بسم الله الرحمن الرحيم

الحلول المختارة لطلاب الهندسة والعمارة

تحليل دوائر (٢)

إعداد

سلمان محمد القاسمي

الأستاذ المشارك بقسم الهندسة الكهربية والحاسبات

بجامعة أم القرى

الطبعة الثانية

رمضان ١٤٢٢هـ - تشرين ثان ٢٠٠١

تمهيد

الحمد لله رب العالمين، والصلاة والسلام على سيد المرسلين، وعلى آله وصحبه أجمعين، وبعد: فهذه مجموعة من المسائل المحلولة في مادة تحليل دوائر(٢) (EE302) لطلبة قسم الهندسة الكهربية بكلية الهندسة والعمارة الإسلامية بجامعة أم القرى، في طبعتها الثانية، حيث أُضيف إلى طبعتها الأولى ما تحصل لدي من أعمال خلال الفترة بين الطبعتين،

وإنني إذْ أشكر للطلاب التالية أسماؤهم:

١) فواز إبراهيم أحمد الزايدي (٤١٨٠٢٣٥٦)،

٢) موسى حسن عوض القثامي (٤١٨٠٢٥٥٠)،

٣) نزار حسين عبدالله بانبيلة (٤١٨٠٠٥٢٨)،

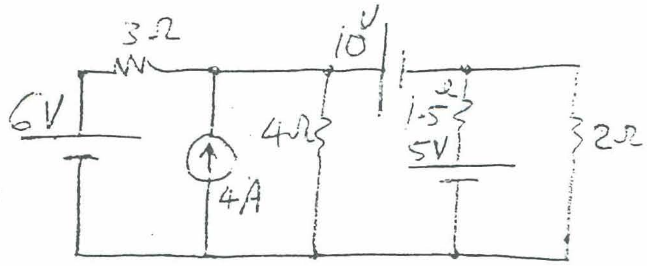
إذْ أشكر لهم مساعدتي في تبويب مسائل الطبعتين كلتيهما وفهرستها، لتسهل المراجعة فيها، وتعم الفائدة منها، لأسأل الله عز وجلّ أن ينفع بهذا العمل كل من يطالعه، وأن لا يحرمني ومن أعانني من مثوبته إنه سميع مجيب.

سلمان محمد القاسمي

في الثلاثاء: ١٤٢٢/٩/٥هـ - ٢٠٠١/١١/٢٠م

الفهرس

To find currents & voltages in every branch of the shown network:



Results should be as follows:

$I_{2\Omega} = 1$ Amp (↓)

$V_{2\Omega} = 2$ Volts $\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right)$

$I_{1.5\Omega} = 2$ Amp (↑)

$V_{1.5\Omega} = 3$ Volts $\left(\begin{smallmatrix}-\\+\end{smallmatrix}\right)$

$I_{10V} = 1$ Amp (←)

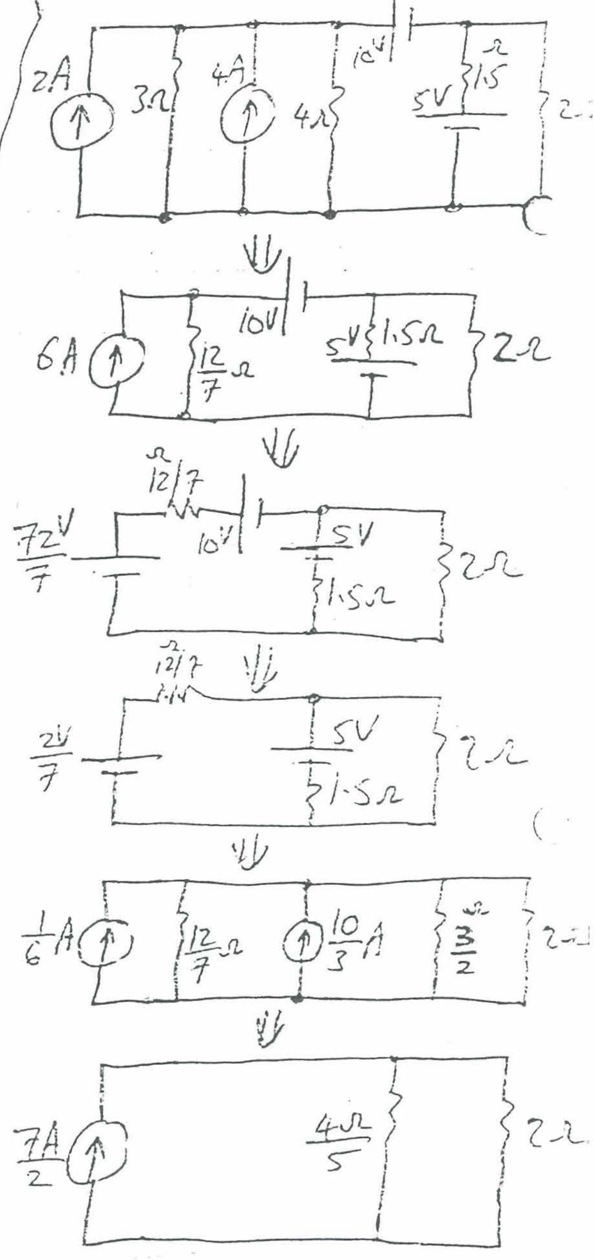
$V_{4\Omega} = 12$ Volts $\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right) = V_{4A}$

$I_{4\Omega} = 3$ Amps (↓)

$I_{3\Omega} = 2$ Amps (⟵) $= I_{6V}$

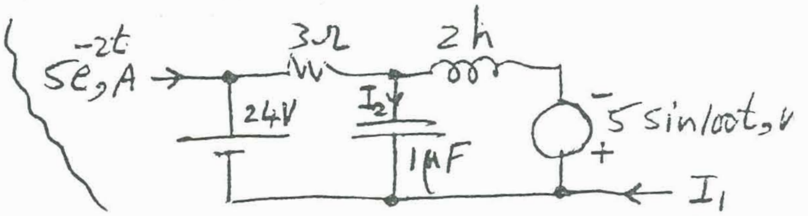
$V_{3\Omega} = 6$ Volts (− +)

To Check



$\therefore V_{2\Omega} = 2 * \frac{7}{2} * \frac{4/5}{2+4/5} = 2V$

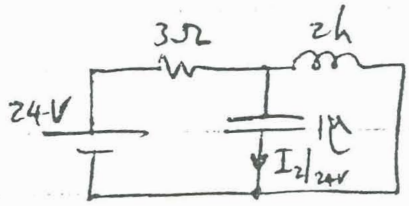
To find I_1 & I_2 of the shown circuit, the whole circuit could be considered as a junction where KCL gives $I_1 = -5e^{-2t}, A$.



As for I_2, it can be obtained using superposition as:

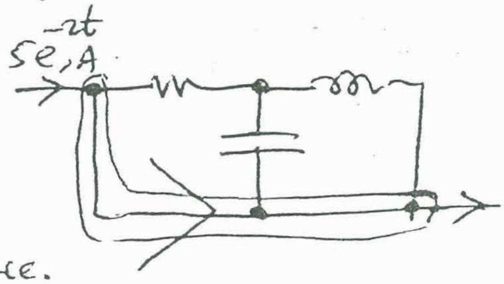
$$I_2 = I_2\Big|_{24V} + I_2\Big|_{5e^{-2t}A} + I_2\Big|_{5\sin 100t, V}$$

For $I_2\Big|_{24V}$, circuit looks like:



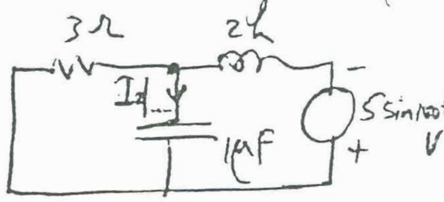
$\therefore$ $I_2\Big|_{24V} = 0$, since no dc current through C.

For $I_2\Big|_{5e^{-2t}A}$, circuit looks like:



$\therefore$ $I_2\Big|_{5e^{-2t}A} = 0$, since there is other path with zero impedance.

For $I_2\Big|_{5\sin 100t, V}$, circuit looks like:

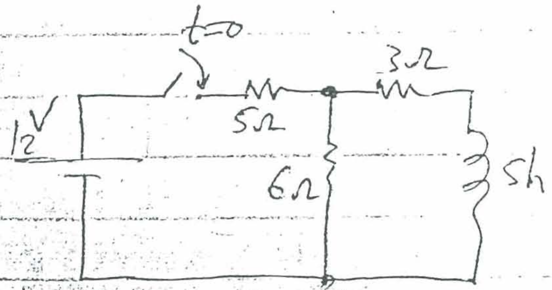


$$\therefore\ I_2\Big|_{5\sin 100t, V} = \frac{-5\angle 0}{\left(3 \,\|\, \frac{1}{j1\mu 100}\right) + j100(2)} \cdot \frac{3}{3 + \frac{1}{j100(1\mu)}} =$$

$$= \frac{-15}{\cancel{\frac{3}{1+j300\mu}} + j200} \cdot \frac{1}{\cancel{3} - j10K} = \frac{-15}{2000K} = -7.5\mu\ Amp$$

$\therefore$ $I_2 = 0 + 0 + (-7.5\mu \sin 100t) = -7.5\mu A \sin 100t.$

For the shown circuit, the initial inductor current is zero.



$\therefore\ i_{5\Omega}(0^+) = i_{6\Omega}(0^+) = \frac{12}{11}$ Amp

& $i_{3\Omega}(0^+) = i_{5h}(0^+) = 0$ Amp

& $V_{5h}(0^+) = V_{6\Omega}(0^+) = 12 \cdot \frac{6}{11} = \frac{72}{11}$ Volts

$\therefore\ i'_{3\Omega}(0^+) = i'_{5h}(0^+) = \frac{V_{5h}(0^+)}{5h} = \frac{72}{11(5)} = \frac{72}{55}$ Amp/sec

& $(12)' = 5\, i'_{5\Omega}(0^+) + 6\, i'_{6\Omega}(0^+) \Rightarrow i'_{5\Omega}(0^+) = -\frac{6}{5}\, i'_{6\Omega}(0^+) \longrightarrow (1)$

& $i'_{5\Omega}(0^+) = i'_{6\Omega}(0^+) + i'_{3\Omega}(0^+) \longrightarrow (2)$. Solving (1) & (2)

$\therefore\ -\frac{6}{5}\, i'_{6\Omega}(0^+) = i'_{6\Omega}(0^+) + \frac{72}{55} \Rightarrow i'_{6\Omega}(0^+) = \frac{-72}{121}$ Amp/sec

$\therefore\ i'_{5\Omega}(0^+) = \frac{432}{605}$ Amp/sec.

& The time constant of the circuit is $\tau = L$ / resistance seen by L

$= \frac{L}{5\|6+3} = \frac{5}{\frac{30}{11}+3} = \frac{55}{63}$ sec

& At steady state:

$$i_{5\Omega} = \frac{12}{5+6\|3} = \frac{12}{5+\frac{18}{9}} = \frac{12}{7} \text{ Amps.}$$

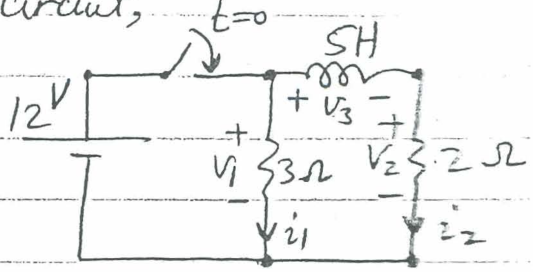
$$\& \quad i_{6\Omega} = i_{5\Omega} \cdot \frac{3}{6+3} = \frac{12}{7} \cdot \frac{1}{3} = \frac{4}{7} \text{ Amps.}$$

$$\& \quad i_{3\Omega} = \frac{8}{7} \text{ Amps.}$$

3

To solve at 0+ for

i_1, i_2', v_1'' & v_2''' of the shown circuit,



a) ∴ $i_1(0^+) = 12/3 = 4$ Aps,

$i_2'(0^+) = 12/5 = 2.4$ Ap/sec,

$v_1''(0^+) = (12)'' = 0$

$v_2'''(0^+) = 2\, i_2'''(0^+) = \frac{2}{5}\, v_3''(0^+)$.

But $12 = v_3 + 2 i_2$ ∴ $v_3' = -2 i_2' = -v_2'$

∴ $v_2'''(0^+) = \frac{2}{5}(-2 i_2')' = -\frac{4}{5}\cdot i_2'' = -\frac{4}{5^2}\cdot v_3' = \frac{8}{25}\, i_2'(0^+) = \frac{8}{25}\cdot\frac{12}{5} = \frac{96}{125} = .768$ V/s^3.

