بسم الله الرحمن الرحيم

الحلول المختارة لطلاب الهندسة والعمارة
الكهارب الصناعية

إعداد

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الأستاذ المشارك بقسم الهندسة الكهربية والحاسبات

بجامعة أم القرى

الطبعة الثالثة

جمادى الآخرة ١٤٢٠هـ - أيلول ١٩٩٩

تمهيد

الحمد لله رب العالمين، والصلاة والسلام على سيد المرسلين، وعلى آله وصحبه أجمعين، وبعد: فهذه مجموعة من المسائل والحلول في مادة الكهارب الصناعية (EE411) لطلبة قسم الهندسة الكهربية والحاسبات بكلية الهندسة والعمارة الإسلامية بجامعة أم القرى، إخترتها لجمعها المنهج المقرر، إبان تدريسي لهذه المادة. وقد قمت بمراجعتها وتبويبها وفهرستها لتسهل المراجعة فيها وتعم الفائدة منها وأخرجت طبعتها الأولى في شعبان ١٤١٠هـ.

ثم زدت فيها حصيلة أخرى، مدرجا مسائل الأبواب المقررة أولا ثم متبعا إياها بالحلول، وبوبتها ووضعت فهرسا لكل من المسائل والحلول وساعدني مشكورا – جزاه الله خيرا – في دمجها وتبويبها وفهرستها المهندس عبدالباسط زيد عابد، فخرجت طبعتها الثانية في شوال ١٤١٤هـ.

ثم زدت فيها حصيلة ثالثة على نفس النظام، وساعدني مشكورين – جزاهما الله خيرا – في دمجها وتبويبها وفهرستها المهندسان خالد صدقة عتيق ورامي عبدالعزيز إكرام، فخرجت في طبعتها الثالثة هذه.

والله أسأل أن ينفع بهذا العمل كل من يطالعه، وأن لا يحرمني ومن أعانني من مثوبته في الآخرة والأولى، إنه سميع قريب مجيب.

فهرس المصطلحات

مصطلحات الكهارب الصناعية

العنوان بالعربي	العنوان بالإنجليزي
مفتاح (ف)	Thyristor
مقاومة حرارية (مح)	Thermal resistance
درجة مئوية (°م)	Degree Centigrade(°C)
مُثبت	Mounted
مبدد سخونة	Heat sink
حرارة منطقة الالتحام	Junction temperature
ظاهريا	Ambient
الساقط	Drop
مقوم سيليكوني محكوم(مسم)	SCR
ناقل(ت)	Transistor
التيار الفعال	i_{rms}
الجهد الفعال	v_{rms}
ت ع	i_{max}
ت و	i_{min}
ت ط	i_{av}
الفترة(ن)	Period(t)
التردد(د)	Frequency(f)
هي منطقة التلاحم بين السيليكون الغني بالبروتونان (الفقير من الكهارب)،والسيليكون الفقير من البروتونات (الغني بالكهارب)	Junction

فهرس المسائل

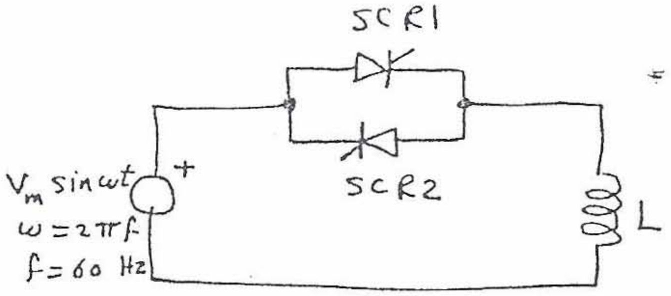
فهرس الحلول

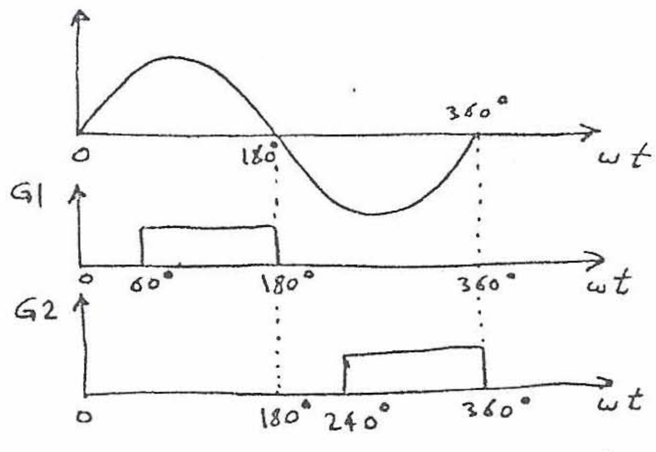
Set One: Ratings & Protection.

1–1 A thyristor having a thermal resistance of 0.5 °C/W is mounted on a heatsink of thermal resistance 1.°C/W. A current I flows through the thyristor for one third of the cycle only. Calculate the maximum value of I if the junction temperature is not to exceed 125 °C in an ambient of 35 °C. The voltage drop across the thyristor equals

$1 + .01\, I$, Volts.

1–2 A sinusoidal supply whose frequency is 60 Hz is connected to an ideal inductor through the two SCR's 1 and 2 as shown. The gate signals G1 and G2 are applied to SCR1 and SCR2 respectively. Determine the turn off times offered to SCR1 and SCR2. Sketch the voltage waveform across SCR1.





Repeat the above if G1 is applied to SCR2 and G2 is applied to SCR1.

1-3 A thyristor, Q, has the following ratings:

a) $i_{F_{av}} = 10$ Amps,

b) $i_{F_{max}} = 100$ Amp,

c) $V_{RB} = 300$ Volts,

d) $V_{FB} = 100$ Volts,

e) $V_{F_{on}} = 1$ Volt,

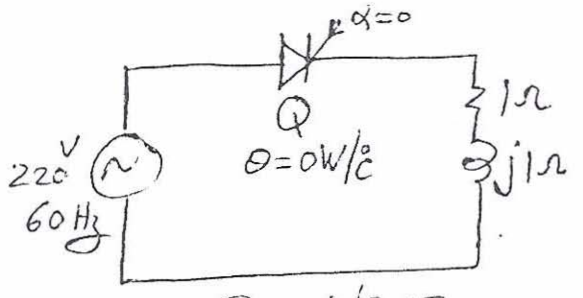
f) $dV/dt\big|_{max} = 10\ V/\mu sec$,

g) $di/dt\big|_{max} = 1\ KAmp/\mu sec$,

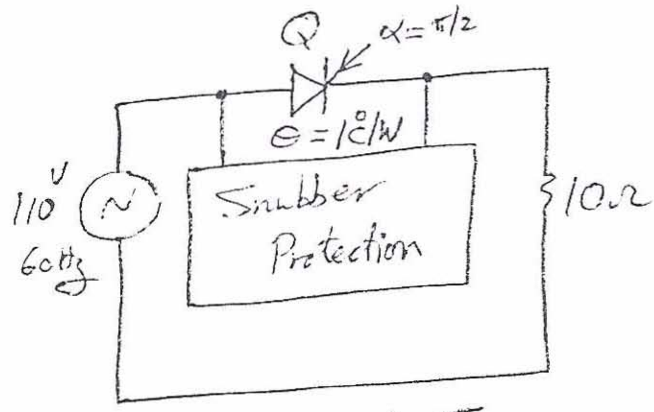
h) $t_q = 200$ ns,

i) $t_{on} = 10$ ns &

j) $T_j = 150\ ^\circ C$.



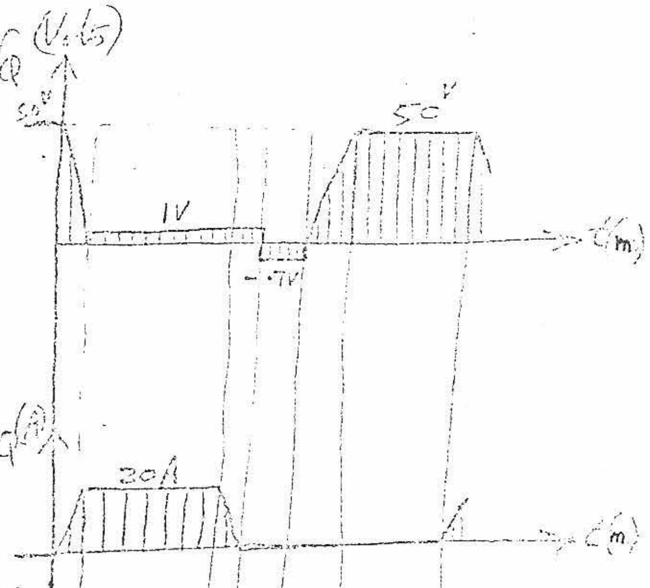
Circuit I



Circuit II

Will the two shown circuits, that use the above thyristor, operate satisfactorily? If not why?

1-4 In the Fig is shown the voltage, v_Q, across a thyristor carrying a current i_Q during one cycle. The thermal resistance of thyristor-heat-sink combination is $10°C/W$.



In what range shall the following ratings be:

a) V_{FB}, the forward blocking voltage,

b) V_{RB}, " reverse " ",

c) V_F, " forward on-state ",

d) $i_{F_{max}}$, " " maximum current,

e) $i_{F_{rms}}$, " " rms ",

f) $i_{F_{av}}$, " " average ",

g) t_{on}, the turn-on time,

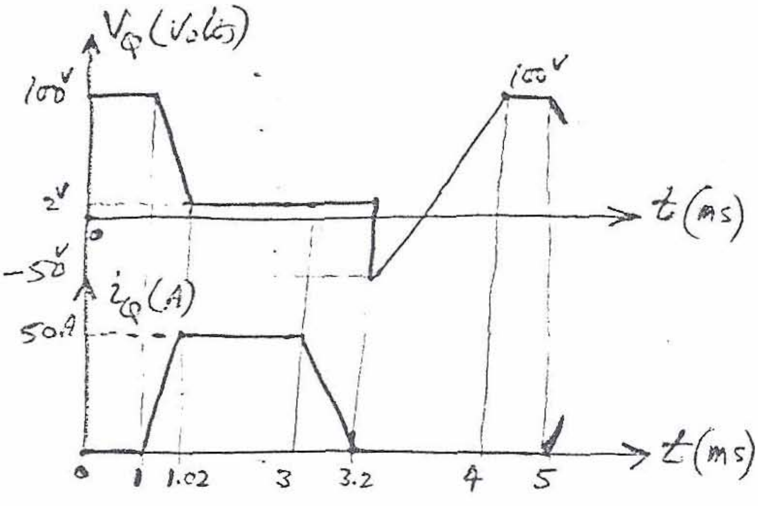
h) t_q, " recovery time,

i) dV/dt, max. rate of change of forward voltage,

j) di/dt, " " " " " current "

k) T_j, the junction temperature. Assume ambient of 30 °C

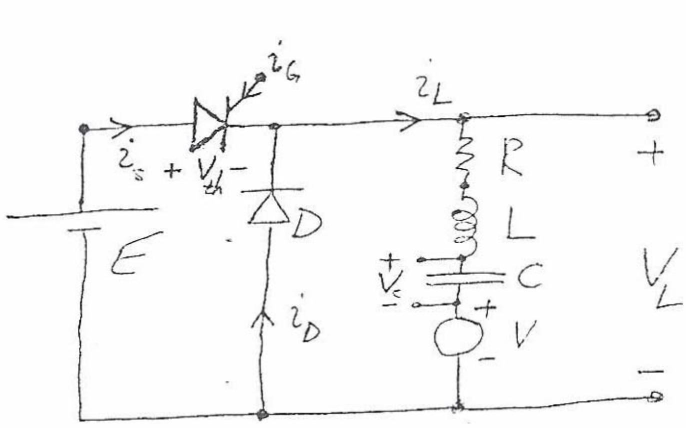
1-5 In the figure opposite, the voltage V_Q accross a thyristor with current I_Q are shown for one cycle. The thermal resistance of the thyristor-heat-sink combination is $5\ °C/W$. In what range shall the following thyristor ratings be:

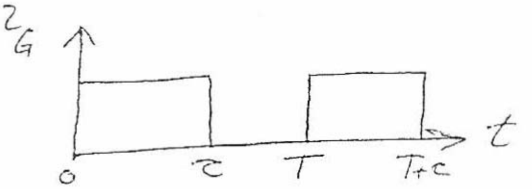


a) V_{FB}, the forward blocking voltage,
b) V_{RB}, " reverse " " ,
c) V_F, " forward on-state " ,
d) $i_{F\max}$, " " max. current,
e) i_{Frms}, " " rms " ,
f) i_{Fav}, " " average " ,
g) t_{on}, " turn-on time,
h) t_q, " recovery time,
i) dV/dt, max. rate of change of forward voltage,
j) di/dt, " " " " " current,
k) T_j, the junction temperature assuming $25°C$ ambient.

