بسم الله الرحمن الرحيم

الحلول المختارة لطلاب الهندسة والعمارة

علم السكون

إعداد

سلمان محمد القاكي

الاستاذ المساعد بكلية العلوم التطبيقية والهندسية بجامعة ام القرى

الفهرس

صفحة

تمهيد

الحمد لله وحده والصلاة والسلام على من لا نبي بعده سيدنا محمد وعلى آله وصحبه

وبعد، فهذه مجموعة من المسائل المحلولة في علم السكون لطلاب الهندسة

والعمارة أقترح إبان تدريسي لهذه المادة لجميع شتات المنهج المقرر وهي

مأخوذة عن كتاب "Vector Mechanics for Engineers, Statics"

لمؤلفيه Beer و Johnston الطبعة الثالثة ١٩٧٧م

وقد قمت بتبويبها وفهرستها لتسهيل المراجعة فيها وتعمها، واكتفيت

عن ذكر المسألة بذكر رقمها والصفحة التي وردت فيها في الكتاب المذكور اعلاه

المقرر لمادة السكون لطلاب الهندسة في كلية العلوم التطبيقية والهندسية

بجامعة ام القرى.

والله أسأل ان يجعل هذا العمل حافزاً للطلاب للسير قدماً في الدراسة

والتحصيل لنفع البلاد والعباد، كما وأسأله أن لا يحرمني اجره في الآخرة والاولى

سامي شاكر

١٩/٨/١٤٠٨هـ

بسم الله الرحمن الرحيم

2.30 / 40

$$T_{CA} * \frac{4}{\sqrt{4^2+5^2}} + T_{CB} * \frac{4}{\sqrt{4^2+3^2}} = 200 \quad (1) \quad \& \quad T_{CA} \cdot \frac{5}{\sqrt{5^2+4^2}} = T_{CB} \cdot \frac{3}{\sqrt{3^2+4^2}}$$

$\therefore$ (2) in (1) $\therefore T_{CA} * .6247 + T_{CA} * 1.301 * .8 = 200 \quad \therefore T_{CA} = 120$ lb

$\therefore T_{CB} = 1.301 * T_{CA} = 156$ lb

—

2.31 / 40

$T_{CA} \cos 40 = T_{CB} \cos 20 \quad \therefore T_{CA} = T_{CB} \cdot \frac{\cos 20}{\cos 40}$

$\&$ $T_{CA} \sin 40 + T_{CB} \sin 20 = 300 \quad \therefore T_{CB} \cdot (\cos 20 \tan 40 + \sin 20) = 300 \quad \therefore T_{CB} = 265.4$ lb

$\therefore T_{CA} = 265.4 \cdot \frac{\cos 20}{\cos 40} = 325.5$ lb.

—

2.33 / 40

$T_{CA} \cos 60 = T_{CB} \cos 20 \Longrightarrow T_{CA} = T_{CB} \cdot \frac{\cos 20}{\cos 60}$

$\&$ $T_{CA} \sin 60 = 50 * 9.81 + T_{CB} \sin 20$

$\therefore \left(T_{CB} \cdot \frac{\cos 20}{\cos 60}\right) \sin 60 = 490.5 + T_{CB} \sin 20$

$$\therefore T_{CB} = \frac{490.5}{\cos 20 \tan 60 - \sin 20} = 382 \text{ N}$$

$\&$ $T_{CA} = 382 \cdot \frac{\cos 20}{\cos 60} = 718$ N

—

2.45 / 42

$$2W * \frac{h}{\sqrt{h^2+d^2}} = P$$

$$\therefore W = P \cdot \frac{\sqrt{h^2+d^2}}{2h}$$

—

2.46 / 42

$\therefore 80 = 10 \cdot \frac{1}{2} \cdot \sqrt{1+\left(\frac{d}{h}\right)^2} \quad \therefore \frac{d}{h} = 15.969 \quad \therefore h = \frac{20}{15.969} = 1.252$ in.

—

2.48 / 42

$$30 * \frac{c}{\sqrt{c^2+12^2}} = P$$

$\therefore$ (a) when $c = 9$ in $\quad \therefore P = 18$ lb.

$\&$ (b) when $c = 16$ in $\quad \therefore P = 24$ lb.

بسم الله الرحمن الرحيم

$\boxed{\frac{2.49}{43}}$ Let the tension in ACB be T

$\therefore (T+P) * \frac{1.6}{\sqrt{1.6^2+1.2^2}} = T * \frac{3.2}{\sqrt{3.2^2+1.2^2}} \Rightarrow T+P = T * 1.1704$

$\therefore P = 0.1704\, T$

$\uparrow (T+P) * \frac{1.2}{\sqrt{1.6^2+1.2^2}} + T * \frac{1.2}{\sqrt{3.2^2+1.2^2}} = 150 * 9.8 \Rightarrow (1.1704T) * .6 + T * .3511$

$= 1471.5 \qquad \therefore T = \frac{1471.5}{.3511 + .6 * 1.1704} = 1397\ N$

$\therefore P \doteq 238\ N$

$\boxed{\frac{2.70}{54}}$ $\cancel{T_{DC}} \cdot \frac{\langle -1.5, -2, 0 \rangle}{2.5} + T_{DA} \cdot \frac{\langle -1.5, 1, 3 \rangle}{\sqrt{1.5^2+1^2+3^2}} + T_{DB} \cdot \frac{\langle -1.5, 1, -3 \rangle}{\sqrt{1.5^2+1^2+3^2}} + 15.6 \cdot \frac{\langle 6, -2, 1.5 \rangle}{6.5} + P\langle 0,1,0 \rangle = 0$

$\therefore T_{DA} \langle -.4286, .2857, .8571 \rangle + T_{DB} \cdot \langle -.4286, .2857, -.8571 \rangle + \langle 14.4, -4.8, 3.6 \rangle$
$+ P\langle 0, 1, 0 \rangle = 0$

$\therefore \begin{bmatrix} -.4286 & -.4286 & 0 \\ .2857 & .2857 & 1 \\ .8571 & -.8571 & 0 \end{bmatrix} \begin{bmatrix} T_{DA} \\ T_{DB} \\ P \end{bmatrix} = \begin{bmatrix} -14.4 \\ 4.8 \\ -3.6 \end{bmatrix} \quad \therefore P = \frac{3.526}{-.4286 * .8571 * 2} = -4.80\,kN$

بسم الله الرحمن الرحيم

$\boxed{\frac{2.71}{54}}$ $18\langle 0,1,0\rangle + T_{DA}\cdot\frac{\langle 8,-12,0\rangle}{\sqrt{8^2+12^2+0^2}} + T_{DB}\cdot\frac{\langle -4,-12,3\rangle}{\sqrt{4^2+12^2+3^2}} + T_{DC}\cdot\frac{\langle -4,-12,-3\rangle}{\sqrt{4^2+12^2+3^2}} = 0$

$\therefore \langle 0,18,0\rangle + T_{DA}\cdot\langle 0.5547,-.8321,0\rangle + T_{DB}\cdot\langle -.3077,-.9231,.2308\rangle$
$+T_{DC}\cdot\langle -.3077,-.9231,-.2308\rangle = 0$

$$\therefore \begin{bmatrix} .5547 & -.3077 & -.3077 \\ -.8321 & -.9231 & -.9231 \\ 0 & +.2308 & -.2308 \end{bmatrix}\begin{bmatrix} T_{DA} \\ T_{DB} \\ T_{DC}\end{bmatrix} = \begin{bmatrix} 0 \\ -18 \\ 0\end{bmatrix}$$

$\therefore \Delta = .3545,\ \Delta_{T_{DA}} = 2.557,\ \Delta_{T_{DB}} = 2.304,\ \Delta_{T_{DC}} = 2.304$

$\therefore T_{DA} = \frac{\Delta_{T_{DA}}}{\Delta} = 7.21\ lb,\ T_{DB} = T_{DC} = 6.50\ lb.$

Another way:

By symmetry $\therefore T_{DB} = T_{DC}$, and since both are in the plane BCD,
$\therefore$ Their resultant is $2T_{DB} * \frac{\sqrt{12^2+4^2}}{\sqrt{12^2+4^2+3^2}}$ and lies in the plane ACD

$\therefore$ The system reduces to three forces in the plane ACD concurrent at D

$\therefore \left[2T_{DB} * \sqrt{\frac{12^2+4^2}{12^2+4^2+3^2}}\right] * \frac{4}{\sqrt{12^2+4^2}} = T_{DA} * \frac{8}{\sqrt{8^2+12^2}} \longrightarrow T_{DB} = .9014\, T_{DA}$

$\&\ \left[2T_{DB} * \sqrt{\frac{12^2+4^2}{12^2+4^2+3^2}}\right] * \frac{12}{\sqrt{12^2+4^2}} + T_{DA} * \frac{12}{\sqrt{8^2+12^2}} = 18 \Rightarrow 1.846\, T_{DB} + .8321\, T_{DA} = 18$

Solving: $\therefore T_{DA} = \frac{18}{.8321 + 1.846 * .9014} = 7.21\ lb\ \&\ T_{DB} = T_{DC} = 6.50\ lb.$

$\boxed{\frac{2.77}{55}}$ $T_{DA}\cdot\langle -1,0,0\rangle + T_{DB}\langle\frac{.75,-3,-1}{3.25}\rangle + T_{DC}\langle\frac{1,-3,1.5}{\sqrt{1^2+3^2+1.5^2}}\rangle + 30\langle 0,1,0\rangle = 0$

$\therefore T_{DA}\langle -1,0,0\rangle + T_{DB}\langle .231,-.923,-.308\rangle + T_{DC}\langle .286,-.857,.429\rangle + 30\langle 0,1,0\rangle = 0$

$$\begin{bmatrix} -1 & .231 & .286 \\ 0 & -.923 & -.857 \\ 0 & -.308 & .429 \end{bmatrix}\begin{bmatrix} T_{DA} \\ T_{DB} \\ T_{DC}\end{bmatrix} = \begin{bmatrix} 0 \\ -30 \\ 0\end{bmatrix} \therefore \Delta = .660,\ \Delta_{T_{DA}} = 5.62,\ \Delta_{T_{DB}} = 12.9,\ \Delta_{T_{DC}} = 9.24$$

$\therefore T_{DA} = 8.52\,KN,\ T_{DB} = 20.0\,KN,\ T_{DC} = 14.0\,KN$

بسم الله الرحمن الرحيم

$\boxed{\frac{2.78}{56}}$

$F_{CB} \cdot \cos 30 \cdot \sin 40 = 2500$

$\therefore F_{CB} = 4.5 \text{ KN}$

$F_{CA} \cdot \cos 60 = F_{CB} \cdot \cos 30 \cdot \cos 40$

$\therefore F_{CA} = 6.0 \text{ KN}$

$\boxed{\frac{2.82}{56}}$

ⓐ $T_{CD} = 600 \text{ lb} \quad , T_{sling} = \dfrac{600}{2 \cos\left(\sin^{-1} \frac{12/2}{15/2}\right)} = 500 \text{ lb.}$

ⓑ $T_{CD} = 600 \text{ lb} \quad , T_{sling} = \dfrac{600}{2 \cos\left(\sin^{-1} \frac{12/2}{20/2}\right)} = 375 \text{ lb.}$

$\boxed{\frac{2.84}{57}}$

$$T_{DA} \cdot \langle 0,-1,0 \rangle + T_{DB} \cdot \left\langle \frac{-6,3,2}{\sqrt{6^2+3^2+2^2}} \right\rangle + T_{DC} \cdot \left\langle \frac{-6,3,-6}{\sqrt{6^2+3^2+6^2}} \right\rangle + 700 \langle 1,0,0 \rangle + 300 \langle 0,0,1 \rangle = 0$$

$$\therefore T_{DA} \cdot \langle 0,-1,0 \rangle + T_{DB} \cdot \langle -.86, .43, .29 \rangle + T_{DC} \cdot \langle -.67, .33, -.67 \rangle = \langle -700, 0, -300 \rangle$$

$$\begin{bmatrix} 0 & -.86 & -.67 \\ -1 & .43 & .33 \\ 0 & .29 & -.67 \end{bmatrix} \begin{bmatrix} T_{DA} \\ T_{DB} \\ T_{DC} \end{bmatrix} = \begin{bmatrix} -700 \\ 0 \\ -300 \end{bmatrix} \therefore \Delta = +.77 , \Delta_{T_{DA}} = 267 , \Delta_{T_{DB}} = 268 , T_{DC} = 461$$

$\therefore T_{DA} = 350 \text{ lb} \quad , T_{DB} = 350 \text{ lb} \quad , T_{DC} = 600 \text{ lb.}$

بسم الله الرحمن الرحيم

$\boxed{\frac{2.88}{57}}$ $T\cos\alpha = 2T\sin 25$ $\therefore \cos\alpha = 2\sin 25 = 0.8452 \therefore \alpha = \pm 32°$

& $T\sin\alpha + 2T\cos 25 = 100 * 9.81$

$\therefore T = \frac{981}{2\cos 25 + \sin\alpha}$

$\therefore$ for $\alpha = +32.3°$ $\therefore T = 418N$

& for $\alpha = -32.3°$ $\therefore T = 767N$

$\boxed{\frac{2.90}{58}}$ Let the ring center be O and OC be x-axis, OB be y-axis & OD be z.

$\therefore T_{DA}\cdot\langle \frac{9}{15}\cos 240, \frac{9}{15}\sin 240, \frac{-12}{15}\rangle + T_{DB}\cdot\langle 0, \frac{9}{15}, \frac{-12}{15}\rangle + T_{DC}\cdot\langle \frac{9}{15}, 0, \frac{-12}{15}\rangle + 60\langle 0,0,1\rangle = 0$

$\therefore T_{DA}\cdot\langle -.3, -.520, -.8\rangle + T_{DB}\cdot\langle 0, .6, -.8\rangle + T_{DC}\langle .6, 0, -.8\rangle = \langle 0, 0, -60\rangle$

$$\begin{bmatrix} -.3 & 0 & .6 \\ -.52 & .6 & 0 \\ -.8 & -.8 & -.8 \end{bmatrix}\begin{bmatrix} T_{DA} \\ T_{DB} \\ T_{DC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -60 \end{bmatrix} \therefore \Delta = .682,\ \Delta_{T_{DA}} = 21.6,\ \Delta_{T_{DB}} = 18.7,\ \Delta_{T_{DC}} = 10.8$$

$\therefore T_{DA} = 32$ lb, $T_{DB} = 27$ lb, $T_{DC} = 16$ lb.

بسم الله الرحمن الرحيم

3.6 / 75

ⓐ $\langle AB\rangle$ is $\langle -15, -6\rangle$ in

Force, F, is $100\langle \cos 160, \sin 160\rangle$ lb

$\therefore$ Moment about A is $|\langle -15,-6\rangle \times 100\langle \cos 160, \sin 160\rangle| =$

$= 100(-15\sin 160 + 6\cos 160) = 1080$ lb.in ↻

ⓑ Moment about A is $100\cos 20 * 6 + 100\sin 20 * 15 = 1080$ lb.in ↻

ⓒ Moment about A is $100\sin\left(20+\tan^{-1}\frac{6}{15}\right) * \sqrt{6^2+15^2} = 1080$ lb.in ↻

3.18 / 76

ⓐ Moment of Q about O is $|\langle OA\rangle \times Q| = \left|\langle 200, 0, 200\rangle \times 450\frac{\langle -25,-100,200\rangle}{\sqrt{25^2+100^2+200^2}}\right|$

$= \left|200\langle 1,0,1\rangle \times \frac{450}{9}\langle -1,-4,8\rangle\right| = 10{,}000|\langle 4,-9,-4\rangle| =$

$= 106000$ Nmm $= 106$ N.m

ⓑ Moment of Q about D is $|\langle DC\rangle \times Q| = |\langle 225, 60, -75\rangle \times 50\langle -1,-4,8\rangle|$

$= |15\langle 15, 4, -5\rangle \times 50\langle -1,-4,8\rangle| = 750|\langle 12,-115,-56\rangle| =$

$= 96400$ Nmm $= 96.4$ N.m

3.19 / 77

ⓐ $M_{P_o} = \langle OA\rangle \times P = \langle 3, 0, 18\rangle \times \overset{20}{\cancel{420}} * \frac{\langle 9, 6, -18\rangle}{\cancel{\sqrt{9^2+6^2+18^2}}} =$

$= 3 * \langle 1,0,6\rangle \times 60 * \langle 3, 2, -6\rangle = 180 * \langle -12, 24, 2\rangle$

$= \langle -2160, 4320, 360\rangle$ lb.in

ⓑ $M_{P_o} = \langle OB\rangle \times P = \langle 12, 6, 0\rangle \times 20\langle 9, 6, -18\rangle =$

$= 6 * \langle 2, 1, 0\rangle \times 60 * \langle 3, 2, -6\rangle = 360 * \langle -6, 12, 1\rangle$

$= \langle -2160, 4320, 360\rangle$ lb.in

3.20 / 77

Moment of P about E is $|\langle EB\rangle \times P| = \left|\langle -200, 300, 0\rangle \times 200\frac{\langle 0,-300,225\rangle}{\sqrt{300^2+225^2}}\right|$

$= \left|100\langle -2, 3, 0\rangle \times \frac{200}{375}\langle 0, -300, 225\rangle\right| = \frac{20{,}000}{375}|\langle 675, 450, 600\rangle|$

$= 53{,}800$ N mm $= 53.8$ N m

✓

بسم الله الرحمن الرحيم

$\boxed{\frac{3.32}{86}}$ (a) $\langle BD\rangle = \langle -6, 3, -6\rangle$, $\langle BA\rangle = \langle -6, -4.5, 0\rangle$

$\therefore \langle BD\rangle \cdot \langle BA\rangle = +36 - 13.5 - 0 = \sqrt{6^2+3^2+6^2}\cdot\sqrt{6^2+4.5^2+0^2}\cdot \cos \widehat{ABD}$

$\therefore +22.5 = 9 * 7.5 * \cos \widehat{ABD} \;\therefore\; \cos\widehat{ABD} = 0.33 \;\therefore\; \widehat{ABD} = 70.5^\circ$

(b) projection of 180 lb on AB $= 180 * \cos \widehat{ABD} = 180 \cos 70.5 = 60$ lb.

$\boxed{\frac{3.42}{87}}$

$$M_{130lb_{AB}} = \left(\langle AC\rangle \times 130 \frac{\langle CD\rangle}{|CD|}\right)\cdot \frac{\langle AB\rangle}{|AB|} =$$

$$= \left(\langle 12, \cancel{60}\rangle \times 130 \frac{\langle -12, 5, 0\rangle}{13}\right)\cdot \frac{\langle 12, -4, 6\rangle}{\sqrt{12^2+4^2+6^2}} =$$

$$= \left(10 \langle 0, 0, 72\rangle\right)\cdot \frac{\langle 12, -4, 6\rangle}{14} = \frac{10}{14} \times 72 \times 6 = 310 \text{ lb in}$$

$\boxed{\frac{3.44}{87}}$

$$M_{546N_{AD}} = \left(\langle AB\rangle \times 546 \frac{\langle BE\rangle}{|BE|}\right)\cdot \frac{\langle AD\rangle}{|AD|} =$$

$$= \left(\langle 450, 0, 0\rangle \times 546 \cdot \frac{\langle -450, 225, 150\rangle}{\sqrt{450^2+225^2+150^2}}\right)\cdot \frac{\langle 0, -125, 300\rangle}{\sqrt{125^2+300^2}} =$$

$$= \left(450\langle 1, 0, 0\rangle \times \frac{546}{525} \times 25 \langle -18, 9, 6\rangle\right)\cdot \frac{25\langle 0, -5, 12\rangle}{325} =$$

$$= \left(11{,}700 \langle 0, -6, 9\rangle\right)\cdot \frac{\langle 0, -5, 12\rangle}{13} = 900(30+108) = 124{,}200 \text{ Nmm}$$

$$= 124 \text{ Nm}$$

بسم الله الرحمن الرحيم

$\boxed{\frac{3.50}{97}}$ ⓐ $F = \frac{975 * 100}{360} = 271\ N$ down at A, up at C

ⓑ $F = \frac{975 * 100}{\sqrt{200^2 + 150^2}} = 390\ N$ normal to BD with $\theta_x = -53^\circ$ at B & $\theta_x = 127^\circ$ at D

ⓒ The smaller the force the larger the distance ∴ It must be applied at AD

∴ $F = \frac{975 * 100}{\sqrt{150^2 + 360^2}} = 250\ N$ normal to AD with $\theta_x = -67^\circ$ at A & $\theta_x = 113^\circ$ at D

$\boxed{\frac{3.52}{97}}$ ∴ Resultant couple acting on the plate $= 6 * 40 = 240$ lb.in ↻

ⓐ Equivalent-couple requires forces at $A \rightarrow$ and $C \leftarrow$ are both of magnitude $= \frac{240}{12} = 20$ lb

ⓑ at A (in the first quarter) & D (in the third) both normal to AD and having magnitude of $\frac{240}{\sqrt{9^2 + 12^2}} = 16$ lb.

