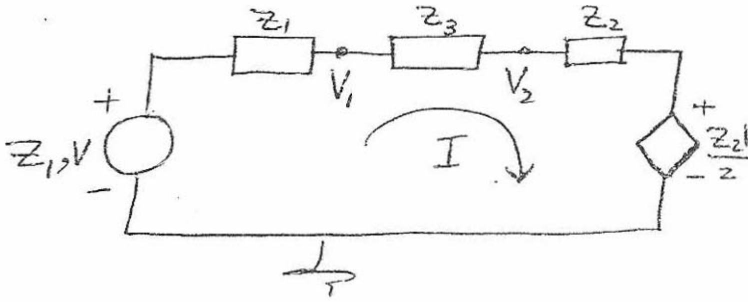
$$\therefore \left(2.6893-j1.1313\right)V_2=j40$$

$$\therefore V_2=\frac{j40}{2.6893-j1.1313}=\frac{40\angle 90^\circ}{2.9175\angle -22.81^\circ}=13.7102\angle 112.81^\circ, V$$

$$\therefore V_2=13.7102\angle 112.81^\circ, \; V$$

3-40 / 153 Solution by Mesh Analysis:

Convert to voltage sources;



$\therefore\ Z_1 = 10 // (-j20)$

$= \dfrac{10(-j20)}{10-j20}$

$= \dfrac{-j200}{10-j20}$

$= \dfrac{200\angle -90^\circ}{22.3607\angle -63.43^\circ}$

$= 8.9443\angle -26.57^\circ,\ \Omega$

$= 8 - j4,\ \Omega$

$\&\ Z_3 = 40 // (-j10) = \dfrac{40 * (-j10)}{40-j10} = \dfrac{-j400}{40-j10} = \dfrac{400\angle -90^\circ}{41.2311\angle -14.04^\circ} = 9.7014\angle -7\ \Omega$

$= 2.3529 - j9.41$

$\&\ Z_2 = 20 + j20 = 28.2843\angle 45^\circ,\ \Omega$

$$\therefore\ I = \frac{Z_1 - (Z_2 V_1/2)}{Z_1 + Z_2 + Z_3} = \frac{Z_1 - V_1}{Z_1} \qquad (1)$$

$$\therefore\ V_1\left(\frac{Z_2/2}{Z_1+Z_2+Z_3} - \frac{1}{Z_1}\right) = \frac{Z_1}{Z_1+Z_2+Z_3} - 1 = \frac{-Z_2-Z_3}{Z_1+Z_2+Z_3}$$

$$\therefore\ V_1 * \left(\frac{Z_2 Z_1 - 2Z_1 - 2Z_2 - 2Z_3}{2Z_1(Z_1+Z_2+Z_3)}\right) = \frac{-Z_2-Z_3}{Z_1+Z_2+Z_3}$$

$$\therefore\ V_1 = \frac{2Z_1(Z_2+Z_3)}{2(Z_1+Z_2+Z_3) - Z_2 Z_1} \qquad (2)$$

$$\therefore\ V_2 = V_1 - Z_3 I = (\text{from } (1))\ V_1 - Z_3(Z_1 - V_1)/Z_1 = V_1\left(1 + \frac{Z_3}{Z_1}\right) - Z_3$$

using (2)

$$\therefore\ V_2 = \frac{2Z_1(Z_2+Z_3)}{2(Z_1+Z_2+Z_3) - Z_1 Z_2} \cdot \frac{Z_1+Z_3}{Z_1} - Z_3 = \frac{2(Z_1+Z_3)(Z_2+Z_3)}{2(Z_1+Z_2+Z_3) - Z_1 Z_2} - Z$$

$$= \frac{2(8-j4+2.3529-j9.4118)(20+j20+2.3529-j9.4118)}{2(8-j4+20+j20+2.3529-j9.4118) - 8.9443\angle -26.57^\circ * 28.2843\angle 45^\circ} -$$

$$= \frac{2(10.3529-j13.4118)(22.3529+j10.5882)}{2(30.3529+j6.5882) - 252.98\angle 18.43^\circ} - Z_3$$

$$= \frac{2 * 16.9428\angle -52.33^\circ * 24.7339\angle 25.35^\circ}{60.7058 + j13.1764 - 240 - j80} - Z_3 = \frac{838.12\angle -26.99^\circ}{-179.29 - j66.824} -$$

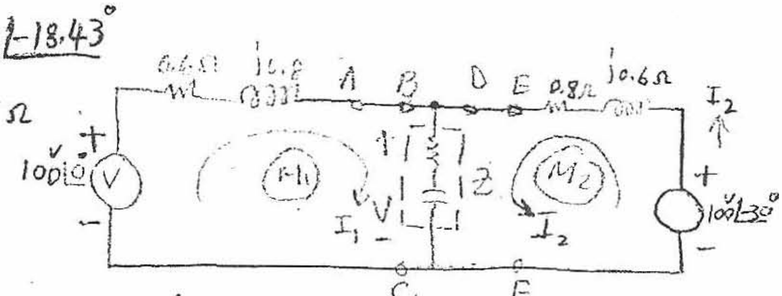
$$= \frac{838.12\angle -26.99^\circ}{191.34\angle -159.56^\circ} - Z_3 = 4.3802\angle 132.57^\circ - Z_3 = -2.9633 + j3.2258 -$$

$= -2.9633 + j3.2258 - 2.3529 + j9.4118 = -5.3162 + j12.$

$= 13.7102\angle 112.81^\circ$

$\therefore\ V_2 = 13.7102\angle 112.81^\circ,\ V$

$\boxed{\frac{3-45}{155}}$ $Z = \frac{10(8-j6)}{10+(8-j6)} = 5.27\angle{-18.43^\circ}$
$= 5-j1.67\,\Omega$



(M_1) $\left[(0.6+j0.8)+(5-j1.67)\right]I_1 + (5-j1.67)I_2 = 100\angle{0^\circ}$ (1)

(M_2) $+(5-j1.67)I_1 + \left[(5-j1.67)+(0.8+j0.6)\right]I_2 = +100\angle{-30}$ (2)

(1) $\rightarrow$ $I_1 = 17.65\angle{8.80^\circ} - 0.93\angle{-9.64^\circ}\, I_2$ (3)

(3) in (2) $+(5-j1.67)(17.65\angle{8.80^\circ}) + \left[+(5-j1.67)(-0.93\angle{-9.64^\circ}) + (5.8-j1.07)\right]I_2$
$= +100\angle{-30^\circ}$

$\therefore$ $I_2 = -18.06\angle{41.52^\circ} = -13.52 - j11.97\ A$

into (3):

$\therefore I_1 = 33.75\angle{20.1^\circ} = 31.70 + j11.57, A$

$\therefore V = (I_1 + I_2)Z = 95.85\angle{-19.70^\circ} = 90.24 - j32.31$, Volts

(a) $P_a = i_{AB} \cdot V_{BC} \cdot \cos \angle^{V_{BC}}_{i_{AB}}$

$\therefore P_a = V I_1 \cos \angle^{I_1}_{V} = 95.85 * 33.75 * \cos(20.1^\circ - (-19.7^\circ)) = 2487\,W$
$= 2.487\,KW$

(b) $P_b = i_{DE} \cdot V_{EF} \cdot \cos \angle^{V_{EF}}_{i_{DE}}$

$\therefore P_b = V I_2 \cos \angle^{-I_2}_{V} = 95.85 * 18.06 * \cos(41.52^\circ + 19.7^\circ) = 833.4\,W$

(c) $P_{100\angle 0^\circ} = I_1 * 100 * \cos(0 - (20.1^\circ)) = 33.75 * 100 * \cos 20.1^\circ = 3170.4\,W = 3.17\,KW$

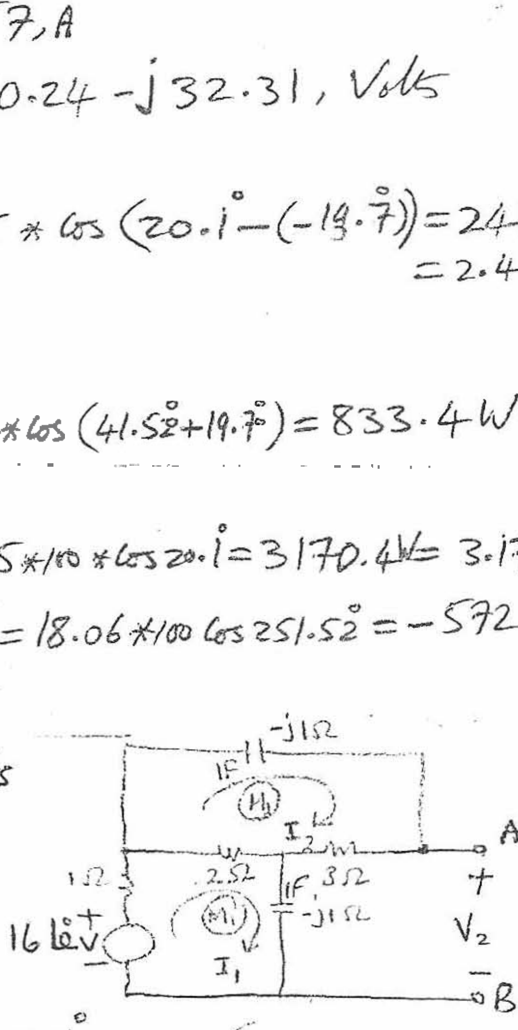
$P_{100\angle 30^\circ} = I_2 * 100 \times \cos(-30^\circ - (180^\circ + 41.52^\circ)) = 18.06 * 100 \cos 251.52^\circ = -572.4\,W$

i.e it absorbs 572.4 W.