(OR: $v_2 = 12(1 - \exp(-.4t))$ ∴ $v_2'''(0^+) = 12(-.4)^3 = -.768$ V/s^3).

b) At steady state:

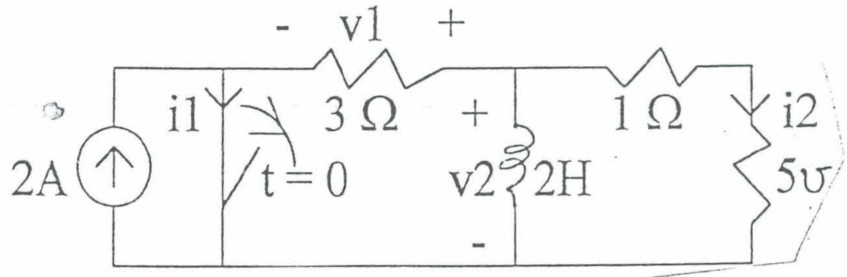
∴ i_1 = as before unaffected = 4 Ap,

$i_2 = 12/2 = 6$ Aps,

v_1 = unaffected 12 Volts &

$v_2' = 0$

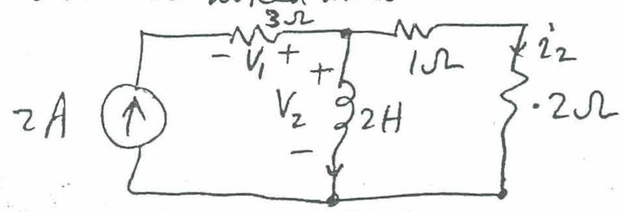
For the Circuit shown,



∵ Circuit was longly having swith on,

∴ It was dead component-wise; till $t=0^+$,

where it looked like:



∴ $i_1(0^+) = 0$,

& $i_2(0^+) = 2$ Amp, giving $V_2(0^+) = 2.4$ Volts

$V_1 = -3 * 2 = -6$ Volts all time ∴ $V_1'(0^+) = 0$

The Circuit time constant is $\frac{2}{1.2} = \frac{5}{3}$ sec

i_2 will decay from 2 Amps at $t=0^+$ to 0 Amps at $t=\infty$ ∴ $i_2(t) = 2e^{-.6t}$, Amps

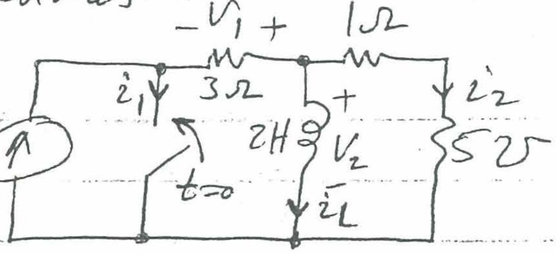
∴ $i_2''(0^+) = 2 * (-.6)^2 = 2 * .36 = .72\ A/s^2$

At $t = \infty$; $V_1 =$ as before -6 Volts

& $V_2(\infty) = 0$ (s/c),

& $i_2'(\infty) = 0$, shorted by L;

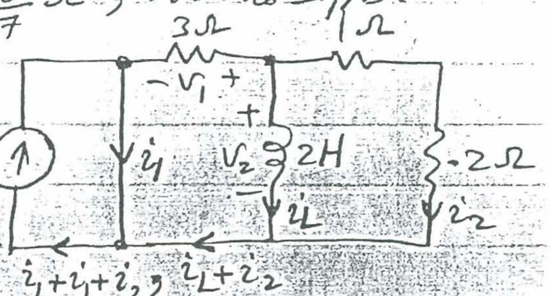
For the shown circuit, the switch was opened for long time,



$\therefore\ i_L(0^-) = 2\ \text{Amps} = i_L(0^+)$

After the switch is closed at $t=0^+$ the circuit looks like:

$\therefore\ i_L$ will see $3 // 1.2\ \Omega = \frac{3*1.2}{4.2} = \frac{6}{7}\ \Omega$, with no supply.



$\therefore\ i_L(t) = 2\,e^{-\frac{t}{2/(6/7)}} = 2e^{-\frac{6t}{14}}$, Ap
$= 2\,e^{-3t/7}$, Ap

$\therefore\ V_2(t) = 2\,i_L' = 4*\left(\frac{-3}{7}\right)e^{-3t/7}$, Volts
$= -\frac{12}{7}\cdot e^{-3t/7}$, Volts.

$\therefore\ i_2(t) = \frac{V_2}{1.2} = -\frac{10}{7}\,e^{-3t/7}$, Ap

$\therefore\ i_1(t) = 2 - i_L - i_2 = 2 - 2e^{-3t/7} + \frac{10}{7}e^{-3t/7} = 2 - \frac{4}{7}e^{-3t/7}$, Ap

$\&\ V_1(t) = -3(i_L + i_2) = -3\left(2e^{-3t/7} - \frac{10}{7}e^{-3t/7}\right) = -\frac{12}{7}e^{-3t/7}$, Volts

Note: after $t=0$ $V_1 = V_2$ as they are parallel.

$\#\ \therefore\ i_1(0^+) = 2 - \frac{4}{7} = \frac{10}{7}$ Amp &

$\#\ \ i_2'(0^+) = -\frac{10}{7}*\left(\frac{-3}{7}\right) = \frac{30}{49}$ Ap/sec &

$\#\ \ V_1''(0^+) = -\frac{12}{7}*\left(-\frac{3}{7}\right)^2 = \frac{-108}{343}$ Volt/sec^2 &

$\#\ \ V_2'''(0^+) = -\frac{12}{7}\left(-\frac{3}{7}\right)^3 = \frac{324}{2401}$ Volts/sec^3.

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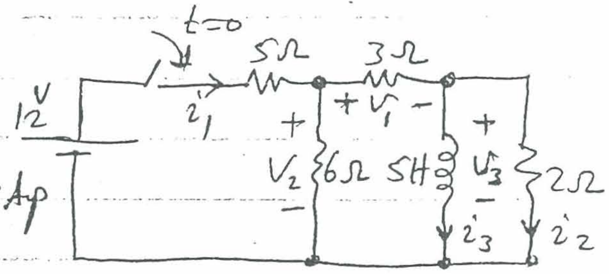
$\#$ At steadystate when $t=\infty$: $i_1 = 2$ Ap & $i_2 = V_1 = V_2 = 0$

To find the terms at 0^+ of $i_1, i_2', V_1'' \,\&\, V_2'''$, then:



a) $i_1(0^+) = \dfrac{12}{5+6\|(2+3)} = \dfrac{132}{85} = 1.553\,Ap$

$\therefore\ i_2(0^+) = i_1(0^+)\cdot\dfrac{6}{6+(3+2)} = \dfrac{72}{85} = .8471\,A$ { Switch was open for long time then put on at $t=0$. ∴ Circuit at 0^- was dead

$\therefore\ V_3(0^+) = 2\cdot i_2(0^+) = \dfrac{144}{85} = 1.694$ Volts

($\therefore\ i_3'(0^+) = \dfrac{V_3(0^+)}{5} = \dfrac{144}{425} = .3388$ Amp/sec & $i_3(0^+) = 0\,A$.

But $12 = 5i_1 + 3(i_2+i_3) + 2i_2 = 5i_1 + 5i_2 + 3i_3$

$\therefore\ 5i_1'(0^+) + 5i_2'(0^+) + 3i_3'(0^+) = 0$ (1)

& $i_1 = i_2 + i_3 + \dfrac{V_2}{6}$ $\therefore\ V_2' = 6(i_1' - i_2' - i_3')$ (2)

& $2i_2 = 5i_3'$ $\therefore\ i_3' = \frac{2}{5} i_2$ (3)

& $6(i_1 - i_2 - i_3) = 3(i_2+i_3) + 2i_2$ $\therefore\ 6i_1' = 11i_2' + 9i_3'$ (4)

Solving (1) & (4) ∴ $55i_2' + 45i_3' + 30i_2' + 18i_3' = 0$ ∴ $i_2' = \dfrac{-63}{85}i_3'$ (5)

into (4) ∴ $6*85i_1' = -693i_3' + 765i_3' = 72i_3'$ ∴ $i_1' = \dfrac{12}{85}i_3'$ (6)

into (2) ∴ $V_2' = \dfrac{6}{85}(12+63-85)i_3' = \dfrac{6}{85}*(-10)i_3' = \dfrac{-12}{17}i_3'$ (7)

(& $V_1' = 3(i_2'+i_3') = (3/85)(-63+85)i_3' = \dfrac{3*22}{85}i_3' = \dfrac{66}{85}i_3'$ (8)

b) $\therefore\ i_2'(0^+) = $ (from (5)) $-\dfrac{63}{85} * \dfrac{144}{425} = \dfrac{-9072}{36125} = -.2511$ Ap/sec (9)

into (3) ∴ $i_3''(0^+) = .4\,i_2'(0^+) = -.1005$ Ap/sec^2 (10)

c) into (8) ∴ $V_1''(0^+) = \dfrac{66}{85} i_3''(0^+) = -.07800$ Volt/sec^2

From (3, 5 & 10) ∴ $i_3''' = .4\,i_2''$ ∴ $i_3'''(0^+) = .4\,i_2''(0^+) = .4\left(\dfrac{-63}{85}\right)\cdot i_3''(0^+)$

$= .4\left(\dfrac{-63}{85}\right)(-.1005) = .02978$ Ap/s^3

d) into (7) ∴ $V_2'''(0^+) = \left(\dfrac{-12}{17}\right) * .02978 = -.02102\,V/s^3$ 7

(Note: It may be better to get $i_3(t) = \dfrac{12}{5+6\|3}\cdot\dfrac{6}{6+3}\left(1-\exp\left[-\frac{t}{5}\{(5\|6+3)\|2\}\right]\right)$

∴ $i_3(t) = \frac{8}{7}\left[1-\exp(-126t/425)\right]$, A. Then $i_1(t) = [i_3 + (5i_3'/2)](1+\frac{3}{6}) + \frac{5}{6}i_3' =$

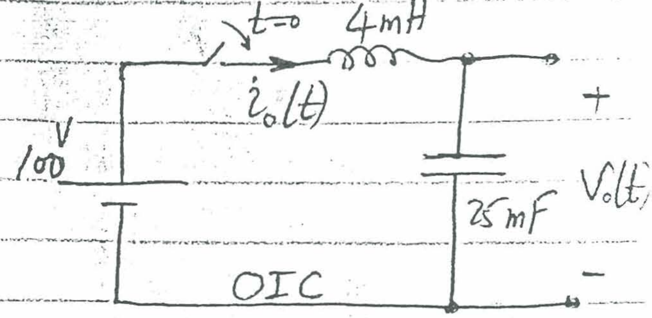
($= 1.5i_3 + \frac{55}{12}\cdot i_3'\Big|_{0^+} = 0 + \frac{55}{12}\cdot\frac{8}{7}\cdot\frac{126}{425} = 1.553\,A$ ∴ OK (a) & $i_2'(t) = \frac{5i_3''}{2}\Big|_{0^+} = \frac{5}{2}\cdot\frac{-8}{7}\cdot\left(\frac{126}{425}\right)^2 = -.25$ A/s ∴ (b)

& $V_1'' = 3(i_3'' + \frac{5}{2}i_3''')\Big|_{0^+} = 3\cdot\frac{8}{7}\cdot\left(\frac{126}{425}\right)^2\left(-1+\frac{5}{2}\cdot\frac{126}{425}\right) = -.07800\,V/s^2$ ∴ OK (c)

& $V_2''' = 3(i_3''' + \frac{5}{2}i_3'''') + 5i_3''''\Big|_{0^+} = \frac{8}{7}\cdot\left(\frac{126}{425}\right)^3\left(3 - \frac{25}{2}\cdot\frac{126}{425}\right) = -.02102\,V/s^3$ ∴ OK (d)).

e) At $t=\infty$ ∴ $i_1 = \frac{12}{5+6\|3} = \frac{12}{7} = 1.714\,A$, $i_2 = 0\,A$, $V_1 = 12 * \frac{6\|3}{5+6\|3} = 12\cdot\frac{2}{7} = 3.429$ Volts & $V_2' = 0$.

a. To find first & second rates of i_o & V_o at 0^+:



∴ $\dot{i}_o(0^+) = \frac{100-0}{4m} = 25\ KA/s$,

& $\dot{V}_o(0^+) = \frac{0}{25m} = 0\ V/s$,

& $\ddot{i}_o(0^+) = \frac{(100-V_o)'}{4m}\Big|_{0^+} = \frac{-\dot{V}_o(0^+)}{4m} = 0\ A/s^2$,

& $\ddot{V}_o(0^+) = \frac{\dot{i}_o(0^+)}{25m} = \frac{25K}{25m} = 1\ MV/s^2$.

b. The circuit is in the oscilatory mode at freq $= \frac{1}{2\pi\sqrt{100\mu}}$ ⇒

∴ Oscilation frequency is 100 rad/sec = 15.9 Hz & period = 20π ms

c. Solving the circuit ⇒ $100 = 4m(25m)\ddot{V}_o + V_o = (.1mD^2+1)V_o$

∴ $V_o(t) = \frac{1}{.1mD^2+1}\cdot(100) + A\sin 100t + B\cos 100t =$

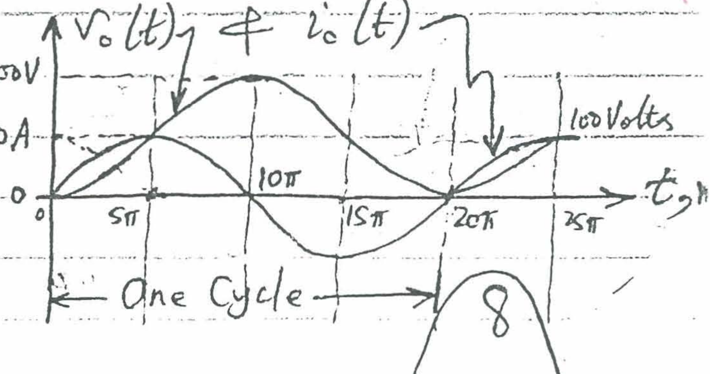
$= 100 + A\sin 100t + B\cos 100t$. But $V(0^+) = V(0^-) = 0$ (OI

∴ $V_o(t) = 100(1-\cos 100t) + A\sin 100t$. But $\dot{V}(0^+) = \frac{i(0^+)}{25m} = \frac{i(0^-)}{25m} = 0$

∴ $A=0$ ⇒ $V_o(t) = 100(1-\cos 100t)$, Volts. Max $V_o = 200V$ at odd $\frac{\pi}{100}$

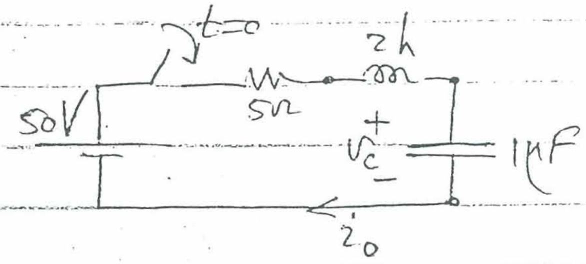
∴ $i_o(t) = C\dot{V}_o(t) = 250\sin 100t$, Amp. Max $i_o = 250$ Amp at odd $\frac{\pi}{200}$ sec

d. The two responces are plotted as shown.