ⓒ The smallest possible forces are those involving the longest distance between them which is obviously CB

∴ at B (in the fourth quadrant) & C (in the second) both normal to BC and having magnitude of $\frac{240}{\sqrt{12^2 + 16^2}} = 12$ lb.

$\boxed{\frac{3.55}{98}}$ The single equivalent couple is the sum =

$$= M \cdot \frac{\langle CD \rangle \times \langle CA \rangle}{|\langle CD \rangle \times \langle CA \rangle|} + M \cdot \frac{\langle BC \rangle \times \langle BA \rangle}{|\langle BC \rangle \times \langle BA \rangle|} = M \cdot \left[\frac{\langle 0, 0, -a \rangle \times \langle -3a, 2a, -a \rangle}{|\langle CD \rangle \times \langle CA \rangle|} + \frac{\langle 3a, 0, 0 \rangle \times \langle 0, 2a, -a \rangle}{|\langle BC \rangle \times \langle BA \rangle|} \right]$$

$$= M \cdot \left[\frac{\langle 2a^2, 3a^2, 0 \rangle}{\sqrt{13}\, a^2} + \frac{\langle 0, 3a^2, 6a^2 \rangle}{3\sqrt{5}\, a^2} \right] = M \cdot \left[\left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0 \right\rangle + \left\langle 0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle \right]$$

$$= M \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} + \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle = \langle 0.555, 1.28, 0.894 \rangle M.$$

$\boxed{\frac{3.56}{98}}$ Resultant Couple $= 3.6 \langle 1, 0, 0 \rangle + 6 \langle 0, \sin 40, \cos 40 \rangle + 6 \langle 0, \sin 40, -\cos 40 \rangle$

$= \langle 3.6, 12 \sin 40, 0 \rangle = \langle 3.6, 7.7, 0 \rangle$ KN.m

بسم الله الرحمن الرحيم

3.58
99

ⓐ The force will transfer to A by a force of 400 N $\langle \sin 30, -\cos 30 \rangle$ and a couple of $\langle .34, -.2, 0 \rangle \times 400 \langle \sin 30, -\cos 30, 0 \rangle$

$= 400 \langle 0, 0, -.34\cos 30 + .2\sin 30 \rangle = \langle 0, 0, -77.8 \rangle$ N.m

or 77.8 N.m ↻

ⓑ The force will transfer to B by a force of 400 N $\langle \sin 30, -\cos 30 \rangle$ and a couple of $\langle .24, -.14, 0 \rangle \times 400 \langle \sin 30, -\cos 30, 0 \rangle$

$= 400 \langle 0, 0, -.24\cos 30 + .14\sin 30 \rangle = \langle 0, 0, -55.1 \rangle$ N.m

or 55.1 N.m ↻

3.72
101

The equivalent force at E is $40 \text{ lb.} \left\langle \frac{12, 6, -8}{\sqrt{12^2+6^2+8^2}} \right\rangle$

$= 40 \cdot \langle 0.768, 0.384, -.512 \rangle = \langle 31, 15, -20 \rangle$ lb

& The equivalent moment is:

$$200 \left\langle \frac{-12, 6, -8}{\sqrt{12^2+6^2+8^2}} \right\rangle + \langle -6, +3, +4 \rangle \times 40 \cdot \langle .768, .384, -.512 \rangle$$

$$= \langle -153.6, +76.82, -102.4 \rangle + 40 \langle -3.072, 0, -4.608 \rangle$$

$$= \langle -153.6, 76.82, -102.4 \rangle + \langle -122.9, 0, -184.3 \rangle$$

$$= \langle -276.5, 76.82, -286.7 \rangle \text{ lb.in}$$

بسم الله الرحمن الرحيم

3.73 / 113 Transfer all forces to the right edge:

$\therefore$ (a) 500 N Down & 1500 N.m
& (b) 500 N Up & 2500 N.m
& (c) 500 N Down & 500 N.m
& (d) 500 N Up & 1500 N.m
& (e) 500 N Down & 1500 N.m
& (f) 500 N Down & 500 N.m
& (g) 500 N Up & 1500 N.m
& (h) 500 N Down & 2500 N.m

Forces Up are in (b), (d) & (g) but their moments are not identical.
Forces down are in (a), (c), (e), (f) & (h) and in these, moments are clockwise for (a), and counterclockwise for (c), (e), (f) & (h) with only (c) & (f) equal in magnitude

$\therefore$ (c) & (f) are equivalent.

3.78 / 114 Resultant of loads is 12 kips x ft from A. Take moments at A

$\therefore$ $3 \times 8 + 4 \times 16 + 5 \times 24 = 12 \cdot x$ $\quad\therefore x = \frac{24+64+120}{12} = 17\frac{1}{3}$ ft

3.81 / 114 Assume A to be the origin & the point $P(x,y)$ to be the point of application of the resultant force. Hence, transforming all forces to P gives

$100\langle 0,-1\rangle + 600\langle \cos 30, \sin 30\rangle + 120\langle -1, 0\rangle$ = resultant $= \langle 400, 200\rangle$ N

$\therefore$ Resultant is $\langle 400, 200\rangle$ N $= 447$ N $\angle 26.6^\circ$

& $-37 - 100(.200 - x) + 600\cos 30 \cdot y + 600 \sin 30 \cdot (.250 - x) + 120 \cdot (.100 - y) = 0$

$y(600\cos 30 - 120) + x(100 - 600\sin 30) = 37 + 20 - 150\sin 30 - 12$

$y(400) + x(-200) = -30$ $\therefore$ Equation of line of action is $40y - 20x = -3$

To find intersection solve equations together.

Line AC is $y = \frac{100}{250}x$ or $5y = 2x$

∴ $40y - 20x = -3$ intersect this at $4(2x)*2 - 20x = -3$ or $x = \frac{3}{4}$ m

with $y = \frac{2x}{5} = \frac{2*3}{5*4}$ m = 300 mm

∴ point of intersection with line AC is at (750, 300) mm

(a) Line AB is $y = 0$ m ∴ $x = \frac{3}{20}$ m = 150 mm ∴ Intersection at (150, 0) mm

(b) Line BC is $x = .250$ m ∴ $40y = (20*\frac{1}{4} - 3)$ m = 2 m ∴ Intersection at (250, 50) mm

3.82 / 114 Let A be the origin, ∴ The resultant is $\langle 320, 120-360\rangle = \langle 320, -240\rangle$ N

= 400 N ↘ 36.9°. Let its line of action intersect with AC at x from A

∴ $400 * \sin 36.9 * x = 360 * .075 + 27 - 320 * .075$ ∴ $x = .125$ m = 125 mm

∴ The line of action of the resultant intersect AC at 125 mm right to A

& 〃 〃 〃 〃 〃 〃 〃 〃 CD at $75 \tan 36.9 = 56.3$ mm below C.

3.83 / 115 (a) Equivalent force at D $= (200+400+400+200)\left[\frac{\langle 10,-30\rangle}{\sqrt{10^2+30^2}} + \frac{\langle 10,30\rangle}{\sqrt{10^2+30^2}}\right] =$

$= 1200 \cdot \frac{\langle 20, 0\rangle}{\sqrt{1000}} = 759$ lb →

Equivalent couple at D $= \Sigma M_D = \frac{10}{\sqrt{1000}} \cdot \left[-200*20 - 400*(20+\frac{10}{3}) - 400*(20+\frac{20}{3}) - 200*30\right.$
$\left. - 200*30 - 400*(20+\frac{20}{3}) - 400*(20+\frac{10}{3}) - 200*20\right] + \frac{30}{\sqrt{1000}} \cdot \left[0 - 400*10 - 400*20 - 200*30\right.$
$\left. + 200*30 + 400*40 + 400*50 + 200*60\right] = \frac{10}{\sqrt{1000}} * (-60000) + \frac{30}{\sqrt{1000}} * (36000) = 15179$ lb·ft ↺

(b) Assume the resultant to be y ft above D. Hence, its moment about D is $-759 * y$. This must equal 15179 lb ft

∴ $y = -\frac{15179}{759}$ ft = -20 ft

∴ The resultant is acting 20 ft below D, with magnitude of 759 lb →.

Another way of solution:

By symmetry of loading on AB, ∴ loading on AB is 1200 lb ↓ mid normal on AB

〃 〃 〃 〃 BC, ∴ 〃 〃 BC is 1200 lb ↑ 〃 〃 BC

∴ The two loading are concurrent at $\frac{1}{2}\sqrt{10^2+30^2} / \sin(\tan^{-1}\frac{10}{30}) = 50$ ft vertically below B

& resultant $= 1200\left[\frac{\langle 10,-30\rangle + \langle 10,30\rangle}{\sqrt{10^2+30^2}}\right] = \frac{1200\langle 20,0\rangle}{\sqrt{1000}} = 759$ lb → 20 ft below D

& $M_D = 759 * 20 = 15179$ lb.ft ↺.

بسم الله الرحمن الرحيم

3.84 / 115 . Assume B the origin and transfer all forces to

$\therefore$ Resultant is $30\langle 1,0\rangle + 85\langle 0,-1\rangle + 50\langle 1,0\rangle + 25\langle 0,1\rangle = \langle 80,-60\rangle$ lb

$\therefore$ Resultant is $\langle 80,-60\rangle$ lb $= 100$ lb $\angle -37^\circ$

$-30(6-y) + 85x - 200 - 400 + 50y + 25(12-x) = 0$

$\therefore y(30+50) + x(85-25) = 180 + 600 - 300 = 480$

$\therefore 80y + 60x = 480$ or $4y + 3x = 24$ is the line of application.

(a) This intersect AB at $y = \frac{24}{4} = 6$ i.e $(0,6)$ in

(b) And intersect BC at $x = \frac{24}{3} = 8$ i.e $(8,0)$ in

(c) And intersect CD at $y = \frac{24-3*12}{4} = -3$ i.e $(12,-3)$ in

3.88 / 116 Equivalent Force is $100\langle 0,-1,0\rangle + \cancel{140}^{10} * \frac{\langle -12,4,-6\rangle}{\sqrt{12^2+4^2+6^2}} =$

$= \langle 0,-100,0\rangle + \langle -120,40,-60\rangle$

$= \langle -120,-60,-60\rangle$ lb.

Equivalent Couple is $\langle 6,0,0\rangle \times 100\langle 0,-1,0\rangle + \langle 12,0,0\rangle \times 10\langle -12,4,-6\rangle$

$= \langle 6,0,0\rangle \times [\langle 0,-100,0\rangle + 2*10*\langle -12,4,-6\rangle]$

$= \langle 6,0,0\rangle \times \langle -240,-20,-120\rangle = \langle 0,720,-120\rangle$ lb.ft

بسم الله الرحمن الرحيم

3.100 / 118 ⓐ Resultant at O is $P\left\langle \frac{a,0,a}{\sqrt{2}a}\right\rangle + P\frac{\langle a,0,-a\rangle}{\sqrt{2}a} = \left\langle \frac{2Pa}{\sqrt{2}a},0,0\right\rangle$

$= \sqrt{2}\frac{aP}{a}\langle 1,0,0\rangle = \sqrt{2}P\langle 1,0,0\rangle = \sqrt{2}P$ along OE

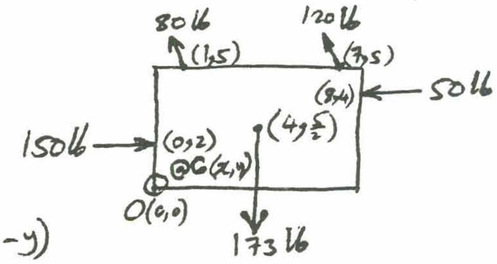
and a moment $= \langle OG\rangle \times P\frac{\langle 1,0,1\rangle}{\sqrt{2}} + \langle OA\rangle \times P\frac{\langle 1,0-1\rangle}{\sqrt{2}} =$
$\langle 0,a,0\rangle \times P\left\langle\frac{1,0,1}{\sqrt{2}}\right\rangle + \langle 0,0,a\rangle \times P\frac{\langle 1,0,-1\rangle}{\sqrt{2}} =$
$= \frac{Pa}{\sqrt{2}}\langle 1,0,-1\rangle + \frac{Pa}{\sqrt{2}}\langle 0,1,0\rangle = \frac{Pa}{\sqrt{2}}\langle 1,1,-1\rangle$

$= Pa.\sqrt{\frac{3}{2}}.\left\langle\frac{1,1,-1}{\sqrt{3}}\right\rangle = Pa\sqrt{\frac{3}{2}}$ along AF

ⓑ The wrench can only eliminate the y & z components of the resultant because it is directed along the x-axis.
To eliminate the y-component we move the resultant along OA by $\frac{Pa/\sqrt{2}}{\sqrt{2}P} = \frac{a}{2}$
To eliminate the z-component we move the resultant along OG by $\frac{-(-Pa/\sqrt{2})}{\sqrt{2}P} = \frac{a}{2}$

$\therefore$ Wrench-equivalent is $\sqrt{2}P\langle 1,0,0\rangle$ at $\left(\frac{a}{2},\frac{a}{2}\right)$ on yz plane plus moment of $Pa/\sqrt{2}$ about the x-axis.

3.109 / 119 Resultant is $173\langle 0,-1\rangle + 150\langle 1,0\rangle$
$-50\langle 1,0\rangle + 80\langle -\sin 30,\cos 30\rangle$
$+120\langle -\sin 30,\cos 30\rangle = \langle 0,0.21\rangle$ lb



& For moments: $-173(4-x) - 150(2-y) + 50(4-y)$
$+ 80\cos 30(1-x) + 80\sin 30(5-y) + 120\cos 30(7-x) + 120\sin 30(5-y) = 0$
$y(150-50-40-60) + x(173 - 80\cos 30 - 120\cos 30) = 173\times 4 + 300 - 200 - 80\cos 30$
$-400\sin 30 - 840\cos 30 - 600\sin 30 \Rightarrow 0y + x(-.21) = -505$ lb.in
$\therefore x = \frac{-505}{-.21} = 2405$ in $= 200$ ft $= 66.8$ Yds
$\therefore$ The resultant of all forces is 0.21 lb upwards acting at $x = 66.8$ Yds.
Or a couple of 505 lb.in ↻.

تابع

3.119 / 121

The resultant is $Q - Q + P = P = 400 \langle 1,0,0 \rangle$ N for all θ

For moments to be zero; $\therefore \langle -x, -y, -z \rangle \times 800 \langle 0,1,0 \rangle + \langle 200\cos\theta - x, -y, 200\sin\theta - z \rangle \times \langle 400, -800, 0 \rangle = 0$

$\therefore 800 \langle z, 0, -x \rangle + \langle 800(200\sin\theta - z), 400(200\sin\theta - z), \underset{+400y}{-800}(200\cos\theta - x) \rangle$

$= \langle 800(0z + 200\sin\theta), +400(200\sin\theta - z), \underset{+400y}{800}(-200\cos\theta + 0x) \rangle = 0$

It is seen that we can only zero two components of the moment and the third would be left unchanged no matter where the transfer is.

$\therefore \; 200\sin\theta - z = 0 \quad \therefore z = 200\sin\theta$, i.e leave z as it was.

$\therefore \; 800(-200\cos\theta) + 400y = 0 \Rightarrow y = 400\cos\theta$ mm

and we will be left with a couple about x of $800 * 200\sin\theta$ N.mm

$\therefore$ a) for $\theta = 30°$ the wrench would be 400 N parallel to x-axis with point of application at $(x, 346, 100)$ mm + 80 N.m couple about x.

& b) for $\theta = 90°$ the wrench would be 400N parallel to x-axis with point of application at $(x, 0, 200)$ mm + 160 N.m couple about x.

3.120 / 121.

$R = 5 \cdot \frac{\langle -4, -3, 0 \rangle}{\sqrt{4^2 + 3^2 + 0^2}} + 7 \cdot \frac{\langle +3, -6, -2 \rangle}{\sqrt{3^2 + 6^2 + 2^2}} = \langle -1, -9, -2 \rangle$ KN

$M = \langle 0, 3, 0 \rangle \times \langle -4, -3, 0 \rangle + \langle 0, 6, 0 \rangle \times \langle +3, -6, -2 \rangle =$

$= \langle 0, 3, 0 \rangle \times [\langle -4, -3, 0 \rangle + 2\langle +3, -6, -2 \rangle] = \langle 0, 3, 0 \rangle \times \langle 2, -15,$

$= \langle -12, 0, -6 \rangle$ KN.m

بسم الله الرحمن الرحيم

4.3 / 135 $T_{CD} * 250 * \sin 55 = 400 * 160 \cos 20$ $\therefore T_{CD} = 183.5$ N, the tension in CD.

$\therefore B_x = T_{CD} \sin 55 = 183.5 * \sin 55 = 150.4\ N \rightarrow$

& $B_y = 400 + T_{CD} \cos 55 = 400 + 183.5 * \cos 55 = 505.3\ N \uparrow$

$\therefore$ the reaction at B is $\langle 150.4, 505.3 \rangle\ N = 527.2\ N \angle 73.4°$.

4.6 / 135 $125 * 3 = 50 * 15 * \cos\theta$ $\therefore \cos\theta = 0.5$ $\therefore \theta = \pm 60°$.

4.13 / 136 (a) Analize forces on the direction of AD & normal to it. Assume upward compon

$\therefore 200 \cos(\sin^{-1}\frac{12}{15}) = R_D * \sin(\sin^{-1}\frac{12}{15})$

$\therefore 200 * .60 = R_D * .80$

$\therefore R_D = 150\ lb \uparrow$

$\sum M_B = 0$ $\therefore 200 * 12 * \frac{5}{15} = 5 * R_C + \frac{10}{15} * \sqrt{15^2 - 12^2} * R_D$

$\therefore R_C = -20\ lb \nearrow$ $\therefore R_C = 20\ lb \swarrow$ normal to AD

$\therefore 200 \sin(\sin^{-1}\frac{12}{15}) + R_D \cos(\sin^{-1}\frac{12}{15}) - 20 + R_B = 0$

$\therefore R_B = -230\ lb \nearrow$ $\therefore R_B = 230\ lb \swarrow$ normal to AD

(b) The roller reacting, for R_B is 2, and for R_C is 4

$\therefore$ Rollers 1 & 3 can be safely removed.

4.14 / 137 Assume all reactions to have upward components.

$\therefore R_D + (R_C + R_B) \cdot \frac{3}{5} = 200$ & $(R_C + R_B)\frac{4}{5} = 0$ & $5R_C + 10R_B = 200 * 9$

$$\begin{pmatrix} 1 & .6 & .6 \\ 0 & .8 & .8 \\ 0 & 5 & 10 \end{pmatrix}\begin{pmatrix} R_D \\ R_C \\ R_B \end{pmatrix} = \begin{pmatrix} 200 \\ 0 \\ 1800 \end{pmatrix}$$ $\therefore \Delta = 4,\ \Delta_{R_D} = 800,\ \Delta_{R_C} = -1440,\ \Delta_{R_B} = 1440$

(a) $\therefore R_B = 360\ lb$, $R_C = -360\ lb$ & $R_D = 200\ lb$

(b) R_C is exerted by roller 4 & R_B by 1 $\therefore$ Rollers 2 & 3 can be removed.