$\boxed{\frac{3-47}{155}}$ $v = 16\sqrt{2}\cos t\ V \Rightarrow \omega = 1\ rad/s$

(a) (M_1) $(3-j)I_1 - 2I_2 = 16\angle{0^\circ}$ (1)

(M_2) $-2I_1 + (5-j)I_2 = 0$ (2)



(2) $\rightarrow$ $I_1 = (2.5 - j0.5)I_2 = 2.55\angle{-11.31^\circ} I_2\ A$ (3)

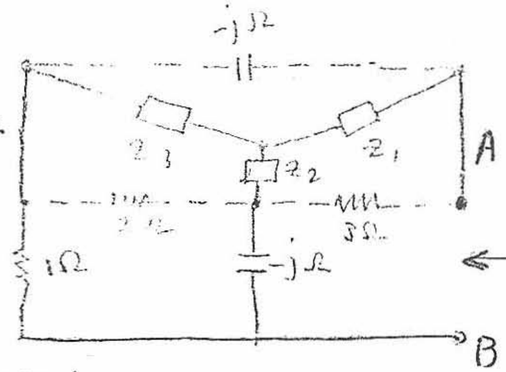
③ in ① $[(3-j)(2.55\angle{-11.31^\circ}) - 2]\, I_2 = 16$

$\therefore I_2 = 2.5\angle{38.7^\circ} = 1.95 + j\,1.56$ A

$\therefore I_1 = 6.37\angle{27.3^\circ} = 5.66 + j\,2.93$ A

$\therefore V_{oc} = V_2 = 3I_2 + (-j)I_1 = 8.83\angle{-6.34^\circ} = 8.78 - j\,0.98$, Volts

To determine Z_o, suppress the voltage source and use $\Delta \rightarrow Y$ transformation.



$\Sigma Z = 2 + 3 - j = 5 - j = 5.1\angle{-11.31^\circ}\ \Omega$

$Z_1 = \dfrac{3(-j)}{\Sigma Z} = 0.59\angle{-78.69^\circ}$

$= 0.12 - j\,0.58\ \Omega$

$Z_2 = \dfrac{2 \times 3}{\Sigma Z} = 1.18\angle{+11.31^\circ} = 1.15 + j\,0.23\ \Omega$

$Z_3 = \dfrac{2(-j)}{\Sigma Z} = 0.39\angle{-78.69^\circ} = 0.077 - j\,0.385\ \Omega$

$Z_o = Z_1 + [(Z_2 - j) \,||\, (Z_3 + 1)] = Z_1 + \dfrac{(1.15 - j.77)(1.077 - j.385)}{1.15 - j.77 + 1.077 - j.385}$

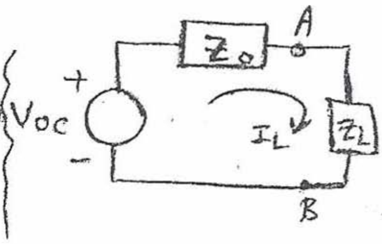
$= .12 - j.58 + (.57 - j.28)$

$\therefore Z_o = .683 - j.854 = 1.09\angle{-51.34^\circ}\ \Omega$

ⓑ For Max. Power output,

$Z_L = (Z_o)^* = 0.683 + j.854\ \Omega$ (inductive)

$= 1.09\angle{51.34^\circ} = R_L + jX_L$



ⓒ $I_L\Big|_{P_{max}} = \dfrac{V_{oc}}{Z_o + Z_o^*} = \dfrac{8.83\angle{-6.34^\circ}}{.683 + .683} = 6.47\angle{-6.34^\circ}$, Amp

$\therefore P_{max} = |I_L|^2 R_L = (6.47)^2(.683) = 28.6$ W.

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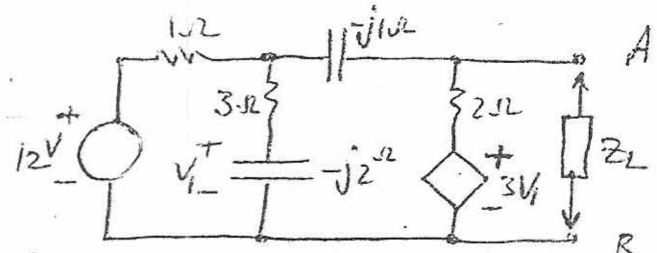
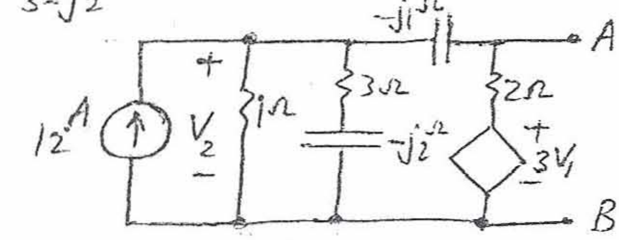
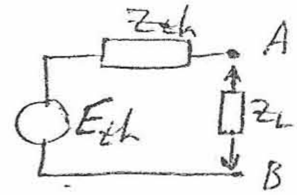
a) Converting to phasor voltages and impedences the circuit becomes as shown opposite.



Using simplification:

$\therefore\ V_1 = V_2 \cdot \dfrac{-j2}{3-j2}$ (1)

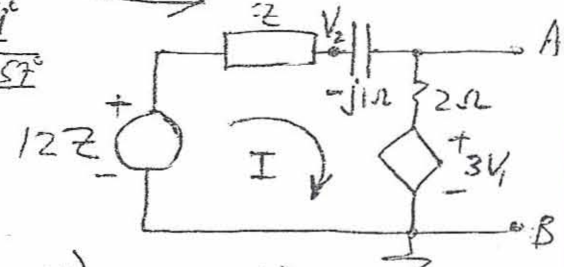




$\therefore\ Z = 1 \,||\, (3-j2)$

$= \dfrac{1*(3-j2)}{1+3-j2} = \dfrac{3-j2}{4-j2} = \dfrac{3.6056\angle -33.69^\circ}{4.4721\angle -26.57^\circ}$

$= 0.8062\angle -7.13^\circ = .8 - j.1,\ \Omega$ (2)



$\therefore\ I = \dfrac{12Z - V_2}{Z} = \dfrac{V_2 - 3V_1}{2-j1} = \text{(using (1))}$

$= \dfrac{V_2 - 3V_2(-j2/(3-j2))}{2-j1} = \dfrac{V_2}{(3-j2)(2-j)}\left[3-j2+j6\right] = V_2\left(\dfrac{3+j4}{4-j7}\right)$

$\therefore\ 12 = \dfrac{V_2}{Z} + V_2\left(\dfrac{3+j4}{4-j7}\right) \therefore\ V_2 = \dfrac{12}{\frac{1}{Z}+\frac{3+j4}{4-j7}} = \dfrac{12Z(4-j7)}{4-j7+(3+j4)Z} = \text{using}$ [illegible]

$= \dfrac{12 * .8062\angle -7.13^\circ * 8.0623\angle -60.26^\circ}{4-j7+5.0\angle 53.13^\circ * .8062\angle -7.13^\circ} = \dfrac{78\angle -67.38^\circ}{4-j7+4.0311\angle 46.01^\circ}$

$\therefore\ V_2 = \dfrac{78\angle -67.38^\circ}{4-j7+2.8+j2.9} = \dfrac{78\angle -67.38^\circ}{6.8-j4.1} = \dfrac{78\angle -67.38^\circ}{7.9404\angle -31.09^\circ} = 9.8232\angle -36.29^\circ,$

$\therefore\ V_{AB} = V_2 + jI = \text{(using (3))}\ V_2 + j\left(12 - \dfrac{V_2}{Z}\right) = +j12 + V_2\left(1 - \dfrac{j}{Z}\right) = +j12 + V_2\left(1 - \dfrac{1\angle 90^\circ}{.8062}\right.$ [illegible]

$= +j12 + V_2\left(1 - 1.2403\angle 97.13^\circ\right) = +j12 + V_2\left(1 + .1538 - j1.2308\right) =$

$= +j12 + V_2\left(1.1538 - j1.2308\right) = +j12 + 9.8232\angle -36.29^\circ * 1.6871\angle -46.8$ [illegible]

$= +j12 + 16.5722\angle -83.14^\circ = +j12 + 1.9794 - j16.4536 = 1.9794 - j4.45$ [illegible]

$= 4.8737\angle -66.0$ [illegible]

$\therefore\ E_{th} = V_{AB} = 4.8737\angle -66.04^\circ,\ V$

& $\dfrac{E_{th}}{Z_{th}} = I_{AB} = \dfrac{3V_1}{2} + \dfrac{12Z}{Z-j1} = \dfrac{3}{2}V_2\left(\dfrac{-j2}{3-j2}\right) + \dfrac{12Z}{Z-j} = \left(\dfrac{-j3}{3-j2}\right)\left(\dfrac{-j}{Z-j}\right)12Z + \dfrac{12Z}{Z-j}$ [illegible]

$= \dfrac{12Z}{(Z-j)(3-j2)} \cdot \left[3-j2-3\right] = \dfrac{12 * .8062\angle -7.13^\circ * 2\angle -90^\circ}{(.8-j.1-j) * 3.6056\angle -33.69^\circ} =$

$$= \frac{19.3494 \angle -97.13^\circ}{(.8 - j1.1) * 3.6056 \angle -33.69^\circ} = \frac{5.3666 \angle -63.43^\circ}{1.3601 \angle 53.97^\circ} = 3.9456 \angle -9.46^\circ, A$$

$$\therefore Z_{th} = \frac{E_{th}}{I_{AB}} = \frac{4.8737 \angle -66.04^\circ}{3.9456 \angle -9.46^\circ} = 1.2352 \angle -56.58^\circ, \Omega = .6804 - j1.0309, \Omega$$

b) $\therefore$ Maximum power in Z_L is obtained when $Z_L = \bar{Z}_{th} = .6804 + j1.0309, \Omega$

c) Value of max. power $= \left(\frac{E_{th}}{Z_{th} + \bar{Z}_{th}}\right)^2 * R_L = \left(\frac{4.8737}{2 * .6804}\right)^2 * .6804 = 8.\overline{72}$ Watt

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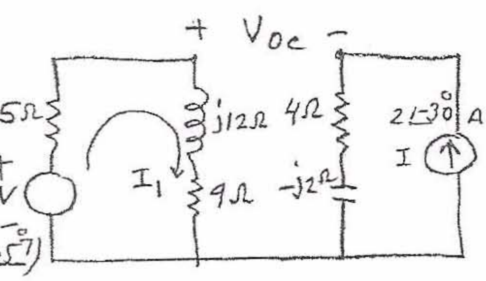
ⓐ $I_1 = \frac{10 \angle 0^\circ}{5 + (9 + j12)}$ A