Set Two: Circuit Revision.

2-1 a) Find condition for the critical mode of current through the load shown in the Fig. asside.
Draw waveforms of
i, i_D, i_L, V_{th}, V_L & V_c.

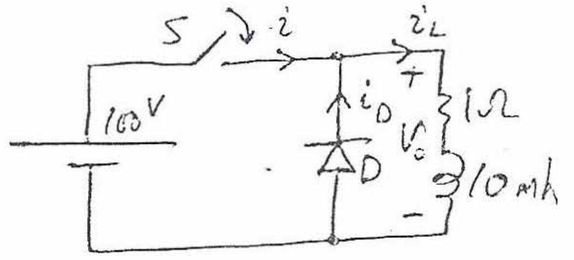




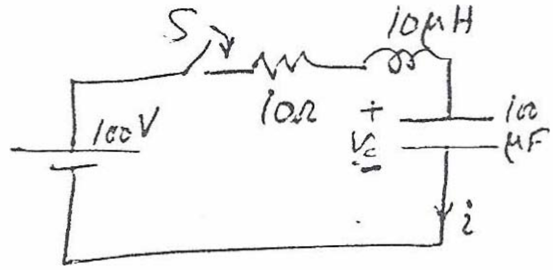
b) What is $\tau_{critical}$ for the above part if $R = 1\Omega$, $L = 0.1$ $T = 1sec$, $C = \infty$ & $V = 0$. Draw the corresponding waves and find $i_{L_{av}}$, $i_{L_{rms}}$, $V_{L_{av}}$ & $V_{L_{rms}}$.

c) Find Condition & sketch waveforms when $V = 0$.

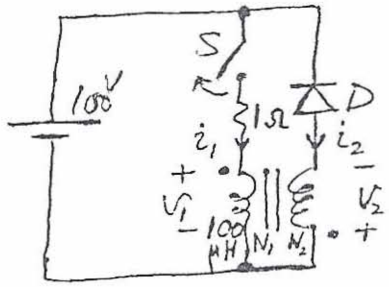
2-2 In the circuit shown, S is switched at $t=0$ when i_L was also zero. At $t = 10\ ms$, S is opened again. Sketch to scale and find the expressions for: i, i_L, i_D & V_o.



2-3 Determine the value of $\frac{di}{dt}$ immeditely after S is put on if the initial conditions were $i = 0$ & $V_c = 50$ Volt.



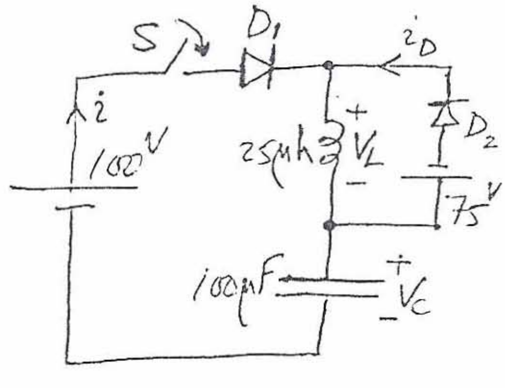
2-4 In the circuit shown, the transformer is assumed ideal with $\frac{N_1}{N_2} = \frac{100}{300}$. S was on for very long time, then at $t = 0$ it is switched off. Sketch and find the expressions for i_1, i_2, V_1 & V_2.



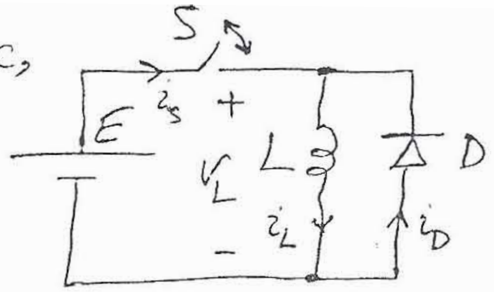
2-5 In the circuit shown, S is switched at $t = 0$, with currents in both diodes initially zero and with $-50V$ initial capacitor charge.

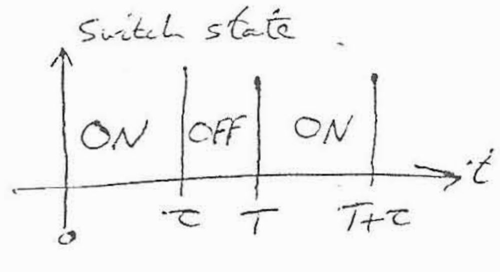
a) Sketch and find expression for i, V_L, V_c & i_D

b) Modify the circuit so that the energy supplied to the 75^V source is pumped back to the 100^V source without changing the time variations of i.

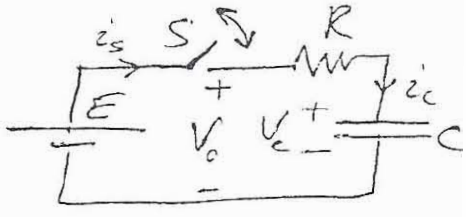


2-6 For the circuit shown, with OIC, the switch is put on & off repeatedly as shown. Sketch the waveforms for i_S, i_L, V_L & i_D. Is the operation of such circuit practical? Why?

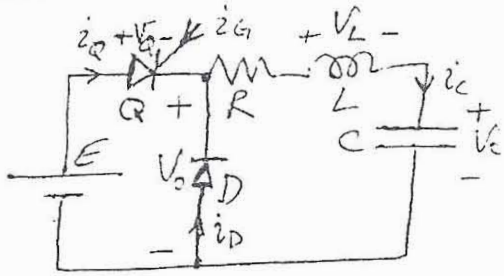


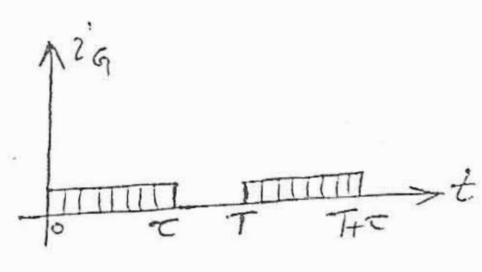


2-7 Repeat the above problem for the circuit shown with waveforms of i_S, i_C, V_o & V_C.



2-8 For the circuit shown, with OIC, the gate current of Q is given as shown, where the end of gate pulse indicates forced commutation. Sketch waveforms for i_Q, V_Q, V_o, V_L, V_C, i_D & i_S in the case when:



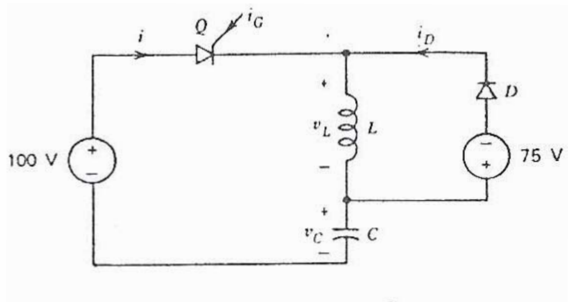


I) $R > 2\sqrt{L/C}$,
II) $R = 2\sqrt{L/C}$, and
III) $R < 2\sqrt{L/C}$. For each case, identify the condition for current mode to give:

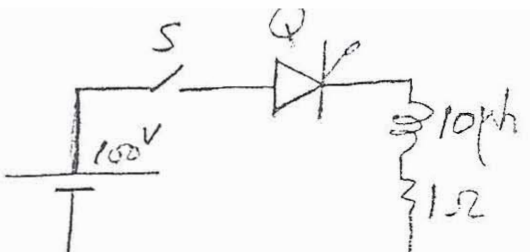
a) Contineous pattern,
b) Critical pattern, and
c) Discontineous pattern. Is the circuit applicable in practise? why?

2-9 Repeat the above problem for the case when $R = 0$ (oscilatory case).

2-10 In the circuit of Fig shown, $L = 30\ \mu H$, $C = 120\ \mu F$. If thyristor Q is turned on at $t = 0$, sketch to scale the subsequent time variations of i, v_L, v_C, and i_D. Capacitor C is initially charged to $V_{CO} = -75$ V.

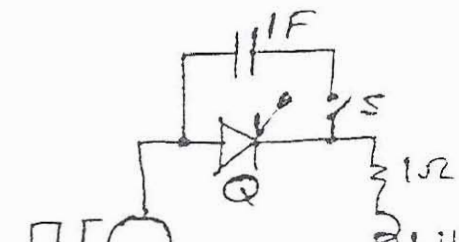


2-11 In Fig the thyristor Q is not gated and have an off-state resistance of 10 KΩ in the forward direction and a dV/dt limit of 50 KV/μsec. Determine, when the switch S is closed, whether or not the thyristor will ON; and if it does, why & what rate of di/dt does it starts passing?



Fig

2-12 In Fig, the source is a square waveform alternating between +100 Volts & −100 Volts at 6 rpm. Sketch the waveforms when no gate is applied for Q if

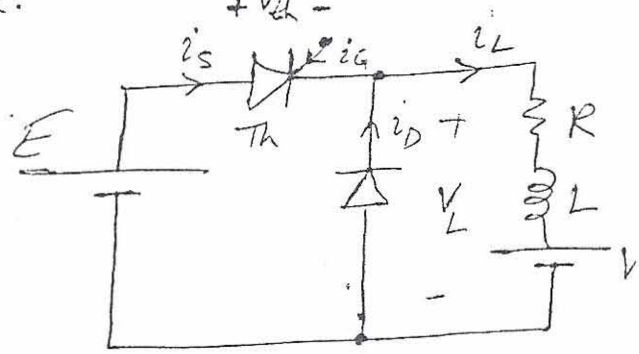


Fig

I) S is put on, and
II) S is put off. What difference does S makes to Q?

Set Three: Choppers

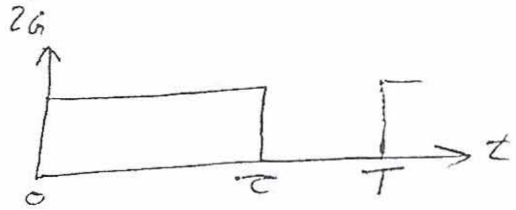
3-1 Analyze the circuit shown in the figure for discontinuous mode.



Sketch waveforms for:

i_S, i_L, i_D, V_{th} & V_L.

Find, also, expressions for $i_{L_{av}}$, $i_{L_{rms}}$ & power delivered to the load battery.



3-2 In the above figure:

$R = 10\ \Omega$,

$L = 10\ mh$,

$\tau = 3\ ms$,

$E = 10$ Volts &

$V = 5$ Volts.

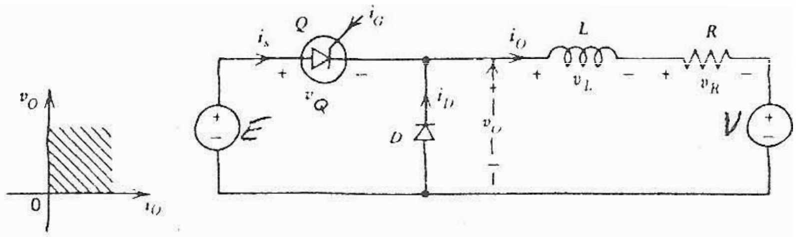
a) Find $T_{critical}$,

b) IF T was made $= (T_{critical} + \tau)/2$,

find $\%e$ in $i_{L_{AV}}$ due to linearization.

3-3 Repeat 3-1 for continuous mode.