بسم الله الرحمن الرحيم

4.20 / 137 $P\cos\alpha + R_D = (R_B+R_c)\cos 30$ & $P\sin\alpha + R_B\sin 30 = R_c \sin 30$ &

moment about circle centre is $0 = R_D * R - P\cos\alpha * R$ where R is its radius

$$\therefore \begin{pmatrix} 1 & -\cos 30 & -\cos 30 \\ 0 & \sin 30 & -\sin 30 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} R_D \\ R_c \\ R_B \end{pmatrix} = \begin{pmatrix} -P\cos\alpha \\ P\sin\alpha \\ P\cos\alpha \end{pmatrix} \therefore \Delta = \sin 60,\ \Delta = P\cos\alpha\sin 60,$$

$\Delta R_c = P(\cos 30 \sin\alpha + 2\sin 30\cos\alpha)$, $\Delta R_B = P(2\cos\alpha\sin 30 - \sin\alpha\cos 30)$

(a) At $\alpha = 0$ $\therefore R_B = 1.155P$, $R_c = 1.155P$ & $R_D = P$

(b) At $\alpha = 30°$ $\therefore R_B = 0.5P$, $R_c = 1.5P$ & $R_D = 0.866P$

4.22 / 138 $R_A + R_B = 60 + P$ & $30*2 + 30*1 + R_A*4.5 = 3P$

$$\begin{pmatrix} 1 & 1 \\ 4.5 & 0 \end{pmatrix}\begin{pmatrix} R_A \\ R_B \end{pmatrix} = \begin{pmatrix} 60+P \\ 3P-90 \end{pmatrix} \therefore \Delta = -4.5,\ \Delta R_A = -1*(3P-90),\ \Delta R_B = 3P-90-4.5(60+P)$$

$\therefore R_A = \frac{2}{3}P - 20$ & $R_B = 60 + P - \frac{3P-90}{4.5} = \frac{P}{3} + 80$

$\therefore R_A \geq 0$ $\therefore P \geq \frac{20*3}{2} = 30$ $\therefore P \in [30, \infty)$ KN

& $R_A \not> 150$ $\therefore 150 \geq \frac{2}{3}P - 20$ $\therefore P \leq 255$ KN $\therefore P \in (-\infty, 255]$ KN

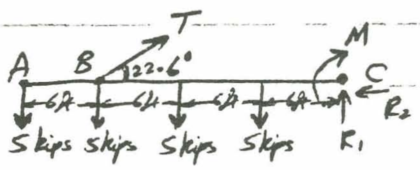
& $R_B \not> 150$ $\therefore 150 \geq \frac{P}{3} + 80$ $\therefore P \leq 210$ KN $\therefore P \in (-\infty, 210]$ KN

$\therefore$ The range of P is $[30, 210]$ KN.

4.29 / 140

$20 = R_1 + T\sin 22.6$ (1)

$R_2 = T\cos 22.6$ (2)



$M_B = 0$ $\therefore 5*6 - 5*6 - 5*12 + R_1*18 - M = 0$

or $M = 18R_1 - 60$ (3)

(a) $T = 39$ Kips $\therefore$ from (1) $20 = R_1 + 39*\sin 22.6$ $\therefore R_1 = 5$ Kips.

$\therefore$ from (2) $R_2 = 39*\cos 22.6 = 36$ Kips , $\therefore R_2 = 36$ Kips

from (3) $\therefore M = 18*5 - 60 = 30$ Kips.ft. , $\therefore M = 30$ Kip.ft.

(b) $M = 0$ $\therefore$ from (3) $R_1 = \frac{60}{18} = 3.3$ Kips , $\therefore R_1 = 3.3$ Kips.

$\therefore$ from (1) $20 = 3.3 + T\sin 22.6$ $\therefore T = 43.4$ Kips , $\therefore T = 43.4$ Kips

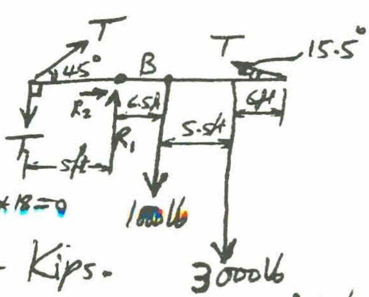
$\therefore$ from (2) $R_2 = 43.4\cos 22.6 = 40$ Kips. , $\therefore R_2 = 40$ Kips

---

4.32 / 141 $\therefore T\cos 45 + R_2 = T\cos 15.5$ or $R_2 = 0.2564T$ (1)

& $T\sin 45 - T + R_1 - 4000 + T\sin 15.5 = 0$

or $R_1 = 4000 + 0.0257T$ (2)



& $M_B = 0$ $(T - T\sin 45)*5 - 6.5*1000 - 12*3000 + T\sin 15.5*18 = 0$

or $T = 6.77$ Kips $\therefore R_1 = 4.17$ Kips & $R_2 = 1.74$ Kips.

بسم الله الرحمن الرحيم

4.38 / 142

ⓐ Cut the rope at B and take $\Sigma M_c = 0$

$\therefore P * K \cos\theta = W * K \cos\frac{\theta}{2}$ $\therefore P\left(2\cos^2\frac{\theta}{2} - 1\right) = W\cos\frac{\theta}{2}$ Let $\cos\frac{\theta}{2} = u$

$\therefore P(2u^2 - 1) = W \cdot u$ $\therefore 2Pu^2 - Wu - P = 0$

$\therefore u = \frac{W \pm \sqrt{W^2 - 4(2P)(-P)}}{4P} = \frac{W \pm \sqrt{W^2 + 8P^2}}{4P} = \cos\frac{\theta}{2}$ ($-$ rejected since $\theta \in [0, \frac{\pi}{2}]$)

$$\therefore \theta = 2 * \cos^{-1}\left[\frac{W + \sqrt{W^2 + 8P^2}}{4P}\right].$$

ⓑ When $P = 2W$ $\therefore \theta = 2 * \cos^{-1}\left(\frac{1 + \sqrt{1 + 8*4}}{4*2}\right) = 65.1^\circ$

4.46 / 146

Assume reaction at A to be R_A in the first quadrant with components R_x & R_y $\therefore 500 = R_y$ & $R_E = R_x$ & $M_A = 0$

$\therefore 500 * .15 = R_E * .4$ $\therefore R_E = 188\,N$ $\therefore R_x = 188\,N$ & $R_y = 500\,N$

$\therefore R_A = 534\,N \angle 69.4^\circ$

4.48 / 146

$\Sigma M_B = 0$ $\therefore A_x * \sqrt{17^2 - 4^2} = 40 * 2$ $\therefore A_x = 7.07$ lb $\rightarrow$.

$\Sigma F_x = 0$ $\therefore B_x = 7.07$ lb $\leftarrow$. & $\Sigma F_y = 0$ $\therefore B_y = 40$ lb $\uparrow$.

4.55 / 148

$T_{BA}\cos\beta = T_c$ (1) & $T_{BA}\sin\beta = W$ (2) & $M_B = 0$

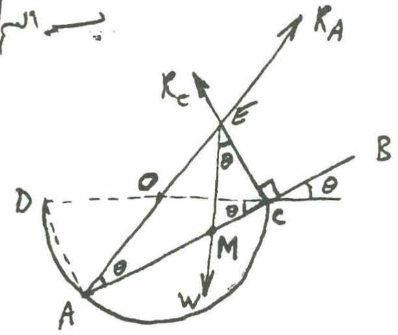
$\therefore W \cdot \frac{L\cos\theta}{2} = T_c * L\sin\theta$, or, $T_c = \frac{W\cot\theta}{2}$.

But from (1) & (2) $\therefore \tan\beta = \frac{W}{T_c} = 2\tan\theta$

$\therefore$ for $\theta = 30^\circ$, $\therefore \beta = 49.1^\circ$ $\therefore T_{BA} = 1.32 * W$.

$\frac{4.58}{149}$

The system is concurrent at E with R_A passing through O, $R_c \perp AB$ & W in mid AB, i.e. acting at M.



$\therefore \Delta$ DCA right at A $\therefore AC = 2R\cos\theta$ (1)

& Δ ECA right at C $\therefore EC = AC\tan\theta = 2R\cos\theta\tan\theta = 2R\sin\theta$

& Δ ECM right at C $\therefore MC = EC\cdot\tan\theta = 2R\sin\theta\cdot\tan\theta$ (2)

$\therefore$ From (1) & (2) $\therefore AM = 2R\cos\theta - 2R\sin\theta\tan\theta = \frac{2R}{\cos\theta}(\cos^2\theta - \sin^2\theta)$

$= \frac{2R}{\cos\theta}(2\cos^2\theta - 1)$. But $AM = \frac{3R}{2}$

$\therefore \frac{3\cancel{R}}{2} = \frac{2\cancel{R}}{\cos\theta}\cdot(2\cos^2\theta - 1)$ $\therefore 3\cos\theta = 4(2\cos^2\theta - 1)$

$\therefore 8\cos^2\theta - 3\cos\theta - 4 = 0$ $\therefore \cos\theta = \frac{3 \pm \sqrt{9 - 4(8)(-4)}}{16} = \frac{3 \pm \sqrt{137}}{16}$

$\because \theta \in [0, 90^\circ]$ $\therefore \cos\theta = \frac{3 + \sqrt{137}}{16} = 0.919$

$\therefore \theta = 23.2^\circ$

$\frac{4.62}{57}$

The three forces at B are concurrent with the resultant, R, given as

$R = 820\langle 0, -1, 0\rangle + T_{BD}\cdot\frac{\langle -6, 3, -6\rangle}{\sqrt{6^2+3^2+6^2}} + T_{BC}\cdot\frac{\langle -6, 2, 3\rangle}{\sqrt{6^2+2^2+3^2}} =$

$= \langle 0, -820, 0\rangle + T_{BD}\cdot\langle -.667, .333, -.667\rangle + T_{BC}\cdot\langle -.857, .286, .429\rangle$

$= \langle -.667\,T_{BD} - .857\,T_{BC},\ -820 + .333\,T_{BD} + .286\,T_{BC},\ -.667\,T_{BD} + .429\,T_{BC}\rangle$.

$\therefore$ The reaction at A, R_{AB} with R make a two force system on member AB. Hence they must be acting along AB and cancelling each other.

$\therefore R_{AB}\frac{\langle 6, 4.5, 0\rangle}{6^2+4.5^2+0^2} + R = R_{AB}\cdot\langle 0.8, 0.6, 0\rangle + R = 0$

$\therefore \begin{bmatrix} -.667 & -.857 & +.8 \\ .333 & .286 & +.6 \\ -.667 & .429 & 0 \end{bmatrix}\begin{bmatrix} T_{BD} \\ T_{BC} \\ R_{AB} \end{bmatrix} = \begin{bmatrix} 0 \\ 820 \\ 0 \end{bmatrix}$, $\therefore \Delta = +.782$, $\Delta_{T_{BD}} = +281$, $\Delta_{T_{BC}} = +438$, $\Delta_{R_{AB}} = 703$

$\therefore T_{BD} = 360$ lb, $T_{BC} = 560$ lb, $R_{AB} = 900$ lb.

بسم الله الرحمن الرحيم

$\boxed{\frac{4.66}{158}}$ $T_{DB} * \frac{\langle 1.5, -3, -1 \rangle}{\sqrt{1.5^2+3^2+1^2}} + T_{DC} * \frac{\langle 0, -3, 1 \rangle}{\sqrt{3^2+1^2}} + T_{DA} * \frac{\langle -1.5, -3, -1 \rangle}{\sqrt{1.5^2+3^2+1^2}} + 600 * 9.81 * \langle 0, 1, 0 \rangle = 0$

$\therefore T_{DA} * \langle -.4286, -.8571, -.2857 \rangle + T_{DB} * \langle .4286, -.8571, -.2857 \rangle +$
$T_{DC} * \langle 0, -.9487, .3162 \rangle = \langle 0, -5886, 0 \rangle$

$\therefore \begin{bmatrix} -.4286 & .4286 & 0 \\ -.8571 & -.8571 & -.9487 \\ -.2857 & -.2857 & .3162 \end{bmatrix} \begin{bmatrix} T_{DA} \\ T_{DB} \\ T_{DC} \end{bmatrix} = \begin{bmatrix} 0 \\ -5886 \\ 0 \end{bmatrix} \therefore \Delta = .4647, \Delta_{T_{DA}} = 797.7, \Delta_{T_{DB}} = 797.7$
$\& \ \Delta_{T_{DC}} = 1441.5$

$\therefore T_{DA} = 1717 \text{ N}, \quad T_{DB} = 1717 \text{ N} \quad \& \ T_{DC} = 3102 \text{ N}$

$\boxed{\frac{4.74}{159}}$ $M_x = 0 \quad \therefore P * .2 = 800 * .1 \quad \therefore P = 400 \text{ N} \langle 0, -\sin 60, -\cos 60 \rangle$

$M_y = 0$ @ $A \therefore -B_z * .5 + 400 \cos 60 * .8 = 0 \quad \therefore B_z = 320 \text{ N}$

$M_z = 0$ @ $A \therefore -800 * .25 + B_y * .5 - 400 * \sin 60 * .8 = 0 \quad \therefore B_y = 954 \text{ N}$

$F_y = 0 \quad \therefore A_y - 800 + B_y - 400 \sin 60 = 0 \quad \therefore A_y = 192 \text{ N}$

$F_z = 0 \quad \therefore A_z + B_z - 400 \cos 60 = 0 \quad \therefore A_z = -120 \text{ N}$

$\therefore$ Reaction at A is $\langle 0, 192, -120 \rangle$ N & at B is $\langle 0, 954, 320 \rangle$ N

$\boxed{\frac{4.80}{161}}$ $M_y = 0 \quad \therefore P * (1.2 - .15) = 15 * 9.81 * 1.2 \cos \frac{90}{2} \quad \therefore P = 119 \text{N} \langle 0, 0, -1 \rangle$

$M_x = 0$ @ $A \therefore 15 * 9.81 * \cos 45 * .25 + 119 * .85 - B_z * 1.5 = 0 \quad \therefore B_z = 84.7 \text{ N}$

$M_z = 0$ @ $A \therefore 15 * 9.81 * \sin 45 * .25 - 20 * 9.81 * .6 + B_x * 1.5 = 0 \quad \therefore B_x = 61.1 \text{ N}$

$F_y = 0 \quad \therefore -20 * 9.81 + B_y = 0 \quad \therefore B_y = 196 \text{ N}$

$F_x = 0 \quad \therefore B_x + A_x - 15 * 9.81 * \sin 45 = 0 \quad \therefore A_x = 43.0 \text{ N}$

$F_z = 0 \quad \therefore B_z - 119 + A_z + 15 * 9.81 * \cos 45 = 0 \quad \therefore A_z = -69.8 \text{ N}$

$\therefore$ Reaction at A is $\langle 43.0, 0, -69.8 \rangle$ N and at B is $\langle 61.1, 196, 84.7 \rangle$ N.

4.88 / 162

Take ΣM about $BC = 0$ $\therefore$ $A_y = 0$

Take ΣM about $AE = 0$ $\therefore$ $T_{CF} = 0$

Take ΣM about $E = 0$

$\therefore \Sigma M_{E_x} = 0$ $\therefore 5 * .75 + A_z * 1.25 = 0$ $\therefore A_z = -3$ KN

& $\Sigma M_{E_z} = 0$ $\therefore 5 * 3 = A_x * 1.25$ $\therefore A_x = 12$ KN

Take $\Sigma M_{C_x} = 0$ $\therefore 5 * .75 = T_{BE} * \frac{1.25}{\sqrt{1.25^2 + 3^2}} * 1.5$ $\therefore T_{BE} = 6.5$ KN

& Take $\Sigma F_y = 0$ $\therefore 5 = T_{BE} * \frac{1.25}{\sqrt{1.25^2+3^2}} + T_{CE} * \frac{1.25}{\sqrt{1.25^2+1.5^2+3^2}}$ $\therefore T_{CE} = 7.16$ KN

$\therefore$ Reaction at A is $\langle 12, 0, -3 \rangle$ KN and $T_{BE} = 6.5$KN, $T_{CE} = 7.16$KN

4.92 / 162

Cut DE and take $\Sigma M_{AC} = 0$

$\therefore \left[\langle AD \rangle \times T_{DE} * \frac{\langle -6, 3, -6 \rangle}{\sqrt{6^2+3^2+6^2}} + \langle AG \rangle \times 210 \langle 0, -1, 0 \rangle\right] \cdot \frac{\langle 8, 0, -6 \rangle}{\sqrt{8^2+0^2+6^2}} = 0$

$\therefore \left[\langle 6, 0, 0 \rangle \times T_{DE} * \frac{\langle -2, 1, -2 \rangle}{3} + \langle 8, 0, -3 \rangle \times 210 \langle 0, -1, 0 \rangle\right] \cdot \langle 4, 0, -3 \rangle = 0$

$\therefore \left[\frac{T_{DE}}{3} \cdot \langle 0, 12, 6 \rangle + 210 \cdot \langle -3, 0, -8 \rangle\right] \cdot \langle 4, 0, -3 \rangle = 0$

$\therefore \langle -630, 4T_{DE}, 2T_{DE} - 1680 \rangle \cdot \langle 4, 0, -3 \rangle = 0$

$\therefore -630 * 4 + 0 - 3(2T_{DE} - 1680) = 0$

$\therefore T_{DE} = \frac{-630 * 4 + 3 * 1680}{6} = 420$ lb.

4.96 / 163

The reaction at D is $\langle D_x, 0, 0 \rangle$.

Hence, taking the moments AB :

$\therefore (\langle AD \rangle \times \langle D_x, 0, 0 \rangle + \langle AE \rangle \times 400 \langle 0, -1, 0 \rangle) \cdot \langle AB \rangle = 0$

$\therefore (\langle -.125, .25, -.25 \rangle \times \langle D_x, 0, 0 \rangle + \langle -.025, .125, -.175 \rangle \times \langle 0, -400, 0 \rangle) \cdot \langle .15, 0, -.2 \rangle = 0$

$\therefore (\langle 0, -.25D_x, -.25D_x \rangle + \langle -70, 0, 10 \rangle) \cdot \langle .15, 0, -.2 \rangle = 0$

$\therefore \langle -70, -.25D_x, 10 - .25D_x \rangle \cdot \langle .15, 0, -.2 \rangle = 0$

$\therefore -70 * .15 + 0 - .2 * (10 - .25D_x) = 0$ $\therefore D_x = 250$ N

$\therefore$ The reaction at D is $250\text{N} \langle 1, 0, 0 \rangle$.

C1

4.98 / 163

$\Sigma M_B = 0 \quad \therefore A_x * 10 = 240 * 6 \quad \therefore A_x = 144 \text{ lb} \rightarrow$

$\Sigma F_x = 0 \quad \therefore B\cos 30 + A_x = 0 \quad \therefore B = -\frac{144}{\cos 30} = -166.3 \text{ lb} \nearrow = 166.3 \text{ lb} \swarrow$

$\Sigma F_y = 0 \quad \therefore B\sin 30 + A_y = 0 \quad \therefore A_y = -B\sin 30 = 166 \sin 30 = 83.14 \text{ lb} \uparrow$

$\therefore$ The reaction at A is $\langle 144, 83.1\rangle$ lb $= 166.3$ lb $\angle 30^\circ$

& The reaction at B is 166.3 lb $\swarrow 30^\circ = \langle -144, -83.1\rangle$ lb.

4.104 / 165

$\Sigma M_A = 0 \quad \therefore T * 6 = 500 * \sqrt{4^2 - 2^2} \quad \therefore T = 289 \text{ lb}, \therefore A_x = 289 \text{ lb} \rightarrow, A_y$

4.106 / 165

$\Sigma M_{A_y} = 0 \quad \therefore T_{EF} * \frac{.6}{\sqrt{.6^2 + .9^2 + 1.8^2}} * 1.8 = T_{BG} * \frac{2.4}{\sqrt{2.4^2 + 1.2^2 + 2.4^2}} * 2.4$

& $\Sigma M_{A_z} = 0 \quad \therefore T_{EF} * \frac{.9}{\sqrt{.6^2 + .9^2 + 1.8^2}} * 1.8 + T_{BG} * \frac{1.2}{\sqrt{2.4^2 + 1.2^2 + 2.4^2}} * 2.4 = 120 * 9.81$

(1) in (2) $\therefore 3.111 T_{BG} * .7714 + T_{BG} * .8 = 1413 \quad \therefore T_{BG} = 441.5 \text{ N}$

$\therefore T_{EF} = 3.111 * T_{BG} = 1373 \text{ N}$

$\therefore \Sigma F = 0 \quad \therefore \langle A_x, A_y, A_z\rangle + 1373 * \frac{\langle -1.8, .9, .6\rangle}{\sqrt{1.8^2 + .9^2 + .6^2}} + 441.5 * \frac{\langle -2.4, 1.2, -2.4\rangle}{\sqrt{2.4^2 + 1.2^2 + 2.4^2}} +$

$120 * 9.81 * \langle 0, -1, 0\rangle = 0 \quad \therefore A_x = 1471 \text{ N}, A_y = 441.6 \text{ N}, A_z = -98.0 \text{ N}$

Hence, tension in BG is 441.5 N & in EF is 1373 N

& Reaction at A is $\langle 1471, 441.6, -98\rangle$ N.