For the shown circuit,

$V_c(0^-) = -50$ Volts.



a) To find Mode of operation:

$\therefore (LCD^2 + RCD + 1)V_c = 0$

$$\therefore D = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC} = \frac{-5\mu \pm \sqrt{(5\mu)^2 - 4(2)(1\mu)}}{2(2)(1\mu)} = \frac{-5 \pm \sqrt{25 - 8M}}{4}$$

$\simeq -1.25 \pm j707$ rad/s

$\therefore$ Two complex roots $\Rightarrow$ Oscilatory Mode (Underdamped)

b) Hence, $i_o(t) = (A \sin 707t + B \cos 707t)\, e^{-1.25t}$, Amp

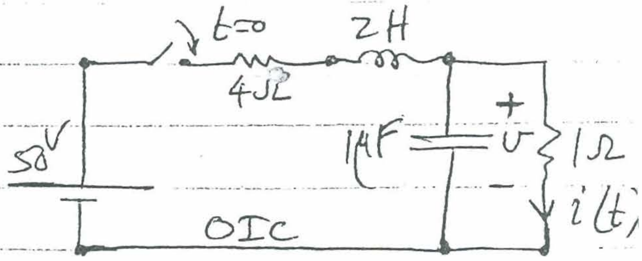
But $i_o(0^-) = 0 = i_o(0^+)$ (open circuit) $\Rightarrow B = 0$

& $V_c(0^-) = -50V = V_c(0^+) \Rightarrow V_L(0^+) = 100$ Volts $\Rightarrow i_o'(0^+) = \frac{100}{2} = 50$ A/s

$\therefore 50 = i_o'(0^+) = 707A \Rightarrow A = \frac{50}{707} = 70.7$ mA

$\therefore i_o(t) = 70.7 \sin 707t \cdot e^{-1.25t}$, mA.

To solve for $i(t)$:



$\therefore$ $50 = 4(i + 1\mu v') +$
$+ 2(i + 1\mu v')' + v$,
where $i = \frac{v}{1} = v$

$\therefore$ $50 = 4(v + 1\mu v') + 2(v' + 1\mu v'') + v$
$= (2\mu D^2 + 2.000004 D + 5) v$

$\therefore$ $D = \frac{-2.000004 \pm \sqrt{2.000004^2 - 40\mu}}{4\mu} = -2.5, -999999.5$
$\approx -2.5, -1M$; rad/sec

$\therefore$ $v(t) = A e^{-2.5t} + B e^{-1Mt} + \frac{50}{5}$.

$\because$ OIC $\therefore$ $v(0) = 0$ & $\overset{0}{\cancel{i(0)}} + 1\mu v'(0) = 0$

$\therefore$ $A + B + 10 = 0$

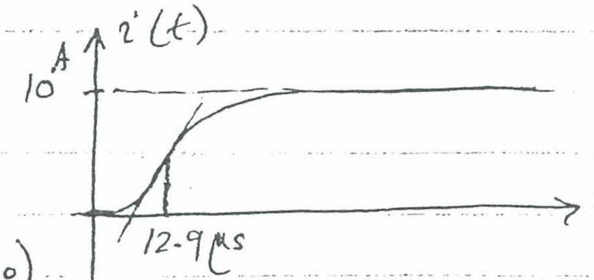
$-2.5A - 1MB = 0$ $\therefore$ $A = -400{,}000 B$

$\therefore$ $(1 - 400000) B = -10$ $\therefore$ $B \approx 25\mu$ Volt & $A \approx -10$ Volts

$\therefore$ $v(t) = 10 - 10e^{-2.5t} + 25\mu e^{-1Mt}$, Volts

$\therefore$ $i(t) = 10 - 10e^{-2.5t} + 25\mu e^{-1Mt}$, Amps $\approx 10(1 - e^{-2.5t})$ Amp

a) The circuit mode is overdamped, and it can be approximated by a first order 5Ω-2H Circuit, except for that its start-off is of zero slope.



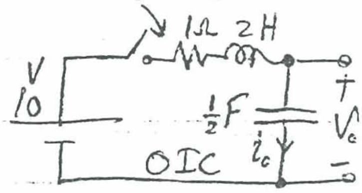
b) $\therefore$ $i(t)$ is as shown

$\therefore$ $i'(t)$ = current rise rate =
$= +25e^{-2.5t} - 25e^{-1Mt}$ $(@\ t=0, i'=0)$
i' is max @ $-2.5e^{-2.5t} + 1M e^{-1Mt} = 0$ $\therefore$ $2.5\mu \simeq e^{-1Mt}$

c) $\therefore$ $t = 1\mu s \cdot \ln(400{,}000) = 12.9\ \mu sec$ when i' is at max.

d) $\therefore$ $i'_{max}(12.9\mu s) = 25(e^{-2.5 \times 12.9\mu} - e^{-12.9}) = 24.9991$ Amp/sec

To plot the responses $V_o(t)$ & $i_o(t)$ of the shown network:



$\therefore V_o + 2\left(\frac{1}{2}V_o''\right) + 1\left(\frac{1}{2}V_o'\right) = 10$

$\therefore (2D^2 + D + 2)V_o = 20$

$\therefore D = \frac{-1 \pm \sqrt{1-16}}{4} = -.25 \pm j.968$, rad/sec

$\therefore V_o(t) = 10 + e^{-.25(t-1)} \cdot [A\sin .968(t-1) + B\cos .968(t-1)]$

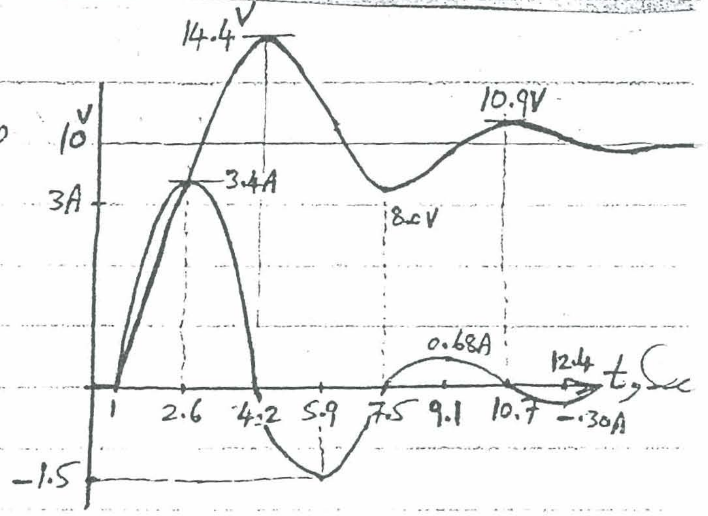
OIC $\Rightarrow$ $B = -10$ & $A = -10\left(\frac{.25}{.968}\right)$

$\therefore V_o(t) = 10\left[1 - e^{-.25(t-1)} * \frac{\cos\{.968(t-1) - \tan^{-1}\left(\frac{.25}{.968}\right)\}}{.968 / \sqrt{.25^2 + .968^2}}\right]$

$\therefore V_o(t) = 10\left[1 - 1.033e^{-.25(t-1)} \cdot \cos\{.968(t-1) + .253\}\right]$, V

& $i_o(t) = 5.164\, e^{-.25(t-1)} * \sin\{.968(t-1)\}$, Amp.

$\therefore$ Period $= \frac{2\pi}{.968} = 6.489$ sec.,
Settlement values 10V & 0 Ap
Maxima values are:
3.442 Amps at $t = 2.622$ sec
& 14.443 Volts at $t = 4.245$ sec
Envelope eq. is $e^{-.25(t-1)}$,
with time constant of 4 sec.
These are plotted as shown.



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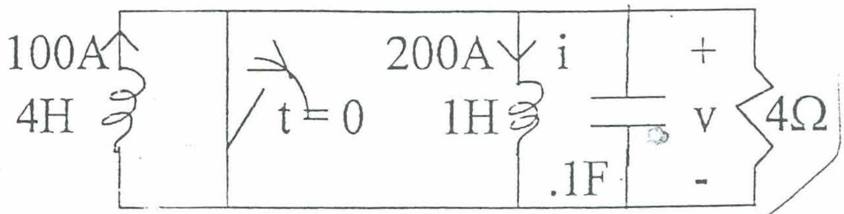
For the Circuit shown:



The circuit longly switched on, will have $v(0^-) = 0$ & the shown inductor currents. At $t \geq 0$

$\therefore \; v = 1 \cdot i' = i'$ & $v = -4\left(i + .1v' + \frac{v}{4}\right)'$
$= -4i' - .4v'' - v'$

$\therefore$ Considering i, the differential eq. is:

$i' = -4i' - .4i''' - i'' \Rightarrow D(D^2 + 2.5D + 12.5)i = 0$

$\therefore$ Diff. eq. order 3,

But considering v, the diff. eq. is:

$v = -4v - .4v'' - v' \Rightarrow (D^2 + 2.5D + 12.5)v = 0,$

the diff. eq. has been reduced to order 2,

$\therefore D = \frac{-2.5 \pm \sqrt{6.25 - 50}}{2} = -1.25 \pm j3.31$, rad/sec $\therefore$ Voltage Under damped

v will settle at 0 Volts,

$\therefore v(t) = e^{-1.25t}(A \sin 3.31t + B \cos 3.31t)$, Volts.

putting $v(0^+) = v(0^-) = 0 \Rightarrow v(t) = A e^{-1.25t} \cdot \sin 3.31t$, Volts

Integrating: $\therefore i = \int v\,dt = C + A e^{-1.25t} \cdot \frac{D + 1.25}{D^2 - 1.25^2} \cdot \sin 3.31t$

$\therefore i(t) = C + A e^{-1.25t} \cdot \frac{3.31 \cos 3.31t + 1.25 \sin 3.31t}{-12.5}.$

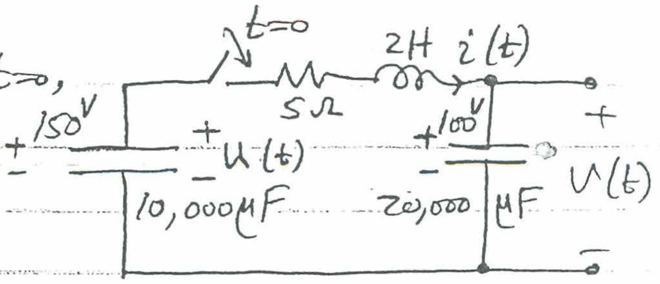
$i(0^+) = 200 = C + A\left(\frac{3.31}{-12.5}\right)$ & $i''(0^+) = v'(0^+) = \frac{100 - 200 + 0}{.1}.$

$\therefore i''(0^+) = -1000 = A e^{-1.25(0)} \cdot \left(\frac{-1}{12.5}\right) \cdot \left[-3.31^3 + 1.25^2 * 3.31 - 2.5 * 1.25 * 3.31\right]$

$\therefore A = -302.37$ Volts

& $C = 120$ Amps (or equally $= \frac{100 * 4 + 200 * 1}{4 + 1}$),

To solve the problem after $t=0$,



$\therefore i = .02\,v'$,

$-i = .01\,u' = .01\,(v + 2i' + 5i)'$

$\therefore -.02\,v' = .01(v' + 2 * .02\,v''' + 5 * .02\,v'')$

$\therefore 2v' + v' + .04\,v''' + .1\,v'' = 0 \Rightarrow v''' + 2.5\,v'' + 75\,v' = 0$

$\therefore (D^3 + 2.5\,D^2 + 75\,D)v = 0 \Rightarrow D\,(D^2 + 2.5\,D + 75)\,v = 0$

$\therefore$ Quadratic term gives:

$D = \frac{-2.5 \pm \sqrt{6.25 - 300}}{2} = -1.25 \pm j1.25\sqrt{47} = -1.25(1 \pm j\sqrt{47})$

$= -1.25 \pm j\,8.57$, rad/sec

$\therefore v = A + e^{-1.25t}(B_1 \sin 8.57t + B_2 \cos 8.57t)$, Volts, $t > 0$

($\therefore$ a) Circuit Mode is underdamped.

b) Applying the initial conditions: $v(0^+) = 100V$, $u(0^+) = 150V$ & $i(0^+) = 0$

$\therefore 100 = A + B_2$ (1), $150 = 100 + 2 * .02\,v''(0^+)$ (2) & $0 = v'(0^+)$ (3)

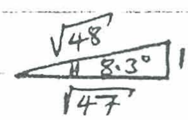
But: $v'(0^+) = -1.25\,B_2 + 8.57\,B_1 \overset{(3)}{\Rightarrow} B_2 = \sqrt{47}\,B_1 = 6.86\,B_1$ (4)

& $v''(0^+) = -8.57^2\,B_2 - 2.5 * 8.57\,B_1 + 1.25^2\,B_2 \overset{(2)}{\Rightarrow} 800 = -46\,B_2 - 2\sqrt{47}\,B_1$ (5)

Putting (4) in (5) $\Rightarrow 800 = -46\,B_2 - 2\,B_2 = -48\,B_2 \Rightarrow B_2 = -800/48 = -50/3$ Volts

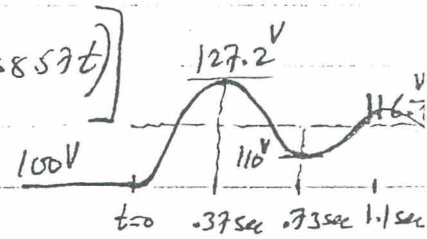
$\therefore B_1 = \frac{-50}{3\sqrt{47}} = -2.43$ Volts & $A = 100 + \frac{50}{3} = \frac{350}{3}V$

$\therefore v = \frac{350}{3} + e^{-1.25t}\left(\frac{-50}{3\sqrt{47}} \sin 8.57t - \frac{50}{3} \cos 8.57t\right)$



$= \frac{50}{3}\left[7 - \sqrt{\frac{48}{47}} \cdot e^{-1.25t} \cdot \left(\frac{1}{\sqrt{48}} \sin 8.57t + \sqrt{\frac{47}{48}} \cos 8.57t\right)\right]$

$= \frac{50}{3}\left[7 - 1.01\,e^{-1.25t} \cdot \cos(8.57t - 8.3^\circ)\right]$, Volts



$= 116.7 - 16.8\,e^{-1.25t} \cdot \cos(8.57t - 8.3^\circ)$, Volts; plotted as shown.

c) $\therefore i = .02\,v' = +.337\,e^{-1.25t} \cdot (1.25 \cos(8.57t - 8.3^\circ) + 8.57 \sin(8.57t - 8.3^\circ))$, Amps

$= \frac{20}{\sqrt{47}}\,e^{-1.25t} \cdot \sin 8.57t$, Amps $= 2.9173\,e^{-1.25t} \cdot \sin 8.57t$, Amps.