3-4 In the circuit shown,



$E = 600$ Volt,

$V = 200$ Volt,

$L = 1mh$, $R = 1.5\,\Omega$, $\tau = 2.5\,ms$ & $T = 4\,ms$. Show that the output current i_o is discontinuous

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3-5 Draw circuit for bidirectional-current chopper-converter with positive output voltage. If the load is RL with:

$R = 1\,\Omega$,

$L = 1mh$,

$T = 2msec$,

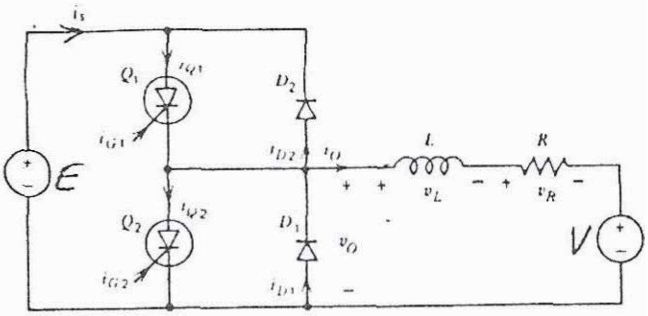
$\tau = 0.8\,ms$ &

$E = 100$ Volts;

Find, $i_{o_{av}}$, state current mode & commutation type. Justify any assumptions. Can the above mentioned load condition reduce the circuit? Explain.

Sketch your waveforms for i_G's, i_Q's, i_D's, v_o & i_o.

3-6 In the circuit shown, $E = 600$ Volt, $V = 200$ Volt, $L = 4$ mh, $R = 1.5\,\Omega$, $\tau = 2.5$ ms & $T = 4$ ms.



(a) Calculate the average output current I_O and the average output voltage V_O.
(b) Calculate I_{max} and I_{min}.
(c) Calculate the average value of the source current i_s.
(d) Sketch to scale the time variations of v_O, i_O, i_{Q1}, i_{Q2}, i_{D1}, i_{D2}, i_s.
(e) Explain in what way the time variations of v_O and i_O differ from those that would be obtained with the type of chopper circuit giving positive V_o & i_o.

3-7 Sketch the circuit of a full bridge chopper feeding RL load, showing the waveforms of i_G's, V_o, i_o, i_D's, i_Q's & i_s. Analyze the circuit & give expressions for: $V_o(t)$, $i_o(t)$, $i_{o_{av}}$, $i_{o_{rms}}$, $i_{s_{av}}$ & $i_{s_{rms}}$ assuming large L.

3-8 Draw a half bridge chopper circuit feeding RLV load.
If $E = 200$ volts, $R = 20\,\Omega$, $L = 0.1$ h, $V = 50$ Volts, $\tau = 7$ ms & $T = 10$ ms, then sketch:
i_G's, i_Q's, i_D's, i_s, i_o & V_o.
Find, $i_{o_{av}}$, $i_{o_{rms}}$, K_i, $V_{o_{av}}$, $V_{o_{rms}}$, K_v, power delivered to V, power supplied by E, diode ratings for $i_{F_{max}}$, $i_{F_{rms}}$ & $i_{F_{av}}$.
justify your assumptions & mention your mode of operation.

3-9 a) Sketch the circuit of a first quadrant chopper given the following data for an RLV load:

$E = 80$ Volts,

$R = 10\,\Omega$,

$L = 50$ mh,

$\tau = 15$ ms,

$T = 25$ ms &

$V = 30$ Volts.

$$I_{min} = -\frac{V}{R} + \frac{E}{R} \cdot \frac{e^{R\tau/L} - 1}{e^{RT/L} - 1}$$

$$I_{max} = -\frac{V}{R} + \frac{E}{R} \cdot \frac{1 - e^{-R\tau/L}}{1 - e^{-RT/L}}$$

b) Sketch the waveforms of i_G, i_Q, i_D, i_s, i_o, V_o & V_Q indicating the mode of current.

c) Find $V_{o_{av}}$, $i_{o_{av}}$, $V_{o_{rms}}$ & K_v.

3-10 Draw a Quarter Chopper to feed LV load with: $E = 200$ Volts, $\tau = 20$ ms, $T = 60$ ms, $L = 100$ mh and $V = 80$ Volts. Indicate your current mode of operation and sketch the waveforms of: i_G's ; i_Q's ; i_D's ; i_s, i_o, V_Q, V_D & V_o. Find: $i_{o_{av}}$, $i_{o_{rms}}$, K_i, $V_{o_{av}}$, $V_{o_{rms}}$, K_v, power delivered to V, power supplied by E; diode ratings for $i_{F_{max}}$, $i_{F_{av}}$, $i_{F_{rms}}$. Is τ, limited operationally?

3-11 a) Draw a half bridge chopper circuit feeding an RLV load with:

$E = 200$ Volts,

$\tau = 7ms$,

$T = 10ms$,

$R = 20\Omega$,

$L = 1.0\,hf$

$V = 50$ Volts.

Note:

$$I_{min} = -\frac{V}{R} + \frac{E}{R} \cdot \frac{e^{R\tau/L} - 1}{e^{RT/L} - 1}$$

$$I_{max} = -\frac{V}{R} + \frac{E}{R} \cdot \frac{e^{R\tau/L} - 1}{e^{RT/L} - 1} \cdot e^{\frac{R(T-\tau)}{L}}$$

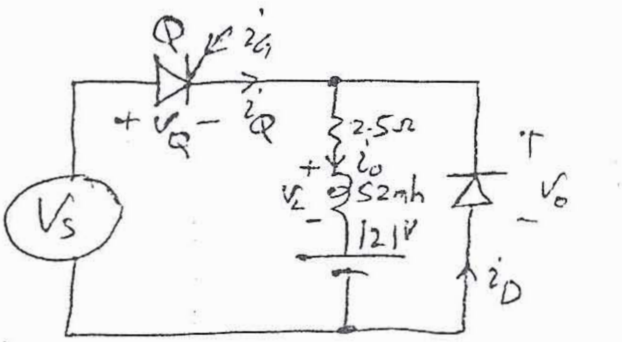
b) Sketch to scale the waveforms of:

i_G's, i_Q's, i_D's, i_S, i_o, v_o & voltage v_L, across L.

c) Find $i_{o_{av}}$, $i_{o_{rms}}$, K_i, $v_{o_{av}}$, $v_{o_{rms}}$, K_v, power delivered to V, power supplied by E, diode ratings for $i_{F_{max}}$, $i_{F_{rms}}$ & $i_{F_{av}}$. Justify your assumptions if any.

d) What mode of operation is obtained?

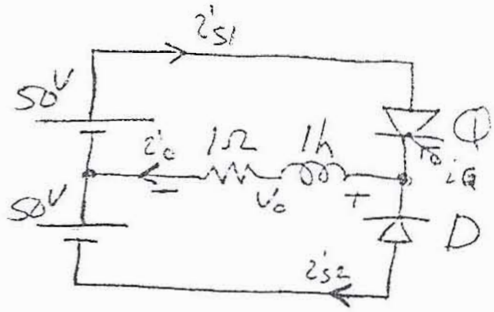
3-12 For the circuit shown, V_s is a 341 DCV voltage source, and Q is put on for $\tau = 7.00$ ms. The current was noted to be contineous with min. value of $I_{min} = 40.0$ Amp.



1. What is the specific function of the circuit?
2. Sketch i_G, i_Q, i_o, i_D, V_o, V_Q & V_L.
3. Find the period, T, of operation, the max. current, I_{max}, $V_{o_{av}}$, $i_{o_{av}}$, $V_{o_{rms}}$, $i_{o_{rms}}$, K_v, K_i, P_R, P_V & P_s.
4. What type of commutation is used here?
5. For Q, what are the ratings of I_{av}, I_{max}, I_{rms}, V_{FB}, V_{RB} ?
6. For D, what are the ratings of I_{av}, I_{max}, I_{rms}, V_{max}?
7. Can the current mode be discontineous? If yes, how should L be varied?

3-13 For the circuit shown

a) Classify the function,
b) Mention the operation quadrant of i_o & V_o;
c) For an on state duration, $\tau = 10$ ms, period, $T = 30$ ms, sketch, i_G, i_{S1}, i_{S2}, i_o, V_o, V_Q, V_D and find $V_{o_{av}}$, $V_{o_{rms}}$, $i_{o_{av}}$, K_v & state mode of operation.



3-14 Draw a full bridge chopper feeding a 5Ω–12V, R.V load from a 100 Volt supply at a chopping rate of 1KHz. duty ratio will you require to supply 500 W average power to the 5Ω load? What power does the battery recieve? What power does the source supply on average? Also find the ripple factor. Draw all associated waveforms.

Set Four: Inverters

4-1 In a sinusoidal pulse width modulated (SPWM) inverter, a triangular waveform with symmetrical peaks, P, was applied to the negative terminal of a comparator. The input signal applied to the positive terminal of the comparator is given by:

$V_{in} = A_o + A \sin \omega t$, where

$A_o, A > 0$ & $A_o + A \leq P$ & $\omega = \frac{2\pi}{NT}$ &

T is the triangular period.

The comparator output was used to gate the full bridge inverter switches giving a powerful output voltage V_o saturating at $\pm E$, where E is the dc supply voltage.

I) Find $V_o(t)$

II) Find Fourier Components of $V_o(t)$

III) Find the distortion factor, μ.

4-2 Given $A_o = 0$ & $A = P$ in the above problem; Find:

I) Fourier Components of $V_o(t)$

II) Distortion factor, μ. And

III) Plot the comparator inputs & output for $N = 10$

4-3 Deduce the spectrum of V_o & the distortion factor μ for:

I) $A_o = 0$, $A << P$ & $N \geq 10$

II) $N = \infty$

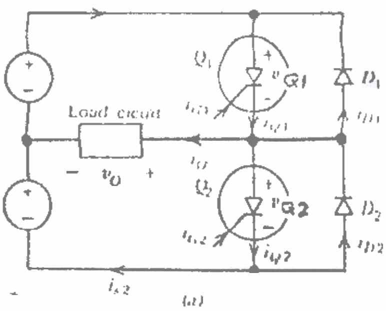
$\boxed{4\text{-}4}$ Solve $\boxed{4\text{-}2}$ for $N=2$ and phase $=0$.

$\boxed{4\text{-}5}$ Repeat $\boxed{4\text{-}4}$ for the case when the input signal lags the triangular by τ seconds, and check whether $\tau = 0$ gives solution of $\boxed{4\text{-}4}$. What should be the value of τ for n th harmonic component of zero.

$\boxed{4\text{-}6}$ Sketch circuit for full bridge chopper inverter using a two-wire battery of 100 Volts, $T = 1$ msec, $\tau = 750$ μsec, when load is RLV with:
$R = 1\Omega$, $L =$ very large & $V = 20$ Volts.
Sketch: i_G's, i_Q's, i_D's, v_o, i_o & i_s.
Find power delivered to V & ripple factor K_r.

$\boxed{4\text{-}7}$ If SPWM was used in $\boxed{4\text{-}6}$ with $V=0$ sketch circuit & waveforms and find the fundamental output voltage & its distortion factor.

$\boxed{4-8}$ In the inverter shown $V = 500$ V and $T = 2000\ \mu s$. The load is an RLC series circuit for which $R = 1.2\ \Omega$, $\omega L = 10\ \Omega$, $1/\omega C = 10\ \Omega$. The gating signals are as shown.

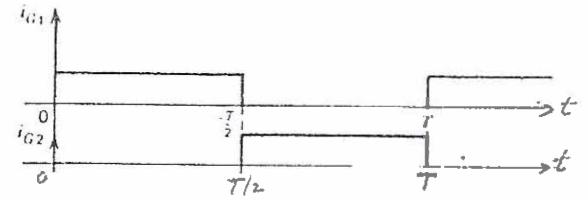


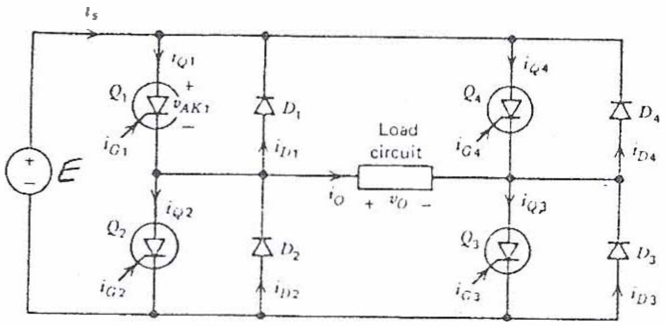
(a) Sketch to scale the waveform of v_O, i_O, i_{Q1}, i_{D1} and v_{Q1}. Higher harmonics than the fundamental component may be neglected.