بسم الله الرحمن الرحيم

S.1 / 178

$A = 100*120 + 60*120/2 = 1.56*10^4\ mm^2$

$\therefore M_x = 100*120*(60+50) + 60*\frac{120}{2}*(\frac{2}{3}*60) = 1.464*10^6\ mm^3$ $\quad \therefore \bar{y} = 93.85\ mm$

& $M_y = 100*120*(\frac{1}{2}*120) + 60*\frac{120}{2}*(\frac{1}{3}*120) = 8.64*10^5\ mm^3$ $\quad \therefore \bar{x} = 55.38\ mm$

$\therefore$ The centroid is at $(55.38, 93.85)$ mm

S.4 / 178

$A = \frac{1}{2}*4 + 2.5*\frac{1}{2} = 3.25\ in^2$, $M_x = \frac{1}{2}*4*2 + 2.5*\frac{1}{2}*(4-\frac{1}{4}) = 8.6875\ in^3$

$M_y = \frac{1}{2}*4*(3-\frac{1}{4}) + 2.5*\frac{1}{2}*\frac{2.5}{2} = 7.0625\ in^3$ $\therefore \bar{X} = \frac{7.0625}{3.25} = 2.173$ in &

$\bar{y} = \frac{8.6875}{3.25} = 2.673$ in $\quad \therefore$ The centroid is at $(2.173, 2.673)$ in..

S.8 / 178

$A = \pi*100^2 - \frac{\pi}{2}*75^2 - 2\left|\int_{-100}^{-50} x\,dy\right| = 2.258*10^4 - 2\int_{-\pi/2}^{-\sin^{-1}.5} 100\cos\theta\, d\,100\sin\theta = 2.258\times10^4 - 100^2(\theta + \frac{\sin 2\theta}{2})\Big|_{-\pi/2}^{-\sin^{-1}.5} = 1.64*10^4\ mm^2$

$M_x = 2*\left[\left|\int_{-\sin^{-1}.5}^{\pi/2}(\frac{100^2}{2}d\theta)*\frac{2}{3}*100*\sin\theta\right| - \left|\frac{50*100.\sin(\cos^{-1}.5)}{2}*\frac{2}{3}*50\right| - \left|\int_0^{\pi/2}(\frac{75^2}{2}d\theta)*\frac{2}{3}*75*\sin\theta\right|\right] =$

$= 2\left[\left|\frac{100^3}{3}.(-\cos\theta\Big|_{-\sin^{-1}.5}^{\pi/2}\right| - \left|7.217*10^4\right| - \left|\frac{75^3}{3}(-\cos\theta\Big|_0^{\pi/2}\right|\right] = 1.518*10^5\ mm^3$, $M_y = 0$ by symmetry.

$\therefore \bar{y} = \frac{1.518*10^5}{1.64*10^4} = 9.26$ mm $\quad \therefore$ The centroid is at $(0, 9.26)$ mm

---

S.10 / 178

$\therefore$ Equation of parabola is $y - 2 = c(x-6)^2$ and $5 - 2 = c(12-6)^2$ $\therefore c = \frac{3}{36} = \frac{1}{12}$

$\therefore$ The parabola is $y = 2 + \frac{(x-6)^2}{12}$ $\quad \therefore A = \int_0^{12} y\,dx = 2x + \frac{(x-6)^3}{12*3}\Big|_0^{12} = 36\ in^2$

$M_x = \int_0^{12}(y\,dx).\frac{y}{2} = \int_0^{12}\frac{y^2}{2}dx = \frac{1}{2}\int_0^{12}[4 + \frac{(x-6)^4}{12^2} + \frac{(x-6)^2}{3}]dx = \frac{1}{2}\left[4x + \frac{(x-6)^5}{12^2*5} + \frac{(x-6)^3}{3*3}\right]_0^{12} = 58.8\ in^3$

$\therefore \bar{y} = \frac{58.8}{36} = 1.633$ in & by symmetry $\bar{x} = 6$ in, $\therefore$ The centroid is at $(6, 1.633)$ in

---

S.24 / 179

Assume C to be the origin $\therefore \bar{x} = 0$, $\therefore M_y = 0$, $\therefore 10*5 + L*(10 - \frac{L}{2}\cos 30) = 0$

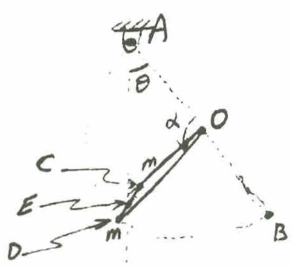
$\therefore 50 + 10L - 0.433L^2 = 0$ $\quad \therefore L = \frac{-10 \pm \sqrt{10^2 - 4(-.433)(50)}}{2(-.433)} = -4.2$ in or 27.3 in $\quad \therefore L = 27.3$ in

---

S.25 / 180

The centroid, C, of a semicircular rod is on the radius $\perp$ to the diameter joining both ends and off the center by : $\frac{\int_0^{\pi}(r\,d\theta)\,r\sin\theta}{\pi r} = \frac{r^2}{\pi r}.(-\cos\theta)_0^{\pi} = \frac{2r}{\pi}$.

$\therefore$ The problem simplifies to the concentrated masses shown with:



$OC = \frac{2r}{\pi}$, $OD = r$, $OA = r$, E (centroid of the problem) is in the middle of CD and vertically below A.

$\therefore \Delta OCD \quad \therefore \overline{CD}^2 = (\frac{2r}{\pi})^2 + r^2 - 2.(\frac{2r}{\pi}).r.\cos(\alpha - \frac{\pi}{2})$ $\quad$ (1)

& $\widehat{OCD}(=\beta)$ is given by : $\frac{r}{\sin\beta} = \frac{\overline{CD}}{\sin(\alpha - \frac{\pi}{2})} \Rightarrow \overline{CD}\sin\beta = -r\cos\alpha$ $\quad$ (2)

$\therefore \Delta OCE \quad \therefore \overline{OE}^2 = (\frac{2r}{\pi})^2 + (\frac{\overline{CD}}{2})^2 - 2(\frac{2r}{\pi}).(\frac{\overline{CD}}{2}).\cos\beta$ $\quad$ (3)

& $\widehat{COE}(=\gamma)$ is given by $\frac{\overline{CD}/2}{\sin\gamma} = \frac{\overline{OE}}{\sin\beta} \Rightarrow \overline{OE}\sin\gamma = \frac{\overline{CD}.\sin\beta}{2} = (\text{from }(2))\ \frac{-r\cos\alpha}{2}$ $\quad$ (4)

$\therefore \Delta AOE \quad \therefore \dfrac{\overline{OE}}{\sin\theta} = \dfrac{r}{\sin(\pi - \theta - (\frac{\pi}{2}+\gamma))} \quad \therefore r\sin\theta = \overline{OE}\sin(\frac{\pi}{2}-\theta-\gamma) = \overline{OE}\cos(\theta+\gamma)$

$= \overline{OE}(\cos\theta\cos\gamma - \sin\theta\sin\gamma) = \overline{OE}.\cos\theta.\cos\gamma - (\text{from }(4))\sin\theta\left(\frac{-r\cos\alpha}{2}\right)$

$\therefore r.\sin\theta\left(1-\frac{\cos\alpha}{2}\right) = \overline{OE}.\cos\theta.\cos\gamma$, or $\therefore r\tan\theta.\left(1-\frac{\cos\alpha}{2}\right) = \overline{OE}\cos\gamma$

$\therefore r^2\tan^2\theta\left(1-\frac{\cos\alpha}{2}\right)^2 = \overline{OE}^2.\cos^2\gamma = \overline{OE}^2.(1-\sin^2\gamma) = \overline{OE}^2 - (\text{from }(4))\ \frac{r^2\cos^2\alpha}{4}$

$\therefore r^2\tan^2\theta\left(1-\frac{\cos\alpha}{2}\right)^2 + r^2\frac{\cos^2\alpha}{4} = (\text{from }(3))\ \left(\frac{2r}{\pi}\right)^2 + \left(\frac{\overline{CD}}{2}\right)^2 - \frac{2r}{\pi}.\overline{CD}.\cos\beta$

$\therefore r^2\left[\tan^2\theta + \frac{\cos^2\alpha}{4}(\tan^2\theta+1) - \cos\alpha.\tan^2\theta - \frac{4}{\pi^2}\right] = (\text{from }(1))\ \frac{1}{4}\left[\left(\frac{2r}{\pi}\right)^2 + r^2 - \frac{4r^2}{\pi}\sin\alpha\right] - \frac{2r}{\pi}.\overline{CD}.\cos\beta$

$\therefore r^2.\left[\tan^2\theta + \frac{\cos^2\alpha}{4\cos^2\theta} - \cos\alpha\tan^2\theta - \frac{4}{\pi^2} - \frac{1}{\pi^2} - \frac{1}{4} + \frac{1}{\pi}.\sin\alpha\right] = -\frac{2r}{\pi}.\overline{CD}.\cos\beta$

$\therefore r^4.\left[\tan^2\theta + \frac{\cos^2\alpha}{4\cos^2\theta} - \cos\alpha.\tan^2\theta - \left(\frac{5}{\pi^2}+\frac{1}{4}\right) + \frac{\sin\alpha}{\pi}\right]^2 = \frac{4r^2}{\pi^2}.\overline{CD}^2.\cos^2\beta = \frac{4r^2}{\pi^2}.\overline{CD}^2.(1-\sin^2\beta)$

$= \frac{4r^2}{\pi^2}.\left[\overline{CD}^2 - \overline{CD}^2.\sin^2\beta\right] = \frac{4r^2}{\pi^2}\left[(\text{from }(1))\ \frac{4r^2}{\pi^2} + r^2 - \frac{4r^2}{\pi}\sin\alpha - (\text{from }(2))\ r^2\cos^2\alpha\right] =$

$= \frac{4r^4}{\pi^2}.\left(\frac{4}{\pi^2}+1-\frac{4\sin\alpha}{\pi} - \cos^2\alpha\right) = \frac{4r^4}{\pi^2}\left(\frac{4}{\pi^2} - \frac{4\sin\alpha}{\pi} + \sin^2\alpha\right) = \frac{4r^4}{\pi^2}\left(\frac{2}{\pi} - \sin\alpha\right)^2$

$\therefore \tan^2\theta + \frac{\cos^2\alpha}{4\cos^2\theta} - \cos\alpha.\tan^2\theta - \left(\frac{5}{\pi^2}+\frac{1}{4}\right) + \frac{\sin\alpha}{\pi} = -\frac{2}{\pi}\left(\frac{2}{\pi} - \sin\alpha\right)$

$\therefore \sin^2\theta + \frac{\cos^2\alpha}{4} - \cos\alpha\sin^2\theta - \left(\frac{1}{\pi^2}+\frac{1}{4}\right)\cos^2\theta + \frac{\sin\alpha\cos^2\theta}{\pi} = \frac{2}{\pi}\sin\alpha\cos^2\theta$

$\therefore \sin^2\theta.(1-\cos\alpha) + \cos^2\theta.\left(\frac{\sin\alpha}{\pi} - \frac{1}{\pi^2} - \frac{1}{4} - \frac{2\sin\alpha}{\pi}\right) + \frac{\cos^2\alpha}{4} = 0$

$\therefore \sin^2\theta.\left(1-\cos\alpha + \frac{1}{\pi^2} + \frac{1}{4} + \frac{\sin\alpha}{\pi}\right) + \frac{\cos^2\alpha}{4} - \frac{1}{\pi^2} - \frac{1}{4} - \frac{\sin\alpha}{\pi} = 0$

$\therefore$ (a) When $\alpha = 180^\circ$ $\quad \therefore \sin^2\theta.\left(1+1+\frac{1}{\pi^2}+\frac{1}{4}+0\right) + \frac{1}{4} - \frac{1}{\pi^2} - \frac{1}{4} - 0 = 0 \quad \therefore \theta = 12.0^\circ$

& (b) When $\alpha = 90^\circ$ $\quad \therefore \sin^2\theta.\left(1-0+\frac{1}{\pi^2}+\frac{1}{4}+\frac{1}{\pi}\right) + 0 - \frac{1}{\pi^2} - \frac{1}{4} - \frac{1}{\pi} = 0 \quad \therefore \theta = 39.3^\circ$

5.31 / 181

Area = (by counting the no of squares * area of a square) $\simeq 27 * (20)^2 = 10800\ mm^2$

$\therefore M_y = (11*8.5 + 7.5*8.3 + 5.5*8.2 + 3.0*8.7)*(20)^3 = 1815600\ mm^3$

$\therefore \bar{X} = 168.1\ mm = 8.41$ divisions

$\therefore$ The x-coordinates of the area is $168.1\ mm$

5.33 / 188

$A = bh/2$

$\therefore M_x = \int_0^h [(b-x)dy]\,y = b\frac{y^2}{2}\Big|_0^h - \int_0^h\left(\frac{b}{h}y\right)y\,dy = \frac{bh^2}{2} - \frac{b}{h}.\frac{y^3}{3}\Big|_0^h = \frac{bh^2}{2} - \frac{bh^3}{3h} = bh^2/6 \quad \therefore \bar{y} = \frac{bh^2/6}{bh/2}$

& $M_y = \int_0^b (y\,dx)x = \int_0^b\left(\frac{h}{b}x\,dx\right)x = \frac{h}{b}.\frac{x^3}{3}\Big|_0^b = \frac{h}{b}.\frac{b^3-0}{3} = \frac{hb^2}{3} \quad \therefore \bar{x} = \frac{hb^2/3}{hb/2} = \frac{2}{3}b.$

$\therefore$ The centroid is at $\left(\frac{2b}{3}, \frac{h}{3}\right)$.

بسم الله الرحمن الرحيم

5.38 / 188

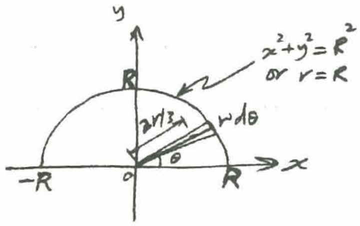
$dA = \frac{r\cdot r\,d\theta}{2} = \frac{r^2}{2}d\theta \therefore M_x = \int_0^\pi \left(\frac{r^2}{2}d\theta\right)\frac{2r}{3}\sin\theta$

$= \int_0^\pi \frac{r^3}{3}\sin\theta\,d\theta = \frac{R^3}{3}\int_0^\pi \sin\theta\,d\theta = \frac{R^3}{3}(-\cos\theta)_0^\pi = \frac{2R^3}{3}$

But $A = \int_0^\pi \frac{R^2}{2}d\theta = \frac{R^2}{2}\theta\Big|_0^\pi = \frac{\pi R^2}{2}$

$\therefore \bar y = \frac{M_x}{A} = \frac{2R^3/3}{\pi R^2/2} = \frac{4R}{3\pi}$. And by symmetry $\bar x = 0$

$\therefore$ The centroid is at $\left(0, \frac{4R}{3\pi}\right)$.



5.44 / 188

$A = \frac12\int_{-\pi/4}^{\pi/4} r^2\,d\theta = \int_0^{\pi/4}(R\cos 2\theta)^2\,d\theta = R^2\int_0^{\pi/4}\frac{1+\cos4\theta}{2}\,d\theta = \frac{R^2}{2}\left[\theta+\frac{\sin4\theta}{4}\right]_0^{\pi/4} = \frac{\pi R^2}{8}$

$M_y = \frac12\int_{-\pi/4}^{\pi/4}(r^2 d\theta)\left(\frac23 r\cdot\cos\theta\right) = \int_0^{\pi/4}\frac23 r^3\cos\theta\,d\theta = \frac23\int_0^{\pi/4}(R\cos2\theta)^3\,d\sin\theta =$

$= \frac{2R^3}{3}\int_0^{\pi/4}(1-2\sin^2\theta)^3\,d\sin\theta = \frac{2R^3}{3}\cdot\left[\sin\theta - 6\cdot\frac{\sin^3\theta}{3} + 12\cdot\frac{\sin^5\theta}{5} - 8\cdot\frac{\sin^7\theta}{7}\right]_0^{\pi/4} =$

$= \frac{2R^3}{3}\cdot\left[\cancel{\frac{1}{\sqrt2}} - \cancel{2\cdot\frac{1}{2\sqrt2}} + \frac{\cancel{12}^{\,3}}{5}\cdot\frac{1}{\cancel{2\cdot2}\cdot\sqrt2} - \frac{\cancel{8}}{7}\cdot\frac{1}{\cancel{2\cdot2\cdot2}\sqrt2}\right] = \frac{2R^3}{3}\cdot\left(\frac{3}{5\sqrt2}-\frac{1}{7\sqrt2}\right) =$

$= \frac{2R^3}{3\sqrt2}\left(\frac{21-5}{35}\right) = \frac{\sqrt2 R^3}{3}\cdot\frac{16}{35} = \frac{16\sqrt2}{105}R^3 = A\bar x = \frac{\pi R^2}{8}\bar x$

$\therefore \bar x = \frac{16\sqrt2}{105}R^3\cdot\frac{8}{\pi R^2} = \frac{128\sqrt2}{105\pi}\cdot R = 0.5488\,R$

$\bar y$ (by symmetry) $= 0$.

$\therefore$ The centroid is at $(0.5488, 0)R$.

5.48 / 189

$\therefore A = \int_a^{3a} y\,dx = \int_a^{3a}\frac{a^2}{x}\,dx = a^2\ln x\Big|_a^{3a} = a^2\ln\frac{3a}{a} = a^2\ln3$.

$\therefore M_x = \int_a^{3a}(y\,dx)\cdot\frac y2 = \int_a^{3a}\frac{y^2}{2}\,dx = \frac12\int_a^{3a}\frac{a^4}{x^2}\,dx = \frac{a^4}{2}\left(\frac{x^{-1}}{-1}\right|_a^{3a} = -\frac{a^4}{2}\cdot\left(\frac{1}{3a}-\frac1a\right) = -\frac{a^4}{\cancel2}\cdot\frac{-\cancel2}{3a} = \frac{a^3}{3}$

$\therefore M_y = \int_a^{3a}(y\,dx)\,x = \int_a^{3a}\frac{a^2}{x}\cdot x\cdot dx = a^2x\Big|_a^{3a} = a^2(3a-a) = 2a^2\cdot a = 2a^3$

$\therefore \bar X = \frac{M_y}{A} = \frac{2a^3}{a^2\ln3} = \frac{2a}{\ln3}$ & $\bar y = \frac{M_x}{A} = \frac{a^3/3}{a^2\ln3} = \frac{a}{3\ln3}$ $\therefore$ Centroid is at $\left(\frac{2a}{\ln3}, \frac{a}{3\ln3}\right)$.

5.50 / 189

Using the above answers, and using the theorem of Pappus,

$\therefore$ (a) $V_x = 1.56*10^4*2\pi*(100+60-93.85) = 6.484*10^6$ mm³.

& (b) $V_y = 1.56*10^4*2\pi*(55.38) = 5.428*10^6$ mm³.

Co

بسم الله الرحمن الرحيم

5.51 / 189

$$A_s = \int_0^{2\pi} (r\,d\theta)\cdot\pi\cdot(R + r\cos\theta) = \pi r \int_0^{2\pi} (R + r\cos\theta)\,d\theta = \pi r\Big[R\theta + r\sin\theta\Big]_0^{2\pi} = \pi r\cdot R\cdot 2\pi = 2\pi^2 rR$$

$$V = 2*\int_0^{\pi} (r\sin\theta\cdot d\,r\cos\theta) * \pi(R + r\cos\theta) = 2\pi r^2 \int_0^{\pi} \sin\theta(-\sin\theta)\,d\theta\cdot(R + r\cos\theta) = -2\pi r^2 \int_0^{\pi} \sin^2\theta(R + r\cos\theta)\,d\theta$$

$$= -2\pi r^2\cdot\left[R\int_0^{\pi}\frac{1-\cos 2\theta}{2}\,d\theta + r\,\frac{\sin^3\theta}{3}\Big|_0^{\pi}\right] = \left|-2\pi r^2\left[R\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)_0^{\pi} + 0\right]\right| = 2\pi r^2 R\,\frac{\pi}{2} = \pi^2 r^2 R$$

OK by P-G theorem $\therefore A_s = 2\pi r\cdot\pi R = 2\pi^2 rR$ & $V = \pi r^2 * \pi R = \pi^2 r^2 R$.