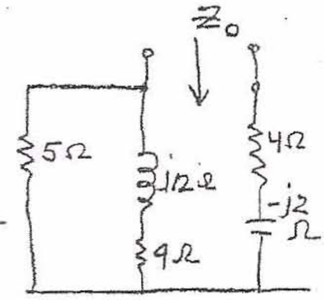


KVL $\longrightarrow V_{oc} = (9 + j12) I_1 - (4 - j2) I$

$$\therefore V_{oc} = \frac{(10)(9 + j12)}{14 + j12} - (2 \angle -30^\circ)(4.47 \angle -26.57^\circ)$$

$$= (7.94 + j1.76) - (4.93 - j7.46) = 3.01 + j9.23 = 9.71 \angle 71.92^\circ V$$

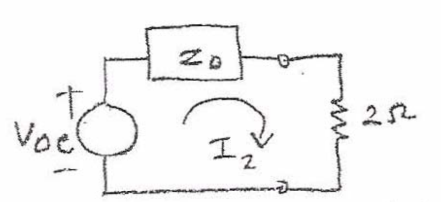
To determine Z_o, suppress the sources



$$Z_1 = 5 // (9 + j12) = \frac{5(9 + j12)}{5 + (9 + j12)} = 3.97 + j0.88 \Omega$$

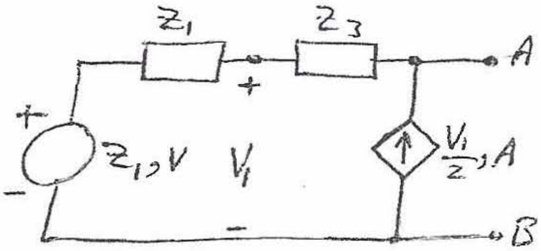
$$Z_o = Z_1 + (4 - j2) = 7.97 - j1.12 = 8.05 \angle -8.0^\circ \Omega$$

ⓑ $I_2 = \frac{V_{oc}}{Z_o + 2} = \frac{9.7 \angle 71.92^\circ}{10.03 \angle -6.4^\circ}$



$I_2 = 0.97 \angle 78.3^\circ$ A

$\boxed{\dfrac{3\text{-}50}{156}}$ a) Take away the load at AB and convert current source to voltage, then circuit is as shown here.



Z_1 (as in $\boxed{\dfrac{3\text{-}40}{153}}$) $= 8.9443\,\Omega \angle -26.57^\circ$
$= 8 - j4,\ \Omega$

& Z_3 (" " ") $= 9.7014\,\Omega \angle -75.96^\circ$
$= 2.3529 - j9.4118,\ \Omega$

$$\therefore\ V_1 = Z_1 + Z_1\left(\frac{V_1}{2}\right) \Rightarrow V_1 = \frac{Z_1}{1 - Z_1/2} = \frac{2Z_1}{2 - Z_1}$$

$$\therefore\ V_{AB} = V_1 + \left(\frac{V_1}{2}\right)Z_3 = V_1\left(1 + \frac{Z_3}{2}\right) = \frac{2Z_1}{2 - Z_1}\cdot\frac{2 + Z_3}{2} = \left(\frac{2 + Z_3}{2 - Z_1}\right)\cdot Z_1$$

$$= \frac{2 + 2.3529 - j9.4118}{2 - 8 + j4}\cdot Z_1 = \frac{4.3529 - j9.4118}{-6 + j4}\cdot Z_1 = \frac{10.3696\angle -65.18^\circ}{7.2111\angle 146.31^\circ}$$

$$= (1.4380\angle -211.49^\circ)\ 8.9443\angle -26.57^\circ = 12.8620\angle 121.95^\circ,\ V$$

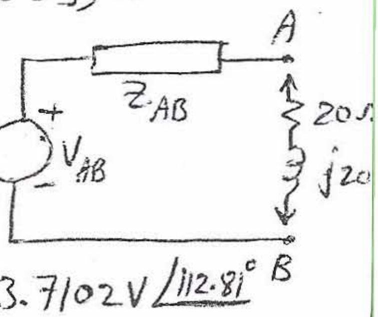
$$\&\ I_{AB} = \frac{V_1}{2} + \frac{Z_1}{Z_1 + Z_3} = \frac{1}{2}\left(Z_1\cdot\frac{Z_3}{Z_1 + Z_3}\right) + \frac{Z_1}{Z_1 + Z_3} = \frac{Z_1(2 + Z_3)}{2(Z_1 + Z_3)}$$

$$= \frac{Z_1(2 + 2.3529 - j9.4118)}{2(8 - j4 + 2.3529 - j9.4118)} = \frac{Z_1(4.3529 - j9.4118)}{2(10.3529 - j13.4118)} =$$

$$= \frac{8.9443\angle -26.57^\circ * 10.3696\angle -65.18^\circ}{2 * 16.9428\angle -52.33^\circ} = 2.7371\angle -39.41^\circ,\ A$$

$$\therefore\ Z_{AB} = \frac{V_{AB}}{I_{AB}} = \frac{12.8620\angle 121.95^\circ}{2.7371\angle -39.41} = 4.6991\angle 161.36^\circ,\ \Omega$$
$$= -4.4525 + j1.5023,\ \Omega$$

$\therefore$ Thevenin Equivalent at AB is as shown.



b) $$\therefore\ V_2 = V_{AB}\cdot\frac{20 + j20}{Z_{AB} + 20 + j20} = \frac{12.8620\angle 121.95^\circ * 20\sqrt{2}\angle 45^\circ}{-4.4525 + j1.5023 + 20 + j20} =$$

$$= \frac{363.791\angle 166.95^\circ}{15.5475 + j21.5023} = \frac{363.791\angle 166.95^\circ}{26.5344\angle 54.13^\circ} = 13.7102\,V\angle 112.81^\circ$$

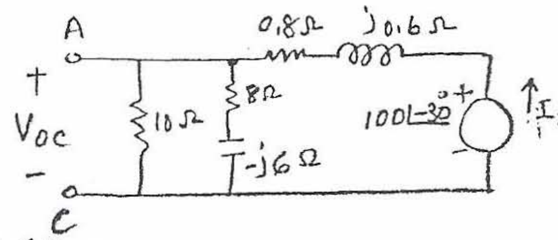
\# Since Z_{AB} has a negative real part due to nature of controlled sources then the power becomes infinite when load at AB $= -Z_{AB} = 4.4525 - j$

—

$\boxed{\dfrac{3-51}{156}}$

(a) $Z_1 = (10) \| (8-j6)$

$= \dfrac{(10)(8-j6)}{10+8-j6}$



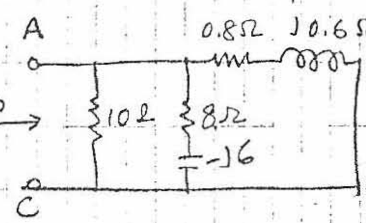
$= 5.27\angle{-18.43^\circ} = 5.0 - j1.67\ \Omega$

$$I_2 = \frac{100\angle{-30^\circ}}{(0.8+j0.6)+Z_1} = \frac{100\angle{-30^\circ}}{5.90\angle{-10.42^\circ}} = 16.96\angle{-19.58^\circ}, Ap$$

$$\therefore V_{oc} = I_2 Z_1 = 16.96\angle{-19.58^\circ} \times 5.27\angle{-18.43^\circ} = 89.37\angle{-38.01^\circ}$$

$= 70.41 - j55.04\ V$

To determine $\hat{Z}_o$, suppress the source

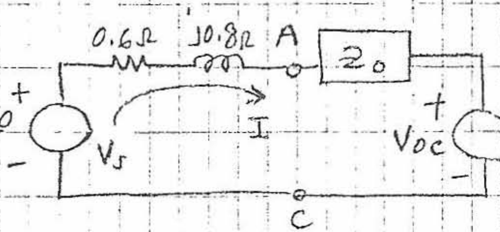


$$Z_o = Z_1 \| (0.8+j0.6) = \frac{(5.27\angle{-18.43^\circ})(1\angle{36.9^\circ})}{(5.0-j1.67)+(0.8+j0.6)}$$

$$= \frac{5.27\angle{18.43}}{5.90\angle{-10.42^\circ}} = 0.89\angle{28.86^\circ} = 0.78 + j0.43\ \Omega$$

(b) $$I = \frac{(100\angle{0^\circ}) - (89.37\angle{-38.01^\circ})}{(0.6+j0.8)+(0.78+j0.43)}$$

$$= \frac{62.49\angle{61.74^\circ}}{1.85\angle{41.68^\circ}} = 33.75\angle{20.05^\circ}\ A$$



$$\therefore P_s = |I||V_s| \cos(\angle V_s - \angle I) = 33.75 \times 100 \cos(0 - 20.05^\circ) = 3170.4\ W$$

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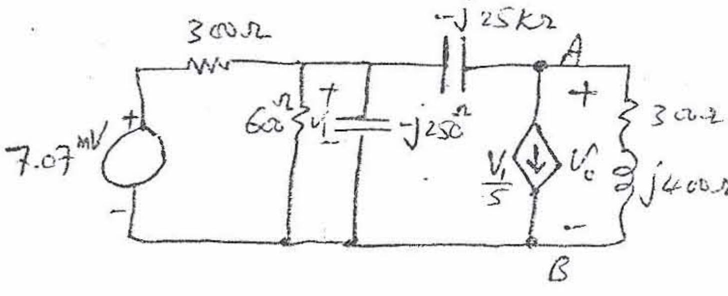
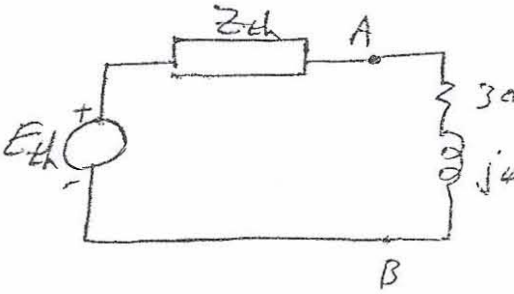
Solution by Thevenin at AB:

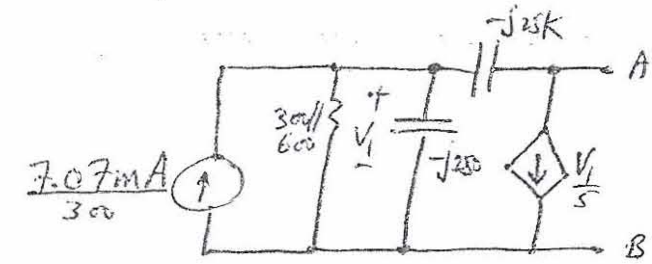
Remove $300 + j400, \Omega$
and find E_{th} & Z_{th}