(b) Calculate the rms and average thyristor and diode currents.

(c) If the turn-off time of the thyristors is 50 μs, state whether forced commutation will be required and determine the thyristor current at the instant of commutation.

$\boxed{4-9}$ The bridge inverter shown is controlled by MPWM. Sketch gate signals for N pulses per half cycle and indicate the path of the output current, i_O. Also sketch V_o. Analize the circuit and find the Fourier Components of V_o.





$\boxed{4-10}$ (a) Calculate the rms values of the fundamental, fifth, and seventh harmonic components of the output voltage for a pulse width of 90°.

(b) Sketch the required gating waveforms if the output voltage v_O is to be defined under all load conditions for a pulse-width of 90°.

$\boxed{4-11}$ The rms value of the fundamental component of the output in the bridge inverter of $\boxed{4-9}$ must be $0.45E$, when E is the source voltage. Calculate the pulse-width required, sketch to scale the waveform of v_O, and determine the harmonic distortion factor K_v for voltage v_O if:

(a) Voltage control by single-pulse modulation is employed.

(b) Voltage control by multiple-pulse modulation is employed in which there are 10 pulses per half cycle of v_O.

Harmonics higher than the seventh may be neglected.

$\boxed{4-12}$ Sketch a 3φ-inverter circuit feeding a purely inductive symmetrical star load whose common point is the neutral. If 180° scheme is employed, sketch, i_G's, V_A, V_B, V_C, V_{AB}, V_{BC}, V_{CA}, V_N, V_{AN}, V_{BN}, V_{CN}, i_A, i_B, i_C, i_D's, i_Q's & i_s. Show that the average power delivered by the source is zero.

$\boxed{4-13}$ Repeat $\boxed{4-12}$ for 120° scheme (two thyristor gating)

4-14 Analize the circuit for the current inverter shown in Fig.1 aside. The load represents an inductive heater Shunted

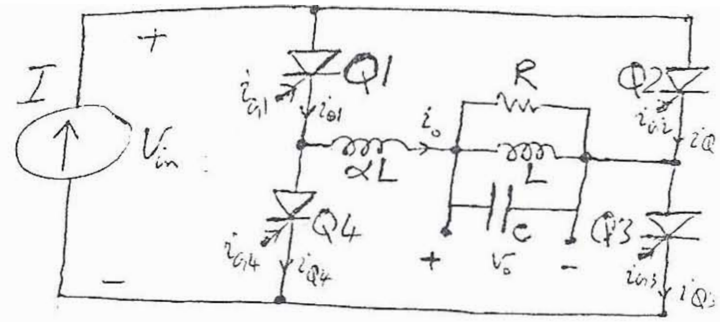


Fig.1. Current Inverter for a Commutated Inductive Heater.

with a commutating capacitor. The connection inductance is αL and $\alpha < 1$. The resonant frequency of the combination facilitates load commutation of the thyristors that are gated by the signals shown in Fig.2. Use your analysis to find expressions for:

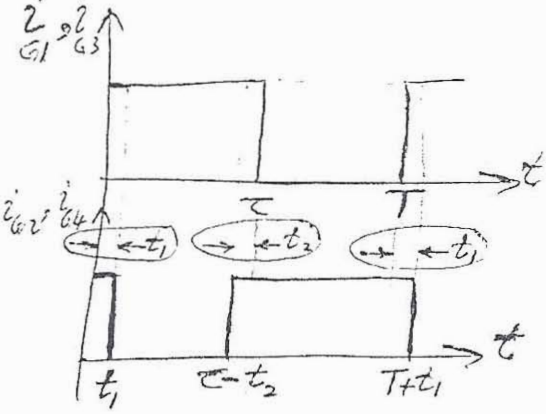


Fig.2 Gate Signals.

I) C, the commutating capacitance giving.

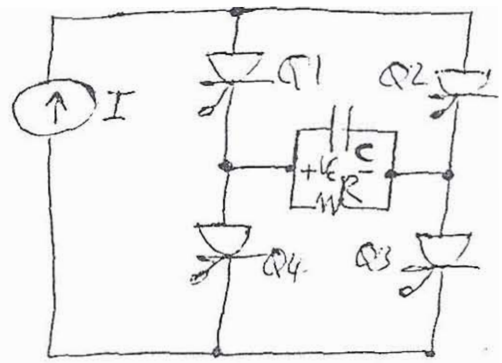
II) Maximum output power, P.

Draw the waveforms and assume equal, t_q, for all thyristors (for i_{Q1}, i_{Q2}, i_{Q3}, i_{Q4}, i_o, v_o & V_{in})

The current source is constant & continuous.

4-15 Sketch circuit & waveforms for 180° scheme, 3φ bridge inverter feeding a symmetrical star resistive load of R Ω / phase, using 100 V source. Find fundamental phase voltage & distortion factor.

4-16 The current inverter shown



is used to feed the RC load with ac power, at 60 Hz. If R = 100 Ω C = 1 μF Q1 & Q3 are turned on for one half of the cycle and Q2 & Q4 for the other half, then show that the four thyristors do not need forced commutation and that as soon as their gate pulses ends they turn off. Sketch to scale the time variation of V_c against i_G's & also sketch thyristor currents, at steady state. What is the time available for turn off for thyristors?

4-17 We have a resistive load and want to supply it with an ac fundamental of 110 volts using a dc supply of 300 volts. Choose one of the following schemes, and for your scheme, sketch circuit, waveforms of $i_{G's}$, $i_{Q's}$, $i_{D's}$, i_s, i_o & v_o:

a) Chopper inverter, giving $\frac{\tau}{T}$ & distortion factor of v_o.

b) Single pulse modulated inverter, giving $\frac{\tau}{T}$ & distortion factor of i_o

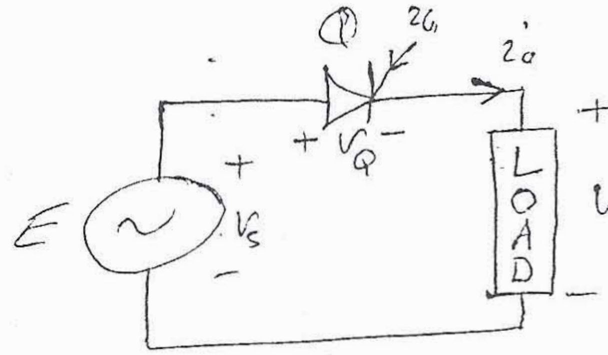
c) Assuming symmetrical star, 3φ inverter with (I) 180° or (II) 120° schemes giving % e in achievement & distortion factor of phase voltage.

4–18 A resistive load with parallel commutating capacitor is fed using current inverter. If $I = 50$ Amp, $R = 10\Omega$, $C = 400\,\mu F$, $f = 200$ Hz, then sketch the circuit showing various waveforms of i_G's ; i_Q's ; i_o ; V_Q ; V_s ; V_o. Find the power delivered to the load, the average supply voltage, $V_{o_{1rms}}$, $V_{o_{av}}$, $V_{o_{rms}}$, μ_V, time available for thyristor recovery ; V_{FB}, V_{RB} & $\frac{dV}{dt}$ ratings of thyristors.

4–19 Draw the circuit of a full bridge current inverter feeding a $5\Omega \,||\, 1\,mF$ load from 100 Amp DC source. If the operating frequency is 50 Hz, draw the associated waveforms, and find, assuming instant turn off, the recovery times & μ_I of load.

Set Five: Rectifiers

$\boxed{5\text{-}1}$ Analize the circuit shown here for the HWCR with:



a) RLV load &

b) RL load &

c) R load.

Sketch the waveforms of:

V_s, i_G, i_o, V_o, V_Q and

find expressions for:

$i_o(t)$, $V_o(t)$, β, $i_{o_{av}}$, $V_{o_{av}}$, $i_{o_{rms}}$, $V_{o_{rms}}$, K_i & K_v & P_{av}.

where β is the extinction angle obtained through natural commutation

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$\boxed{5\text{-}2}$ Repeat $\boxed{5\text{-}1}$ for the case of force commutation at β. Mention the required modification to your load circuit if any. Sketch your circuit.

$\boxed{5\text{-}3}$ Repeat $\boxed{5\text{-}1}$ parts ⓐ & ⓑ when L is very large

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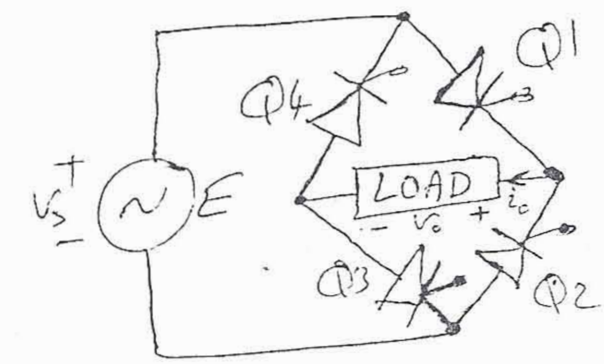
$\boxed{5\text{-}4}$ Repeat $\boxed{5\text{-}2}$ parts ⓐ & ⓑ when L is very large.

$\boxed{5\text{-}5}$ Analize the FWCR shown here for the case when Q1 & Q3 are both fired at α & Q2 & Q4 are both fired at $\pi+\alpha$. All thyristors are line commutated, and current is



a) Discontineous, and

b) Contineous. Load is

I) RLV load,

II) RL load &

III) R load.

For each case, draw: i_G's, i_Q's, V_s, V_o & i_o.

Find expression for: $V_{o_{av}}$, $i_{o_{av}}$ & K_v.

$\boxed{5\text{-}6}$ Repeat $\boxed{5\text{-}5}$ for the case when Q3 & Q4 are replaced by diodes D3 & D4 respectively

$\boxed{5\text{-}7}$ Repeat $\boxed{5\text{-}5}$ when force commutation is used, sketch your circuit and mention any modification.

$\boxed{5\text{-}8}$ Repeat $\boxed{5\text{-}6}$ when force commutation is used, sketch circuit

$\boxed{5\text{-}9}$ For all the preceeding four problems repeat for L very large.

5-10 a) Draw the circuit for 3 ϕ HWCR with RL load, using symmetrical gating at α,

b) Sketch all the associated waveforms, and

c) Give range of α for continuous load current.

d) Find $V_{o_{av}}$ &

e) Find $i_{o_{av}}$.

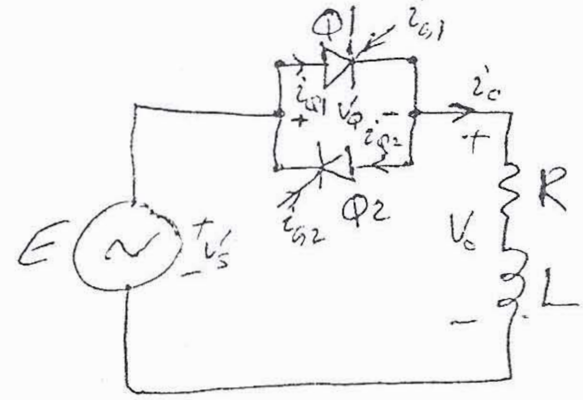
5-11 Repeat 5-10 for 3 ϕ FWCR.

5-12 For the circuit shown V_s is 341 sin $120\pi t$, Volts, and Q is fired at $\alpha = 90°$.