5.57 / 190

Use P-G theorem $\therefore$ Area $= \frac{1}{2}\left[60*63 - 30*45\right] = 1215\ mm^2$

& $M_{AA'} = \frac{1}{2}*60*63*\left(15 + \frac{63}{3}\right) - \frac{1}{2}*30*45*\left(15 + \frac{45}{3}\right) = 47790\ mm^3$, $\therefore \bar{x} = 39\frac{1}{3}\ mm$

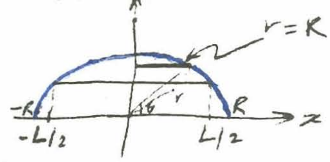
$\therefore V = 1215 *$ distance traveled by centroid $= 1215 * 2\pi * 39\frac{1}{3} = 3.003 * 10^5\ mm^3$

5.64 / 191

Using P-G theorem, $\therefore$ Volume, V, of remaining steel $= 2\pi *$ moment of area about the axis of revolution $= 2\pi * 2\int (r\cos\theta\,d\,r\sin\theta)\cdot r\sin\theta = 4\pi\int_{\cos^{-1}L/2R}^{\pi/2} R^3\cos^2\theta\sin\theta\,d\theta$

$= -4\pi R^3\int\cos^2\theta\,d\cos\theta = -4\pi R^3\,\frac{\cos^3\theta}{3}\Big|_{\cos^{-1}L/2R}^{\pi/2} =$

$= +4\pi\,\frac{R^3}{3}\left[\cos^3\left(\cos^{-1}\frac{L}{2R}\right) - 0\right] = 4\pi\frac{R^3}{3}\cdot\left(\frac{L}{2R}\right)^3 = \frac{\pi L^3}{6}.$

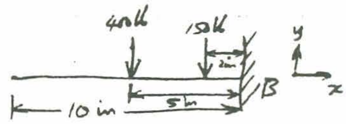


بسم الله الرحمن الرحيم

5.68 / 196

Assume A to be the origin. Hence loading becomes two concentrated loads, one due to rectangle & another due to triangle as shown.

∴ Reaction at B is $B_y = 550$ lb ↑ & $M_B = 300 + 2000 = 2300$ lb in ↺



5.76 / 197

A to be the origin ∴ Area of cross-section of dam $= A = \int_0^{24}(10-x)dy + 7*24 + \frac{9*24}{2}$

But $(x-10) = \frac{-10}{24^2}(y-24)^2$ ∴ $\int_0^{24}(10-x)dy = \int_0^{24}\frac{10}{24^2}(y-24)^2 dy = \frac{10}{24^2}\cdot\frac{(y-24)^3}{3}\Big|_0^{24} = \frac{10}{24^2*3}\cdot(0-(-24)^3) = 80$

∴ $A = 80 + 168 + 108 = 356\ ft^2$ ∴ weight of dam $= 356 * 1 * 150 = 53400$ lb

To find $\bar{x}$, ∴ $M_y = \left|\int_0^{24}(10-x)dy * \left(\frac{10+x}{2}\right)\right| + 7*24*(10+\frac{7}{2}) + \frac{9 \times 24}{2}*(10+7+\frac{9}{3}) =$

$= \left|\int_0^{24} 10\left(\frac{y-24}{24}\right)^2 * \frac{20+(x-10)}{2}\cdot dy\right| + 4428 = \frac{5}{24^2}\cdot\left|\int_0^{24}(y-24)^2\cdot\left(20 - \frac{10}{24^2}(y-24)^2\right)dy\right| + 4428 =$

$= \frac{5}{24^2}\left|20.\frac{(y-24)^3}{3} - \frac{10}{24^2}\cdot\frac{(y-24)^5}{5}\right|_0^{24} + 4428 = \frac{5}{24^2}\cdot\frac{14}{3}\cdot 24^3 + 4428 = 4988\ ft^3$ ∴ $\bar{X} = \frac{4988}{356} = 14.01\ ft$

∴ The weight of the dam is 53400 lb at $\bar{x} = 14.01$ ft.

& The weight of the water over the dam $= \frac{9*24}{2}*1*62.4 = 6739.2$ lb at $\bar{x} = 17 + \frac{9 \times 2}{3} = 23$ ft

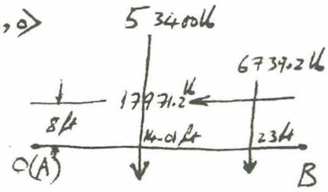
& The water pressure is $62.4*24*1*24*\frac{1}{2} = 17971.2$ lb horizontally at $\bar{y} = \frac{24}{3} = 8$ ft

∴ The total load on AB is $\langle 0, -53400\rangle + \langle 0, -6739.2\rangle + \langle -17971.2, 0\rangle$

$= \langle -17971.2, -60139.2\rangle$ lb acting at x from A such that

$0 = -53400*(14.01-x) - 6739.2*(23-x) + 17971.2*8$

∴ $x = 12.63$ ft



∴ (a) The reaction is $\langle 17971.2, 60139.2\rangle$ lb at $x = 12.63$ ft

& (b) The resultant of forces on BC is: pressure of $\langle -17971.2, 0\rangle$ and water weight of $\langle 0, -6739.2\rangle$ lb $= \langle -17971.2, -6739.2\rangle$ lb at (23, 8) ft on BC.

or $= 19193.3$ lb $\angle 200.6°$ acting at (23, 8) ft on BC.

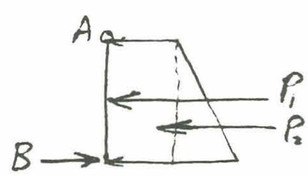
5.77 / 197.

(b) $P_1 = 62.4 * 2 * 3 * 3 = 1123.2$ lb

$P_2 = 62.4 * 3 * 3 * \frac{3}{2} = 842.4$ lb

∴ $\Sigma M_A = 0$ ∴ $1123.2 * 1.5 + 842.4 * 2 = B * 3$

∴ $B = 1123.2$ lb →



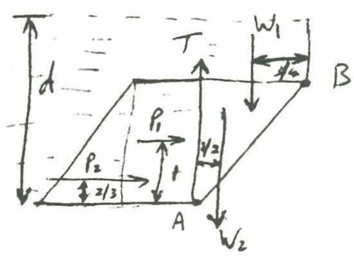
5.82 / 198

$W_1 = (d-3)*1.5*1.75*62.4 = 163.8(d-3)$ lb

$W_2 = 2*1.5*\frac{1}{2}*1.75*62.4 = 163.8$ lb

$P_1 = 62.4*(d-3)*1.75*2 = 218.4(d-3)$ lb

$P_2 = 62.4*2*1.75*2*\frac{1}{2} = 218.4$ lb



تابع

$\therefore \Sigma M_B = 0 \quad \therefore W_1 * \frac{3}{4} + W_2 * 1 + P_1 * 1 + P_2 * \frac{4}{3} = T * 1.5$

$\therefore 163.8(d-3) * \frac{3}{4} + 163.8 * 1 + 218.4(d-3) * 1 + 218.4 * \frac{4}{3} = 800 * 1.5$

$\therefore d = 5.183$ ft.

S.89 / 199 Forces acting on the block of wood are:- (assuming unit width

Its weight, $W_1 = \frac{2}{3} * 2a * h * 1 * \gamma_1 = \frac{4ah\gamma_1}{3} \downarrow$, a from point B

The weight of water, $W_2 = \frac{1}{3} * a * h * 1 * 62.4 = 20.8ah \downarrow$, $(a + \frac{3a}{4})$ from B

The hydrostatic pressure, $P = (62.4 * h * 1) * \frac{h}{2} = 31.2 h^2 \rightarrow$, $\frac{h}{3}$ from B

$\therefore \Sigma M_B = 0 \quad \therefore W_1 * a + W_2 * (a + \frac{3a}{4}) - P * \frac{h}{3} = 0$

$\therefore (\frac{4ah}{3} * 40) * a + (20.8 ah) * \frac{7a}{4} - (31.2 h^2) * \frac{h}{3} = 0$

$\therefore 53.3 a^2h + 36.4 a^2h - 10.4 h^3 = 0$

$\therefore 89.73 a^2h - 10.4 h^3 = 0$

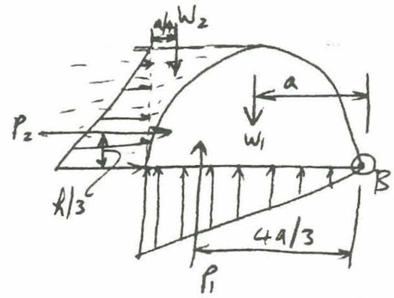
$\therefore 89.73 (\frac{h}{a}) - 10.4 (\frac{h}{a})^3 = 0$

$\therefore (\frac{h}{a}) \cdot [89.73 - 10.4 (\frac{h}{a})^2] = 0$

$\therefore$ either $(\frac{h}{a}) = 0$ or $(\frac{h}{a}) = 2.937$

$\therefore$ The maximum $\frac{h}{a}$ ratio is 2.937

S.90 / 199 Assuming unit thickness:



$\therefore W_1 = 2a * h * \frac{2}{3} * 1 * 40 = 53.33 ah$

& $W_2 = a * h * \frac{1}{3} * 1 * 62.4 = 20.8 ah$

& $P_1 = 62.4 * h * 1 * 2a * \frac{1}{2} = 62.4 ah$

& $P_2 = 62.4 * h * 1 * h * \frac{1}{2} = 31.2 h^2$

$\therefore \Sigma M_B = 0$

$\therefore 0 = W_1 * a + W_2 * (2a - \frac{a}{4}) - P_1 * \frac{4a}{3} - P_2 * \frac{h}{3}$

$\therefore 53.33 ah * a + 20.8 ah * \frac{7a}{4} - 62.4 ah * \frac{4a}{3} - 31.2 h^2 * \frac{h}{3} = 0$

$\therefore a^2h (53.33 + 20.8 * \frac{7}{4} - 62.4 * \frac{4}{3}) - 31.2 \frac{h^3}{3} = 0$

$\therefore 6.533 a^2h - 10.4 h^3 = 0 \qquad \therefore 6.533 = 10.4 \frac{h^3}{a^2h}$

$\therefore \frac{6.533}{10.4} = (\frac{h}{a})^2 \qquad \therefore \frac{h}{a} = 0.7926$

$\therefore$ The maximum value of $\frac{h}{a}$ that will not cause overturn is 0.793

بسم الله الرحمن الرحيم

5.96 / 207 This solid is made of two parts: $\frac{\pi a^2 h}{2}$ with $\bar{z} =$ (like semicircle) $-\frac{4a}{3\pi}$

& $-\frac{\pi a^2 h}{6}$ with $\bar{z} =$ (like sample problem 5.13) $= -\frac{a}{\pi}$

$\therefore$ Its z-coordinates of centroid is $\bar{z} = \dfrac{\frac{\pi a^2 h}{2} * (-\frac{4a}{3\pi}) - \frac{\pi a^2 h}{6} * (-\frac{a}{\pi})}{\frac{\pi a^2 h}{2} - \frac{\pi a^2 h}{6}} = \dfrac{\pi a^3 h}{\pi a^2 h} \cdot \dfrac{(-\frac{2}{3\pi} + \frac{1}{6\pi})}{\frac{1}{2} - \frac{1}{6}} = a \cdot \dfrac{-\frac{1}{2\pi}}{\frac{1}{3}}$

$= -\frac{3a}{2\pi}$ $\therefore$ $\bar{z} = -3a/2\pi$.

Or, by integration, $\bar{z} = \dfrac{M_y}{V} = \dfrac{\int_0^a (y\,dx) \cdot \pi x * (\frac{-2x}{\pi})}{\int_0^a (y\,dx) \cdot \pi x} = \dfrac{-\int_0^a 2x^2 y\,dx}{\int_0^a \pi x y\,dx} =$ (since $y = \frac{hx}{a}$)

$= \dfrac{-\int_0^a 2x^2 \cdot \frac{hx}{a} dx}{\int_0^a \pi x \frac{hx}{a} dx} = \dfrac{-\frac{2h}{a} \cdot \frac{x^4}{4}\Big|_0^a}{\frac{\pi h}{a} \cdot \frac{x^3}{3}\Big|_0^a} = \dfrac{-3 \cdot a^4}{2\pi \cdot a^3} = -3a/2\pi$.

5.98 / 207 By symmetry $\therefore$ $\bar{z} = 0$, $\bar{x} = \dfrac{M_y}{V} = \dfrac{6*3*1*(\frac{3}{2}+\frac{1}{2})}{6*3*1+6*5*1+1*\frac{\pi}{2}*3^2-\pi*1^2*1} = \dfrac{36}{59.0} = 0.61$ in

& $\bar{y} = \dfrac{M_x}{V} = \dfrac{-6*3*1*(\frac{1}{2}+4) - 6*5*1*(\frac{5}{2}) + 1*\frac{\pi}{2}*3^2*(\frac{4*3}{3*\pi}) + 0}{59.0} = \dfrac{-138}{59.0} = -2.34$ in

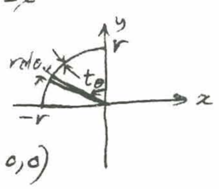
$\therefore$ The centroid is at $(0.61, -2.34, 0)$ in

5.106 / 208 By symmetry, $\therefore$ $\bar{y} = \bar{z} = 0$ $\therefore$ $\bar{x} = \dfrac{M_y}{V} = \dfrac{\frac{4}{3}\pi \frac{(r+t)^3}{2} * \frac{-3(r+t)}{8} - \frac{4}{3}\pi \frac{r^3}{2} * \frac{-3r}{8}}{\frac{4}{3}\pi((r+t)^3 - r^3)} =$

$= \dfrac{(r+t)^4 - r^4}{\cancel{r^3} + 3r^2 t + 3rt^2 + t^3 - \cancel{r^3}} \cdot \dfrac{(-3)}{8} = \dfrac{-3}{8} \cdot \dfrac{\cancel{r^4} + 4r^3 t + 6r^2 t^2 + 4rt^3 + t^4 - \cancel{r^4}}{t(3r^2 + 3rt + t^2)} =$ (if t is $<< r$)

$= \dfrac{-3}{8} \cdot \dfrac{\cancel{t}(4r^3 + 6r^2 t + 4rt^2)}{\cancel{t}(3r^2 + 3rt)} = \dfrac{-3}{8} \cdot \dfrac{\cancel{r}(4r^2 + 6rt + 4t^2)}{\cancel{r}(3r + 3t)} = \dfrac{-\cancel{3}}{8} \cdot \dfrac{(4r^2 + 6rt)(r-t)}{\cancel{3}(r+t)(r-t)} = \dfrac{-1}{8} \cdot \dfrac{2r(2r^2 + tr - 3t^2)}{r^2 - t^2} =$

$= \frac{-1}{4}(2r + t) = \frac{-r}{2}$ $\therefore$ The centroid is at $(-\frac{r}{2}, 0, 0)$.



Or, by integration $\therefore$ $\bar{x} = \dfrac{M_y}{V} = \dfrac{-\int_0^{\pi/2} (r\,d\theta)\, t * 2\pi r\cos\theta * r\sin\theta}{\int_0^{\pi/2} (r\,d\theta)\, t * 2\pi r\cos\theta} =$

$= \dfrac{-2\pi t r^3 \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta}{2\pi t r^2 \int_0^{\pi/2} \cos\theta\, d\theta} = -r \cdot \dfrac{\frac{\sin^2\theta}{2}\Big|_0^{\pi/2}}{\sin\theta\Big|_0^{\pi/2}} = -r \cdot \dfrac{1/2}{1} = -r/2$ $\therefore$ Centroid at $(\frac{-r}{2}, 0, 0)$

5.118 / 210 By symmetry, $\therefore$ $\bar{x} = 0$, $\therefore$ $V = \int_0^{2\pi} (a\,d\theta) \cdot t \cdot y = at \cdot \int_0^{2\pi} y\,d\theta = at \cdot \int_0^{2\pi} (h \cdot \frac{a-z}{2a})\, d\theta = \frac{th}{2} \int_0^{2\pi} (a - a\cos\theta)\, d\theta$

$= \frac{ath}{2} \int_0^{2\pi} (1 - \cos\theta)\, d\theta = \frac{ath}{2} \cdot [\theta - \sin\theta]_0^{2\pi} = \pi tha$, $\therefore$ $\bar{z} = \frac{1}{\pi tha} \cdot \int_0^{2\pi} [\frac{ath}{2}(1 - \cos\theta)\, d\theta]\, z =$

$= \frac{1}{2\pi} \int_0^{2\pi} (1 - \cos\theta)\, a\cos\theta\, d\theta = \frac{a}{2\pi}\left[\sin\theta - \frac{\theta + \frac{\sin 2\theta}{2}}{2}\right]_0^{2\pi} = \frac{a}{2\pi}(-\frac{2\pi}{2}) = -a/2$, $\therefore$ $\bar{y} = \frac{1}{\pi tha} \cdot \int_0^{2\pi} [\frac{ath}{2}(1 - \cos\theta)\, d\theta] \frac{y}{2} =$

$= \frac{1}{2\pi} \int_0^{2\pi} (1 - \cos\theta) \cdot \frac{h(a-z)}{2*2a}\, d\theta = \frac{h}{8\pi a} \int_0^{2\pi} (1 - \cos\theta) \cdot (a - a\cos\theta)\, d\theta = \frac{h}{8\pi} \int_0^{2\pi} (1 - \cos\theta)^2\, d\theta = \frac{h}{8\pi}\left[\theta - 2\sin\theta + \frac{\theta + \frac{\sin 2\theta}{2}}{2}\right]_0^{2\pi}$

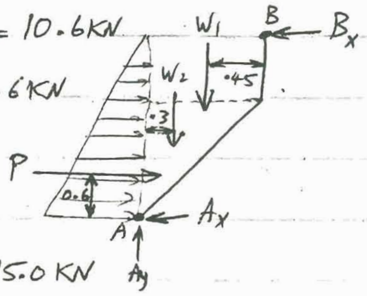
$= \frac{h}{8\pi}\left[2\pi - 0 + \frac{2\pi + 0}{2}\right] = \frac{h}{8\pi} \cdot 3\pi = \frac{3h}{8}$. The required centroid is at $(0, \frac{3h}{8}, -\frac{a}{2})$.

5.124 / 211

$W_1 = 0.6 * 0.9 * 2 * 1.0 * 10^3 * 9.81 = 1.06 * 10^4 \text{ N} = 10.6 \text{ KN}$

& $W_2 = 1.2 * 0.9 * \frac{1}{2} * 2 * 1.0 * 10^3 * 9.81 = 1.06 * 10^4 \text{ N} = 10.6 \text{ KN}$

& $P = 1.0 * 10^3 * 9.81 * 1.8 * 2 * 1.8 * \frac{1}{2} = 3.18 * 10^4 \text{ N} = 31.8 \text{ KN}$



$\therefore \Sigma M_A = 0 \quad \therefore -P * .6 - W_2 * .3 - W_1 * .45 + B_x * 1.8 = 0$

$\therefore B_x = \dfrac{.6P + .3W_2 + .45W_1}{1.8} = \dfrac{.6 * 31.8 + .3 * 10.6 + .45 * 10.6}{1.8} = 15.0 \text{ KN}$

$\therefore A_x = P - B_x = 31.8 - 15.0 = 16.8 \text{ KN}$

& $A_y = W_1 + W_2 = 10.6 + 10.6 = 21.2 \text{ KN}$

$\therefore$ Reaction at A is $27.0 \text{ KN} \angle 128.4^\circ$ & at B is $15.0 \text{ KN} \angle 180^\circ$.

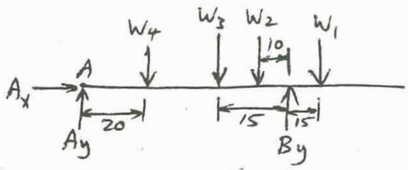
5.126 / 211

$w_1 = 10 * 45 * \frac{1}{2} = 225 \text{ lb}$

$w_2 = (10 - 7.5) * 30 * \frac{1}{2} = 37.5 \text{ lb}$

$W_3 = 7.5 * 30 = 225 \text{ lb}$

$W_4 = 7.5 * 30 * \frac{1}{2} = 112.5 \text{ lb}$



$\therefore \Sigma M_A = 0 \quad \therefore -W_4 * 20 - W_3 * 45 - W_2 * 50 + B_y * 60 - W_1 * 75 = 0$

$\therefore B_y = \frac{1}{60} \cdot (20 W_4 + 45 W_3 + 50 W_2 + 75 W_1) = \frac{1}{60} * (20 * 112.5 + 45 * 225 + 50 * 37.5 + 75 * 225)$

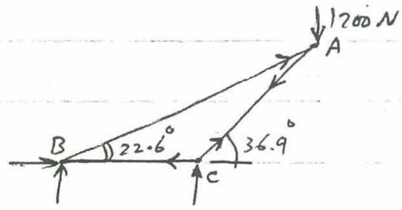
$\therefore B_y = 518.75 \text{ lb}$

$\therefore A_y = W_1 + W_2 + W_3 + W_4 - B_y = 225 + 37.5 + 225 + 112.5 - 518.75 = 81.25 \text{ lb}$

$\therefore A_x = 0$

$\therefore$ Reaction at A is $81.25 \text{ lb} \uparrow$ & at B is $518.75 \text{ lb} \uparrow$.