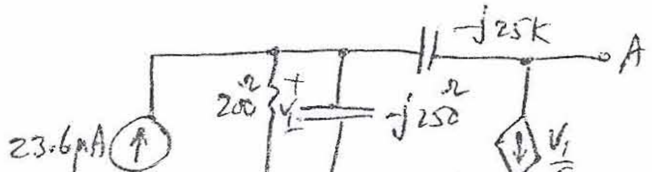


$\therefore E_{th} = V_{AB}$

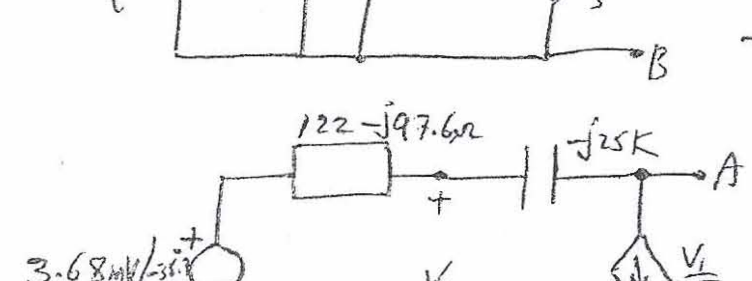
Using simplification:







Let $Z = 200 // -j2$
$= 122 - j97$
$= 156 \angle -38$



$\therefore \frac{V_1}{5} = \frac{3.68\angle -38.7^\circ \text{ mV} - V_1}{122 - j97.6}$ $\quad \therefore V_1 = \frac{3.68\text{m} \angle -38.7^\circ}{\frac{122}{5} - j\frac{97.6}{5} + 1} = .115\text{mV}\angle$

$\therefore V_{AB} = V_1 - \frac{V_1}{5}(-j25K) = V_1(1 + j5K) = .115 \text{ mV} \angle -1.1^\circ * 5K\angle 90^\circ = E_{th}$
$= .575V\angle 8$

& $Z_{th} = \frac{E_{th}}{I_{sc}} \Rightarrow I_{sc} = \frac{3.68\text{mV}\angle -38.7^\circ}{122 - j97.6 - j25K}\left(1 - \frac{1}{5}(-j25K)\right) = 733\mu A \angle 141.0^\circ$

$\therefore Z_{th} = \frac{.575\angle 88.9^\circ}{733\mu \angle 141.0^\circ} = 784\Omega\angle -52.1^\circ = 482 - j619, \Omega$

$\therefore V_o = \frac{E_{th}(300 + j400)}{Z_{th} + 300 + j400} = \frac{.575\angle 88.9^\circ * 500\angle 53.1^\circ}{782 - j219} = 0.354 V \angle 157.$

$\therefore$ OK

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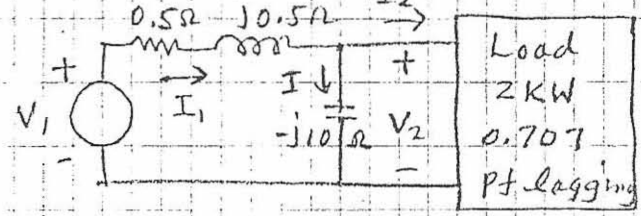
$V_2 = 200\angle 0^\circ$ V

$\theta_2 = \cos^{-1} 0.707 = 45^\circ$

Lagging pf $\Rightarrow$ I_2 Lags V_2

$\therefore \angle^{V_2}_{I_2} = \angle -45^\circ$



$P_{Load} = 2000 = |V_2||I_2| \cos\theta_2 = 200 \times 0.707 I_2$

(a) $\therefore |I_2| = 14.14$ A $\Rightarrow I_2 = 14.14\angle -45^\circ$ A $= 10 - j10$ A

(b) $I = \dfrac{V_2}{-j10} = \dfrac{200}{10\angle -90^\circ} = j20$ A $= 20\angle 90^\circ$ A

KCL $\longrightarrow I_1 = I_2 + I = (10 - j10) + (j20) = 14.14\angle 45^\circ$ A
$= 10 + j10$ A

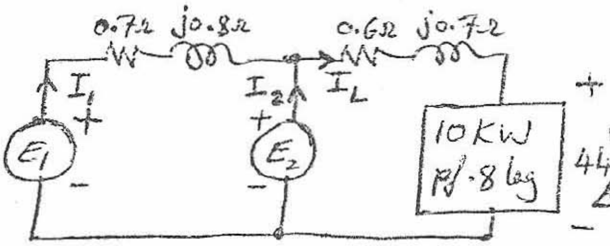
KVL $\rightarrow V_1 = (0.5 + j0.5) I_1 + V_2 = (10\angle 90) + 200\angle 0 = 200.25\angle 2.86^\circ$

(c) $P = |V_1||I_1| \cos \angle^{I_1}_{V_1} = \underset{|V_1|}{200.25} \times \underset{|I_1|}{14.14} \cos(2.86 - 45)$
$= 200.25 \times 14.14 \times 0.742 = 2100.0$ W $= 2.1$ KW

$Q = |V_1||I_1| \sin \angle^{I_1}_{V_1} = 200.25 \times 14.14 \sin(-42.14) = -1900.0$ Vars (Capaciti

$VA = |V_1||I_1| = 200.25 \times 14.14 = 2831.96$ VA

$\boxed{\frac{3\text{-}59}{158}}$ $I_L = \dfrac{10K}{440 * .8} \angle{-\cos^{-1} 0.8} =$



$= 28.4\overline{09}\,A \angle{-36.87°} =$

$= 22.7\overline{27} - j\,17.0\overline{45},\ A$

$\therefore\ E_2 = (.6 + j.7)\,I_L + 440 =$

$= 0.92195 \angle{49.40°} * 28.4\overline{09} \angle{-36.87°} + 440 =$

$= 26.192 \angle{12.53°} + 440 = 25.568 + j\,5.68\overline{18} + 440 =$

$= 465.57 + j\,5.68\overline{18} = 465.60 \angle{0.70°},\ V$

$\therefore\ I_2 = \dfrac{5K}{465.60 * .6} \angle{-\cos^{-1} 0.6 + 0.70°} = 17.898\,A \angle{-53.13° + 0.70}$

$= 17.898\,A \angle{-52.43°} = 10.913 - j\,14.186,\ A$

$\therefore\ I_1 = I_L - I_2 = 22.7\overline{27} - j\,17.0\overline{45} - 10.913 + j\,14.186 =$

$= 11.814 - j\,2.8592,\ A = 12.156 \angle{-13.60°},\ A$

$\therefore\ E_1 = (.7 + j.8) * I_1 + E_2 = 1.0630 \angle{48.81°} * 12.156 \angle{-13.60°} + E_2 =$

$= 12.922 \angle{35.21°} + E_2 = 10.558 + j\,7.4502 + 465.57 + j\,5.68\overline{18} =$

$= 476.13 + j\,13.132,\ V = 476.31 \angle{1.58°},\ V$

a) Hence, the terminal voltage of generator 1 is 476.31 Volts, & that of generator 2 is 465.60 Volts.

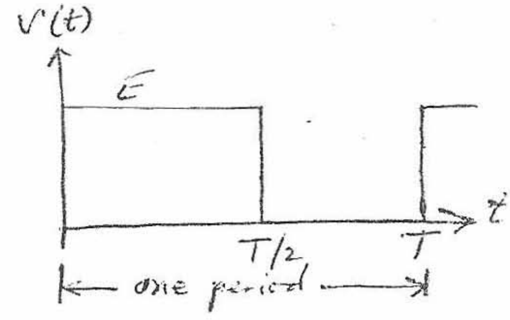
b) $S_{E_1} = 12.156 * 476.31 = 5789.8\ VA = 5.7898\ KVA$

& pf at $E_1 = \cos \angle^{E_1}_{I_1} = \cos(1.58° + 13.60°) = \cos 15.18° = 0.96509$ lag

\# Line losses $= |I_1|^2 * 0.7 + |I_2|^2 * 0.6 = 12.156^2 * .7 + 17.898^2 * .6 =$

$= 103.43 + 192.20 = 295.63$ Watts.

To find Fourier Series of the waveform shown, assuming periodic.



$\therefore$ Av. $= \frac{E \cdot T/2}{T} = \frac{E}{2}$

Fundamental frequency $= \frac{2\pi}{T}$ rad/sec $= \omega$

$$\therefore V_o(t) = \frac{E}{2} + \sum_{n=1}^{\infty} a_n \sin n\omega t + b_n \cos n\omega t$$

$$\therefore a_n = \frac{2}{T}\int_0^T V_o(t) \sin n\omega t \, dt = \frac{2}{T}\int_0^{T/2} E \sin n\omega t \, dt = \frac{2E}{T} \cdot \left.\frac{-\cos n\omega t}{n\omega}\right|_0^{T/2} =$$

$$= \frac{2E}{n\omega T}\left[1 - \cos(n\omega T/2)\right] = \frac{2E}{n 2\pi}\left(1 - \cos(n 2\pi/2)\right) = \frac{E}{n\pi}\left(1 - \cos n\pi\right)$$

$$= \frac{2E}{n\pi} \text{ for odd } n \ \ \& \text{ zero for even } n$$

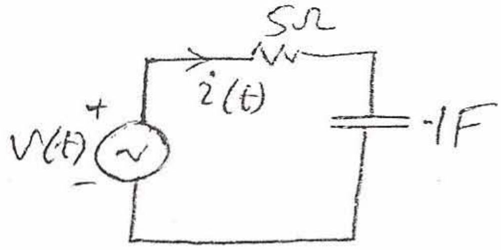
$$\& \ b_n = \frac{2}{T}\int_0^T V_o(t) \cos n\omega t \, dt = \frac{2}{T}\int_0^{T/2} E \cos n\omega t \, dt = \frac{2E}{T} \cdot \left.\frac{\sin n\omega t}{n\omega}\right|_0^{T/2} =$$

$$= \frac{2E}{n\omega T}\left(\sin(n\omega T/2) - 0\right) = \frac{2E}{n 2\pi}\left(\sin(n 2\pi/2) - 0\right) = \frac{E}{n\pi}\left(\sin n\pi - 0\right) = 0$$

$$\therefore V_o(t) = \frac{E}{2} + \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{2E}{n\pi} \sin n\omega t = \frac{E}{2} + \frac{2E}{\pi} \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi t}{T}\right)$$

$$\therefore V_o(t) = \frac{E}{2} + \frac{2E}{\pi}\left(\sin\left(\frac{2\pi t}{T}\right) + \frac{1}{3}\sin\left(\frac{6\pi t}{T}\right) + \frac{1}{5}\sin\left(\frac{10\pi t}{T}\right) + \cdots + \underset{n \text{ odd}}{\frac{1}{n}} \sin\left(\frac{2n\pi t}{T}\right) \ldots\right)$$