1. What is the specific function of the circuit? What is the control range of α?
2. Sketch V_s, i_G, i_Q, i_c, i_D, V_o & V_Q for the current mode of operation. Mention it.
3. What is the conduction times of both Q & D? What type of commutation is used?
4. Find $V_{o_{av}}$, $i_{o_{av}}$, $V_{o_{rms}}$ & power delivered to V.
5. For Q find t_q, V_{FB} & V_{RB} & for D find P_J + T_J given 25°C ambient, 0.5°C/W capacity & V_F = 1

Set Six : Converters

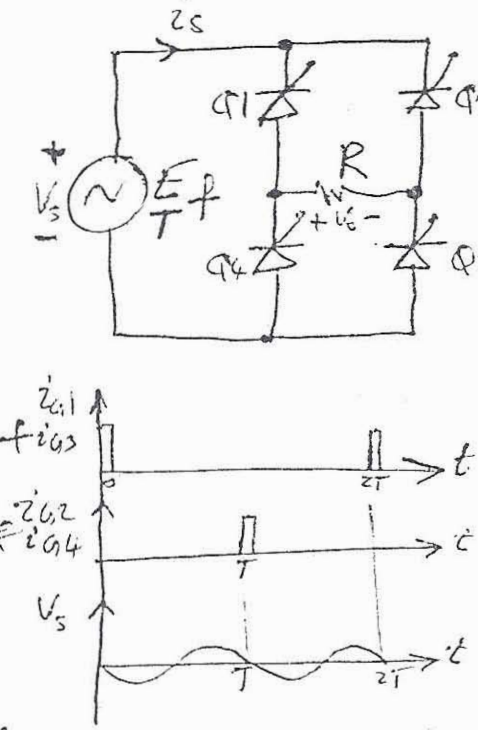
6-1 Analize the converter shown for RL load. Give range of α, assuming symmetrical gating. Sketch: V_s, i_G's, i_Q's, V_o, i_o & V_Q.



Find expressions for: $V_{o_{rms}}$, $i_{o_{rms}}$, $P_{o_{av}}$ & pf.

6-2 If α was $90°$ in the above problem, sketch the above mentioned waveforms and find values for the above mentioned expresions.

Set Seven: Cycloconverters.



7-1 a) Name the function of the circuit shown for the given gate signals.

b) Sketch all the waveforms

c) Find the fundamental output current,

d) Find the power,

e) Find the power factor,

f) Find the fundamental power factor,

g) Find the distortion factor of the output.

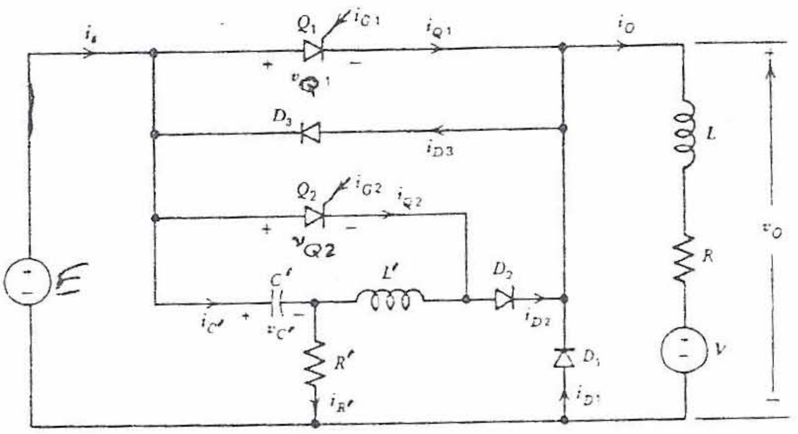
h) What is the disadvantage of the circuit from the supply point of view?

7-2 Draw a circuit that can supply a resistive load by controlled ac supply at 300 Hz using 3ϕ supply at 60 Hz. Sketch the waveforms of i_G's i_Q's, i_o & v_o for symmetrical gating at α. Mention control range of α and the corresponding value of the fundamental output voltage. Also get expression for the voltage distortion factor.

Set Eight: Firing & Commutation Circuits.

8-1 Draw a circuit that can generate a train of pulses using: 741 amplifier, 311 Comparator & discrete Capacitors & resistors. Analize your circuit showing the relationships governing the period of your train & width of pulse. How can you change these two parameters. Sketch any relevent waveforms.

8-2 In the Circuit shown, $E = 600$ Volts, $L' = 42\ \mu h$, $C' = 6\ \mu F$, $T = 2.5$ ms & i_o at instant of commutation is 150 Amp. Explain the Circuit in general and then, for the given values:



(a) Calculate the time t_q available for turn-off of thyristor Q_1.
(b) Calculate the total commutation interval t_C.
(c) Sketch to scale the time variations of i_C, i_{Q2}, i_{D2}, i_{D3}, i_{Q1}, i_{D1}, v_C, v_{Q1} & v_{Q2}.
(d) Determine the time available for turn-off of thyristor Q_2.

8-3 For the circuit shown in **8-2** C' was $1\ \mu F$. What would happen, show the corresponding curves & figs.

1-1 Total thermal resistance $= 0.5 + 1 = 1.5\ °C/W$

$$\therefore P_{thyristor\,av} = \frac{1}{T}\int_0^T i_{th} \cdot v_{th}\, dt = \frac{1}{T}\int_0^{T/3} I \cdot (1 + .01 I)\, dt =$$

$$= I(1 + .01 I)\, \frac{t}{T}\Big|_0^{T/3} = I(1 + .01 I)/3$$

$$\therefore \left[(I + .01 I^2)/3\right] * 1.5 = 125 - 35 = 90$$

$$\therefore I + .01 I^2 = 90 * 3 / 1.5 = 180$$

$$\therefore I^2 + 100 I - 18000 = 0$$

$$\therefore I = \frac{-100 \pm \sqrt{100^2 + 4(18000)}}{2} = \frac{-100 \pm \sqrt{82{,}000}}{2} = 93.2 \text{ OR } -193.2$$

$\therefore$ Maximum value of I is 93.2 Amps since thyristor can't allow negative currents.

$\boxed{1-3}$ ③ The output is half a sinusoid, approximately.

a) $\because i_{o_{av}} \simeq \frac{2 * \sqrt{2}E * \pi}{2\pi^2 R} = \frac{\sqrt{2}E}{\pi R} = \frac{\sqrt{2} * 220}{\pi * 1} = 99.035$ Amps > 10 Amp $(\therefore X)$

b) $\because i_{o_{max}} = i_o(\gamma)$ where $\dot{i}_o(\gamma) = 0$.

$\because i_o(\omega t) = \frac{\sqrt{2}E}{Z}\left[\sin(\omega t - \phi) + \sin\phi \cdot e^{-\omega t/\tan\phi}\right]$

$\therefore \dot{i}_o(\gamma) = 0 = \frac{\sqrt{2}E}{Z}\left[\cos(\gamma - \phi) + \sin\phi\, e^{-\gamma/\tan\phi} \cdot (-\cot\phi)\right]$

$\therefore \cos(\gamma - \phi) = \cos\phi \cdot e^{-\gamma/\tan\phi}$

$\because \phi = \tan^{-1}\frac{1}{1} = 45° = \frac{\pi}{4}$ rad $\therefore \sqrt{2}\, e^{\gamma} \cos(\gamma - \pi/4) - 1 = 0 \quad \therefore \gamma = 2.2841$ rad

$\therefore i_{o_{max}} = i_o(\gamma) = \frac{\sqrt{2} * 220}{\sqrt{2}}\left[\sin(2.2841 - \pi/4) + \frac{1}{\sqrt{2}} e^{-2.2841}\right] = 235.3$ Amps

$\therefore i_{o_{max}} = 235.3$ Amps > 100 Amps $\quad (\therefore X)$

c) Q will have to withstand the negative half cycle of supply.

$\therefore V_{R_{max}} = \sqrt{2}E = \sqrt{2} * 220 = 311.13$ Volts > 300 Volts $(\therefore X)$

d) Q is blocking no forward voltage since $\alpha = 0 \quad (\therefore OK)$

e) (OK)

f) $\frac{dV}{dt} = 0$ (since $\alpha = 0$) $< 10\, V/\mu sec \quad (\therefore OK)$

g) $di/dt|_{max} = \dot{i}_o(0) = \frac{\sqrt{2}E}{Z}\left(\cos\phi + \sin\phi\, e^{0}(-\cot\phi)\right) = 0 \quad (\therefore OK)$

h) Q will extinct at β given by:

$\sqrt{2}\, \epsilon^{\beta} \sin(\beta - \pi/4) + 1 = 0$ $\qquad \therefore \beta = 3.9407\ rad$

$\therefore$ Q will remain reversed biased between β & 2π

$\therefore$ Should recover in $\frac{2\pi - \beta}{\omega} = 6.214 * 10^{-3}$ sec $= 6.214$ msec

$\therefore t_{off} = 6.214 > t_q = 200ns$ $\qquad (\therefore OK)$

i) $f = 60$ Hz $\Rightarrow T = 16.\bar{6}$ msec $>>> t_{on}$ $\quad (\therefore OK)$

j) power dissapated in $Q \simeq \frac{1}{2\pi} * 1V * 235.3 * \frac{2}{\pi} * \pi A = 74.9W$

$\therefore$ Temperature rise $\simeq \frac{75}{0} = \infty$ $\quad \therefore$ Q will boil mad. $(\therefore X)$

$\therefore$ Q will not operate satisfactorily, since it fails in:

a, b, c & j.

(II) The output is exactly one quarter sinusoid.

a) $\therefore i_{o_{av}} = \frac{2\sqrt{2} E\, \pi/2}{2\pi^2 R} = \frac{E}{\sqrt{2}\pi R} = \frac{110}{\sqrt{2}\, \pi * 10} = 2.476$ Amps < 10 $(\therefore OK)$

b): $i_{o_{max}} = \sqrt{2}E/R = \frac{\sqrt{2} * 110}{10} = 15.56$ Amps < 100 $\quad (\therefore OK)$

c) Q will have to withstand the negative half of supply cycle.

$\therefore V_{R_{max}} = \sqrt{2}E = 155.6$ Volts < 300 Volts $\quad (\therefore OK)$

d) Q will have to withstand half the positive half cycle.

$\therefore V_{F_{max}} = 155.6$ Volts > 100 Volts $\qquad (\therefore X)$

e) (OK)

f) $\frac{dV}{dt}$ is worst (at $\omega t = 0$) $= \sqrt{2} E \omega = \sqrt{2} * 110 * 2\pi * 60$

$= 58.65$ KV/sec < 10 V/μsec (∴ OK)

(no need for the snubber circuit).

g) $di/dt = \infty$ (∴ X)

h) Q will extinct at π ∴ It has $\frac{2\pi - \pi}{\omega}$ to recover $= 8.3\bar{3}$ ms

∴ $t_{off} = 8.3\bar{3}$ ms $>$ 200 ns (∴ OK)

i) $f = 60$ Hz & $T = 16.\bar{6}$ ms $>>>$ 10 ns (∴ OK)

j) Power dissapcated in Q $\approx \frac{1}{2\pi} * 1V *$, ... 2 π

∴ Temperature rise $= 2.5 W * \frac{1\ °C}{W} = 2.5\ °C$

∴ junction temperature = Ambient $+ 2.5 \approx 30 + 2.5 = 32.5°C$

∴ $32.5°C < 150°C$ (∴ OK).

∴ Q will not operate satisfactorily, since it fails in:

d & g.

1-4 a) V_{FB} = forward blocking voltage = 50 Volts

$\therefore$ V_{FB} rating of the thyristor must be ≥ 50 Volts

b) V_{RB} = reverse blocking voltage = 0.7 Volts

$\therefore$ V_{RB} rating of the thyristor must be ≥ 0.7 Volts

c) V_F = forward on-state voltage = 1 Volt

d) $i_{F_{max}}$ = maximum forward current = 20 Amp

$\therefore$ $i_{F_{max}}$ rating of the thyristor must be ≥ 20 Amp

e) $i_{F_{rms}}$ = forward rms current $= \sqrt{\frac{1}{20}\left(\frac{1}{3} * 20^2 * .001 + 20^2 * 9.989 + \frac{1}{3} * 20^2 * .01\right)}$

$= \sqrt{199.853} = 14.137$ Amp. (Note: $i_{F_{rms}} \approx 20 * \sqrt{\frac{t_{on}}{T}} = \frac{20A}{\sqrt{2}}$).