بسم الله الرحمن الرحيم



6.4 / 226

$T_{BA}\cos 22.6 = T_{AC}\cos 36.9 \quad \therefore T_{BA} = 0.8665\,T_{AC}$

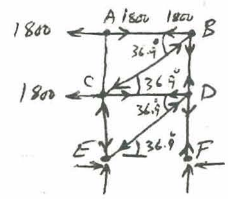
$\&\ T_{BA}\sin 22.6 = 1200 + T_{AC}\sin 36.9$

$\therefore .8665\,T_{AC}\cdot\sin 22.6 = 1200 + T_{AC}\sin 36.9$

$\therefore T_{AC} = -4487\ N$ $\quad\therefore$ Member AC has a compressive force of 4.5 KN

$\therefore T_{BA} = .8665 * T_{AC} = -3888\ N$ $\quad\therefore$ Member BA has a tensive force of 3.9 KN

$\therefore T_{CB} = T_{AC}\cos 36.9 = -4487 * \cos 36.9 = -3588 N$ $\quad\therefore$ Member BC has a compressive force of 3.6 KN.



6.8 / 226

Joint A $\therefore$ Force on member AB is 1800 lb T & Force on member AC is zero.

& B, Force on member BC is $\frac{1800}{\cos 36.9} = 2250$ lb C &

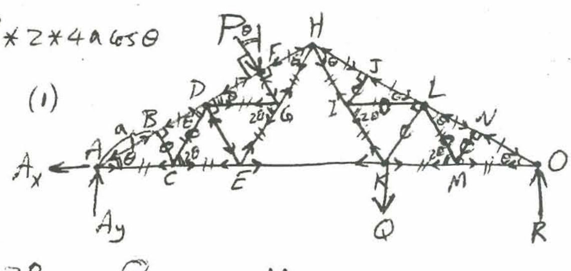
$\therefore$ Force on member BD is $2250\sin 36.9 = 1351$ lb T

& C, $\therefore$ Force on member CD is $1800 + 2250\cos 36.9 = 3600$ lb T &

Force on member CE is $2250\sin 36.9 = 1351$ lb C

& D, $\therefore$ Force on member ED is $3600/\cos 36.9 = 4500$ lb C &

Force on member DF is $1351 + 4500\sin 36.9 = 4053$ lb T.



6.14 / 227

$\Sigma M_A = 0 \therefore P * 3a + Q * \left(2 * 4a\cos\theta - \frac{2a}{\cos\theta}\right) = R * 2 * 4a\cos\theta$

$\therefore R = \frac{3P + (8\cos\theta - 2\sec\theta)Q}{8\cos\theta} = \frac{3P}{8\cos\theta} + Q - \frac{Q}{4\cos^2\theta} \quad (1)$

$\therefore A_x = P\sin\theta\ (2)\ \&\ A_y = Q + P\cos\theta - R$

$\therefore A_y = \cancel{Q} + P\cos\theta - \frac{3P}{8\cos\theta} - \cancel{Q} + \frac{Q}{4\cos^2\theta} = P\cos\theta - \frac{3P}{8\cos\theta} + \frac{Q}{4\cos^2\theta} \quad (3)$

Joint A

$\therefore BA\sin\theta = A_y \quad \therefore BA = \frac{A_y}{\sin\theta} = \frac{P\cos\theta}{\sin\theta} - \frac{3P}{8\cos\theta\sin\theta} + \frac{Q}{4\cos^2\theta\sin\theta}$ C $\quad (4)$

$\&\ AC = A_x + BA\cos\theta = P\sin\theta + \frac{P\cos^2\theta}{\sin\theta} - \frac{3P}{8\sin\theta} + \frac{Q}{4\cos\theta\sin\theta} = \frac{8P\sin^2\theta + 8P\cos^2\theta - 3P}{8\sin\theta} + \frac{Q}{4\sin\theta\cos\theta}$

$= \frac{5P}{8\sin\theta} + \frac{Q}{4\sin\theta\cos\theta}$ T $\quad (5)$

Joint B

$\therefore DB = AB = \frac{P\cos\theta}{\sin\theta} - \frac{3P}{8\sin\theta\cos\theta} + \frac{Q}{4\sin\theta\cos^2\theta}$ C $\quad (6)$

f $BC = 0$ (7)

Joint C

$\therefore CD \sin 2\theta = 0 \qquad \therefore CD = 0$ (8)

$\therefore CE = CA = \dfrac{5P}{8\sin\theta} + \dfrac{Q}{4\sin\theta\cos\theta}$ T (9)

Joint F

$\therefore GF = P$ C (10)

f $DF = HF$

Joint G

$\therefore FG\cos\theta = GD\sin 2\theta \qquad \therefore GD = \dfrac{FG\cos\theta}{\sin 2\theta} = \dfrac{P\cos\theta}{2\sin\theta\cos\theta} = \dfrac{P}{2\sin\theta}$ T (11)

f $GD\cos 2\theta + FG\sin\theta + GE = GH \qquad \therefore GH = \dfrac{P}{2\sin\theta}\cdot\cos 2\theta + P\sin\theta + GE =$

$= \dfrac{P(\cos 2\theta + 2\sin^2\theta)}{2\sin\theta} + GE = \dfrac{P}{2\sin\theta} + GE$

Joint D

$\therefore DG\sin\theta = ED \qquad \therefore ED = \dfrac{P}{2\sin\theta}\cdot\sin\theta = \dfrac{P}{2}$ C (12)

f $BD + DG\cos\theta = FD \qquad \therefore FD = \dfrac{P\cos\theta}{\sin\theta} - \dfrac{3P}{8\sin\theta\cos\theta} + \dfrac{Q}{4\sin\theta\cos^2\theta} + \dfrac{P}{2\sin\theta}\cdot\cos\theta =$

$= \dfrac{8P\cos^2\theta - 3P + 4P\cos^2\theta}{8\sin\theta\cos\theta} + \dfrac{Q}{4\sin\theta\cos^2\theta} = \dfrac{3P\cdot(4\cos^2\theta - 1)}{8\sin\theta\cos\theta} + \dfrac{Q}{4\sin\theta\cos^2\theta}$ C (13)

$\therefore HF = \dfrac{3P(4\cos^2\theta - 1)}{8\sin\theta\cos\theta} + \dfrac{Q}{4\sin\theta\cos^2\theta}$ T (14)

Joint E

$\therefore DE\cos\theta = EG\sin 2\theta \qquad \therefore EG = \dfrac{DE\cos\theta}{\sin 2\theta} = \dfrac{(P/2)\cdot\cos\theta}{2\sin\theta\cos\theta} = \dfrac{P}{4\sin\theta}$ T (15)

$\therefore GH = \dfrac{P}{2\sin\theta} + GE = \dfrac{P}{2\sin\theta} + \dfrac{P}{4\sin\theta} = \dfrac{3P}{4\sin\theta}$ T (16)

f $EK = EC - DE\sin\theta - EG\cos 2\theta = \dfrac{5P}{8\sin\theta} + \dfrac{Q}{4\sin\theta\cos\theta} - \dfrac{P}{2}\cdot\sin\theta - \dfrac{P}{4\sin\theta}\cdot\cos 2\theta =$

$= \dfrac{5P - 4P\sin^2\theta - 2P\cos 2\theta}{8\sin\theta} + \dfrac{Q}{4\sin\theta\cos\theta} = \dfrac{P(5-2)}{8\sin\theta} + \dfrac{Q}{4\sin\theta\cos\theta} = \dfrac{3P}{8\sin\theta} + \dfrac{Q}{4\sin\theta\cos\theta}$ T (17)

Joint O

$\therefore NO\cdot\sin\theta = R = \dfrac{3P}{8\cos\theta} + Q - \dfrac{Q}{4\cos^2\theta} \qquad \therefore NO = \dfrac{3P}{8\sin\theta\cos\theta} + \dfrac{Q(4\cos^2\theta - 1)}{4\sin\theta\cos^2\theta}$ C (18)

f $OM = NO\cdot\cos\theta = \dfrac{3P}{8\sin\theta} + \dfrac{Q\cdot(4\cos^2\theta - 1)}{4\sin\theta\cos\theta}$. T (19)

Joint N

$\therefore LN = ON = \dfrac{3P}{8\sin\theta\cos\theta} + \dfrac{Q\cdot(4\cos^2\theta - 1)}{4\sin\theta\cos^2\theta}$ C (20)

f $NM = 0$ (21)

\# Joint M

$\therefore ML \sin 2\theta = 0 \qquad \therefore ML = 0 \qquad (22)$

$\& \quad MK = MO = \dfrac{3P}{8\sin\theta} + \dfrac{Q(4\cos^2\theta - 1)}{4\sin\theta\cos\theta} \quad T \qquad (23)$

\# Joint J

$\therefore IJ = 0 \qquad (24)$

$\& \quad HJ = LJ$

\# Joint I

$\therefore IL \sin 2\theta = 0 \qquad \therefore IL = 0 \qquad (25)$

$\& \quad IH = IK$

\# Joint L

$\therefore LK = 0 \qquad (26)$

$\& \quad JL = NL = \dfrac{3P}{8\sin\theta\cos\theta} + \dfrac{Q(4\cos^2\theta - 1)}{4\sin\theta\cos^2\theta} \quad C \qquad (27)$

$\therefore HJ = \dfrac{3P}{8\sin\theta\cos\theta} + \dfrac{Q(4\cos^2\theta - 1)}{4\sin\theta\cos^2\theta} \quad C \qquad (28)$

\# Joint K

$\therefore KI \sin 2\theta = Q \qquad \therefore KI = \dfrac{Q}{2\sin\theta\cos\theta} \quad T \qquad (29)$

$\& \text{(check)} \quad KE = KM - KI\cos 2\theta = \dfrac{3P}{8\sin\theta} + \dfrac{Q(4\cos^2\theta - 1)}{4\sin\theta\cos\theta} - \dfrac{Q}{2\sin\theta\cos\theta} \cdot \cos 2\theta =$

$= \dfrac{3P}{8\sin\theta} + \dfrac{Q(4\cos^2\theta - 1 - 2\cos 2\theta)}{4\sin\theta\cos\theta} = \dfrac{3P}{8\sin\theta} + \dfrac{Q}{4\sin\theta\cos\theta} \quad T$ (just like (17))

$\therefore IH = IK = \dfrac{Q}{2\sin\theta\cos\theta} \quad T \qquad (30)$

\# Check joint H

$\therefore FH\cos\theta - HG\cos 2\theta + HI\cos 2\theta - JH\cos\theta = (FH - JH)\cos\theta + (HI - HG)\cos 2\theta =$

$= \left(\dfrac{3P(4\cos^2\theta - 1)}{8\sin\theta\cos\theta} + \dfrac{Q}{4\sin\theta\cos^2\theta} - \dfrac{3P}{8\sin\theta\cos\theta} - \dfrac{Q(4\cos^2\theta - 1)}{4\sin\theta\cos^2\theta}\right)\cos\theta + \left(\dfrac{Q}{2\sin\theta\cos\theta} - \dfrac{3P}{4\sin\theta}\right)\cos 2\theta =$

$= \left(\dfrac{3P \cdot 2\cancel{\cos 2\theta}}{8\sin\theta\cos\theta} + \dfrac{Q \cdot \cancel{(-2\cos 2\theta)}}{4\sin\theta\cos^2\theta}\right)\cos\theta + \left(\dfrac{\cancel{Q}}{2\sin\theta\cos\theta} - \dfrac{3\cancel{P}}{4\sin\theta}\right) \cdot \cos 2\theta = 0$ (right)

$\& \quad FH\sin\theta - HG\sin 2\theta + JH\sin\theta - HI\sin 2\theta = (FH + JH)\sin\theta - (HI + HG)\sin 2\theta =$

$= \left(\dfrac{3P(4\cos^2\theta - 1)}{8\sin\theta\cos\theta} + \dfrac{Q}{4\sin\theta\cos^2\theta} + \dfrac{3P}{8\sin\theta\cos\theta} + \dfrac{Q(4\cos^2\theta - 1)}{4\sin\theta\cos^2\theta}\right)\sin\theta - \left(\dfrac{Q}{2\sin\theta\cos\theta} + \dfrac{3P}{4\sin\theta}\right)\sin 2\theta =$

$= \left(\dfrac{3}{2}P\cancel{\cos\theta} + \cancel{Q}\right) - \left(\dfrac{\cancel{Q}}{\cancel{\sin 2\theta}} + \dfrac{3P\cancel{\cos\theta}}{2\cancel{\sin 2\theta}}\right)\sin 2\theta = 0$ (right.

$\therefore$ The zero-force members are BC, CD, JI, IL, LK, LM & MN

6.24 / 234 Cut through BD, DC, CE & consider the left half.

$\therefore \Sigma M_D = 0 \quad \therefore \quad 20*2+30*4 = F_{EC}*\left(\frac{4}{\sqrt{4^2+.5^2}}*1.5+\frac{.5}{\sqrt{4^2+.5^2}}*2\right)$

$\therefore F_{EC} = 92.14$ KN C.

& $\Sigma M_B = 0 \quad \therefore \quad 30*2+F_{CD}*\frac{2}{\sqrt{2^2+1.5^2}}*1.5 = 92.14*\frac{4}{\sqrt{4^2+.5^2}}*1.5$

$\therefore F_{CD} = 64.29$ KN T

6.28 / 234 Section through the named members. Consider the upper half.

$\therefore \Sigma M_F = 0 \quad \therefore \quad 20*6+20*3 = EG*4 \quad \therefore EG = 45$ KN T

$\therefore FE = 45+10 = 55$ KN C & $FD = 45$ KN C

6.29 / 235 Section through the named members & consider the left half.

$\therefore \Sigma M_G = 0 \quad \therefore \quad 1*24+2*16+2*8 = FH*\frac{1}{\sqrt{2}}*6 \quad \therefore FH = 17$ Kip T.

$\therefore HG = FH*\frac{1}{\sqrt{2}} = 12$ Kips C & $IG = 1+2+2+1+FH*\frac{1}{\sqrt{2}} = 6+12 = 18$ Kips C

6.34 / 235 $\Sigma M_L = 0 \quad \therefore \quad 2P*d+3P*d+J_y*2d = 0 \quad \therefore J_y = -\frac{5P}{2}$ & $J_x = -2P$

Consider the left part of the section shown.

$\therefore \Sigma M_K = 0 \quad \therefore \quad -\frac{5P}{2}*d+AB*3d = 0 \quad \therefore AB = \frac{5P}{6}$ T

$\therefore AB+J_x = LK \quad \therefore LK = \frac{5P}{6}-2P = \frac{-7P}{6} \quad \therefore LK = \frac{7P}{6}$ T

6.36 / 236 Consider the upper half of the section shown.

$\therefore \Sigma M_E = 0 \quad \therefore \quad 15*4+15*2 = JG*4 \quad \therefore JG = 22.5$ KN C

6.50 / 245 $\Sigma M_B = 0 \quad \therefore \quad 560*.1 = D_y*.2 \quad \therefore D_y = 280$ N ↑ (Force at D)

$\therefore B_y = 280$ N ↓ & $B_x = 560$ N ← $\quad \therefore$ Force at B is $\langle -560, -280\rangle$ N $= 626$ N $\angle 207°$

Cut joint A and ball C & consider the member ABCD

$\therefore \Sigma M_A = 0 \therefore (-560-280)*.05+C_y*.175+280*.250 = 0 \quad \therefore C_y = -160$ N $= 160$ N ↓ (Force at C)

$\therefore A_x = 560$ N → & $A_y = 280+160-280 = +160$ N $\quad \therefore$ Force at A $= \langle 560, 160\rangle$ N $= 582$ N $\angle 16°$.

6.53 / 246 (a) Cut CD & take $\Sigma M_A = 0 \quad \therefore CD*15 = 90000*9.81*38 \quad \therefore CD = 2.237*10^6$ N T $= 2.237$ MN T

(b) Cut through DC, CA, AB & take $\Sigma M_B = 0 = -90000*9.81*3+2.237*10^6*50-AC\cdot\left(\frac{18*50+15*40}{\sqrt{18^2+15^2}}\right)$

$\therefore AC = 1.706*10^6$ N C $= 1.706$ MN C. & Cut joint A and + CD take $\Sigma M_C = 0$

$\therefore$ (c) $A_x*18+A_y*15 = 90*10^3*9.81*53$. But $A_y = 2.237*10^6+90*10^3*9.81 = 3.120*10^6$ N ↑

تابع

$\therefore A_x = -350\ N$ (it should be zero and this small figure is due to accuracy)

$\therefore$ Force at A acting on AB is $< 0\ , 3.120 > MN + CA =$

$= < 0\ , 3.120 > MN + 1.706\ MN < \frac{15, -18}{\sqrt{15^2+18^2}} > = < 1.092, 1.809 > MN$

$= 2.113\ MN \angle 58.9°$

6.62
247

Weight $= 40 * 30 = 1200\ lb = 1.2$

$\therefore A_x = 0 \quad \therefore C_x = 0 \quad \therefore$ By symmetry $A_y = C_y = 0.6$ Kips & $T_{DE} = \frac{0.6}{\sin\theta}$

Consider member ADB, $\therefore \Sigma M_B = 0 \quad \therefore 0.6 * 8 = T_{DE} * (\sin\theta * \frac{6*16}{14*2} + 6\cos\theta)$

ⓐ $\therefore .6 * 8 = \frac{\sin\theta}{\sin\theta} * (\frac{24}{7} . \sin\theta + 6\cos\theta) \Rightarrow 8 = \frac{24}{7} + 6\cot\theta \Rightarrow \theta = 52.7°$

ⓑ $\therefore$ Force at A is 0.6 Kips $\uparrow$ & Force at D is $\frac{.6}{\sin 52.7} . < \cos 52.7, -\sin 52.7 >$

$= < .457, -.6 >$ Kips $= 0.754$ Kips $\angle -52.7°$

and force at $B = -[< .457, -.6 > + < 0, .6 >] = < -.457, 0 >$ Kips $= .457$ Kips $\leftarrow$

6.64
248

ⓐ $\Sigma M_A = 0 \quad \therefore 2B_y * 7.1 = 10 * 10^3 * 9.81 * 8.6 + 50 * 10^3 * 9.81 * 3.4$

$\therefore B_y = 176.9 * 10^3\ N = 176.9\ KN \uparrow \quad \therefore 2A_y = 10 * 10^3 * 9.81 + 50 * 10^3 * 9.81 - 2 * 176.9 * 10^3$

$\therefore A_y = 117.4 * 10^3\ N = 117.4\ KN \uparrow$

$\therefore$ Each of the front wheels (B) is reacted upon by 176.9 KN $\uparrow$ and of back ones (A) by 117.4 KN $\uparrow$

ⓑ Cut at C & D and consider the right part. Since C is free to move up & down

$\therefore C_y = 0 \quad \Sigma M_D = 0 \quad \therefore C_x * .75 + 10 * 10^3 * 9.81 * 2.1 - 2 * 176.9 * 10^3 * .6 = 0$

$\therefore C_x = 8.36 * 10^3\ N = 8.36\ KN \rightarrow \quad \therefore D_x = 8.36\ KN \leftarrow$

& $D_y = 10 * 10^3 * 9.81 - 2 * 176.9 * 10^3 = -2.557 * 10^5\ N \uparrow = 255.7\ KN \downarrow$.

6.70
249

The load P can be supported for all values of a which result in positive reactions at both A & B. Cut joint C & walls and take $\Sigma M_C = 0$

$\therefore R_A * \frac{200 * 250}{500} + R_B * \frac{300 * 250}{500} = 600 * (a - 200)$

$\therefore 2R_A + 3R_B = 12\ (a - 200)$. Taking the $\Sigma M_D = 0$ of the original assembly,

$\therefore R_A * 250 = P * a = 600a \quad \therefore R_A = \frac{12a}{5}, \therefore 3R_B = 12(a - 200) - \frac{24a}{5}$

$\therefore R_B = \frac{36a}{5} - 2400 \quad \therefore R_A \geq 0$ for $a \geq 0$ & $R_B \geq 0$ for $a \geq 333.3mm \quad \therefore a \in [333.3, 500]\ mm$.

تابع

6.74 / 250

(a) Cut at B & take $\Sigma M_c = 0$ of right half, $\therefore B_x * 2.5 + B_y * 5 = 0 \quad \therefore B_x = -2B_y$

& $C_x = -B_x = 2B_y$ & $C_y = -B_y$.

Considering the left half, $\therefore \Sigma M_A = 0 \quad \therefore 20 * 3 = B_x * 2.5 - B_y * 5 =$

$= -2B_y * 2.5 - B_y * 5 = -10 B_y \quad \therefore B_y = -6\ KN \quad \therefore B_x = -2 * -6 = 12\ KN$

$\therefore C_x = -B_x = -12\ KN$ & $C_y = -B_y = 6\ KN$

$\therefore$ Reaction at C is $\langle -12, 6 \rangle KN = 13.42\ KN \angle 153^\circ$

& (b). Force exerted at B on AB $= \langle B_x, -B_y \rangle = \langle 12, 6 \rangle KN = 13.42\ KN \angle 153^\circ$.