To find the current $i(t)$ for the circuit shown given that:



$$V(t) = 12 + 3\cos t - 2\sin t + 1.5\cos 2t - \sin 3t \text{ , V;}$$

we apply the superposition principle.

$$\therefore\ i(t) = i(t)\Big|_{\text{due } 12,V} + i(t)\Big|_{\text{due } 3\cos t,V} + i(t)\Big|_{\text{due } -2\sin t,V} + i(t)\Big|_{\text{due } 1.5\cos 2t,V} + i(t)\Big|_{\text{due } -\sin 3t,V}$$

$i(t)\Big|_{12V} = 0$ Amp since the $.1F$ capacitor looks like open circuit for dc.

$$i(t)\Big|_{V\cos\omega t} = \frac{V \angle \tan^{-1}\frac{1}{\omega CR}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\omega t = \frac{\omega C V}{\sqrt{1+(\omega C R)^2}} \cos(\omega t + \phi)$$

$$= \frac{\omega C V}{\sqrt{1+\omega^2C^2R^2}}\left(\cos\omega t \cos\phi - \sin\omega t \sin\phi\right)$$

$\sqrt{1+(\omega CR)^2}$, 1, ϕ, ωCR

$$= \frac{\omega C V}{\sqrt{1+\omega^2C^2R^2}}\left(\cos\omega t \cdot \left(\frac{\omega CR}{\sqrt{1+\omega^2C^2R^2}}\right) - \sin\omega t \cdot \left(\frac{1}{\sqrt{1+\omega^2C^2R^2}}\right)\right)$$

$$= \frac{\omega C V}{1+\omega^2C^2R^2}\left(\omega CR \cos\omega t - \sin\omega t\right)$$

$$\therefore\ i(t)\Big|_{\substack{3\cos t,V\\ \omega=1\\ V=3}} = \frac{1*.1*3}{1+1^2*.1^2*5^2}\left(1*.1*5\cos t - \sin t\right) = \frac{.3}{1.25}(.5\cos t - \sin t)$$

$$= .12\cos t - .24\sin t \text{ , A}$$

$$\& \ i(t)\Big|_{-2\sin t,V} = i(t)\Big|_{\substack{-2\cos(t-90^\circ),V\\ \omega=1\\ V=-2}} = \frac{1*.1*(-2)}{1.25}\left(.5\cos(t-90^\circ) - \sin(t-90^\circ)\right)$$

$$= -.08\sin t - .16\cos t \text{ , A}$$

$$\& \ i(t)\Big|_{\substack{1.5\cos 2t,V\\ \omega=2\\ V=1.5}} = \frac{2*.1*1.5}{1+2^2*.1^2*5^2}\left(2*.1*5\cos 2t - \sin 2t\right) = .15\cos 2t - .15\sin 2t \text{ , A}$$

$$+ \ i(t)\Big|_{-\sin 3t,V} = i(t)\Big|_{\substack{\cos 3(t+90^\circ)\\ \omega=3\\ V=1}} = \frac{3*.1*1}{1+3^2*.1^2*5^2}\left(3*.1*5\cos 3(t-90^\circ) - \sin 3(t-90^\circ)\right) =$$

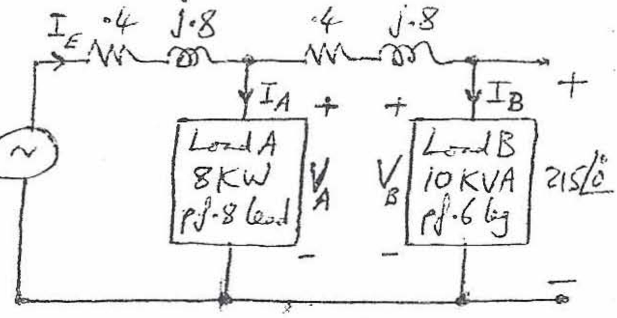
$$= -.1385\sin 3t - .09231\cos 3t \text{ , A}$$

$$\therefore\ i(t) = -.04\cos t - .32\sin t + .15\cos 2t - .15\sin 2t - .09231\cos 3t - .1385\sin 3t \text{ , A}$$

$$\therefore\ P = (5/2) * \left[.04^2 + .32^2 + .15^2 + .15^2 + .09231^2 + .1385^2\right] = 0.44173 \text{ Watt.}$$

#) $|V_B||I_B| = 10\,KVA$

Let $V_B = 215\angle 0^\circ$, V



$\therefore |I_B| = \frac{10000}{215} = 46.512\,A$

$\angle_{I_B}^{V_B} = \cos^{-1}.6 = 53.13^\circ$

$\therefore I_B = 46.512\angle -53.13^\circ$, A

$\therefore V_A = V_B + (.4 + j.8)\,I_B =$

$= 215 + .8944\angle 63.43^\circ * 46.512\angle -53.13^\circ =$

$= 215 + 41.601\angle 10.30^\circ =$

$= 215 + 40.930 + j7.4419 = 255.930 + j7.4419$, V

$= 256.04\angle 1.67^\circ$, V

a) $\therefore$ The terminal voltage of load A is 256.04 Volts

$\because |I_A||V_A|\cos\angle_{I_A}^{V_A} = 8\,KW$

$\therefore |I_A| = \frac{8000}{256.04 * .8} = 39.057\,A$

$\angle_{V_A}^{I_A} = \cos^{-1}.8 = 36.87^\circ$

$\therefore I_A = 39.057\angle 1.67 + 36.87^\circ = 39.057\angle 38.54^\circ$, A

$\therefore I_E = I_A + I_B = 39.057\angle 38.54^\circ + 46.512\angle -53.13^\circ =$

$= 30.551 + j24.332 + 27.907 - j37.209 =$

$= 58.458 - j12.877 = 59.859\angle -12.42^\circ$, A

$\therefore E = V_A + (.4 + j.8)I_E = V_A + .8944\angle 63.43^\circ * 59.859\angle -12.42^\circ =$

$= V_A + 53.540\angle 51.01^\circ = 255.930 + j7.4419 + 33.685 + j41.616 =$

$= 289.62 + j49.057 = 293.74\angle 9.61^\circ$, V

b) $\therefore$ The terminal voltage of E is 293.74 Volts

$\angle_{I_E}^{E} = 9.61 - (-12.42) = 22.04^\circ$

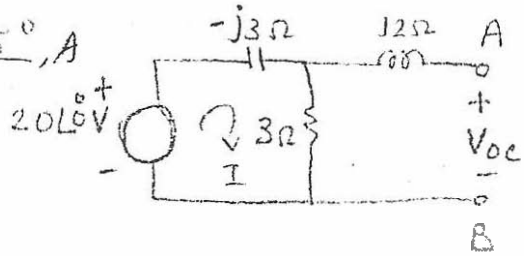
c) $\therefore$ pf at $E = \cos(22.04^\circ) = 0.9269$ lagging

d) $S_E = |E|\cdot|I_E| = 293.74 * 59.859 = 17583\,VA = 17.583\,KVA$

e) Line losses $= .4|I_E|^2 + .4|I_B|^2 = .4[59.859^2 + 46.512^2] = 2298.6\,W$

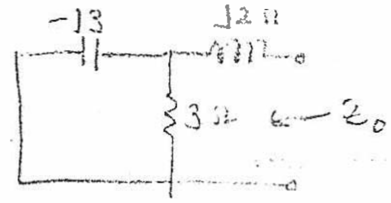
$\therefore$ Line losses $= 2.2986\,KW$

(a) $I = \frac{20\angle 0^\circ}{3-j3} = \frac{20}{4.24\angle -45^\circ} = 4.71\angle 45^\circ, A$



$V_{oc} = 3I = 14.14\angle 45^\circ$ V
$= 10 + j10$ V

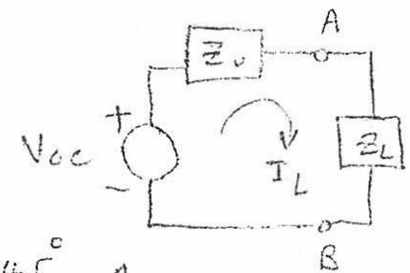
determine Z_o, suppress the voltage source:



$Z_o = j2 + \frac{3(-j3)}{3-j3} + \frac{3\angle -90^\circ}{\sqrt{2}\angle -45^\circ} = j2 + 2.12\angle -45^\circ$

$= j2 + 1.5 - j1.5 = 1.5 + j0.5 = 1.58\angle 18.4^\circ\ \Omega$

(b) For max. power transfer,
$Z_L = Z_o^* = 1.5 - j0.5\ \Omega = 1.58\angle -18.4^\circ\ \Omega$

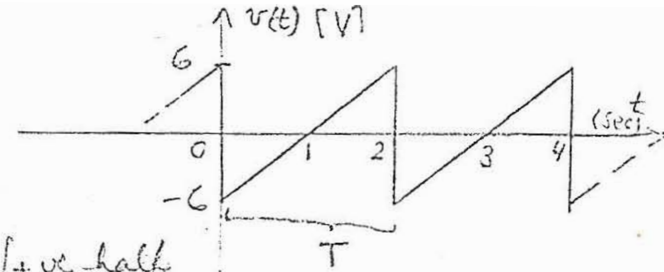


(c) $I_L\Big|_{P_{max}} = \frac{V_{oc}}{Z_o + Z_o^*} = \frac{14.14\angle 45^\circ}{1.5+1.5} = 4.71\angle 45^\circ$ A