$\therefore$ $i_{F_{rms}}$ rating of the thyristor must be ≥ 14.137 Amp.

f) $i_{F_{av}}$ = average forward current $= \frac{1}{20}\left(\frac{1}{2} * 20 * .001 + 20 * 9.989 + \frac{1}{2} * 20 * .01\right)$

$= 9.9945$ Amp (Note $i_{F_{av}} \approx 20 * \frac{t_{on}}{T} = 10$ Amp)

$\therefore$ $i_{F_{av}}$ rating of the thyristor must be ≥ 9.9945 Amp.

g) t_{on} = turn on time = .001 ms = 1 μsec

$\therefore$ t_{on} rating of the thyristor must be ≤ 1 μsec

h) t_q = recovery time = 0.3 ms = 300 μsec

∴ t_q rating of the thyristor must be ≤ 300 μs

i) $\left.\frac{dV}{dt}\right|_{max}$ = maximum rate of change of forward voltage = $\frac{50V}{.5ms}$ = 100 KV/s

∴ $\left.\frac{dV}{dt}\right|_{max}$ rating of the thyristor must be ≥ 100.0 KV/sec.

j) $\left.\frac{di}{dt}\right|_{max}$ = maximum rate of change of forward current = $\frac{20A}{.001\,ms}$ = 20 KA/ms

∴ $\left.\frac{di}{dt}\right|_{max}$ rating of the thyristor must be ≥ 20 KA/ms.

k) Power consumed by thyristor $= \frac{1}{T_o}\int^{T} i_Q \cdot v_Q \, dt =$

$$= \frac{1}{20}\cdot\left[\int_0^{.001}\left(\frac{20}{.001}t\right)\cdot\left(50-\frac{49}{.001}t\right)\cdot dt + 20 * 1 * 9.989 + \int_{9.99}^{10}\frac{20(10-t)}{.01} * 1\, dt\right]$$

$$= \frac{1}{20}\cdot\left[\int_0^{.001}(1M\,t - 980M\,t^2)dt + 199.78 + \left[\frac{2K\,(10-t)^2}{-2}\right]_{9.99}^{10}\right] =$$

$$= \frac{1}{20}\cdot\left[1M\frac{t^2}{2} - 980M\frac{t^3}{3}\Big|_0^{.001} + 199.78 + K * .01^2\right] =$$

$$= \frac{1}{20}\cdot\left[1M * \frac{.001^2}{2} - 980M * \frac{.001^3}{3} + 199.78 + .1\right] =$$

$$= \frac{1}{20}\cdot\left[0.5 - .32\bar{6} + 199.88\right] = \frac{1}{20}\cdot\left(0.17\bar{3} + 199.88\right) = 10.003\,Watt$$

(Note ≈ $20 * 1 * \frac{t_{on}}{T}$ = 10 Watts).

∴ $\frac{T_j - 30}{10.003} = 10 \implies T_j = 30 + 100.03 = 130.03\,°C$

∴ T_j = junction temperature = 130.03 °C.

∴ T_j rating of the thyristor must be ≥ 130.03 °C

$\boxed{1\text{-}5}$ a) $V_{FB} \geq 100$ Volts

b) $V_{RB} \geq 50$ Volts

c) $V_F = 2$ Volts

d) $i_{F_{max}} \geq 50$ Amp

e) $i_{F_{rms}} \geq \sqrt{\frac{1}{4}\int_0^4 i_Q^2\,dt = \frac{1}{4}\cdot\left[\frac{1}{3}*.02*50^2+1.98*50^2+\frac{1}{3}*.2*50^2+0\right]} = 35.8$ Amp

$\therefore\ i_{F_{rms}} \geq 36$ Amp

f) $i_{F_{av}} \geq \frac{1}{4}\left(\frac{1}{2}*.02*50+1.98*50+\frac{1}{2}*.2*50\right) = 26.13$ Amp

$\therefore\ i_{F_{av}} \geq 26$ Amp.

g) $t_{on} \leq .02$ ms $= 20\,\mu$sec

h) $t_q \leq \frac{50}{100+50}*(4-3.2) = .2\bar{6}$ ms $= 267\,\mu$sec.

i) $dV/dt\big|_{max} \geq \frac{150}{4-3.2}\cdot\frac{V}{ms} = 187.5$ KV/sec

j) $di/dt\big|_{max} \geq \frac{50A}{.02ms} = 2.5$ MA/sec $= 2.5$ A/μsec

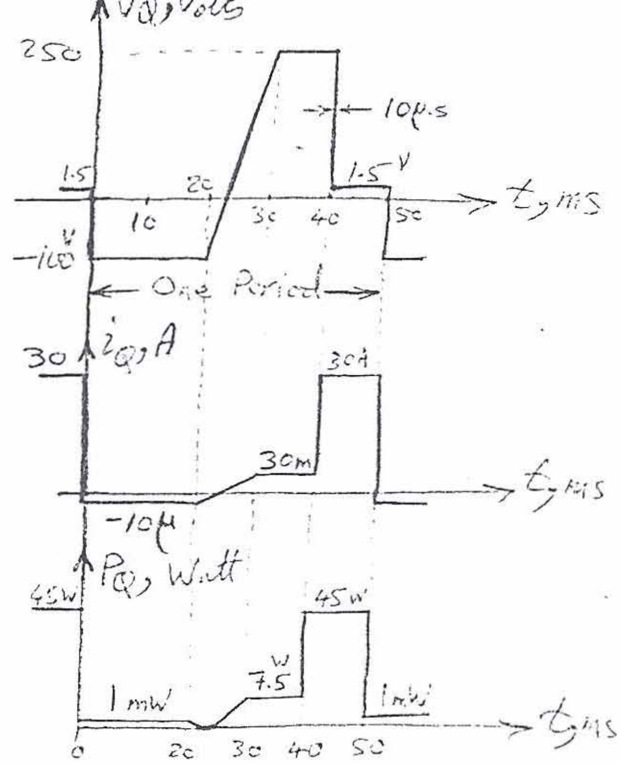
k) $P_Q = \frac{1}{4}\int_0^4 i_Q V_Q\,dt \simeq \frac{1}{4}\cdot\left[0+\frac{2}{3}*.02*\frac{100}{2}*\frac{50}{2}+1.98*2*50+\frac{1}{2}*.2*2*50\right]$
$= \frac{1}{4}*(16.\bar{6}+198+10) = 56.1\bar{6}$ Watt $= \frac{\Delta T_j}{5}$

$\therefore\ \Delta T_j = 5*56.1\bar{6} \doteq 280.8\bar{3}$ °C $= T_j - 25$°C

$\therefore\ T_j \geq 306$°C, max. junction-temperature rating of thyristor.

33A

To find the ratings of Q surviving the shown voltage and current waveforms:



$\therefore$ $I_{Q_{max}} > 30\,A$,

$I_{Q_{AV}} > \frac{30}{5} = 6\,Amp$,

$I_{Q_{rms}} > 30\sqrt{\frac{1}{5}} = 13.4\,A$,

$I_{leak} = \left\langle \begin{matrix} -10\mu A \ @ \ -100V \\ 30mA \ @ \ 250V \end{matrix} \right\}$,

$V_{QFB} > 250\,V$,

$V_{QRB} > 100\,V$,

$V_{Q_{on}} = 1.5\,V$, $\quad \left.\frac{dV_Q}{dt}\right|_{max} > \frac{250+100}{10\,m} = 35\,V/ms$

$t_q < 20 + 10 \times \frac{100}{350} = 22.86\,ms$

$t_{on} < 10\,\mu s$

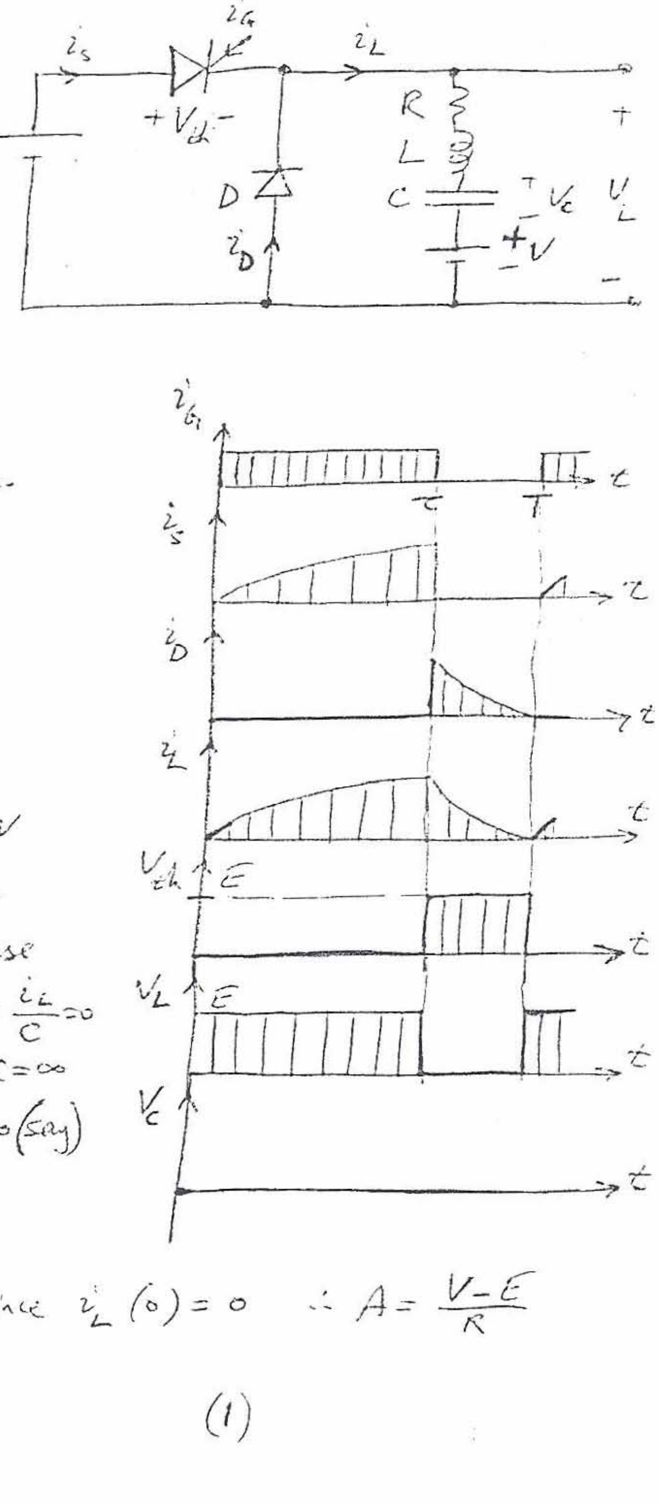
$\left.\frac{di_Q}{dt}\right|_{max} > \frac{30A}{10\mu s} = 3\,A/\mu s$

$P_{Q_{av}} > \frac{45}{5} + \frac{7.5}{5} = 10.5\,W$

$\therefore$ $T_j > \frac{10.5}{1} + 25 = 35.5\,^{\circ}C$

33B

2-1 a) Critical mode of current occures when the current, i_L, starts from zero, at the instant the thyristor is fired on, and increases till it is shut down whereby it will freewheel through the diode till it reaches zero again just at the instant the thyristor is fired again for the next cycle. The associated waveforms are shown in the figure asside for one steady-state cycle.



Note that i_L is always positive & hence V_c is going to charge up in every cycle till it reaches $E-V$ whereby no current will flow in the thyristor. To avoid this case C must be ∞ such that $\frac{dV_c}{dt} = \frac{i_L}{C} = 0$

$\therefore$ For critical mode to happen, $C = \infty$ i.e. short-circuited. Hence, $V_c = 0$ (say) as shown.