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$\because$ Current rise is i',

$\therefore i' = 2.9173\,e^{-1.25t} \cdot (8.57 \cos 8.57t - 1.25 \sin 8.57t) = 25.265\,e^{-1.25t} \cdot \cos(8.57t + 8.3^\circ)$ A/sec

This is highest when $i'' = 0 \Rightarrow 0 = -1.25 \cos(8.57t + 8.3) - 8.57 \sin(8.57t + 8.3)$

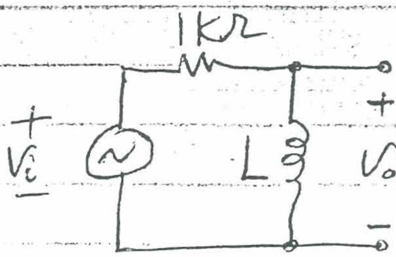
Or when $\sin 8.57t = 0 \Rightarrow t = 0$ (max) or $\frac{\pi}{8.57}$ (min) ......

$\therefore i'$ is max at start $\Rightarrow i$ has highest rise at start ($t=0$).

d) $\therefore i'_{max} = i'(0) = 25$ A/sec. [Note: $(150 - 100)/2 = 50V/2H = 25$ Amp/sec $\therefore$ ok

a. The filter type of the circuit shown is:

High Pass with 0 gain at low frequency & Unity gain at high frequency.



b. $\left|\frac{V_o}{V_i}\right| = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}$. Hence, corner frequency is ω_c

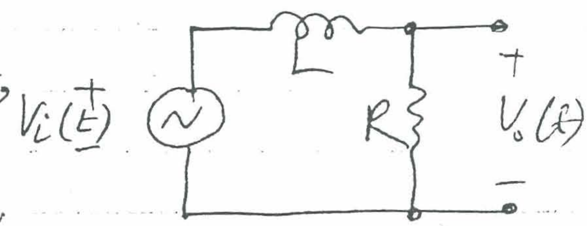
at $\frac{R}{\omega_c L} = 1$ (giving $\frac{1}{\sqrt{2}}$ gain) $\Rightarrow \omega_c = \frac{R}{L} = \frac{1K\Omega}{L}$.

Cut rate (at ω small) = Cut rate of $\frac{\omega L}{R}$ = 20 db/decade.

c. To get $f_c = 1MHz \Rightarrow \omega_c = 2\pi M\,rad/s = \frac{1K\Omega}{L} \Rightarrow L = \frac{1}{2\pi K} H = 15$[illegible]

d. $\angle^{V_o}_{V_i} = 90° - \tan^{-1}\left(\frac{\omega L}{R}\right)$. Hence quadrature is obtained between V_i & V_o at low frequency.

For the circuit shown:



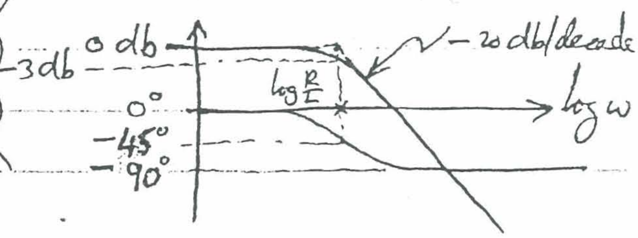
a) It functions as Low Pass Filter,

b) $\frac{V_o}{V_i} = \frac{R}{R+sL} = \frac{R}{R+j\omega L} = \frac{1}{1+j\omega L/R}$,

c) dc gain $= 1 \equiv 0$ db

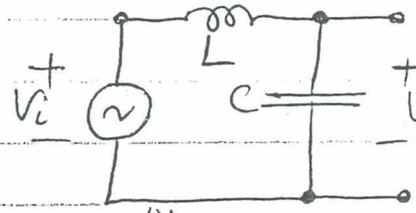
hf gain $= 0 \equiv -\infty$ db

cut-off rate @ -20 db/decade



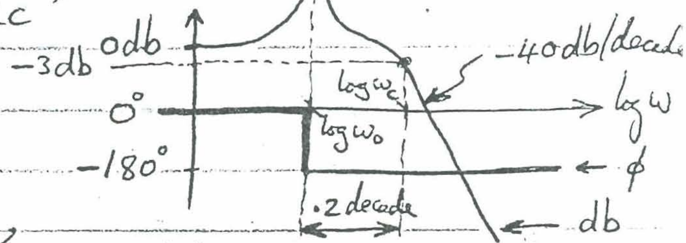
d) Corner frequency = BW = $\frac{R}{2\pi L}$.

For the circuit shown:



a) It functions as second-order low pass filter,

b) $\frac{V_o}{V_i} = \frac{1/sC}{sL + 1/sC} = \frac{1}{1+s^2LC} = \frac{1}{1-\omega^2 LC}$, plotted as shown!



c) dc gain $= 1 \equiv 0$ db,

hf gain $= 0 \equiv -\infty$ db,

cut-off rate $= -40$ db/decade,

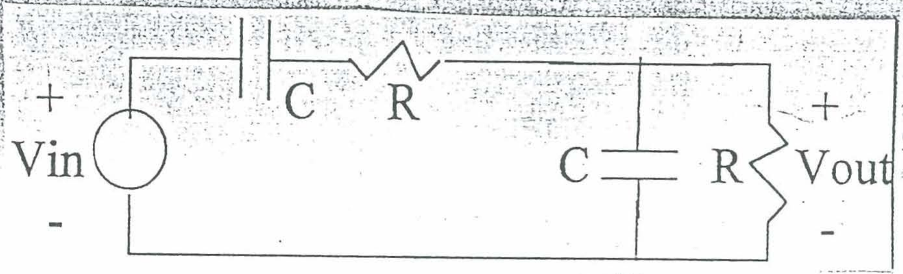
d) half-power frequency is at ω_c: $\frac{1}{-\sqrt{2}} = \frac{1}{1-\omega_c^2 LC} \Rightarrow -1 + \omega_c^2 LC = \sqrt{2}$

$\therefore \omega_c^2 LC = \sqrt{2} + 1 = 2.4142 \therefore \omega_c = 1.5538/\sqrt{LC} = 1.5538\omega_o$ where $\omega_o = \frac{1}{\sqrt{LC}}$

$\therefore BW = f_c = \frac{\omega_c}{2\pi} = .2473\,\omega_o$.

e) At resonance gain goes to ∞ & ϕ reverses. This happens when $\omega = \omega_o$.

For the filter shown,



The above is a second order filter,

It blocks dc & hf signals. $\therefore$ It is Band Pass filter,

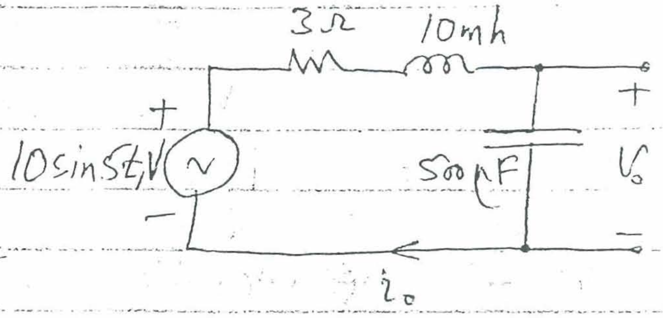
$\therefore$ Cutting-in rate is at 20 db/dec,

& Cutting-out rate is at -20 db/dec,

dc-gain is zero,

& hf-gain is also zero,

For the circuit shown:



1. The resonant frequency is at $\omega_o = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{(10m)(500\mu)}} \text{ rad/sec}$$

∴ $\omega_o = 447.2$ rad/sec

Hence, the operational frequency is below resonance.

2. $i_o \cong \frac{10\angle 0}{3 + j5(10m) - j\left(\frac{1}{5}\right)\left(\frac{1}{500\mu}\right)} \cong \frac{10}{\cancel{3} - j400} \cong .025\angle 90^\circ$, Amp

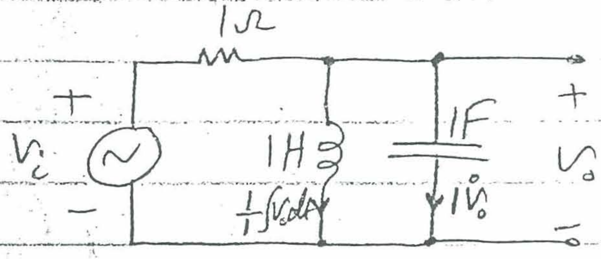
∴ 25 mAmp would flow at 90° lead with supply.

3. $V_o(t) \cong .025\angle 90 \, (-j400) \cong 10\angle 0^\circ \simeq 10 \sin 5t$, Volts.

(circuit looks open at this frequency)

4. For V_o to be $\perp$ supply ∴ Supply frequency have to be equal to resonant $\Rightarrow$ ω must equal $\omega_o = 447.2$ rad/sec.

For the circuit shown



a) The dc gain is zero, since, V_o is shorted by 1H.

b) The high frequency gain is zero, because V_o is shorted by 1F. Hence, the network is acting as a band pass filter.

c) The differential equation is: $V_i = V_o + 1\left(1\dot{V}_o + \frac{1}{1}\int V_o dt\right) \Rightarrow$

$$\dot{V}_i = \ddot{V}_o + \dot{V}_o + V_o \Rightarrow DV_i = (D^2 + D + 1)V_o \Rightarrow \frac{V_o}{V_i} = \frac{D}{D^2+D+1}$$

$$\therefore G(j\omega) = \left|\frac{V_o}{V_i}\right| = \left|\frac{j\omega}{1-\omega^2+j\omega}\right| = \frac{\omega}{\sqrt{(1-\omega^2)^2+\omega^2}}.$$

Center frequency at peak gain $\Rightarrow G' = 0$

$$\therefore \frac{1}{\sqrt{(1-\omega^2)^2+\omega^2}} + \omega\left(-\frac{1}{2}\cdot\frac{2(1-\omega^2)(-2\omega)+2\omega}{[(1-\omega^2)^2+\omega^2]\sqrt{(1-\omega^2)^2+\omega^2}}\right) = 0$$

$$\therefore (1-\omega^2)^2+\omega^2-\omega^2\left(1-2(1-\omega^2)\right) = 0 \Rightarrow (1-\omega^2)^2+2\omega^2(1-\omega^2) = 0$$

$$\therefore (1-\omega^2)(1-\omega^2+2\omega^2) = 0 \Rightarrow (1-\omega^2)(1+\omega^2) = 0 \Rightarrow \omega^2 = 1 \therefore \omega$$

$\therefore$ center frequency $= \omega = 1$ rad/sec

d) Hence, peak gain $= G(1) = \frac{1}{\sqrt{0^2+1^2}} = 1$

e) To get the quality factor, we solve $G(\omega) = \frac{G(1)}{\sqrt{2}} = \frac{1}{\sqrt{\ }}$

$$\therefore \frac{\omega}{\sqrt{(1-\omega^2)^2+\omega^2}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\left(\frac{1-\omega^2}{\omega}\right)^2}} \Rightarrow \left(\frac{1-\omega^2}{\omega}\right)^2 = 1$$

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$$\therefore 1+\omega^4-2\omega^2 = \omega^2 \Rightarrow \omega^4-3\omega^2+1 = 0 \quad \therefore \omega^2 = \frac{3\pm\sqrt{9-4}}{2} = \frac{3\pm\sqrt{5}}{2} = \left\langle\right.$$

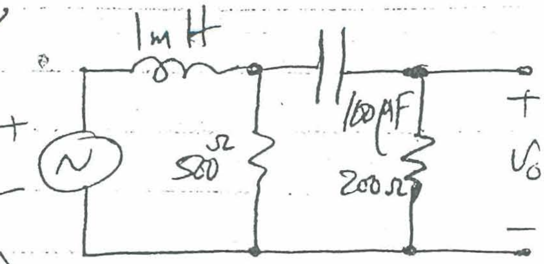
$\therefore \omega = .618$ or 1.618

$\therefore$ Bandwidth $= 1.618 - .618 = 1.000$ rad/sec.

$\therefore$ Quality factor $= \frac{1}{1} = 1$

To Solve the circuit opposite:

a) This circuit functions as a filter, and is categorized as:
Second order Band-pass filter.



b) Gain $= \left|\frac{V_o}{V_i}(j\omega)\right| \Rightarrow$ But

$$\frac{V_o}{V_i} = \frac{200}{200+\frac{1}{j\omega(.0001)}} \cdot \frac{500\,//\left(200+\frac{1}{.0001j\omega}\right)}{.001j\omega+\left[500\,//\left(200+\frac{1}{.0001j\omega}\right)\right]} =$$

$$= \frac{.02j\omega}{1+.02j\omega} \cdot \frac{(1+.02j\omega)500}{(.0001j\omega 500+1+.02j\omega)(.001j\omega)+500(1+.02j\omega)} =$$

$$= \frac{j\omega 10}{(1+j\omega(.07)).001j\omega+500+j\omega 10} = \frac{j\omega 10}{500-.00007\omega^2+j\omega 10.001}$$

$\therefore$ At center frequency ω_c: $500 = .00007\omega_c^2 \Rightarrow \omega_c = \sqrt{\frac{50}{7}}$ Krad/sec $= 2,673$ rad/sec

$\therefore G(\omega_c) = \frac{10}{10.001} \simeq 1 =$ peak gain

$$G(\omega_1,\omega_2) = \frac{1}{\sqrt{2}} = \sqrt{\frac{\omega_1^2 10^2}{(500-.00007\omega_1^2)^2+(\omega_1 10.001)^2}} \Rightarrow$$

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$200\omega_1^2 = (500-.00007\omega_1^2)^2 + 100.02\omega_1^2$ Let $\omega_1^2 = \alpha$

$\therefore 99.98\alpha = 250000 - .07\alpha + 49\times 10^{-10}\alpha^2$

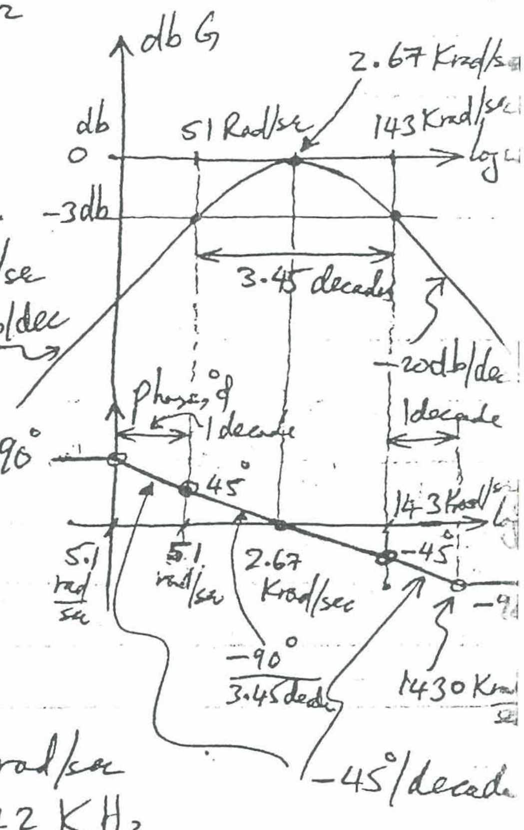
$\therefore 49\alpha^2 - 100.05\times 10^{10}\alpha + 25\times 10^{14} = 0$