6.80 / 251

$\Sigma M_A = 0 \quad \therefore D_x * 3 = 1200 * 8 \Rightarrow D_x = 3200\ lb \rightarrow \quad \therefore A_x = 3200\ lb \leftarrow$ & $A_y = 1200\ lb \uparrow$

Joint D

$\therefore D_y = 0$ & $DE = 3200\ lb$ C (Force in DE)

Joint A

$\therefore AE * \frac{3}{5} = A_y \quad \therefore AE = 1200 * \frac{5}{3} = 2000\ lb$ T (Force in AE)

& $AB = A_x - AE * \frac{4}{5} = 3200 - 2000 * \frac{4}{5} = 1600\ lb$ T (Force in AB)

Joint B

$\therefore BF * \frac{3}{5} = 0 \quad \therefore BF = 0$ (Force in BF)

$\therefore BC = BA = 1600\ lb$ T (Force in BC)

Joint C

$\therefore CG * \frac{4}{5} = CB \quad \therefore CG = 1600 * \frac{5}{4} = 2000\ lb$ T (Force in CG)

& $FC = CG * \frac{3}{5} = 2000 * \frac{3}{5} = 1200\ lb$ C (Force in FC).

6.86 / 252

Reaction at A is vertical due to member AB $\therefore \Sigma M_G = 0 \Rightarrow AB * 560 = 2000 * 280$

$\therefore AB = 1000\ N \quad \therefore G_x = 0$ & $G_y = 1000\ N \uparrow$.

Consider member BCD $\therefore \Sigma M_c = 0 \quad \therefore AB * 200 = D_x * 80 \quad \therefore D_x = \frac{1000 * 200}{80} = 2500 N \rightarrow$

$\therefore C_x = 2500 N \leftarrow$ & $C_y = 1000 N \downarrow$

Consider member EFG $\therefore \Sigma M_F = 0 \quad \therefore 2000 * 80 + 1000 * 200 = E_x * 80 \quad \therefore E_x = 4500 N \rightarrow$

$\therefore F_x = 4500\ N \leftarrow$ & $F_y = 2000 - 1000 = 1000\ N \uparrow$

$\therefore$ Forces on disk are at C $\langle 2500, 1000 \rangle N$ at D $\langle -2500, 0 \rangle N$ at E $\langle -4500, 0 \rangle N$ & at F $\langle 4500, -1000 \rangle N$

تابع

6.94 / 256

Cut BD & C, take $\Sigma M_C = 0$

$\therefore 100 * 70 = BD * \sin\left(25^\circ - \sin^{-1}\left(\frac{45 \sin 25^\circ - 10}{100}\right)\right) * 45 \quad \therefore BD = 458.6\,N$

& $C_x - 100 \sin 25 + BD \cos\left(\sin^{-1}\left(\frac{45 \sin 25^\circ - 10}{100}\right)\right) = 0 \quad \therefore C_x = -414.5N$

(a) $\therefore Q = C_x = 414.5N \leftarrow$

& (b) $BD = 458.6\ N\ T$.

6.102 / 258

Cut A & B and take $\Sigma M_A = 0 \quad \therefore M_A = B * \left[\cos\left(\tan^{-1}\frac{80}{200}\right) * 60*10^{-3} + \sin\left(\tan^{-1}\frac{80}{200}\right) * 80*10^{-3}\right]$

$\therefore B = 234.1\ N$

$\therefore$ (a) Cut C & B and take $\Sigma M_C = 0 \quad \therefore M_C = B * \sqrt{80^2 + 200^2} * 10^{-3} = 50.4\,Nm$

& (b) $C_x + B \cos\left(\tan^{-1}\frac{80}{200}\right) = 0 \quad \therefore C_x = -217.4\,N$

& $C_y = B \sin\left(\tan^{-1}\frac{80}{200}\right) = 86.9\ N$

$\therefore$ Reaction at C is $\langle -217.4, 86.9\rangle\ N$

6.114 / 261

Cut C & F. Since $C_y = 0 \quad \therefore F_y = 5\,KN$

$\Sigma M_C = 0 \quad \therefore 5 * 1 + F_x * .6 = 0 \quad \therefore F_x = -\frac{5}{.6} = -8.33\ KN$

$\therefore C_x = +8.33\,KN$.

Cut BE & A $\therefore \Sigma M_A = 0 \quad \therefore 8.33 * 1.2 = BE * .8$

$\therefore$ (a) $BE = 12.5\ KN\ T$

& (b) Cut BE, D & HE and take $\Sigma M_D = 0$

$\therefore 12.5 * .8 + HE * \left(.8 \cos\left(\tan^{-1}\frac{.8}{1.6}\right) - .4 \sin\left(\tan^{-1}\frac{.8}{1.6}\right)\right) - 8.33 * 1.2 - 5 * 2.4 = 0$

$\therefore HE = 22.4\ KN\ C$

$\therefore$ Force on pin $H = -HE = -22.4 \frac{\langle .8, 1.6\rangle}{\sqrt{.8^2 + 1.6^2}} = \langle -10.0, -20.0\rangle\ KN$

6.116 / 262

$C_2 + C_1 = 4500\ lb$ & Cut D, F & ropes, then consider the bucket.

$\therefore$ By symmetry $FC = DA$ & $4C_1 + 2*FC * \cos\left(\tan^{-1}\frac{25}{50}\right) = 4400$

Cut E, F & 2 ropes, then consider the right part, $\therefore \Sigma M_E = 0$

$\therefore 2000 * 30 = FC * \left(\cos\left(\tan^{-1}\frac{25}{50}\right) * 35 + \sin\left(\tan^{-1}\frac{25}{50}\right) * 10\right) \quad \therefore FC = 1677\ lb\ T$

$\therefore C_1 = 350\ lb \quad \therefore C_2 = 4150\ lb.$

تابع

6.128 / 264 Cut CD, CG & GF, then take $\Sigma M_G = 0$ for the left part.

$\therefore 6 * (6+4+2) = CD * \left(\frac{2}{2.5} * 1.5 + \frac{1.5}{2.5} * 2\right) \quad \therefore CD = 30 \, KN \, T$ (Force in CD)

& $\Sigma F_y = 0 \quad \therefore 18 - 30 * \frac{1.5}{2.5} = GC * \frac{1.5}{2.5} \quad \therefore GC = 0$ (Force in GC)

—

6.134 / 265 Cut F & take $\Sigma M_B = 0 \quad \therefore F * 1 = 700 * \frac{6 * \sin 60}{\sin 30} \therefore F = 7275 \, lb$

$\therefore$ EF must pull F by 7275 lb

Cut DE & take $\Sigma M_A = 0$

$\therefore 700 * \left(12 - \frac{6 \sin 60}{\sin 30}\right) = DE * \left(4 * \cos\left(\tan^{-1}\left(\frac{4-1}{2-1}\right)\right) + 1 * \sin\left(\tan^{-1}\left(\frac{4-1}{2-1}\right)\right)\right)$

$\therefore DE = 508.4 \, lb \, C$

—

بسم الله الرحمن الرحيم

7.10 / 269

Cut A and take $\Sigma M_E = 0$ and since $A_y = 500\ N \uparrow$

$\therefore 500 * 350 - 500 * 200 - A_x * 400 = 0 \quad \therefore A_x = 187.5\ N \rightarrow$

Cut at J & consider the upper part, $\therefore J_x = 187.5\ N \leftarrow$ & $J_y = 500\ N \downarrow$

and $M_J = A_x * .100 = 187.5 * .100 = 18.75$ N.m

$\therefore$ Internal forces at J on AJ are: 187.5 N $\leftarrow$, 500 N T & 18.75 N.m

---

7.12 / 270

$A_y = C_y = \frac{30}{2} = 15\ lb \uparrow$ & $C_x = A_x = 0$.

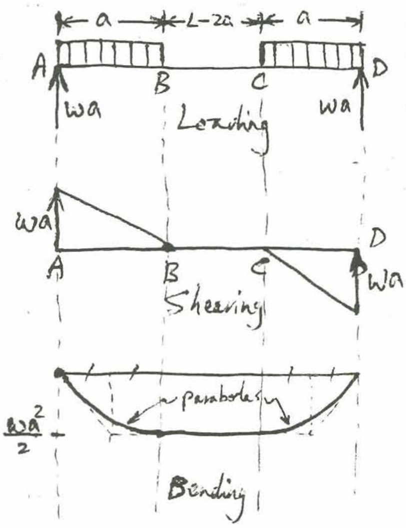
Cut at J and consider the right half. $\therefore M_J = 15 * 10 - \int_0^{\pi/2} \left(30 * \frac{10\,d\theta}{10\pi}\right) * 10 \cos\theta$

$= 150 - \frac{300}{\pi} \int_0^{\pi/2} \cos\theta\, d\theta = 150 - \frac{300}{\pi} \cdot [\sin\theta]_0^{\pi/2} = 150 - \frac{300}{\pi} = 54.5$ lb.in

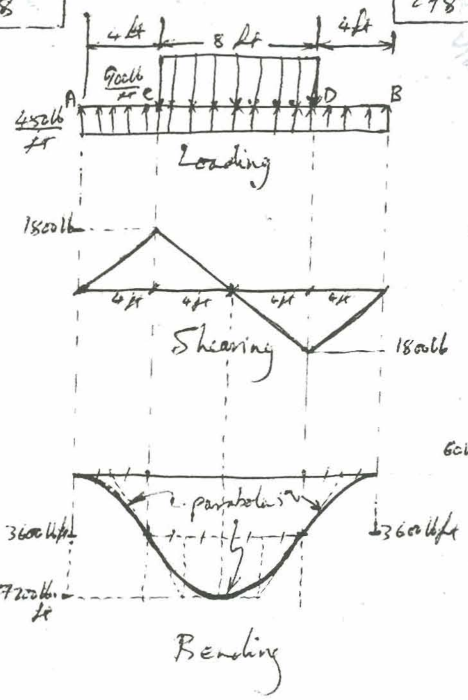
$\therefore$ The bending moment at J on JC (when $\theta = 90°$) is 54.5 lb.in

---

7.22 / 277



7.29 / 278

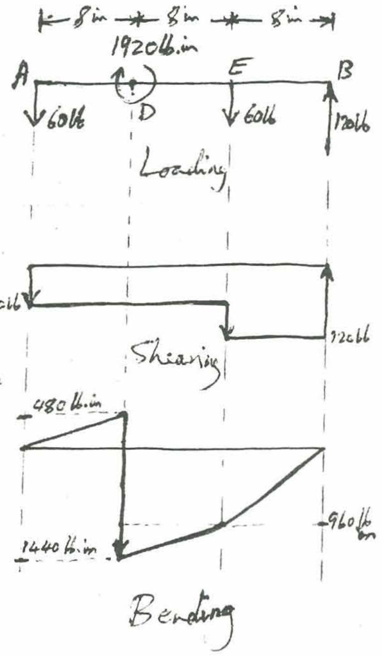


7.32 / 278

Cut CF and take $\Sigma M_B = 0$

$\therefore CF = \frac{60 * (24 + 8)}{3} = 640\ lb$

Cut at D $\therefore M_D = 1920$ lb.in

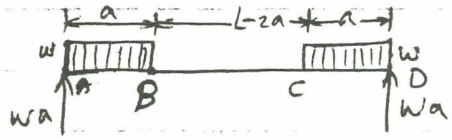


بسم الله الرحمن الرحيم

7.40 / 286

Reaction at A & B is equal to wa

Let A be the origin ∴ for any section x from A the following hold



	$x=0$ (A)	$0<x\le a$	$a<x<L-a$	$L-a\le x<L$	$x=L$ (B)
Loading	wa	w	0	w	wa
Shearing	wa	$wa-wx$	0	$-w(x-L+a)$	wa
Bending	0	$wax-\frac{wx^2}{2}$	$\frac{wa^2}{2}$	$-w\left(\frac{x^2}{2}-Lx+ax+\frac{L^2}{2}-aL\right)$	0

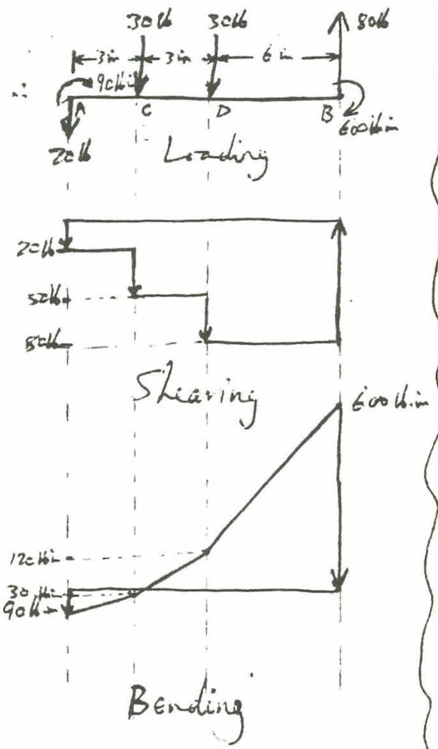
This ties up with the diagrams shown earlier in $\boxed{P\,7.22 / 277}$.

7.48 / 286

$\Sigma M_A=0$ ∴ $B_y=80$ lb ↑

∴ $A_y=20$ lb ↓

∴



7.56 / 287

By symmetry ∴ $A_y=B_y$

$=\frac{1}{2}\int_0^L w_0\sin\frac{\pi x}{L}\,dx=$

$=\frac{w_0}{2}\cdot\frac{-\cos\frac{\pi x}{L}}{\frac{\pi}{L}}\Big|_0^L=\frac{w_0L}{2\pi}(1+1)=$

$=w_0L/\pi$

∴ Shearing $=\frac{w_0L}{\pi}-\int_0^x w_0\sin\frac{\pi x'}{L}\,dx'=$

$=\frac{w_0L}{\pi}+\frac{w_0L}{\pi}\cdot\left(\cos\frac{\pi x}{L}-1\right)$

$=\frac{w_0L}{\pi}\cdot\cos\left(\frac{\pi x}{L}\right)$.

∴ Bending $=C+\int_0^x\frac{w_0L}{\pi}\cos\frac{\pi x'}{L}\,dx'=$

$=\frac{w_0L}{\pi}\cdot\frac{\sin\frac{\pi x'}{L}}{\pi/L}\Big|_0^x=\frac{w_0L^2}{\pi^2}\sin\left(\frac{\pi x}{L}\right)$.

∴ Bending is maximum at

$\frac{\pi x}{L}=\frac{\pi}{2}\Rightarrow x=L/2$

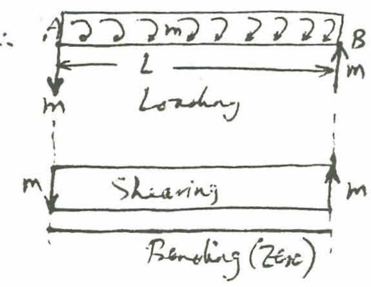
with value of w_0L^2/π^2.

7.60 / 287

(a) $\Sigma M_A=0$ ∴ $B_y*L=mL$

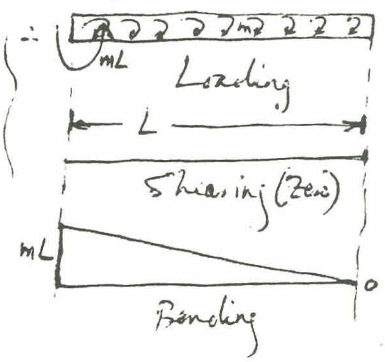
∴ $B_y=m$ ↑ ∴ $A_y=m$ ↓

∴



(b) $A_y=0$ & $M_A=mL$

∴



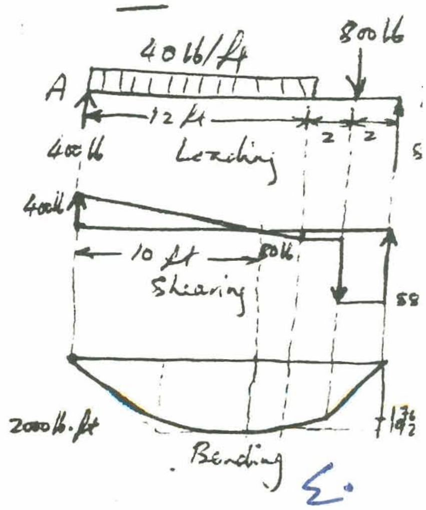
7.50 / 286

$\Sigma M_A=0$ ∴ $B=\frac{40*12*6+800*14}{16}=880$ lb

∴ $A=40*12+800-880=400$ lb

Location of max. bending is at zero shearing, x from A, ∴ $x=\frac{400}{40}=10$ ft

∴ Magnitude of Max. Bending $=\frac{400*10}{2}=2000$ lb.ft



ع٠

بسم الله الرحمن الرحيم

8.2 / 314

(a) $P\cos\beta = 800\sin\beta + \mu_s * (P\sin\beta + 800\cos\beta)$, $\beta = 30$ & $\mu_s = 0.35$

$\therefore P = \dfrac{800\sin 30 + .35 * 800\cos 30}{\cos 30 - .35 * \sin 30} = 929.8\ N \leftarrow$

(b) $P\cos\beta + \mu_s * (P\sin\beta + 800\cos\beta) = 800\sin\beta$, $\beta = 30$ & $\mu_s = 0.35$

$\therefore P = \dfrac{800\sin 30 - .35 * 800\cos 30}{\cos 30 + .35 * \sin 30} = 151.3\ N \leftarrow$

8.4 / 314

(a) $P\cos(\theta - \alpha) = W\sin\alpha + \mu_s * (W\cos\alpha - P\sin(\theta - \alpha))$

$\therefore P = W \cdot \dfrac{\sin\alpha + \mu_s\cos\alpha}{\cos(\theta-\alpha) + \mu_s\sin(\theta-\alpha)}$ minimum when $P' = 0$ or $-\sin(\theta-\alpha) + \mu_s\cos(\theta-\alpha) = 0$

or $\tan(\theta - \alpha) = \mu_s$ $\therefore \theta = \alpha + \tan^{-1}\mu_s = 25 + \tan^{-1} 0.2 = 36.3^\circ$

$\therefore P = 40 \cdot \dfrac{\sin 25 + .2\cos 25}{\cos(36.3-25) + .2\sin(36.3-25)} = 23.7\ lb$ $\therefore$ Min. P is $23.7\ lb \angle 36.3^\circ$.

(b) $P\cos(\theta-\alpha) + \mu_s * (W\cos\alpha - P\sin(\theta-\alpha)) = W\sin\alpha$

$\therefore P = W \cdot \dfrac{\sin\alpha - \mu_s\cos\alpha}{\cos(\theta-\alpha) - \mu_s\sin(\theta-\alpha)}$ minimum when $P' = 0$ or $-\sin(\theta-\alpha) - \mu_s\cos(\theta-\alpha) = 0$

or $\tan(\theta-\alpha) = -\mu_s$ $\therefore \theta = \alpha - \tan^{-1}\mu_s = 25 - \tan^{-1}.2 = 13.7^\circ$

$\therefore P = 40 \cdot \dfrac{\sin 25 - .2\cos 25}{\cos(13.7-25) - .2\sin(13.7-25)} = 9.47\ lb$ $\therefore$ Min. P is $9.47\ lb \angle 13.7^\circ$.

8.14 / 315

$A_y = 200 - B_y$ & $\Sigma M_A = 0$ $\therefore B_y * 5 = 200 * 2.5 + C_x(\leftarrow) * 6$

& $C_x = .15 A_y + .25 B_y = .15 * (200 - B_y) + .25 B_y = 30 + .1 * B_y =$

$30 + .1 * \dfrac{200 * 2.5 + 6C_x}{5} = 30 + 10 + .12 C_x$ $\therefore C_x(1 - .12) = 40$

$\therefore C_x = \dfrac{40}{.88} = 45.5\ lb \leftarrow$

8.18 / 316

Take $\Sigma M_{A'} = 0$ $\therefore W * \frac{L}{2} * \cos\theta = T * (\sin\beta * L\sin\theta + \cos\beta * L\cos\theta) =$

$= T * L * \cos(\theta - \beta)$ $\therefore T = \dfrac{W\cos\theta}{2\cos(\theta-\beta)}$ $\therefore A'_y = W - T\cos\beta$

$\therefore A'_x = \mu_s \cdot A'_y = T\sin\beta$ $\therefore \mu_s * (W - T\cos\beta) = T\sin\beta$ $\therefore \mu_s W = T * (\sin\beta + \mu_s\cos\beta)$

$= \dfrac{W\cos\theta}{2\cos(\theta-\beta)} \cdot (\sin\beta + \mu_s\cos\beta)$ $\therefore$ With $\mu_s = .4$ & $\theta = 30^\circ$ $\therefore .4 = \dfrac{\cos 30 * (\sin\beta + .4\cos\beta)}{2\cos(30-\beta)}$

$\therefore .9238\cos(30 - \beta) = \sin\beta + .4\cos\beta$ $\therefore .9238(\cos 30\cos\beta + \sin 30\sin\beta) = \sin\beta + .4\cos\beta$

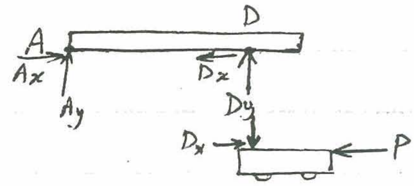
$\therefore \tan\beta = \dfrac{.9238\cos 30 - .4}{1 - .9238\sin 30} = .7433$ $\therefore \beta = 36.6^\circ$

8.19 / 316

$\Sigma M_A = 0 \quad \therefore D_y * 8 = 1200 * 5 \quad \therefore D_y = \frac{6000}{8} = 750\ lb \uparrow$

$\therefore A_y = 1200 - 750 = 450\ lb \uparrow$

$\therefore A_x = .3 * A_y = .3 * 450 = 135\ lb \rightarrow$



& $D_x = .3 * D_y = .3 * 750 = 225\ lb$

$\therefore$ The beam can be moved into the platform when P is slightly greater than 135 lb whereby no slip occure at D because the force P is less than D_x of 225 lb.