Now, during $t \in [0, \tau)$:

$\therefore \quad i_L(t) = \frac{E-V}{R} + A e^{-Rt/L}$ & since $i_L(0) = 0 \quad \therefore A = \frac{V-E}{R}$

$$\therefore \quad i_L(t) = \frac{E-V}{R}\left(1 - e^{-Rt/L}\right) \qquad (1)$$

$$\therefore \quad i_L(\tau) = \frac{E-V}{R}\left(1 - e^{-R\tau/L}\right). \qquad (2)$$

When the thyristor is switched off, D becomes on during $t \in [\tau, T)$. Hence:

$$i_L(t) = \frac{-V}{R} + Be^{-Rt/L} \quad \& \text{ since } i_L(\tau) = \frac{E-V}{R}\left(1-e^{-R\tau/L}\right)$$

$$\therefore \frac{E-V}{R}\left(1-e^{-R\tau/L}\right) = \frac{-V}{R} + Be^{-R\tau/L}$$

$$\therefore B = \left[\frac{E-V}{R}\left(1-e^{-R\tau/L}\right) + \frac{V}{R}\right] \cdot e^{R\tau/L}$$

$$\therefore i_L(t) = \frac{-V}{R} + \left[\frac{E-V}{R}\left(1-e^{-R\tau/L}\right) + \frac{V}{R}\right] e^{-R(t-\tau)/L} \qquad (3)$$

Now the mode of current is critical when $i_L(T) = 0$

$$\therefore 0 = \frac{-V}{R} + \left[\frac{E-V}{R}\left(1-e^{-R\tau/L}\right) + \frac{V}{R}\right] \cdot e^{-R(T-\tau)/L} \qquad (4)$$

$$\text{OR:} \quad 0 = -V + E\left(1-e^{-R\tau/L}\right)e^{-R(T-\tau)/L} + Ve^{-RT/L}$$

$$\therefore V\left(1-e^{-RT/L}\right) = E \cdot e^{-R(T-\tau)/L} * \left(1-e^{-R\tau/L}\right)$$

$$\text{OR } 2V \cdot e^{-RT/2L} \sinh(RT/2L) = 2E e^{-R(T-\tau)/L} \cdot e^{-R\tau/2L} \cdot \sinh(R\tau/2L)$$

$$\text{OR} \quad V e^{(RT/2L)} \sinh(RT/2L) = E e^{(R\tau/2L)} \sinh(R\tau/2L) \qquad (5)$$

$\therefore$ Eq. 5, also, must be satisfied to get critical mode for i_L.

b) If $V = 0$ in (5) then $E e^{(R\tau/2L)} \sinh(R\tau/2L)$ must be zero

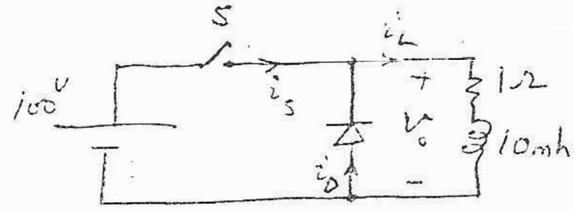
$\therefore R\tau/2L = 0 \quad \therefore \tau = 0$ (limit case, or no operation)

$$\therefore i_{L_{av}} = i_{L_{rms}} = V_{L_{av}} = V_{L_{rms}} = 0$$

c) If $V = 0$, C must be ∞ & hence as before $\underset{\text{Crud}}{\tau = 0}$

$\therefore$ Again we have limit case & $i_L(t) = V_L(t) = 0$

$\boxed{2\text{-}2}$ At $t \in [0, 10)$ msec

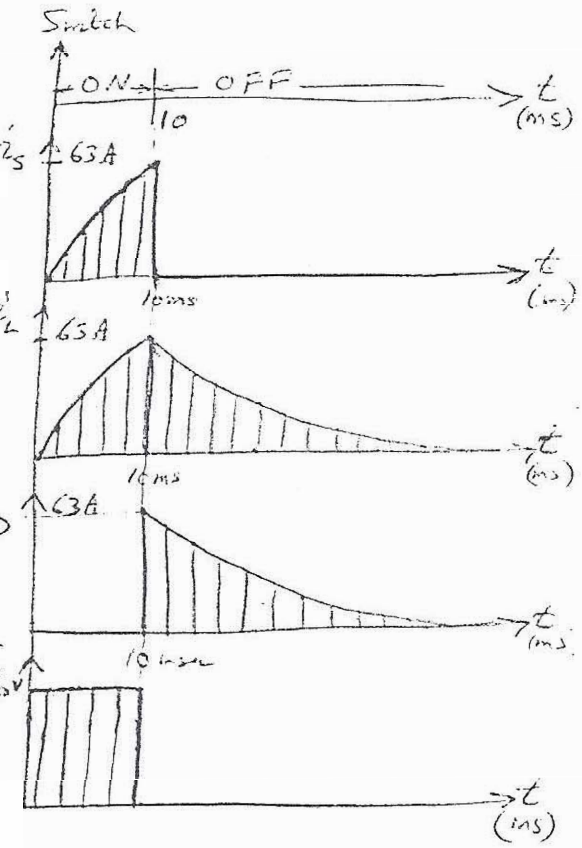


$\therefore \; i_s(t) = i_L(t) = \frac{100}{1} + Ae^{-1t/.01}$

$= 100 + Ae^{-100t}$ Amps

$\because \; i_L(0) = 0 \quad \therefore \; A = -100$

$\therefore \; i_L(t) = 100\left(1 - e^{-100t}\right)$ Amps, $t \in [0, 10)$ms



$\therefore \; i_L(10\text{ msec}) = 100\left(1 - e^{-100 \times 10 \times 10^{-3}}\right) =$

$= 100\left(1 - e^{-1}\right) = 63.2$ Amps

When S is off the diode is on

$\therefore$ at $t \in [10\text{msec}, \infty)$:

$\therefore \; i_L(t) = Be^{-1t/10 \times 10^{-3}} =$

$= Be^{-100t}$

$\because \; i_L(10\text{ ms}) = 63.2$

$\therefore \; 63.2 = Be^{-100(10\text{ ms})}$

$\therefore \; B = 63.2 \cdot e^{100(10\text{ msec})}$

$\therefore \; i_L(t) = 63.2\, e^{-100(t - 10\text{ ms})}$ Amp, $t \in [10\text{ms}, \infty)$

$\boxed{2\text{-}3}$ $100 = 10\mu\, \dot{i} + V_c + 10\, i$

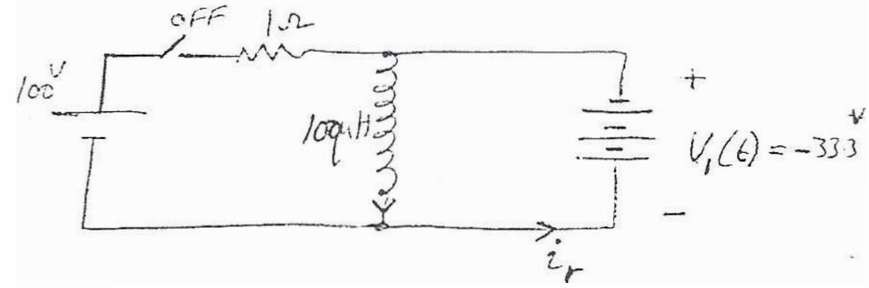
$\because V_c = 50^V$ & $i = 0 \Rightarrow 100 = 10\mu\, \dot{i} + 50 + 0 \Rightarrow \dot{i} = \frac{50}{10\mu} \frac{A}{S}$

$\therefore \; di/dt\big|_{t=0} = 5\, A/\mu s$

$\boxed{2\text{-}4}$ $i_1(0^-) = \frac{100V}{1\Omega} = 100\ \text{Amp}$, $V_1(0^-) = 0$, $V_2(0^-) = 0$, $i_2(0^-) = 0$

At $t = 0^+$, the circuit referred to primary would like:

The reason is due to transforming negative voltage that makes the diode on.



$\therefore\ V_2(t) = -100^V$

$\&\ V_1(t) = \quad * \frac{N_1}{N_2} = -100 * \frac{100}{300} = -\frac{100}{3} = -33.3$ Volts

$\&\ -i_2 = i_r \frac{N_1}{N_2} = i_r * \frac{100}{300} = i_r/3$

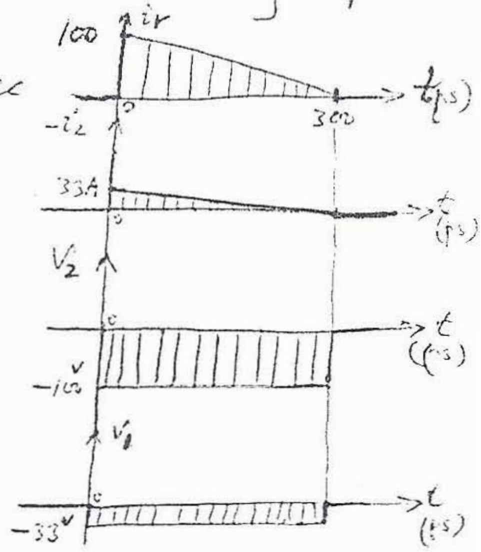
Now $i_r = 100\ \text{Amp} - \frac{33.3^V}{100\ \mu h} t = \left[100 - .333 * 10^6\ t(\text{sec})\right]$ Amps

$\therefore\ i_r$ will stop at $t = \frac{100}{.333 * 10^6} = 300\ \mu\,\text{sec}$

The waveforms are shown here.

Note:

After $t = 300\,\mu s$ Current i_2 tries to reverse. This makes the diode off. Hence, $V_2 = 0$ again. Hence $V_1 = 0$ again. So is i_r. As for i_1 it is zero throughout.



2-5 a) $V_c(t) = \dfrac{\frac{1}{sC}}{\frac{1}{sC} + sL} \cdot 100 = \dfrac{1}{s^2LC+1} \cdot 100$

$= \dfrac{1}{2500\mu^2 s^2 + 1} \cdot 100$

$= 100 + A \sin \dfrac{t}{50\mu} + B \cos \dfrac{t}{50\mu}$

$= 100 + A \sin 20Kt + B \cos 20Kt$

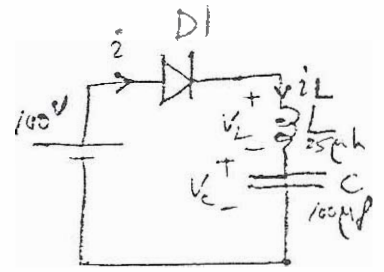


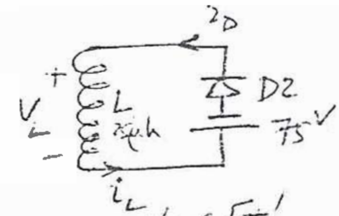
Fig. 1. $t \in [0, t']$

$\because V_c(0) = -50 = 100 + B \qquad B = -150$ volts

$\because i(0) = 0 = C\dot{V}_c(0) = 100\mu * 20K * A \Rightarrow A = 0$

$\therefore V_c(t) = 100 - 150 \cos 20Kt$ volts

$V_L = 100 - V_c(t) = 150 \cos 20Kt$ volts.



$t \in [t'$

& $i(t) = C\dot{V}_c(t) = 100\mu * 150 * 20K * \sin 20Kt$

$\therefore i(t) = 300 \sin 20Kt = i_L$ (Amps), period $= 100\pi$ μsec $= 314.16$ μsec

These expressions are valid as long as $V_L > -75^V$ (i.e D2 is off). (Fig. 1).

i.e till: $150 \cos 20Kt' = -75 \Rightarrow t' = \frac{1}{20K} \cos^{-1} \frac{-1}{2}$

i.e till $t = t' = 104.72$ μsec

After that V_L is clamped to -75 volts and D2 becomes on. (Fig. 2).

& $V_c = 100 + 75 = 175$ volts

$\therefore i = C\dot{V}_c = 0$

& $i_D = i_L$ governed by:

$25\mu\, \dot{i} = -75 \Rightarrow i = \frac{-75}{25\mu}(t - t') + I$

$\because i_L(t') = i(t') = 300 \sin(20K * 104.75\mu) = I$

$\therefore I = 259.81$ Amp

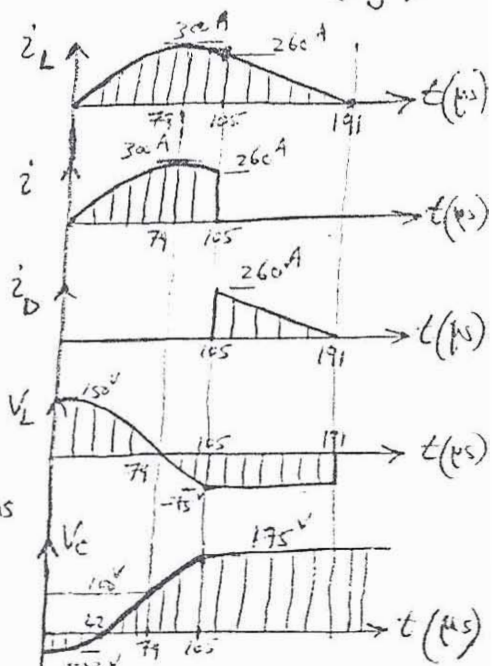
$\therefore i_L(t) = i_D(t) = 259.81 - 3M(t - 104.72\mu)$

& $V_L = -75$ volts & $V_c = 175$ volts, valid for $t \geq t'$, till $t = t''$ where $i_L(t'') = 0 \therefore t'' = 191.32$ μs

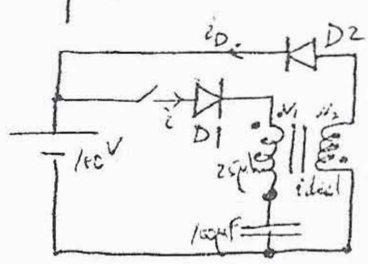
At $t > 191.3$ μs D2 is off, D1 is also off &

$i = i_D = i_L = 0$ & V_c keeps at 175 volts & $V_L = 0$.