$P_{max} = |I_L|^2 R_L = (4.71)^2(1.5) = 33.\overline{3}$ W

\# $v(t) = \frac{6}{1} t - 6 = 6(t-1)\,V$

$T = 2\,sec$



$$v_{av} = \frac{1}{T}\int_0^2 6(t-1)\,dt$$

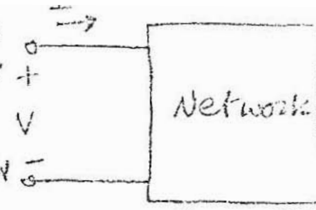
$$= \frac{6}{2}\left[\frac{t^2}{2} - t\right]_0^2 = 0\,V$$ [+ve half = -ve half]

$$v_{rms} = \sqrt{\frac{1}{T}\int_0^T 36(t-1)^2\,dt} = 6\sqrt{\frac{1}{T}\int_0^T (t^2 - 2t + 1)\,dt}$$

$$= 6\sqrt{\frac{1}{T}\left[\frac{t^3}{3} - \frac{2t^2}{2} + t\right]_0^T} = 6\sqrt{\frac{1}{2}\left[\frac{8}{3} - 4 + 2\right]} = 6\sqrt{\frac{1}{2}\times\frac{2}{3}}$$

$$V_{rms} = \frac{6}{\sqrt{3}} = 3.46\,V$$

\# $V = 100\angle 30^\circ\,V$ & $I = \frac{10}{\sqrt{2}}\angle 60^\circ\,A$, $\omega = 377$ rad/s



(a) $\theta = 60^\circ - 30^\circ = 30^\circ$

$\therefore P = VI\cos(30^\circ) = 100 \times \frac{10}{\sqrt{2}} \times 0.866 = 612.4\,W$

$Q = VI\sin(30^\circ) = 100 \times \frac{10}{\sqrt{2}} \times (+0.5) = +353.55$ var (capacitive)

$VA = VI = 707\,VA$

(b) $Z = R + jX = \frac{V}{I} = \frac{100\angle 30^\circ}{(10/\sqrt{2})\angle 60^\circ} = 14.14\angle -30^\circ = 12.25 - j7.07\,\Omega$

(c) Since X is -ve $\Rightarrow$ Capacitive Reactance.

$\therefore X = X_C = \frac{1}{\omega C} = 7.07 \Rightarrow C = \frac{1}{7.07 \times 377} = 375.12\,\mu F$

4-1
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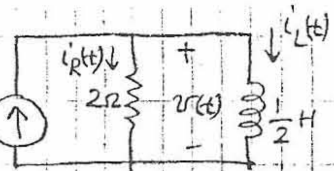
$v_f(t)$ & $i_{L_f}(t) = ?$



KCL $\rightarrow \frac{v}{2} + \frac{1}{L}\int v\,dt = i$

Differentiating : $\frac{1}{2}\frac{dv}{dt} + 2v = \frac{di}{dt}$ (1)

(a) $i = 4$ A $\rightarrow$ Assume $v_f(t) = V_0$ (same form of excitation)

from (1) : $0 + 2V_0 = 0 \Rightarrow v_f(t) = 0$ {L is short for DC}

$\therefore i_{L_f}(t) = i(t) = 4$ A .

(b) $i(t) = 2t$ A $\longrightarrow$ Assume $v_f(t) = V_1 t + V_0$ {excitation + all unique derivatives}

from (1) : $\frac{1}{2}V_1 + 2(V_1 t + V_0) = 2$

Comparing terms: $\frac{V_1}{2} + 2V_0 = 2$ & $2V_1 = 0$

$\therefore V_1 = 0$ & $V_0 = 1$ V

$\therefore v_f(t) = 1$ V , $i_{R_f}(t) = \frac{v_f(t)}{2} = \frac{1}{2}$ A

KCL $\longrightarrow i_{L_f}(t) = i(t) - i_{R_f}(t) = 2t - \frac{1}{2}$ A

(c) $i(t) = 4 + 2t$ A $= i_1(t) + i_2(t)$

$i_1(t) = 4$ A & $i_2(t) = 2t$ A

By superposition principle : {or follow the same Procedure as in (b).

I) for $i_1(t) = 4$ A , from a) $v_{f_1}(t) = 0$ V & $i_{L_{f_1}}(t) = 4$ A

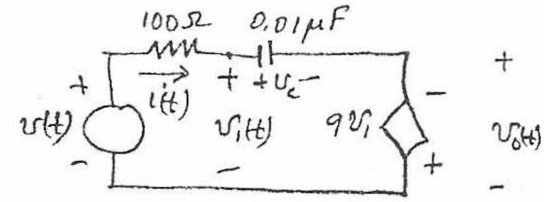
II) for $i_2(t) = 2t$ A, from b) $v_{f_2}(t) = 1$ V & $i_{L_{f_2}}(t) = 2t - \frac{1}{2}$ A

$\therefore v_f(t) = v_{f_1}(t) + v_{f_2}(t) = 1$ V

$i_{L_f}(t) = i_{L_{f_1}}(t) + i_{L_{f_2}}(t) = 2t + 3.5$ A

$\boxed{\frac{4-4}{215}}$ $v_{of}(t)$, $i_f(t) = ?$

KVL → $100i + v_c - 9v_1 = v$



$i = i_c = C\frac{dv_c}{dt}$ & $v_1 = v - i(100)$

∴ $100i + v_c - 9(v - 100i) = v \Rightarrow 1000i + v_c = 10v$

∴ $10^{-5}\frac{dv_c}{dt} + v_c = 10v$ (1)

(a) $v = 0.1$ V , from (1): $10^{-5}\frac{dv_c}{dt} + v_c = 1$

Assume $v_{cf}(t) = V_0 \Rightarrow 0 + V_0 = 1 \Rightarrow V_0 = 1\,V = v_{cf}(t)$

$i_f(t) = C\frac{dv_{cf}}{dt} = 0$ (open ckt for DC excitation)

∴ $v_{of}(t) = -9v_1 = -9v + 900\,i_f = -9(0.1) + 900(0) = -0.9$ V

(b) $v = 0.2e^{-10^5t}\cos 10^5 t$

Assume $v_{cf}(t) = Ae^{-10^5t}\cos 10^5t + Be^{-10^5t}\sin 10^5 t$

from (1)

$$2e^{-10^5t}\cos 10^5t = 10^{-5}\{(-10^5Ae^{-10^5t}\cos 10^5t - 10^5Be^{-10^5t}\sin 10^5t) + (-10^5Ae^{-10^5t}\sin 10^5t + 10^5Be^{-10^5t}\cos 10^5t)\} + Ae^{-10^5t}\cos 10^5t + Be^{-10^5t}\sin 10^5t$$

Comparing terms: $2 = -A + B + A \Rightarrow B = 2$

$0 = -A - B + B \Rightarrow A = 0$

∴ $v_{cf}(t) = 2e^{-10^5t}\sin 10^5t$, V

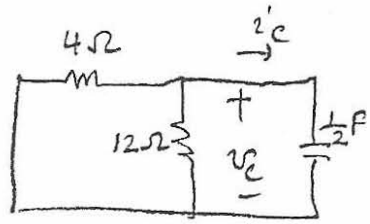
∴ $i_f(t) = C\frac{dv_{cf}}{dt} = 0.01\times10^{-6}\times 2[-10^5e^{-10^5t}\sin 10^5t + 10^5e^{-10^5t}\cos 10^5t]$
$= .002e^{-10^5t}(\cos 10^5t - \sin 10^5t)$ Amp

∴ $v_{of}(t) = -9v_1 = -9v + 900i_f = -1.8e^{-10^5t}\cos 10^5t + 1.8[-e^{-10^5t}\sin 10^5t + 10^{-10^5t}\cos 10^5t]$

$\Rightarrow \boxed{v_{of}(t) = -1.8e^{-10^5t}\sin 10^5t, V}$

$\boxed{\frac{4-8}{216}}$

KCL $\rightarrow \quad \frac{v_c}{4} + \frac{v_c}{12} + \frac{1}{2}\frac{dv_c}{dt} = 0$



(a) $\therefore \frac{2}{3}v_c + \frac{dv_c}{dt} = 0 \quad$ (i)

$v_{cn}(t) = Ae^{st} \quad \Rightarrow \quad s = -\frac{2}{3}$

$\therefore v_{cn}(t) = Ae^{-\frac{2}{3}t}$ & $v_{cf}(t) = 0$ (force free)

$\therefore v_c(t) = v_{cf} + v_{cn} = Ae^{-\frac{2}{3}t}$

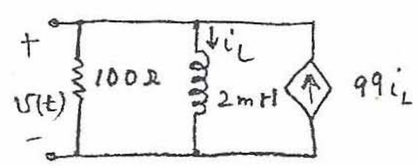
$i_{cn}(t) = \frac{1}{2}\frac{dv_{cn}}{dt} = \frac{-A}{3}e^{-\frac{2}{3}t} = i_c(t)$

(b) $v_c(0) = 6 = A \quad \Rightarrow v_c(t) = 6e^{-\frac{2}{3}t}$ V

$\therefore i_c(0) = C\frac{dv_c}{dt}(0) = \frac{1}{2}\left(\frac{-12}{3}\right) = -2$ A

$\boxed{\frac{4-10}{216}}$

(a) $v_n(t) = ?$ if $i_L(0) = 10$ mA



KCL $\rightarrow 99i_L = i_L + \frac{v}{100} = i_L + \frac{L}{100}\frac{di_L}{dt}$

$\therefore \frac{2\times10^{-3}}{100}\frac{di_L}{dt} - 98i_L = 0$

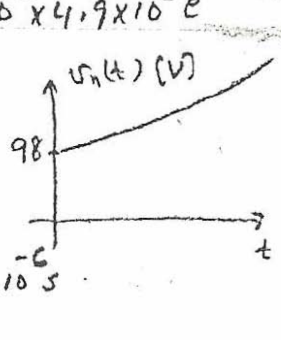
Assume $i_{Ln}(t) = I_0e^{st}$

$\Rightarrow 20\times10^{-6}s - 98 = 0 \quad \Rightarrow s = 4.9\times10^{6}$ /s

$\therefore i_L(t) = I_0e^{4.9\times10^{6}t}, \quad i_L(0) = 10^{-2} = I_0$

$\therefore i_{Ln}(t) = 10^{-2}e^{4.9\times10^{6}t}, \quad v_n(t) = L\frac{di_L}{dt} = 2\times10^{-3}\times10^{-2}\times4.9\times10^{6}e^{4.9\times10^{6}t}$