$\therefore \alpha = \frac{100.05\times 10^{10} \pm \sqrt{(100.05\times 10^{10})^2 - 49\times 10^{16}}}{98}$

$\therefore \alpha = \omega_1^2 = 2.042E10$ OR 2601, rad/sec

$\therefore \omega_1$ & ω_2 are 50.998 & $142,893$ rad/sec

This is plotted as shown:

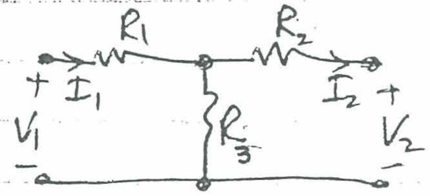


c) dc gain $= 0$
hf gain $= 0$
cutting-in rate $= 20$ db/decade
cutting-out rate $= -20$ db/decade

d) Corner frequencies are:
$\omega_1 = 51$ rad/sec & $\omega_2 = 143$ Krad/sec
OR $f_1 = 8.117$ Hz & $f_2 = 22.742$ KHz

$\therefore$ BW $= 22,742 - 8 = 22.734$ KHz geometrically centered about $f_c = 425$ Hz

To find the A matrix of the shown network, given by:



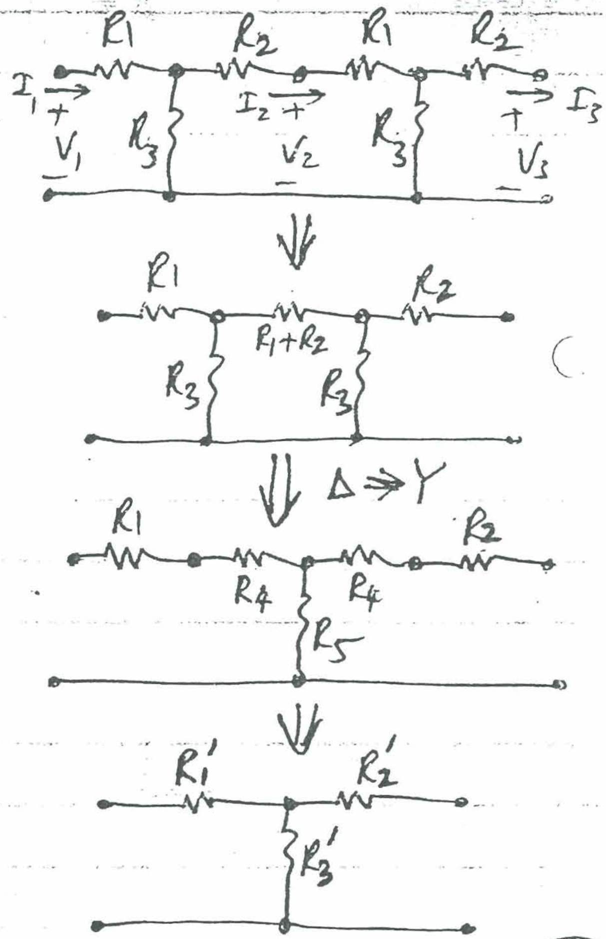
$$\begin{bmatrix} I_1 \\ V_1 \end{bmatrix} = A \begin{bmatrix} I_2 \\ V_2 \end{bmatrix} \quad \text{where:}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \quad \therefore a_{11} = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{R_2+R_3}{R_3} = 1 + \frac{R_2}{R_3}$$

& $a_{12} (\neq a_{21}) = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{R_3}$ & $a_{22} = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{R_1+R_3}{R_3} = 1 + \frac{R_1}{R_3}$

& $a_{21} = \left.\frac{V_1}{I_2}\right|_{V_2=0} = (R_1 + R_2 \| R_3)(I_1/I_2) = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2+R_3} \cdot \frac{R_2+R_3}{R_3} = \frac{R_1R_2+R_1R_3+R_2R_3}{R_3}$

Now, if two such networks are cascaded, then they can be considered as another T' as follows:



where:

$$R_4 = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + 2R_3 + R_2}$$

& $$R_5 = \frac{R_3^2}{R_1 + 2R_3 + R_2}.$$

New values for T' are:

$R_1' = R_1 + R_4$, $R_3' = R_5$ &
$R_2' = R_2 + R_4$. Denoting:

$$\begin{bmatrix} I_1 \\ V_1 \end{bmatrix} = B \begin{bmatrix} I_3 \\ V_3 \end{bmatrix}.$$

Then $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \Rightarrow b_{11} = \frac{R_2' + R_3'}{R_3'}$, $b_{12} = \frac{1}{R_3'}$,

$$b_{21} = \frac{R_1'R_2' + R_2'R_3' + R_3'R_1'}{R_3'} = R_1' + R_2' + \frac{R_1'R_2'}{R_3'} \qquad \& \quad b_{22} = \frac{R_1' + R_3'}{R_3'}$$

To check that $B = A^2$:

$\therefore$ b_{11} must be $= a_{11}^2 + a_{12}a_{21}$, (1)

b_{12} " $= a_{12}(a_{11} + a_{22})$, (2)

b_{21} " $= a_{21}(a_{11} + a_{22})$ & (3)

b_{22} " $= a_{12} \cdot a_{21} + a_{22}^2$ (4)

Let $\alpha = R_1/R_3$ & $\beta = R_2/R_3$

To prove (1):

$$RHS = \left(1 + \frac{R_2}{R_3}\right)^2 + \overbrace{\left[\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3^2}\right]}^{\alpha\beta+\alpha+\beta} = (1+\beta)^2 + \alpha\beta + \beta + \alpha$$

$$LHS = \frac{R_2' + R_3'}{R_3'} = 1 + \frac{R_2 + R_4}{R_5} = 1 + \frac{R_2 + \frac{(\beta+\alpha)R_3}{\beta+\alpha+2}}{\frac{R_1R_3}{\beta+\alpha+2}} = 1 + \beta(\beta+\alpha+2) + \alpha + \beta$$

$\therefore$ RHS of (1) = LHS of (1) $\therefore$ (1) is OK, and $b_{11} = a_{11}^2 + a_{12}a_{21}$

To prove (2):

$$RHS = \frac{1}{R_3} \cdot \left(1 + \frac{R_2}{R_3} + 1 + \frac{R_1}{R_3}\right) = (2 + \alpha + \beta)/R_3$$

$$LHS = \frac{1}{R_3'} = \frac{1}{R_5} = \frac{R_1 + 2R_3 + R_2}{R_3^2} = (2 + \alpha + \beta)/R_3$$

$\therefore$ RHS of (2) = LHS of (2) $\therefore$ (2) is OK & $b_{12} = a_{12}(a_{11} + a_{22})$

To prove (3): $RHS = (\alpha + \beta + \alpha\beta)R_3 \cdot (1 + \alpha + 1 + \beta) = (\alpha + \beta + \alpha\beta)(\alpha + \beta + 2)$

$$LHS = R_1' + R_2' + \frac{R_1'R_2'}{R_3'} = R_1 + R_4 + R_2 + R_4 + \frac{(R_1 + R_4)(R_2 + R_4)}{R_5} = \alpha R_3 + \beta R_3 +$$

$$+ 2 \cdot \frac{\alpha+\beta}{2+\alpha+\beta} \cdot R_3 + \frac{\left(\alpha + \frac{\alpha+\beta}{2+\alpha+\beta}\right)\left(\beta + \frac{\alpha+\beta}{2+\alpha+\beta}\right)}{\frac{R_3}{2+\alpha+\beta}} \cdot R_3^2 = \left[\alpha + \beta + \frac{2(\alpha+\beta)}{2+\alpha+\beta}\right.$$

$$\left. + \alpha\beta(2+\alpha+\beta) + \frac{(\alpha+\beta)^2}{2+\alpha+\beta} + (\alpha+\beta)^2\right] \cdot R_3 = R_3 \cdot \left[\alpha\beta(2+\alpha+\beta) +\right.$$

$$\left. + \alpha + \beta + (\alpha+\beta)^2 + \frac{\alpha+\beta}{\cancel{2+\alpha+\beta}} \cancel{(2+\alpha+\beta)}\right] = R_3 \cdot \left[\alpha\beta(2+\alpha+\beta) +\right.$$

$$\left. + (\alpha+\beta)(1 + \alpha + \beta + 1)\right] = R_3 \cdot (2 + \alpha + \beta)(\alpha + \beta + \alpha\beta) = RHS$$

$\therefore$ (3) is OK for $b_{21} = a_{21}(a_{11}+a_{22})$.

To prove, finally, (4):

$$RHS = \frac{1}{R_3}\cdot(\alpha+\beta+\alpha\beta)\cdot R_3 + (1+\alpha)^2 = 1+3\alpha+\beta+\alpha\beta+\alpha^2$$

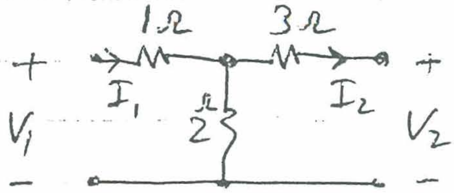
$$LHS = \frac{R_1'+R_3'}{R_3'} = 1+\frac{R_1+R_4}{R_5} = 1+\frac{\left(\alpha+\frac{\alpha+\beta}{2+\alpha+\beta}\right)\cdot R_3}{\frac{R_3}{2+\alpha+\beta}} =$$

$$= 1+(2+\alpha+\beta)\alpha+\alpha+\beta = 1+3\alpha+\beta+\alpha\beta+\alpha^2 = RHS$$

$\therefore$ (4) is OK for $b_{22} = a_{12}a_{21}+a_{22}^2$

$\therefore$ $B = A^2$ for the two cascaded T's.

To get A given as $\begin{bmatrix} I_1 \\ V_1 \end{bmatrix} = A \begin{bmatrix} I_2 \\ V_2 \end{bmatrix}$ of the network shown:



$$\therefore A = \begin{bmatrix} \left.\dfrac{I_1}{I_2}\right|_{V_2=0} = \dfrac{1}{\frac{2}{2+3}} = \dfrac{5}{2} = 2.5 & \left.\dfrac{I_1}{V_2}\right|_{I_2=0} = \dfrac{1}{2} = 0.5\,\mho \\ \left.\dfrac{V_1}{I_2}\right|_{V_2=0} = \left[1+(2//3)\right] \cdot \left.\dfrac{I_1}{I_2}\right|_{V_2=0} = \dfrac{11}{5}\cdot\dfrac{5}{2} = \dfrac{11}{2} = 5.5\,\Omega & \left.\dfrac{V_1}{V_2}\right|_{I_2=0} = \dfrac{1}{\left.\frac{V_2}{V_1}\right|_{I_2=0}} = 1/\frac{2}{2+1} = 1/\frac{2}{3} = \dfrac{3}{2} = 1.5 \end{bmatrix}$$

$\therefore A = \begin{bmatrix} 2.5 & .5\,\mho \\ 5.5\,\Omega & 1.5 \end{bmatrix}$. For two cascaded T's, the equivalent A' matrix is given by $A' = A^2$.

$$\therefore A' = \begin{bmatrix} \frac{5}{2} & \frac{1}{2} \\ \frac{11}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \frac{5}{2} & \frac{1}{2} \\ \frac{11}{2} & \frac{3}{2} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & 1 \\ 11 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 11 & 3 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 36 & 8 \\ 88 & 20 \end{bmatrix} = \begin{bmatrix} 9 & 2\,\mho \\ 22\,\Omega & 5 \end{bmatrix}$$

To find the Laplace of the given functions using rules.

a) $\therefore \alpha(t) = \cos\omega t \Rightarrow \alpha(s) = \int_0^\infty e^{-st} \cos\omega t \, dt =$

$= \frac{1}{D} e^{-st} \cos\omega t = e^{-st} \cdot \frac{1}{D-s} \cdot \cos\omega t = e^{-st} \cdot \frac{D+s}{D^2-s^2} \cdot \cos\omega t =$

$= \frac{e^{-st}}{-\omega^2-s^2} \cdot (-\omega \sin\omega t + s\cos\omega t)\Big|_0^\infty = 0 + \frac{1}{s^2+\omega^2} \cdot (0+s) = \frac{s}{s^2+\omega^2}.$

$\therefore \alpha(s) = \frac{s}{s^2+\omega^2}.$

b) $\beta(t) = t\cos\omega t = t\,\alpha(t) = -(-t\,\alpha(t)).$

But $\mathcal{L}(-t\,\alpha(t)) = \frac{d}{ds}\alpha(s) = \frac{s^2+\omega^2-2s^2}{(s^2+\omega^2)^2} = -\frac{s^2-\omega^2}{(s^2+\omega^2)^2}.$

$\therefore \beta(s) = -\left[-\frac{s^2-\omega^2}{(s^2+\omega^2)^2}\right] = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}.$

c) $\gamma(t) = (t-1)\cos\omega(t-\alpha) = \beta(t-1).$

But $\mathcal{L}(\beta(t-1)) = e^{-s}\beta(s) = \frac{s^2-\omega^2}{(s^2+\omega^2)} \cdot e^{-s}.$

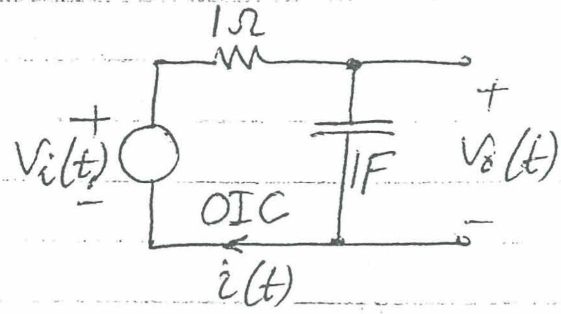
$\therefore \gamma(s) = \frac{s^2-\omega^2}{s^2+\omega^2} \cdot e^{-s}$