8.24 / 317

$A_x = 0.2 * A_y \rightarrow \quad , \quad B_y = 0.2 * B_x \uparrow$ due to slipping.

$\Sigma M_A = 0 \quad \therefore 7.5 * P + 4 * 10 = B_x * 15 + B_y * 8 = 15 B_x + .2 B_x * 8$

$= 16.6 B_x \quad \therefore B_x = \frac{40 + 7.5P}{16.6} = 2.41 + 0.452 P$

& $F_y = 0 \quad \therefore A_y = 10 - B_y = 10 - .2 B_x = 10 - .2(2.41 + .452P) = 9.518 - .0904P$

& $F_x = 0 \quad \therefore P = -A_x + B_x = -.2A_y + B_x = -.2(9.518 - .0904P) + 2.41 + .452P$

$= 0.506 + .47P \quad \therefore P(1 - .47) = 0.506 \quad \therefore P = \frac{.506}{.53} = 0.955\ lb.$

8.27 / 318

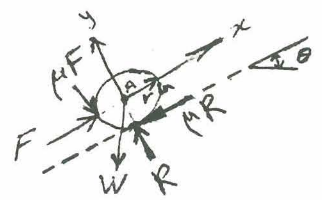
$\Sigma M_A = 0 \quad \therefore 250 * 120 + 5 * 9.81 * (50 + 150) = 200\ C_y \quad \therefore C_y = 199.1\ N \uparrow$

$\therefore C_x = \mu_s * 199.1 \leftarrow$. Cut B and take $\Sigma M_B = 0 \quad \therefore 5 * 9.81 * 50 = 199.1 * 160 - C_x * 500$

$\therefore C_x = 34.91\ N \quad \therefore \mu_s = \frac{34.91}{199.1} = 0.175$

8.40 / 320

$\Sigma F_x = 0 \quad \therefore F - W\sin\theta - \mu R = 0$ (1) Assuming no roll



$\Sigma F_y = 0 \quad \therefore R - W\cos\theta - \mu F = 0$ (2)

(2) in (1) $\therefore F - W\sin\theta - \mu(W\cos\theta + \mu F) = 0$

$\therefore F = W \cdot \frac{\sin\theta + \mu\cos\theta}{1 - \mu^2}$, into (2) $\therefore R = W\cos\theta + \mu \cdot W \cdot \frac{\sin\theta + \mu\cos\theta}{1 - \mu^2} = \frac{W(\cos\theta + \mu\sin\theta)}{1 - \mu^2}$.

For $\mu = 0.2$, $\theta = 12^\circ$, $W = 2\ lb$ $\therefore F = 0.841\ lb$ & $R = 2.12\ lb$

$\therefore \Sigma M_A = r \cdot \mu(2.12 - .841) = .2(1.28)r = .256r\ lb.in$, $\therefore$ can A will roll, $\therefore$ friction will be smaller than μR, assume it R_x $\therefore \Sigma M_A = 0 \therefore r \cdot \mu F = r \cdot R_x \quad \therefore R_x = \mu F$

$\therefore$ (1) modifies to $F - W\sin\theta - R_x = 0 \quad \therefore F - W\sin\theta - \mu F = 0 \quad \therefore F = \frac{W\sin\theta}{1 - \mu} = \frac{2\sin 12}{1 - .2} = 0.520\ lb.$

بسم الله الرحمن الرحيم

9.2 / 357

$$I_y = \int_0^a (y\,dx)\cdot x^2 = \int_0^a x^2\cdot\left(b\sqrt{\tfrac{x}{a}}\right)dx = \frac{b}{\sqrt a}\cdot\frac{x^{7/2}}{7/2}\Big|_0^a = \frac{2b}{7\sqrt a}\cdot a^3\sqrt a = \frac{2a^3 b}{7}.$$

9.14 / 358

Assume the required midpoint to be the origin; $x \parallel$ bottomside & $y \parallel$ leftside.

$\therefore$ (a) $I_x = \int_0^a (2a\,dy)\,y^2 = 2a\,\frac{y^3}{3}\Big|_0^a = 2a\cdot\frac{a^3}{3} = 2a^4/3$

, $I_y = \int_{-a}^{a} (a\,dx)\,x^2 = a\cdot\frac{x^3}{3}\Big|_{-a}^{a} = \frac{a}{3}(a^3+a^3) = 2a^4/3$

$\therefore I_z = I_x + I_y = \frac{4a^4}{3}$ & $k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{4a^4/3}{2a^2}} = \sqrt{2a^2/3} = \sqrt{\frac{2}{3}}\cdot a.$

& (b), $\therefore I_x = \int_{-a/2}^{a/2} (2a\,dy)\,y^2 = 2a\,\frac{y^3}{3}\Big|_{-a/2}^{a/2} = \frac{2a}{3}\left[\frac{a^3}{8}+\frac{a^3}{8}\right] = \frac{2a}{3}\cdot\frac{a^3}{4} = \frac{a^4}{6}$

& $I_y = \int_0^{2a} (a\,dx)\,x^2 = a\frac{x^3}{3}\Big|_0^{2a} = \frac{a}{3}\cdot 8a^3 = \frac{8a^4}{3}$

$\therefore I_z = I_x + I_y = \frac{a^4}{6} + \frac{8a^4}{3} = \frac{17a^4}{6}$ & $k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{17a^4/6}{2a^2}} = \sqrt{\frac{17}{12}}\,a.$

9.16 / 358

(a) $I_z = \iint [(r\,d\theta)(dr)]\,r^2 = \int_0^{2\pi}\int_{R_1}^{R_2} r^3\,dr\,d\theta = \int_0^{2\pi} \frac{r^4}{4}\Big|_{R_1}^{R_2} d\theta = \frac{R_2^4 - R_1^4}{4}\cdot\theta\Big|_0^{2\pi} = \frac{\pi}{2}(R_2^4 - R_1^4)$

$A = \pi(R_2^2 - R_1^2)$

$\therefore k_z^2 = \frac{I_z}{A} = \frac{(\pi/2)(R_2^4 - R_1^4)}{\pi(R_2^2 - R_1^2)} = \frac{1}{2}\cdot\frac{\cancel{(R_2^2 - R_1^2)}(R_2^2 + R_1^2)}{\cancel{(R_2^2 - R_1^2)}} = \frac{R_2^2 + R_1^2}{2}$

But $t = R_2 - R_1$ & $R_m = \frac{R_1 + R_2}{2}$ $\therefore R_1 = R_m - \frac{t}{2}$, $R_2 = R_m + \frac{t}{2}$

$\therefore k_z^2 = \frac{1}{2}\left[\left(R_m + \frac{t}{2}\right)^2 + \left(R_m - \frac{t}{2}\right)^2\right] = \frac{1}{2}\left[R_m^2 + \frac{t^2}{4} + \cancel{tR_m} + R_m^2 + \frac{t^2}{4} - \cancel{tR_m}\right] = R_m^2 + \frac{t^2}{4}$

$\therefore k_z = k_o = \sqrt{R_m^2 + \frac{t^2}{4}} \cong$ (for $t \ll R_m$) R_m

(b) percentage error, e, due to approximation $= \frac{\left(\sqrt{R_m^2 + \frac{t^2}{4}} - R_m\right)\times 100}{\sqrt{R_m^2 + \frac{t^2}{4}}} = \frac{\left(\sqrt{1+\eta^2/4} - 1\right)\times 100}{\sqrt{1+\eta^2/4}}$, where $\eta = t/R_m$

$\therefore$ for $\eta = 1$ $\therefore e = 10.6\ \%$.

& for $\eta = 0.5$ $\therefore e = 2.99\ \%$.

& for $\eta = 0.1$ $\therefore e = 0.125\ \%$.

9.20 / 366. The moment of inertia of a rectangular area $(a \times b)$ about a centroidal axis // to a is $\frac{ab^3}{12}$, and about side a is $\frac{ab^3}{3}$.

$\therefore$ I_x of the shaded area by symmetry = 2 * moment of area above x-axis =

$= 2 * \left[\frac{5^3}{3} * 80 + \frac{45^3}{3} * 10\right] = 0.6142 * 10^6 \text{ mm}^4 = 61.42 \text{ Cm}^4$

& $A = 2 * [5 * 80 + 45 * 10] = 1700 \text{ mm}^2 = 17 \text{ Cm}^2$

$\therefore k_x = \sqrt{\frac{I_x}{A}} = 1.9 \text{ Cm}$

9.28 / 366. Assume point B is the origin, $\therefore \bar{y} = -90$ mm (by symmetry) & $\bar{x} = -\frac{120 * 180 * 60 - 80 * ...}{120 * 180 - 80 * ...} = -76$ mm

$\therefore$ The centroid is at $(-76, -90)$ & the area $= 12000 \text{ mm}^2$.

$\therefore I_{\bar{x}} = 2 * \left(\frac{120 * 90^3}{3} - \frac{80 * 60^3}{3}\right) = 4.68 * 10^7 \text{ mm}^4$

& $I_y = 180 * \frac{120^3}{3} - 120 * \frac{80^3}{3} = 8.32 * 10^7 \text{ mm}^4 = I_{\bar{y}} + A \cdot \bar{x}^2$

$\therefore I_{\bar{y}} = 8.32 * 10^7 - 1.2 * 10^4 * (76)^2 = 1.3888 * 10^7 \text{ mm}^4$

$\therefore$ The centroidal polar moment of inertia $= I_{\bar{x}} + I_{\bar{y}} = 6.0688 \times 10^7 \text{ mm}^4$

9.30 / 367. (a) $I_{z_0} = \iint [(r\,d\theta)(dr)]\, r^2 = \int_0^{\pi/2}\int_{100}^{150} r^3\, dr\, d\theta = \frac{r^4}{4}\Big|_{100}^{150} * \theta\Big|_0^{\pi/2} = \frac{150^4 - 100^4}{4} \cdot \left(\frac{\pi}{2} - 0\right) = 1.595 \times 10^8 \text{ mm}^4$

$\therefore I_{z_0} = 1.595 * 10^8 \text{ mm}^4$

(b) The centroid is on the line of symmetry of $\theta = 45°$ $\therefore \bar{x} = \bar{y}$

$A = \frac{\pi}{4}(150^2 - 100^2) = 9.817 * 10^3 \text{ mm}^2$

$M_x = \int_0^{\pi/2} \left[(150\, d\theta) * \frac{150}{2}\right] * \frac{2}{3} * 150 * \sin\theta - \left[(100\, d\theta) * \frac{100}{2}\right] * \frac{2}{3} * 100 \sin\theta$

$= \int_0^{\pi/2} 1.125 * 10^6 \sin\theta\, d\theta - .333 * 10^6 \sin\theta\, d\theta = .7917 * 10^6 \int \sin\theta\, d\theta = .7917 * 10^6 * (-\cos\theta)\Big|_0^{\pi/2}$

$= 0.7917 * 10^6 \text{ mm}^3$ $\quad \therefore \bar{y} = \frac{.7917 * 10^6}{9.817 * 10^3} = 80.64 \text{ mm} = \bar{x}$

$\therefore$ Using //'l axis theorem,

$\therefore I_{z_{(\bar{x},\bar{y})}} = I_{z_0} - A * (\bar{x}^2 + \bar{y}^2) = 1.595 * 10^8 - 9.817 * 10^3 * 2 * 80.64^2 = 3.182 * 10^7 \text{ mm}^4$

$\therefore I_{z_{(\bar{x},\bar{y})}} = 3.182 * 10^7 \text{ mm}^4$

9.38 / 368. Assume the line AA' to be the x-axis and y is mid normal to it.

$\therefore$ Pressure force, $P = 2 * \int_0^{-h} (x\, dy)\, \rho g\, y = 2\rho g \int_0^{-h} y \cdot \left(\frac{a}{2} + \left(\frac{a}{2} - \frac{b}{2}\right)\frac{y}{h}\right) dy =$

$= \rho g \cdot \left[a\frac{y^2}{2} + \frac{a-b}{h} \cdot \frac{y^3}{3}\right]_0^{-h} = -\rho g\left[0 - \frac{ah^2}{2} + \frac{a-b}{3h}(0 + h^3)\right] = -\rho g\left(-\frac{ah^2}{2} + \frac{a-b}{3} \cdot h^2\right) = \rho g h^2 \cdot \frac{a+2b}{6}$

$\therefore$ Depth, D, of point of application of P * P = $M_x = 2 * \int_0^{-h} [(x\, dy) \rho g y] \cdot y = 2\rho g \int_0^{-h} y^2 \left(\frac{a}{2} + \frac{a-b}{2} \cdot \frac{y}{h}\right) dy =$

$= \rho g \cdot \left[a \cdot \frac{y^3}{3} + \frac{a-b}{h} \cdot \frac{y^4}{4}\right]_0^{-h} = \rho g \cdot \left[-\frac{ah^3}{3} + \frac{a-b}{4h} \cdot h^4\right] = \rho g h^3\left(-\frac{a}{3} + \frac{a}{4} - \frac{b}{4}\right) = -\rho g h^3 \cdot \frac{a+3b}{12} = P \cdot D$

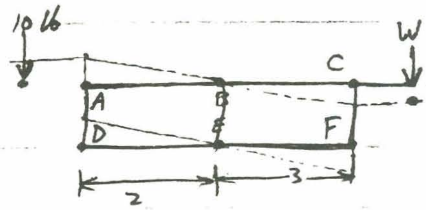
$\therefore D = \frac{M_x}{P} = \frac{-\rho g h^3 (a+3b)/12}{\rho g h^2 (a+2b)/6} = -h \cdot \frac{a+3b}{a+2b} \cdot \frac{1}{2} = -\frac{h}{2} \cdot \frac{a+3b}{a+2b}$

$\therefore$ The point of application of the hydrostatic forces is $\left(\frac{h}{2} \cdot \frac{a+3b}{a+2b}\right)$ below surface of water.

10.2 / 415 Assume W to drop slightly down with angle $d\theta$ made by BC

$\therefore dW = 0 = W * 3\, d\theta - 10 * 2\, d\theta$

$\therefore 3W - 20 = 0 \quad \therefore W = \frac{20}{3} = 6.67$ lb.



10.5 / 415 $P\, d(3l\cos\theta) + Q\, d(2l\sin\theta) = 0 \quad \therefore -3Pl\sin\theta\, d\theta + 2Ql\cos\theta\, d\theta = 0 \quad \therefore Q = \frac{3P}{2}\tan\theta = \frac{3*40\tan 20}{2} = 21.8$ N

10.6 / 415 Assume a change of $d\theta$ in the value of θ.

$\therefore$ D before change is given by $(3l\cos\theta, -l\sin\theta)$ where A is the origin.

& $\therefore$ D after change is given by $\left(3l\cos\theta + d(3l\cos\theta), -l\sin\theta + d(-l\sin\theta)\right)$

$\therefore dW = 0 = P * d(3l\cos\theta) - Q * d(-l\sin\theta) = P*3l(-\sin\theta\, d\theta) + Q\, l\cos\theta\, d\theta$

$\therefore -3P\sin\theta + Q\cos\theta = 0 \quad \therefore Q = 3P\tan\theta$.

10.7 / 416 Let A be the origin, and assume a change $d\theta$ in θ.

$\therefore \widehat{ABC} = \pi - 2\theta \quad \therefore d\widehat{ABC} = -2\, d\theta$

& y-coordinates of $D = y_D = b\sin\theta$

$\therefore dy_D = b\cos\theta\, d\theta$

$\therefore dW = 0 = -P\, dy_D - M\, d\widehat{ABC} = -Pb\cos\theta\, d\theta - M(-2\, d\theta) = (-Pb\cos\theta + 2M)\, d\theta$

$\therefore 2M = Pb\cos\theta \quad \therefore M = \frac{P \cdot b \cdot \cos\theta}{2}$.

10.12 / 416 Assume θ to change by $d\theta$. $\therefore$ C will move by $d(l\sin\theta) \rightarrow$

& a will by $d(l\cos\theta) \downarrow$

$\therefore$ BA will by $d\left(\frac{l\cos\theta}{2l}\right) = \frac{-l\sin\theta\, d\theta}{2l} = \frac{-\sin\theta\, d\theta}{2}$

$\therefore dW = 0 = -P.(d\, l\sin\theta) - M.\left(\frac{-\sin\theta\, d\theta}{2}\right) = -Pl\cos\theta\, d\theta + \frac{M\sin\theta}{2}\, d\theta$

$\therefore Pl\cos\theta = \frac{M\sin\theta}{2} \implies M = 2\, P.l.\cot\theta$.

10.14 / 417 Assume C to change by dc $\therefore$ B moves by $dc \rightarrow$

$\therefore$ A moves by $d\sqrt{18^2 - 8^2 - c^2} \downarrow = d\sqrt{260 - c^2} \downarrow = \frac{-2c\, dc}{2\sqrt{260 - c^2}} \downarrow$

$\therefore dW = 0 = 5.6 * \left(\frac{-c\, dc}{\sqrt{260 - c^2}}\right) + P * (dc) \quad \therefore P = \frac{5.6c}{\sqrt{260 - c^2}}$.

(a) for $c = 14$ in $\therefore P = 9.8$ lb

(b) for $c = 16$ in $\therefore P = 44.8$ lb.

10.24
418

Assume a change of $d\theta$ in the value of θ.

$\therefore$ C will go down by $d(125 \sin\theta)$.

$\therefore$ B will go down by $\frac{200}{200+100} * d(125 \sin\theta) = \frac{2}{3} * 125 \cos\theta \, d\theta = \frac{250 \cos\theta d\theta}{3}$

The elongation of the spring $= 100 * \theta$

$\therefore$ Force of spring $= k * 100 * \theta = \frac{10 * 10^3}{1 * 10^3} * 100 * \theta = 10^3 * \theta$

$\therefore$ Increase of spring elongation $= d(100 * \theta) = 100 \, d\theta$

$\therefore$ $dW = 0 = W * (\frac{250}{3} \cos\theta \, d\theta) - 10^3 * \theta * (100 \, d\theta)$

$\therefore$ $800 * \frac{250}{3} * \cos\theta - 10^5 \theta = 0$

$\therefore$ $\frac{2}{3} * 10^5 \cos\theta - 10^5 \theta = 0$

$\therefore$ $2 \cos\theta - 3\theta = 0$

Let $f(\theta) = 2\cos\theta - 3\theta$

$\therefore$ Using Newton-Raphson method to find its root,

$\therefore$ $\theta_{n+1} = \theta_n - \frac{f(\theta_n)}{f'(\theta_n)} = \theta_n - \frac{2\cos\theta_n - 3\theta_n}{(-2\sin\theta_n - 3)} = \theta_n + \frac{2\cos\theta_n - 3\theta_n}{2\sin\theta_n + 3}$

start with $\theta_0 = 0.0$ rad

$\therefore$ $\theta_1 = 0 + \frac{2\cos 0 - 3*0}{2\sin 0 + 3} = \frac{2}{3} = 0.667$ rad

$\therefore$ $\theta_2 = .667 + \frac{2\cos .667 - 3 * .667}{2\sin .667 + 3} = 0.566$ rad

$\therefore$ $\theta_3 = .566 + \frac{2\cos .566 - 3 * .566}{2\sin .566 + 3} = 0.5636$ rad

$\therefore$ $\theta_4 = 0.5636$ rad (the same as θ_3)

$\therefore$ θ of equilibrium is 0.5636 rad $= 32.3^\circ$