$\boxed{v_n(t) = 98e^{4.9\times10^{6}t}\text{ V}} \quad = 98e^{\frac{t}{\tau}}$ V

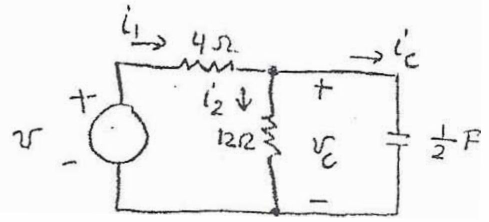


(b) $\tau = \frac{1}{4.9\times10^{6}}, \quad t_1 = 5\tau = \frac{5}{4.9\times10^{6}} = 1.02\times10^{-6}$ s

$\therefore v_n(t_1) = 98e^{5} = 14544.5$ V

$\boxed{\frac{4-13}{217}}$

$$v(t) = 12\text{ V}, \quad t<0$$
$$= 6\cos t, \quad t \geqslant 0$$



$v_c(0^+)$, $i_c(0^+)$, $\frac{dv_c}{dt}(0^+)$ & $\frac{di_c}{dt}(0^+) = ?$

For $t<0$, $i_c(0^-) = 0$ (open ckt for dc)

$\therefore\ i_1(0^-) = i_2(0^-) + i_c(0^-) = i_2(0^-) = \frac{12}{4+12}$ A

$\therefore\ v_c(0^-) = i_1(0^-)(12) = \frac{12\times 12}{16} = 9$ V

From principle of continuity, $v_c(0^+) = v_c(0^-) = 9$ V.

For $t \geqslant 0$

KVL $\rightarrow$ $v(0^+) = (4)\, i_1(0^+) + v_c(0^+) = 6 \Rightarrow i_1(0^+) = \frac{-3}{4}$ A

also KVL $\rightarrow$ $v_c(0^+) = 9 = (12)\, i_2(0^+) \Rightarrow i_2(0^+) = \frac{9}{12} = 0.75$ A

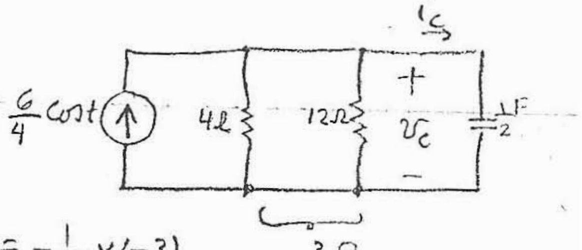
KCL $\rightarrow$ $i_c(0^+) = i_1(0^+) - i_2(0^+) = -0.75 - 0.75 = -1.5$ A

$$i_c(0^+) = C\,\frac{dv_c}{dt}(0^+) \Rightarrow \frac{dv_c}{dt}(0^+) = \frac{i_c(0^+)}{C} = -3.0\text{ V/s}$$

Converting the voltage source to current source;

KCL $\rightarrow$ $\frac{6}{4}\cos t = \frac{v_c}{3} + i_c$

Differenting $\quad -1.5\sin t = \frac{1}{3}\frac{dv_c}{dt} + \frac{di_c}{dt}$



$$\therefore\ \frac{di_c}{dt}(0^+) = -0 - \frac{1}{3}\frac{dv_c}{dt}(0^+) = -\frac{1}{3}\times(-3)$$
$$= +1\text{ A/s}$$

4-17
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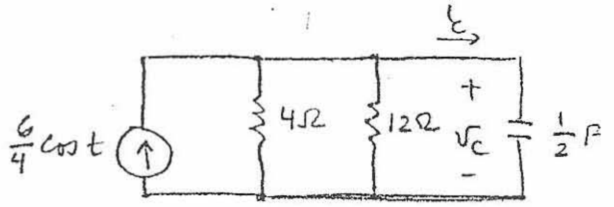
$v(t) = 12\ V \quad , t<0$

$\quad = 6\cos t\ V, \ t \geq 0$

(Notice the source conversion)



KCL $\longrightarrow \quad \frac{6}{4}\cos t = \frac{v_c}{4//12} + i_c = \frac{v_c}{3} + C\frac{dv_c}{dt}$

Assume $v_{cf}(t) = A\cos t + B\sin t$

$\therefore \quad \frac{6}{4}\cos t = \frac{A}{3}\cos t + \frac{B}{3}\sin t + \frac{1}{2}[-A\sin t + B\cos t]$

Comparing terms:

$$\frac{6}{4} = \frac{A}{3} + \frac{B}{2} \quad \& \quad 0 = \frac{B}{3} - \frac{A}{2}$$

$\Rightarrow \quad A = \frac{18}{13} \quad \& \ B = \frac{27}{13}$

$v_{cn}(t) = K e^{-t/\tau} \qquad , \tau = RC = (4//12)(\frac{1}{2}) = \frac{3}{2}$ sec

$\therefore v_c(t) = v_{cn}(t) + v_{cf}(t) = K e^{-\frac{2t}{3}} + \frac{18}{13}\cos t + \frac{27}{13}\sin t$

from problem (4-13), $v_c(0) = 9$ -

$\therefore \quad 9 = K + \frac{18}{13} + 0 \Rightarrow K = \frac{99}{13}$

$\therefore \boxed{v_c(t) = \frac{9}{13}[11 e^{-\frac{2}{3}t} + 2\cos t + 3\sin t]\ V}$

$i_c(t) = C\frac{dv_c}{dt} = \frac{1}{2}\times\frac{9}{13}[-\frac{22}{3}e^{-\frac{2}{3}t} - 2\sin t + 3\cos t]$

$i_c(t) = \frac{9}{26}[3\cos t - 2\sin t - \frac{22}{3}e^{-\frac{2}{3}t}]\ A$

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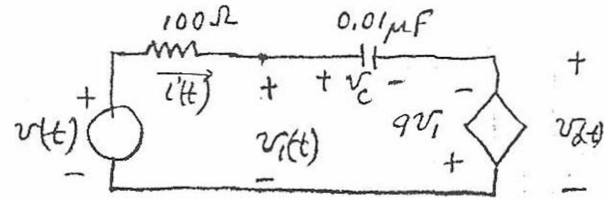
(a) $v(t) = 0 \quad , t < 0$

$\quad = 0.2t \quad , t \geq 0$

$v_0(t) = ? \quad t \geq 0$



KVL $\longrightarrow 10^{-5}\frac{dv_c}{dt} + v_c = 10v$ [see soln of problem 4-4]

OM-

Forced response

Assume $v_{c_f}(t) = V_1 t + V_0$

$\therefore \quad (10^{-5}V_1) + (V_1 t + V_0) = 10(0.2t)$

Comparing terms: $\quad 10^{-5}V_1 + V_0 = 0$ & $V_1 = 2$ V

$\therefore V_0 = -10^{-5}V_1 = -2\times10^{-5}$ V

$\therefore v_{cf}(t) = 2t - 2\times10^{-5}$ V

Natural Response

$v_{cn}(t) = Ke^{st} \longrightarrow 10^{-5}s + 1 = 0 \Rightarrow s = -10^5$ 1/s

$\therefore v_c(t) = v_{cn}(t) + v_{cf}(t) = Ke^{-10^5 t} + 2t - 2\times10^{-5}$

Since $v(t) = 0 \ , t<0 \quad \Rightarrow v_c(0^-) = v_c(0^+) = 0$

$0 = K + 0 - 2\times10^{-5} \Rightarrow K = 2\times10^{-5}$

$\therefore v_c(t) = 2\times10^{-5}(e^{-10^5 t} - 1) + 2t$, V

$i(t) = i_c(t) = C\frac{dv_c}{dt} = 10^{-8}[2 + 2\times10^{-5}\times(-10^5 e^{-10^5 t} - 0)]$

$= 2\times10^{-8}(1 - e^{-10^5 t})$; A

$v_0(t) = -9v_1(t) = -9(v(t) - 100\,i(t)) = -1.8t + 1.8\times10^{-5}(1 - e^{-10^5 t})$ V

(b) $v(t) = 0 \quad , t<0$ & $v(t) = 0.2e^{-10^5 t}\cos 10^5 t \quad , t \geq 0$

From problem (4-4), $v_{cf}(t) = 2e^{-10^5 t}\sin 10^5 t$ V

$\therefore v_c(t) = v_{cn}(t) + v_{cf}(t) = Ke^{-10^5 t} + 2e^{-10^5 t}\sin 10^5 t$

Assume $v_c(0) = 0$ [uncharged capacitor] $\Rightarrow K = 0$

$\therefore v_c(t) = v_{cf}(t) = 2e^{-10^5 t}\sin 10^5 t$

$i(t) = i_c(t) = C\frac{dv_c}{dt} = 2\times10^{-3}e^{-10^5 t}(\cos 10^5 t - \sin 10^5 t)$

$\therefore v_0(t) = -9v_1(t) = -9[v(t) - 100\,i(t)] = -1.8e^{-10^5 t}\sin 10^5 t$, V

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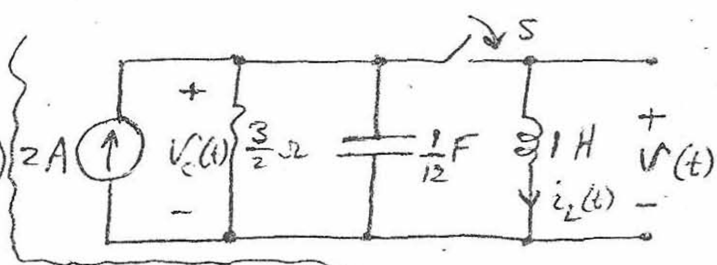
∵ S is open for long time till closed at $t=0$ sec,