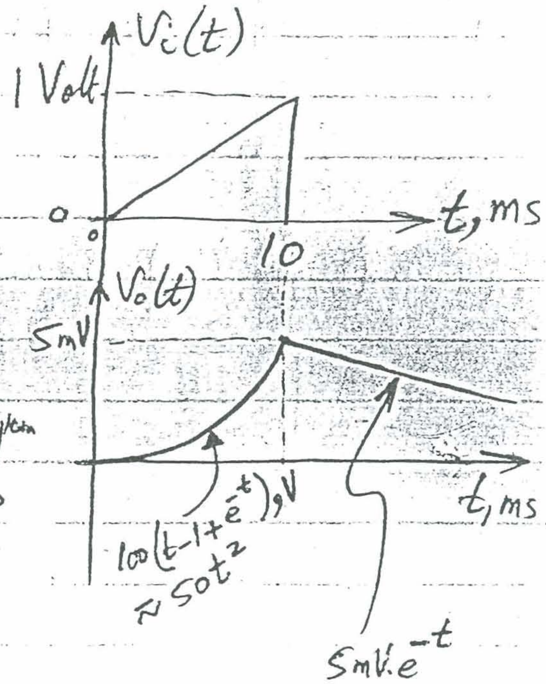
d) $\delta(t) = t \cdot e^{4t} \cdot \cos\omega t = e^{4t} \cdot \beta(t).$

But $\mathcal{L}\left[e^{4t}\beta(t)\right] = \beta(s-4) = \frac{(s-4)^2-\omega^2}{[(s-4)^2+\omega^2]^2}.$

$\therefore \delta(s) = \frac{(s-4)^2-\omega^2}{[(s-4)^2+\omega^2]^2}.$

To find $V_o(t)$ by Laplace transform of the shown network & input $V_i(t)$:





∴ $V_i = 1.1.V_o' + V_o = V_o' + V_o$

Taking the Laplace:

∴ $V_i(s) = \int_0^{10m} 100t.e^{-st}\,dt$

$$= \int 100t.\,d\left(\frac{e^{-st}}{-s}\right) = -100t\,e^{-st}/s\Big|_0^{10m} + \int \frac{e^{-st}}{s}100\,dt = -e^{-.01s}/s + \frac{100}{s}\cdot\frac{e^{-st}}{-s}\Big|_0^{10m}$$

$$= -\frac{e^{-.01s}}{s} - \frac{100}{s^2}\cdot\left(e^{-.01s} - 1\right) =$$

$$= \frac{100}{s^2} - e^{-.01s}\left(\frac{1}{s} + \frac{100}{s^2}\right)$$

& Laplace of RHS $= sV_o(s) + V_o(s) - 0 = (s+1)V_o(s)$

∴ $V_o(s) = \frac{1}{s+1}\cdot V_i(s) = \frac{100}{s^2(s+1)} - e^{-.01s}\left(\frac{1}{s(s+1)} + \frac{100}{s^2(s+1)}\right) =$

$$= 100\cdot\left(\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right) - e^{-.01s}\cdot\left(\frac{100}{s^2} - \frac{\cancel{100}^{99}}{s} + \frac{\cancel{100}^{99}}{s+1} + \cancel{\frac{1}{s}} - \cancel{\frac{1}{s+1}}\right) \Rightarrow$$

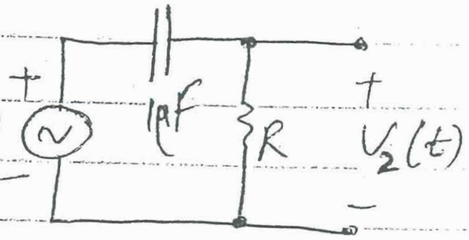
∴ $V_o(t) = 100(t - 1 + e^{-t}).u(t) - u(t-.01).\left(100(t-.01) + 99e^{-t+.01} - 99\right)$

$$= \begin{cases} 100(t-1+e^{-t}), V, & t \in [0, .01] \text{ sec} \\ 100e^{-t} - 99e^{.01}.e^{-t} = 0.00503e^{-t} = 5mVe^{-t}, & t \in [.01, \infty) \text{ sec} \end{cases}$$

$V_o(0.01) = 4.9834$ mV in both expressions. Hence $V_o(t)$ is as shown

For the circuit shown

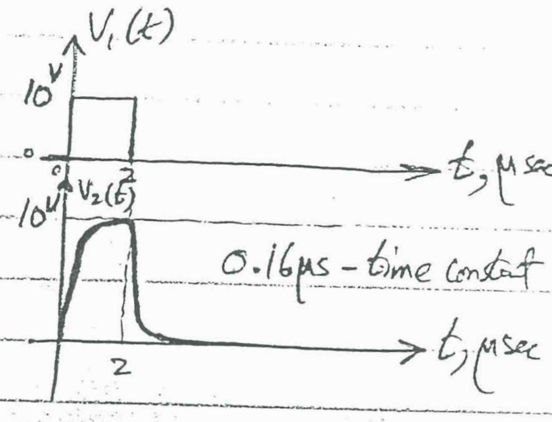
a. The filter is a high-pass one with unity high-frequency gain and zero dc-gain.



b. Corner frequency, f_c, is given by:

$2\pi f_c R * 1\mu = 1 \Rightarrow f_c = \frac{1M}{2\pi R}, Hz;$

with cutting rate of 20 db/decade.



c. For 1 MHz f_c, then $2\pi R = 1$ &

$R = \frac{1}{2\pi}, \Omega = 0.1592\ \Omega$

d. For $V_1(t)$ as shown, with OIC:

then the circuit time constant is $RC = \frac{1}{2\pi}\ \mu sec = .16\ \mu sec$;

and the response is given by:

$V_2(t) = 10(1 - e^{-2\pi t})$, Volts for $t \in (0, 2)\ \mu sec$.

$\therefore V_2(2\mu sec) = 9.999965 \approx 10$ Volts (almost there)

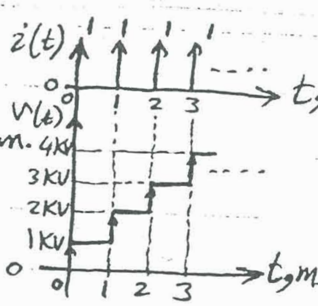
$\therefore V_2(t) = 10\ e^{-2\pi(t - 2\mu)}$, Volts for $t \geq 2\mu sec$.

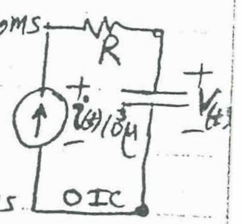
The response is plotted as shown.

To find v(t) for the circuit shown,

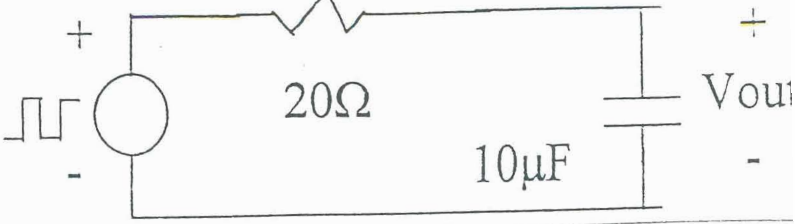
then it is a stair case wave given as shown.

Here; independent of R, every impulse gives a quota of 1 Colm $\equiv \frac{1}{10^3\mu} = 1 KV$ to the capacitor.





For the Circuit shown:



(a) V_{out} has a time constant of $RC = .2\,msec$

$\therefore$ To get square signal $\frac{T}{2} = \frac{1}{2f}$ must be $>> .2\,msec$

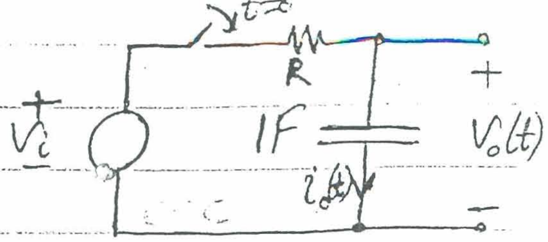
$\therefore$ chose $f << 2.5\,KHz$

(b) To get a triangular signal $\frac{T}{2} = \frac{1}{2f}$ must be $< .2\,ms$

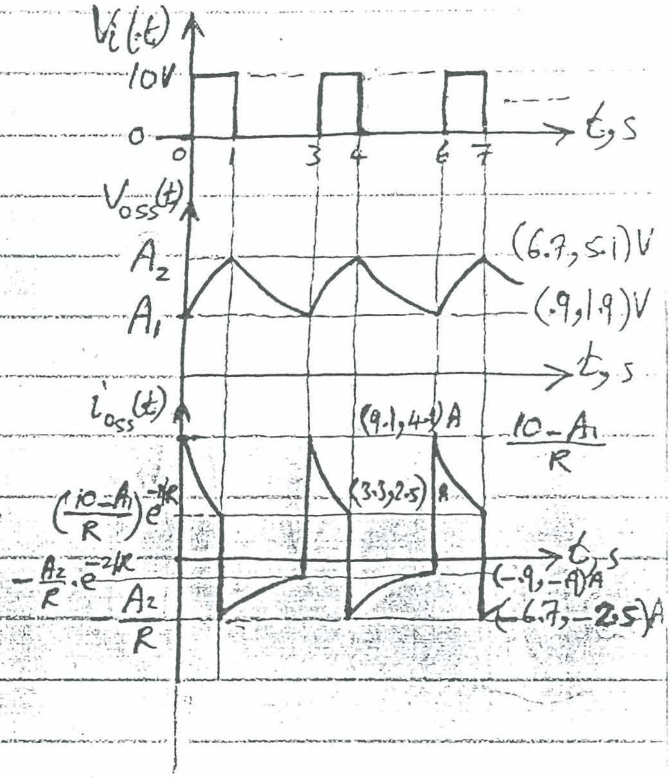
$\therefore$ choose $f > 2.5\,KHz$

For the shown network & signal;

$V_{o_{ss}}(t)$ is a charge-discharge pattern given, during charge, by:



$$V_{o_{ss}}(t) = 10 + A\,e^{-\frac{t}{R}}, \text{ where}$$



$$A_1 = V_{o_{ss}}(0) = 10 + A \Rightarrow A = A_1 - 10$$

$$\therefore V_{o_{ss}}(t) = 10 + (A_1 - 10)\,e^{-t/R}, \text{ Volts}$$

$$\therefore A_2 = V_{o_{ss}}(1) = 10 + (A_1 - 10)\,e^{-1/R} \quad (1)$$

At discharge, this is given as:

$$V_{o_{ss}}(t) = A_2\,e^{-(t-1)/R}$$

$$\therefore A_1 = V_{o_{ss}}(3) = A_2\,e^{-2/R} \quad (2)$$

Solving (1) & (2) gives:

$$A_2 = \frac{10}{1 + e^{-1/R} + e^{-2/R}} \quad \&$$

$$A_1 = \frac{10\,e^{-2/R}}{1 + e^{-1/R} + e^{-2/R}}.$$

The current is respectively given by:

$$i_o(t) = \frac{10 - A_1}{R}\cdot e^{-t/R} \quad \& \quad i_o(t) = -\frac{A_2}{R}\,e^{-(t-1)/R}.$$

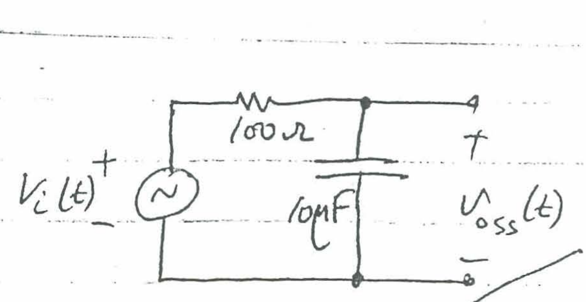
These patterns are then repeated.

For $R = 1$, $\therefore A_1 = 0.900$ Volts & $A_2 = 6.652$ Volts &

For $R = 2$, $\therefore A_1 = 1.863$ Volts & $A_2 = 5.065$ Volts.

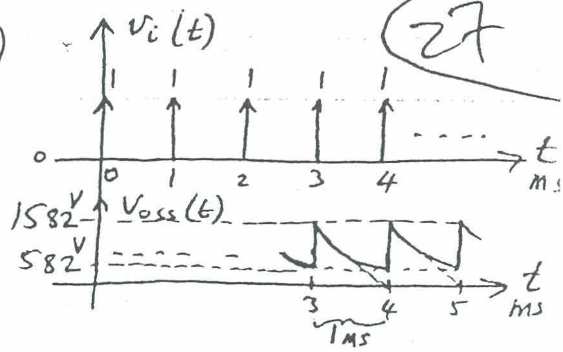
To find $V_{oss}(t)$ for the circuit shown:

then it is a decayed pattern given by:



$$V_{o_{ss}}(t) = 1582\,e^{-\frac{t - k\,ms}{1ms}} = 1582\,V\cdot e^{-\left(\frac{t}{ms} - k\right)}$$

where k is the time of impulse & the equation is valid till next one, provided steady-state is reached.



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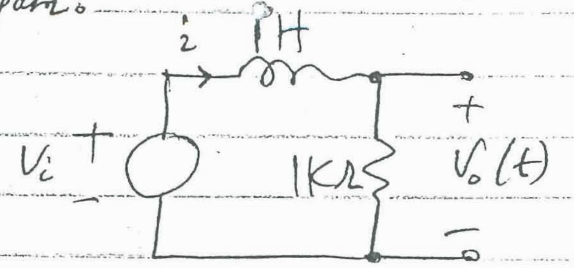
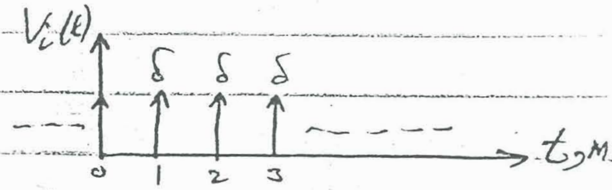
For the circuit shown with $V_i(t)$ being a train of unit impulses spaced one msec apart:



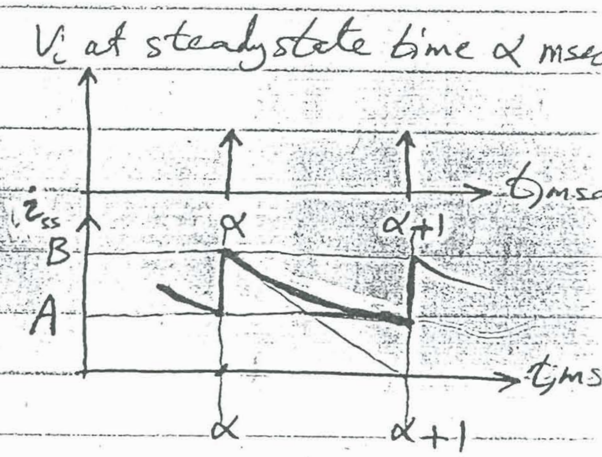
∴ The coil will take the impulse voltage, δ:



$\therefore \delta = 1 \cdot i'$ ⟹ integrating

$\therefore 1 = i(\text{after impulse}) - i(\text{before})$

Assuming any impulse time at steady-state to be α:



$\therefore 1 = i(\alpha^+) - i(\alpha^-)$

Let $i(\alpha^-) = A$ & $i(\alpha^+) = B$

$\therefore 1 = B - A$ (1)

After the impulse has gone, the coil current B will decay through the resistance

$\therefore i(t) = Be^{-\left(\frac{t-\alpha}{1/1}\right)} = Be^{-(t-\alpha)}$, where both t & α are in msec

When steady-state is reached $i(\alpha+1) = A$ again

$\therefore A = Be^{-(\alpha+1-\alpha)} = Be^{-1}$ $\therefore B = Ae$ (2)

(1) & (2) ⟹ $1 = Ae - A = (e-1)A$ $\therefore A = \frac{1}{e-1} = .582$ Aps

$\therefore B = \frac{e}{e-1} = 1.582$ Aps.