∴ $V_c(0^-) = 2 \times \frac{3}{2} = 3$ Volts $= V_c(0^+)$

& $i_L(0^-) = 0$ Amp ∴ $v(0^-) = 0V$

For $t > 0$ ∴ $V_c(t) = v(t) = v_n(t) + v_{ss}(t)$

& $v_n(t)/(\frac{3}{2}) + \frac{1}{12}\dot{v}(t) + \frac{1}{1}\int v(t)\,dt = 0$

∴ $\ddot{v}_n + 8\dot{v}_n + 12 v_n = 0 \Rightarrow m^2 + 8m + 12 = 0$

∴ $(m+6)(m+2) = 0$ ∴ $m = -6$ or -2

∴ $v_n(t) = Ae^{-6t} + Be^{-2t}$

& $v_{ss}(t) = 0$

∴ $v(t) = v_n(t) + v_{ss}(t) = Ae^{-6t} + Be^{-2t}$, V

∵ $v(0^+) = V_c(0^+) = 3$ Volts ∴ $A + B = 3$ (1)

& $i_L(t) = 2 - v(t)/(\frac{3}{2}) - \frac{1}{12}\dot{v}(t)$

$= 2 - \frac{2}{3}(Ae^{-6t} + Be^{-2t}) + \frac{2}{12}(3Ae^{-6t} + Be^{-2t})$

∴ $i_L(0^+) = 2 - \frac{2}{3}(A+B) + \frac{1}{6}(3A+B) = i_L(0^-) = 0$

∴ $12 - 8(A+B) + 2(3A+B) = 0 \Rightarrow 2A + 6B = 12$

∴ $A + 3B = 6$ (2)

∴ (2) − (1) $\Rightarrow$ $2B = 3$ ∴ $B = \frac{3}{2}$

into (1) $\Rightarrow$ $A = 3 - B = 3 - \frac{3}{2} = \frac{3}{2}$

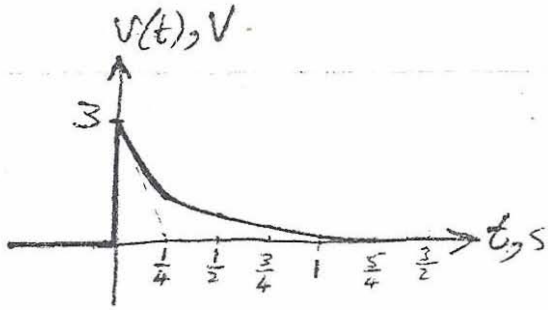
∴ $v(t) = \frac{3}{2}(e^{-6t} + e^{-2t})$, V

Note:

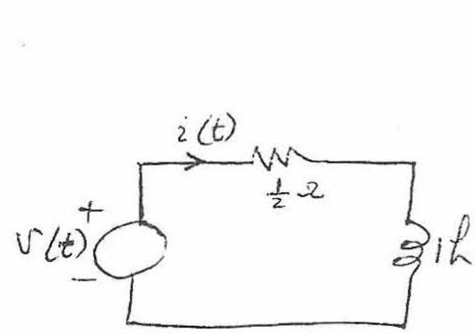
$v(t<0) = 0$, $v(0^+) = 3V$ & $v(\infty) = 0$

& $v'(t) = -9e^{-6t} - 3e^{-2t}$ ∴ $v'(0^+) = -12$ V/s

∴ Plot of $v(t)$ is as shown here.

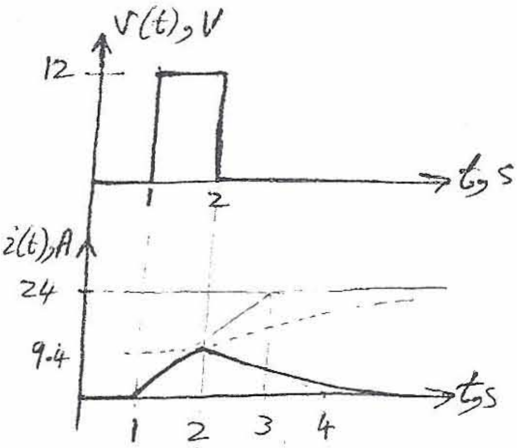


To find $i(t)$ for the given circuit and input;



$\therefore\ i(t<1s) = 0$ Amp

& $i(1_s^-) = i(1_s^+) = 0$ Amp



$\therefore\ i(t\in[1,2]\,\text{sec}) = \frac{12}{1/2} + Ae^{-tR/L}$

$\therefore\ 0 = i(1^+) = 24 + Ae^{-1*\frac{1}{2}/1}$

$\therefore\ A = -24e^{1/2}$

$\therefore\ i(t\in[1,2]\,\text{sec}) = 24\left(1-e^{-(t-1)/2}\right)$ Amp

$\therefore\ i(2^-) = i(2^+) = 24\left(1-e^{-\frac{1}{2}}\right) = 9.4434$ Amp

$\therefore\ i(t\in[2,\infty)\,\text{sec}) = \frac{0}{R} + Be^{-tR/L}$

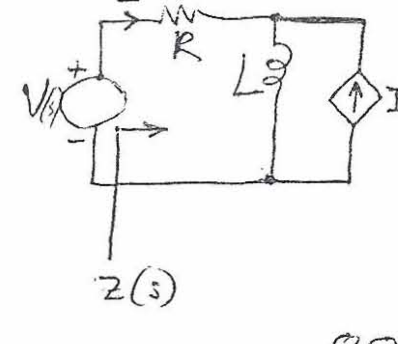
$\therefore\ 9.4434\ \text{Amp} = i(2^+) = \frac{0}{1/2} + Be^{-2*\frac{1}{2}/1} = 0 + Be^{-1} = Be^{-1}$

$\therefore\ B = 9.4434\,e$, Amp

$\therefore\ i(t\in[2,\infty)\,\text{sec}) = 9.4434\,e^{-(t-2)/2}$, Amp [Note: $i(\infty) = 0$ A].

The sketch is as shown above.

To find $Z(s)$ of the given circuit, obtain the source voltage V in terms of supplied current I.



$\therefore\ V = IR + sL(I+I) = I(R+2sL)$

$\therefore\ Z(s) = \frac{V(s)}{I(s)} = R + 2sL.$

#) $V_o(t<0) = 0$

$V_o(t\in[0,1]s) = 12\left(1-e^{-t/RC}\right) = 12\left(1-e^{-t/3}\right), V$ & $i_c(t\in(0,1)s) = 2e^{-t/3}, A$

$\therefore V_o(1) = 12(1-e^{-1/3}) = 3.4016$ Volts

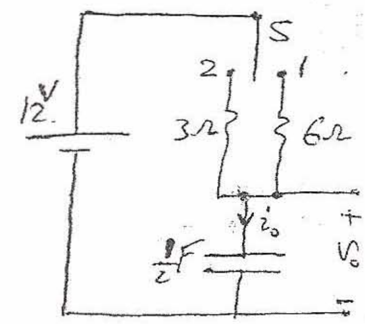
$\therefore V_o(t\in[1,\infty)) = 12 + Ae^{-t/1.5}$

$\therefore V_o(1) = 3.4016 = 12 + Ae^{-1/1.5}$

$\therefore A = (3.4016-12)e^{1/1.5} = -8.5984\, e^{1/1.5}$ Volts

$\therefore V_o(t\in[1,\infty)) = 12 - 8.5984\, e^{-(t-1)/1.5}$, Volts

& $i_o(t\in(1,\infty)\text{sec}) = C\frac{dV_o}{dt} = 2.8661\, e^{-(t-1)/1.5}$, Amp



a) $\therefore$ at $t=0^-$s : $i_{6\Omega}=0A$, $V_{6\Omega}=0V$, $i_{3\Omega}=0A$, $V_{3\Omega}=0V$, $i_c=0$ Amp & $V_c=0V$

& at $t=0^+$s : $i_{6\Omega}=2A$, $V_{6\Omega}=12V$, $i_{3\Omega}=0A$, $V_{3\Omega}=0V$, $i_c=2$ Amp & $V_c=0V$

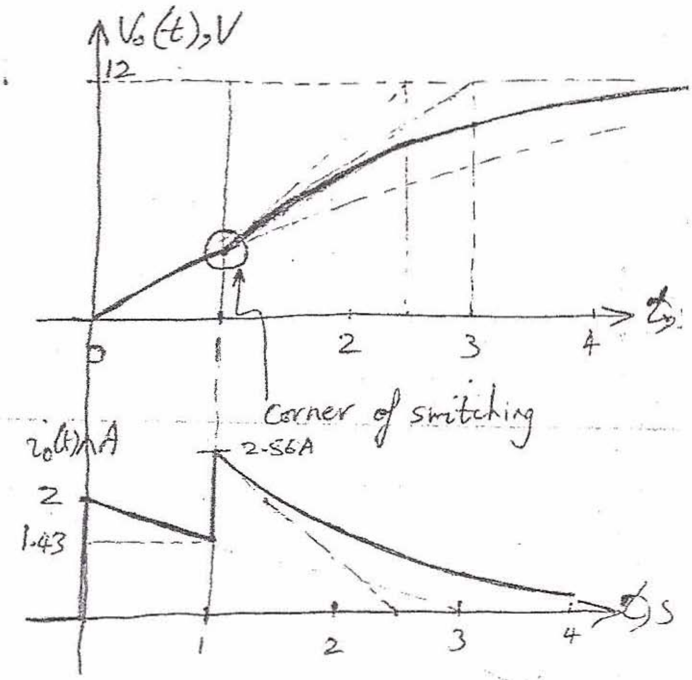
& at $t=1^-$s : $i_{6\Omega}=1.433A$, $V_{6\Omega}=8.598V$, $i_{3\Omega}=0A$, $V_{3\Omega}=0V$, $i_c=1.433A$ & $V_c=3.402$

& at $t=1^+$s : $i_{6\Omega}=0A$, $V_{6\Omega}=0V$, $i_{3\Omega}=2.866A$, $V_{3\Omega}=8.598V$, $i_o=2.866A$ & $V_o=3.4$

b) & $v(t) = \begin{cases} 0V & \text{for } t<0 \text{ sec} \\ 12(1-e^{-t/3}), V & \text{for } t\in[0,1] \text{ sec} \\ 12-8.5984e^{-(t-1)/1.5}, V & \text{for } t>1 \text{ sec} \end{cases}$

c) The sketch is as shown here.

Note:
A corner in $V_o(t)$ plot, corresponds to a spike in $i_o(t)$ plot. This is because $i_c(t)$ is a derivative of $V_o(t)$ and at the corner of $V_o(t)$ there is a change of slope.



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