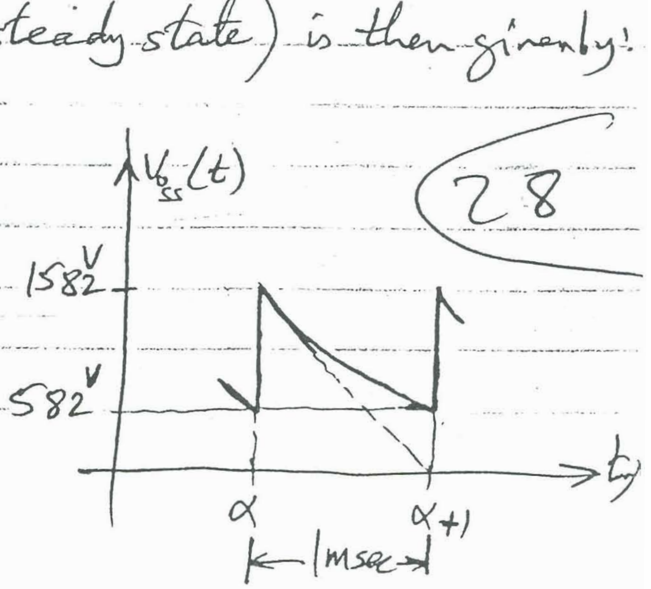
The waveform of $i_{ss}(t)$ (at steady state) is then given by:

$i_{ss}(t) = 1.582\, e^{-(t-\alpha)}$, Aps

∴ $V_{o_{ss}}(t) = 1582\, e^{-(t-\alpha)}$, Volts

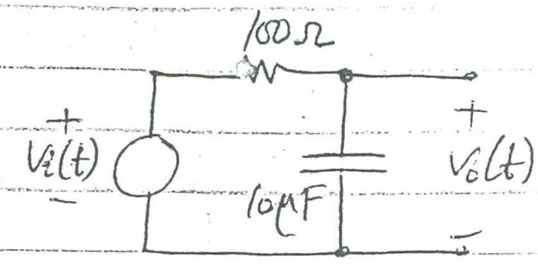
$t \in (\alpha, \alpha+1)$

plotted as shown.



28

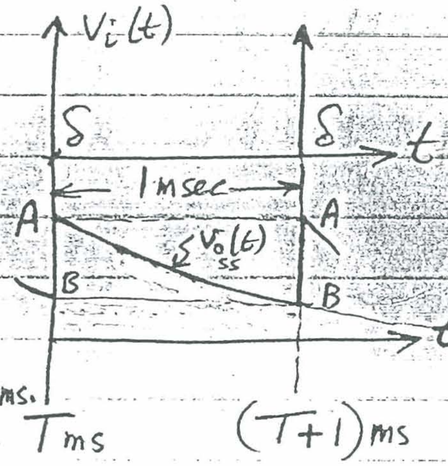
For the circuit shown, $V_i(t)$ is given as a train of unit impulses spaced by 1ms.



$$\therefore V_i(t) = \sum_{k=0}^{\infty} \delta(t - k\,\text{ms})$$

To find $V_o(t)$ at steady-state, $V_{o_{ss}}(t)$:

One $V_{o_{ss}}(t)$ cycle is a charge-discharge pattern of RC. The charge time is infinitismally small, whereas the discharge time is 1msec.
Hence, one steady state responce cycle is as shown.



Here, $V_{o_{ss}}(t)$ is given as:

$$V_{o_{ss}}(t) = A e^{-\left(\frac{t-T}{RC}\right)} = A e^{-\left(\frac{t-T}{1\text{ms}}\right)}, \quad t \in (T, T+1)\,\text{ms}.$$

$$\therefore B = V_o(T+1, \text{ms}) = A e^{-1} \Rightarrow A = e \cdot B \qquad (1)$$

But, $V_i(t) = V_o(t) + 1000\mu\, \dot{V}_o(t)$. Integrating during the infinitismally small charging time:

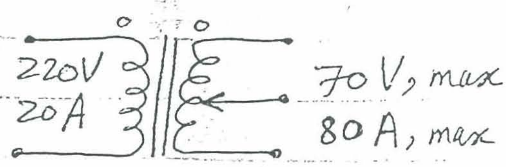
$$\therefore \int_{T^-}^{T^+} \delta \, dt = \int_{T^-}^{T^+} V_o(t)\, dt + 1m\, V_o(t)\Big]_{T^-}^{T^+} \Rightarrow 1 = 0 + 1m(A - B)$$

$$\therefore A = B + 1000 \qquad (2) \qquad \text{Solving (1) \& (2)} \Rightarrow$$

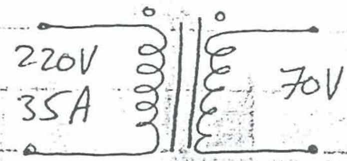
$$B + 1000 = eB \Rightarrow B = \frac{1000}{e-1} = 582 \text{ Volts} \Rightarrow A = 1582 \text{ Volts}$$

To obtain controllable 100 Amp current from 220V mains using the two transformers shown with indicated ratings, then transformer-A can not provide 100 A controlled current whereas it can provide up to 80 A controllable current. To the contrary; transformer-B can provide $\frac{220*35}{70} = 110$ Amp fixed current, with no control.

Transformer-A

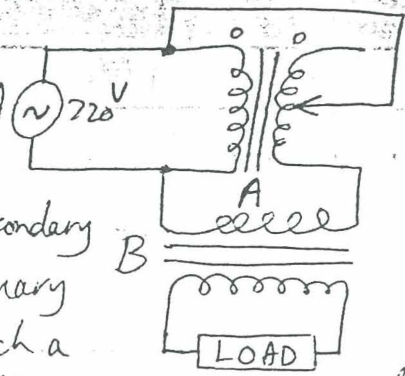


Transformer-B

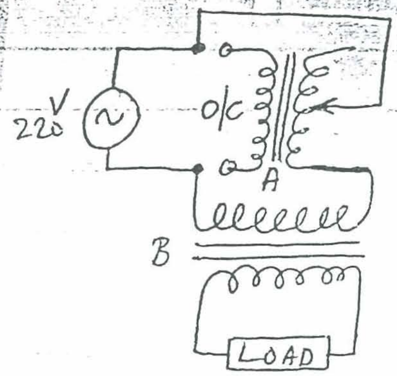


To do the job two schemes are possible as shown:

Scheme-A



Scheme-B

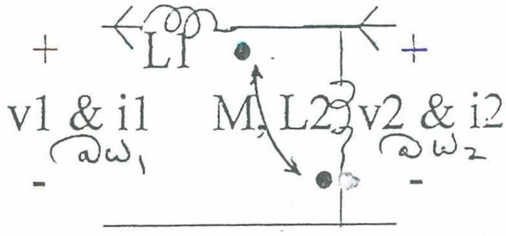


In Scheme-A, Transformer-A is primary-connected to mains, where its secondary is connected to the primary of Transformer-B in such a manner that the voltage across Transformer-B primary is reduced and hence the voltampere input and hence the output current to the load. Here, the dots of Transformer-B are not used.
On the other hand, Scheme-B uses the secondary of Transformer-A as inductance to decrease the voltage input to Transformer-B primary and hence controlling the load current. Here, all dots never mind.

#1 For the two port L-circuit shown, the z-parameters relating v's to i's as:

v1 = z1 * i1 + z2 * i2, and:

v2 = z3 * i1 + z4 * i2, are:



(a)
$$V_1 = L_1(-i_1)' + L_2(i_2 - i_1)' - M i_1' - M(i_1' - i_2')$$
$$= \underbrace{-(L_1+L_2+2M)j\omega_1}_{z_1} i_1 + \underbrace{(L_2+M)j\omega_2}_{z_2} i_2$$

(b)
$$V_2 = L_2(i_2 - i_1)' - M i_1' = \underbrace{-(L_2+M)j\omega_1}_{z_3} i_1 + \underbrace{j\omega_2 L_2}_{z_4} i_2$$

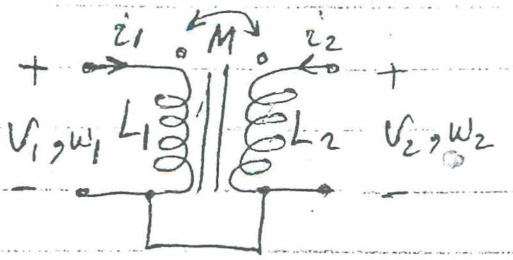
If port 2 was o/c then $i_2 = 0$

∴ $V_1 = z_1 i_1 = L_{eq} \cdot (-i_1)' = -L_{eq} j\omega_1 i_1$

∴ $z_1 = -L_{eq} j\omega_1$ ∴ $L_{eq} = \frac{z_1}{-j\omega_1}$

∴ port 1 will see inductance $L_{eq} = L_1 + L_2 + 2M$

For the circuit shown, to find A given by:



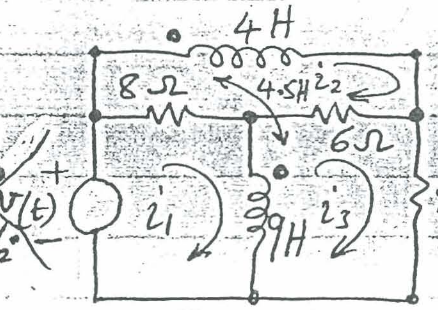
$$\begin{bmatrix} V_2 \\ i_2 \end{bmatrix} = A \begin{bmatrix} V_1 \\ i_1 \end{bmatrix}$$ with sinusoidal excitations at ω_1 & ω_2, then:

$$V_1 = L_1 i_1' + M i_2' = j\omega_1 L_1 i_1 + j\omega_2 M i_2 \Rightarrow i_2 = \frac{V_1}{j\omega_2 M} - \frac{\omega_1 L_1}{\omega_2 M} \cdot i_1$$

& $$V_2 = L_2 i_2' + M i_1' = j\omega_2 L_2 i_2 + j\omega_1 M i_1 =$$
$$= (L_2/M)\, V_1 + j\omega_1 \left[M - (L_1 L_2 / M) \right] i_1$$

$$\therefore A = \frac{1}{j\omega_2 M} \cdot \begin{bmatrix} j\omega_2 L_2 & -\omega_1\omega_2(M^2 - L_1 L_2) \\ 1 & -j\omega_1 L_1 \end{bmatrix}.$$

To solve the circuit opposite:



$$\therefore V = 8(i_1 - i_2) + 9(i_1 - i_3)' + 4.5 i_2',$$
$$0 = 6(i_3 - i_2) + 20 i_3 - 9(i_1 - i_3)' - 4.5 i_2'$$
& $$0 = 8(i_1 - i_2) + 6(i_3 - i_2) - 4.5(i_1 - i_3)' - 4 i_2'$$

OIC

a) $$\therefore \begin{bmatrix} V(t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 & -8 & 0 \\ 0 & -6 & 26 \\ 8 & -14 & 6 \end{bmatrix} \cdot \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} + \begin{bmatrix} 9 & 4.5 & -9 \\ -9 & -4.5 & 9 \\ -4.5 & -4 & 4.5 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}'$$

b) Taking Laplace Transform with OIC:

$$\therefore \begin{bmatrix} V(s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8+9s & -8+4.5s & -9s \\ -9s & -6-4.5s & 26+9s \\ 8-4.5s & -14-4s & 6+4.5s \end{bmatrix} \cdot \begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix}.$$

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$$\therefore i_3(s) = \frac{V(s) \cdot [9s(14+4s) + (6+4.5s)(8-4.5s)]}{(8+9s)(-6-4.5s)(6+4.5s) - (8-4.5s)^2(26+9s) - 81s^2(14+4s) - 9s(6+4.5s)(8-4.5s) - 9s(8-4.5s)(6+4.5s) + (8+9s)(26+9s)(14+4s)}.$$

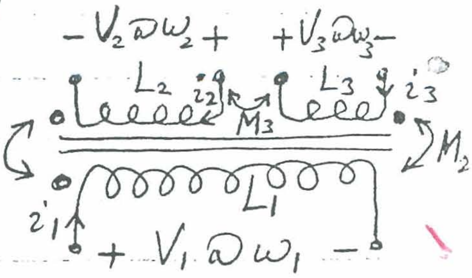
c) Simplified to:

$$i_3(s) = \frac{63s^2 + 540s + 192}{2142s^2 + 1968s + 3840} \cdot V(s).$$ & for $V(t) = u(t) \therefore V(s) = \frac{1}{s}$.

$$\therefore i_3(s) = \frac{63s^2 + 540s + 192}{2142s(s+.2050)(s+8.7436)} = \frac{107.1}{2142s} - \frac{.02238}{s+.2050} + \frac{.0017937}{s+8.7436}.$$

d) $$\therefore i_3(t) = .05 - .02238\, e^{-.2050t} + .0017937\, e^{-8.7436t}, \text{ Amps.}$$

To get the Z-matrix relating the V's of the shown transformer to its i's:



$\therefore$ Let: $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} I_1 \sin \omega_1 t \\ I_2 \sin \omega_2 t \\ I_3 \sin \omega_3 t \end{bmatrix} \therefore \begin{bmatrix} i_1' \\ i_2' \\ i_3' \end{bmatrix} = j \begin{bmatrix} \omega_1 i_1 \\ \omega_2 i_2 \\ \omega_3 i_3 \end{bmatrix}$

(a) $\therefore \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix} = \begin{bmatrix} L_1 i_1' - M_1 i_2' + M_2 i_3' \\ L_2 i_2' - M_1 i_1' - M_3 i_3' \\ L_3 i_3' - M_3 i_2' + M_2 i_1' \end{bmatrix} (1) \quad \therefore \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = Z \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} \Rightarrow Z = \begin{bmatrix} j\omega_1 L_1 & -j\omega_2 M_1 & j\omega_3 M_2 \\ -j\omega_1 M_1 & j\omega_2 L_2 & -j\omega_3 M_3 \\ -j\omega_1 M_2 & j\omega_2 M_3 & -j\omega_3 L_3 \end{bmatrix}$

(b) To get the inductance at port 1 when ports 2 & 3 are shorted: putting $V_2 = V_3 = 0$ in second & third raw of (1):

$\therefore \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} L_2 & -M_3 \\ -M_3 & L_3 \end{bmatrix} \begin{bmatrix} i_2' \\ i_3' \end{bmatrix} + \begin{bmatrix} -M_1 \\ M_2 \end{bmatrix} i_1' \Rightarrow \begin{bmatrix} L_2 & -M_3 \\ -M_3 & L_3 \end{bmatrix} \begin{bmatrix} i_2' \\ i_3' \end{bmatrix} = \begin{bmatrix} M_1 \\ -M_2 \end{bmatrix} i_1'$

$\therefore \begin{bmatrix} i_2' \\ i_3' \end{bmatrix} = \frac{1}{L_2 L_3 - M_3^2} \begin{bmatrix} L_3 & M_3 \\ M_3 & L_2 \end{bmatrix} \cdot \begin{bmatrix} M_1 \\ -M_2 \end{bmatrix} \cdot i_1' = \begin{bmatrix} L_3 M_1 - M_2 M_3 \\ M_1 M_3 - L_2 M_2 \end{bmatrix} \frac{i_1'}{L_2 L_3 - M_3^2}$

Using this in the first raw of (1):

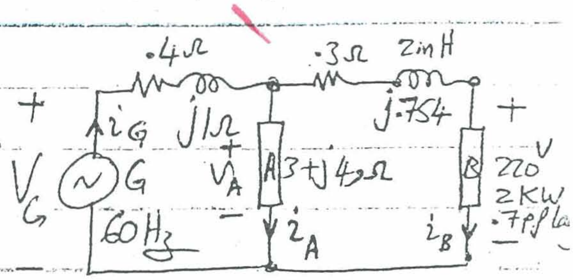
$\therefore V_1 = L_1 i_1' + [-M_1 (L_3 M_1 - M_2 M_3) + M_2 (M_1 M_3 - L_2 M_2)] \frac{i_1'}{L_2 L_3 - M_3^2} =$

$= \frac{i_1'}{L_2 L_3 - M_3^2} \cdot \left[L_1 L_2 L_3 - L_1 M_3^2 - M_1^2 L_3 + M_1 M_2 M_3 + M_1 M_2 M_3 - M_2^2 L_2 \right] =$

$\therefore$ Inductance @ 1 when 2 & 3 are shorted $= \frac{V_1}{i_1'} = \frac{L_1 L_2 L_3 + 2 M_1 M_2 M_3 - (L_1 M_3^2 + L_2 M_2^2 + L_3}{L_2 L_3 - M_3^2}$

For the power circuit shown, to find: V_G, pf_G, i_G, η_G, P_{Line}, V_A, P_A, Q_A & Z_B

The inductance of 2mH makes at 60Hz frequency, an impedence of $j\omega L = j.754\Omega$.
Taking V_B as reference:



$$\therefore\ i_B = \frac{2000}{220 * .7} \angle -\cos^{-1} .7 = 12.987\ Ap \angle -45.573^\circ$$

$$\therefore\ Z_B = \frac{V_B}{i_B} = \frac{220\angle 0}{12.987\angle -45.573^\circ} = 16.94\Omega \angle 45.573^\circ = 11.858 + j\,12.098,\ \Omega$$

$$\therefore\ V_A = 220 + i_B(.3 + j.754) = 229.756\ V \angle 1.016^\circ$$

$$\therefore\ i_A = \frac{V_A}{3+j4} = 45.951\ Ap \angle -52.115^\circ$$

$$\therefore\ P_A = i_A \cdot V_A \cdot \cos\left(\tan^{-1}\frac{4}{3}\right) = 6334.551\ Watts = 6.335\ KW$$

$$\&\ Q_A = i_A \cdot V_A \cdot \sin\left(\tan^{-1}\frac{4}{3}\right) = 8446.068\ VARI = 8.446\ KVARI$$

$$\therefore\ i_G = i_A + i_B = 58.872\ Ap \angle -50.675^\circ$$

$$\therefore\ V_G = V_A + i_G * (.4 + j1) = 291.108\ Volts \angle 4.564^\circ$$

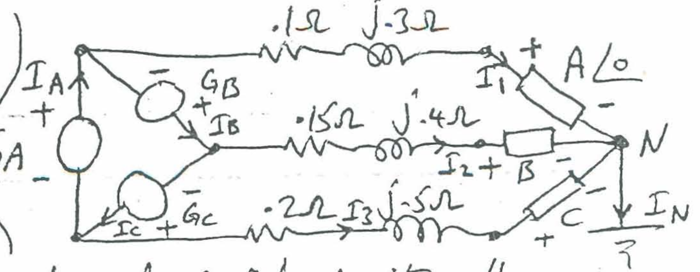
$$\therefore\ P_G = i_G \cdot V_G \cdot \cos \angle_{i_G}^{V_G} = 9771.530\ Watts = 9.772\ KW$$

$$\&\ pf_G = \cos \angle_{i_G}^{V_G} = \cos(55.239^\circ) = .570$$

$$\therefore\ P_{Line} = P_G - P_A - P_B \left(= .4\,|i_G|^2 + .3\,|i_B|^2\right) = 1436.978\ W = 1.437\ KW$$

$$\therefore\ \eta_G = \frac{P_A + P_B}{P_G} * 100 = \left(1 - \frac{P_{Line}}{P_G}\right) * 100 = 85.294\ \%$$

\# To solve the circuit shown when A, B & C are symmetrical loads of 220V, 10KW & .8 pf lagging



∴ Taking A voltage as ref., then:

Each load passes $\frac{10K}{220 \times .8} = 56.82$ Amp at 36.9° lagging its voltage.

∴ $I_1 = 56.82A\angle -36.9^\circ$, $I_2 = 56.82A\angle -156.9^\circ$ & $I_3 = 56.82A\angle +83.1^\circ$

(a) ∴ $G_A = (.1+j.3)56.82\angle -36.9^\circ + 220 - [220\angle 120^\circ + (.2+j.5)56.82\angle 83.1^\circ] = 419.7V\angle -27^\circ$

& $G_B = -[(.1+j.3)56.82\angle -36.9 + 220\angle 0^\circ] + 220\angle -120^\circ + (.15+j.4)56.82\angle -156.9^\circ = 410.8V\angle -146^\circ$

∴ $G_C = -(G_A + G_B) = 420.9V\angle 93.8^\circ$. Generator currents can be assumed symmetrical

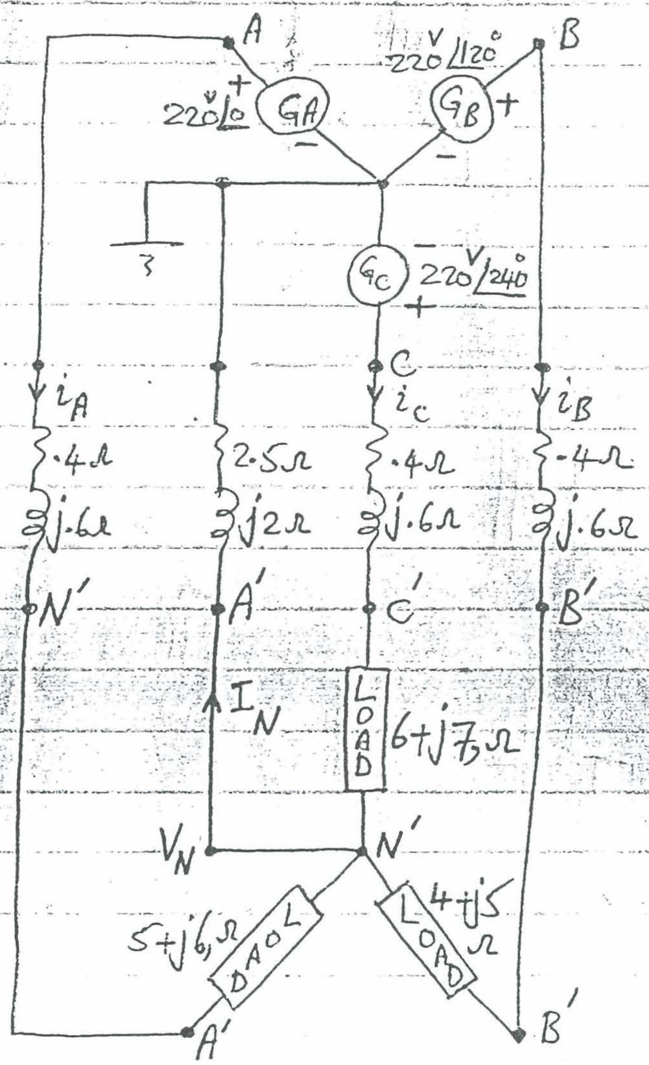
(b) ∴ $I_A = \frac{I_1}{\sqrt{3}\angle 30^\circ} = 32.8A\angle -66.9$, $I_B = 32.8A\angle 173.1^\circ$, $I_C = 32.8A\angle 53.1^\circ$ ∴ pf $G_A = .775$ lag, pf $G_B = .765$ lag & pf $G_C = .758$ lag

(c) Line losses $= 56.82^2(.1+.15+.2) = \underline{1452.7W}$, $I_N = V_N = 0$

To find the neutral voltage and current when sound & broken:

Using superposition

$\therefore V_N = V_{N_A} + V_{N_B} + V_{N_C}$



Let: $Z = 2.5 + j2, \Omega;$

$Z_1 = Z_{A'} + .4 + j.6 = 5.4 + j6.6, \Omega;$

$Z_2 = Z_{B'} + .4 + j.6 = 4.4 + j5.6, \Omega;$

$Z_3 = Z_{C'} + .4 + j.6 = 6.4 + j7.6, \Omega.$

$\therefore$ For sound connection:

$$\therefore V_{N_A} = \frac{220\angle 0 * (Z \| Z_2 \| Z_3)}{Z_1 + (Z \| Z_2 \| Z_3)},$$

$$V_{N_B} = \frac{220\angle 120^\circ * (Z \| Z_1 \| Z_3)}{Z_2 + (Z \| Z_1 \| Z_3)} \quad \&$$

$$V_{N_C} = \frac{220\angle 240^\circ * (Z \| Z_1 \| Z_2)}{Z_3 + (Z \| Z_1 \| Z_2)}.$$

$\therefore V_N = 38.689 \angle -5.50^\circ + 0^\circ + 46.326 \angle -6.63^\circ + 120^\circ + 33.206 \angle -4.68 + 240^\circ =$

$= 38.689 \angle -5.50^\circ + 46.326 \angle 113.37^\circ + 33.206 \angle 235.32^\circ =$

$= 11.581 \text{ Volts} \angle 83.86^\circ$

$\therefore I_N = V_N / Z = 3.617 \text{ Amp} \angle 45.20^\circ.$

& For broken connection; $Z = \infty$ in the above expressions:

$\therefore V_N = 72.004 \angle 0.22^\circ + 0^\circ + 86.218 \angle -0.92 + 120^\circ + 61.799 \angle 1.03^\circ + 240^\circ =$

$= 72.004 \angle .22^\circ + 86.218 \angle 119.08^\circ + 61.799 \angle 241.03^\circ =$

$= 21.553 \text{ Volts} \angle 89.58^\circ.$

& $I_N = 0$ Amps.

$$\therefore\ i'_A = \frac{220\angle 0^\circ - 11.581\angle 83.86^\circ}{5.4 + j6.6} = 25.689\ \text{Ap}\ \angle -53.72^\circ ,$$

$$i'_B = \frac{220\angle 120^\circ - 11.581\angle 83.86^\circ}{4.4 + j5.6} = 29.593\ \text{Ap}\ \angle 70.01^\circ\ \&$$

$$i'_C = \frac{220\angle 240^\circ - 11.581\angle 83.86^\circ}{6.4 + j7.6} = 23.213\ \text{Ap}\ \angle -168.74^\circ .$$

$$\therefore\ P_A = 220 * 25.689 * \cos 53.72^\circ = 3343.9\ W = 3.344\ KW$$

$$P_B = 220 * 29.593 * \cos(120^\circ - 70.01^\circ) = 4186.1\ W = 4.186\ KW$$

$$P_C = 220 * 23.213 * \cos(240^\circ + 168.74^\circ) = 3368.2\ W = 3.368\ KW$$

$$pf_A = \cos 53.72^\circ = 0.592 \text{ lagging},$$

$$pf_B = \cos(120^\circ - 70.01^\circ) = 0.643 \text{ lagging } \&$$

$$pf_C = \cos(240^\circ + 168.74^\circ) = 0.660 \text{ lagging}.$$

$$\text{Line Losses} = 25.689^2 * 0.4 + 29.593^2 * 0.4 + 23.213^2 * 0.4 + 3.617^2 * 0 \ldots$$
$$= 862.52 \text{ Watts}$$

a) $\therefore$ $P_A = 3.344$ KW, $P_B = 4.186$ KW & $P_C = 3.368$ KW.

b) & $pf_A = .592$, $pf_B = .643$ & $pf_C = .660$ all lagging.

c) & Line Losses = 863 Watts.

d) & $I_N = 3.617$ Amps & $V_N = 11.58$ Volts.

e) & $I_{N_{oc}} = 0$ Aps & $V_{N_{oc}} = 21.55$ Volts.

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