The waveforms are as shown.



(b) The energy trapped in the inductance could be returned to the 100V source if a transformer is used as shown,



where the secondary circuit refered to primary is just as before but the 75^V source is now replaced by $100\ N_1/N_2$

$\therefore$ Chose the transformer turns ratio to be $\frac{N_1}{N_2} = \frac{75}{100} = 3/4$.

$\boxed{2\text{-}10}$ $L = 30\mu h$.

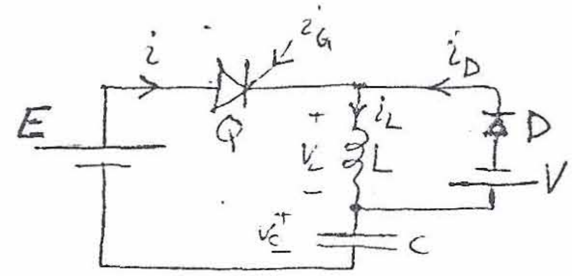
$C = 120\mu f$.

$E = 100$ Volts.

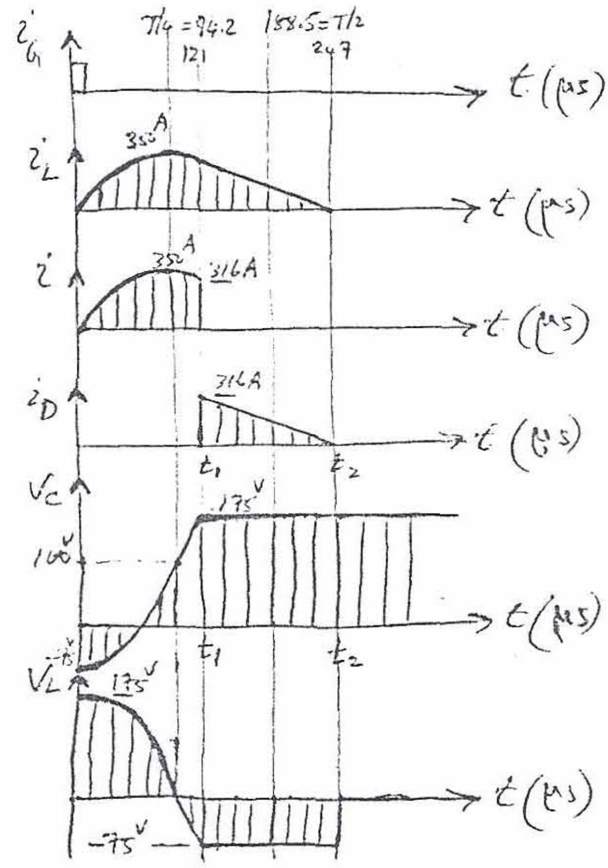
$V = 75$ Volts.

$V_{c_0} = -75$ Volts

This is the same as problem $\boxed{\frac{6}{72}}$ but values are different.



V_L will make part of oscillation till it reaches $-V$ whereby it is held constant at $-V$ and the inductor current decays to 0.



Let $\omega = 1/\sqrt{LC} = 16.\bar{6}$ Krad/sec

$\therefore V_c(t) = E + A\sin\omega t + B\cos\omega t$

& $V_{c_0} = V_c(0) = E + B \Rightarrow B = -E + V_{c_0}$

& $i(0) = 0 = C\dot{V}_c(t)\big|_{t=0} \Rightarrow$

$\therefore \omega A = 0 \Rightarrow A = 0$

$\therefore V_c(t) = E - (E - V_{c_0})\cos\omega t$

$= E(1 - \cos\omega t) + V_{c_0}\cos\omega t$

$= 100 - 175\cos\omega t$.

& $V_L(t) = 100 - V_c = 175\cos\omega t$ & $i_L(t) = C\dot{V}_c(t) = 350\sin\omega t$

$V_L(t_1) = -75 = 175\cos\omega t_1$

$\therefore t_1 = 120.8$ μsec & $i_L(t_1) = 316.2$ Amps & $V_c(t_1) = 175$ Volts

& $T = 2\pi/\omega = 377.0$ μsec.

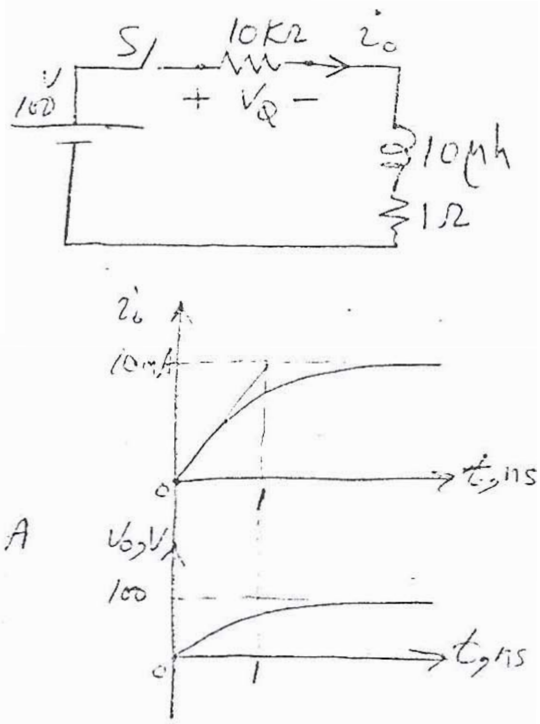
Then i_L decrease linearly to zero whereas $V_L = -75^V$ & $V_c = 175^V$

$\therefore i_L = -\frac{75}{30\mu}(t - t_1) + i_L(t_1) = -2.5\,(t(\mu s) - 120.8) + 316.2$ Amp

$\therefore i_L(t_2) = 0 \Rightarrow t_2 = 247.3$ μsec.

At $t > t_2$ all waveforms are zero except $V_c = 175$ Volts.

2-11 The equivalent circuit at H-state is as shown. Assuming S is put on at $t=0$, then its an RL circuit with time constant $\frac{L}{R} = \frac{10\mu}{10001} \simeq 1.0$ n sec



$\therefore\ i_o(t) = \frac{100}{10{,}001} + A e^{-\frac{t}{1n}}$, Amp

$\because\ i_o(0) = 0 \quad \therefore\ A = -\frac{100}{10{,}001} \simeq 10$ mA

$\therefore\ i_o(t) \simeq 10\left(1 - e^{-t/1n}\right)$, mA

$\therefore\ V_Q(t) = 10K * 10m\left(1 - e^{-t/1n}\right) =$
$= 100\left(1 - e^{-t/1n}\right)$, Volts

$$\therefore\ \frac{dV_Q}{dt}(t) = 100 * \left(-e^{-t/1n}\right) * \left(-\frac{1}{1n}\right)$$
$$= 100G\, e^{-t/1n}, \ V/s$$
$$= 100 e^{-t/1n}, \ KV/\mu s$$

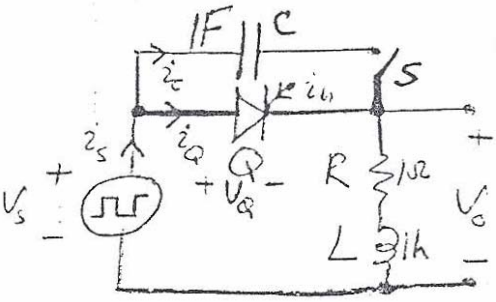
$\therefore$ Worst dV_Q/dt is at start when $t=0$, whereby $\frac{dV_Q}{dt} = 100KV/\mu s$

$\because$ Q can withstand 50 KV/μs, then the value indicated above will cause it to fire, as soon as S is put on. Hence, when S is put on Q becomes on without gating. This is because of the inherited capacitance of Q.

The initial rate of load current is $\frac{di_o}{dt}(0) = \frac{dV_Q}{dt}(0)/10K = 10A/\mu s$

(Note $\frac{di_o}{dt}(0)$ is also obtained by $\frac{100V}{10\mu h} = 10A/\mu s$).

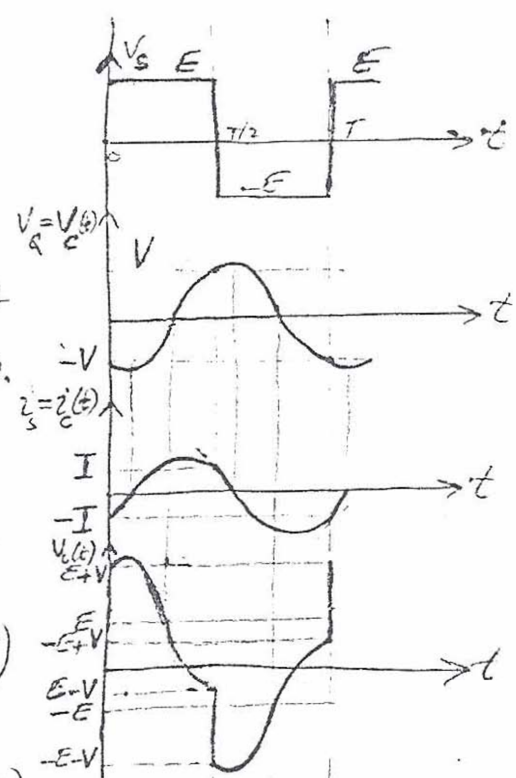
2-12 I) When the thyristors Q is not gated, it remains off. Hence, for the case when the switch S is put on, V_s sees an RLC series circuit.



$$\therefore V_s(t) = \begin{cases} E & \text{for } t \in [0, T/2) \\ -E & \text{for } t \in [T/2, T) \end{cases}$$ where T is the period of V_s

$$\therefore V_c(t) = V_Q(t) = \begin{cases} E + e^{-t/2}\left(A_1 \sin(\sqrt{3}t/2) + A_2 \cos(\sqrt{3}t/2)\right), & t \in [0, \tfrac{T}{2}) \\ -E + e^{-(t-T/2)/2} \cdot \left(B_1 \sin\{\sqrt{3}(t-T/2)/2\} + B_2 \cos\{\sqrt{3}(t-T/2)/2\}\right), & t \in [T/2, T) \end{cases}$$

The waveforms at steady-state are as shown where: $V_c(t)$ is smooth due to continuity of $V_c(t)$ & $\dot{V}_c(t)$; and during one half-cycle of input, V_c makes $\frac{\sqrt{3}/2}{2\pi} \times \frac{60}{6 \times 2} = 0.69$ cycle $> \frac{1}{2}$ cycle. Hence, the RLC dynamic oscilations will not show very clearly during a half cycle input step. Also note that $i_c(t)$ is continuous but not smooth, hence, $V_o(t)$ is not continuous.



Due to symmetry of both half-cycles:

$$\therefore V = V_c(\tfrac{T}{2}) = E + e^{-T/4}\left(A_1 \sin\tfrac{\sqrt{3}T}{4} + A_2 \cos\tfrac{\sqrt{3}T}{4}\right) \quad (1)$$
$$= -E + B_2$$

$$\& \; -V = V_c(0) = V_c(T) = E + A_2 \quad (2)$$
$$= -E + e^{-T/4}\left(B_1 \sin\tfrac{\sqrt{3}T}{4} + B_2 \cos\tfrac{\sqrt{3}T}{4}\right)$$

From (1) $\Rightarrow$ $\therefore B_2 = E + V$ (3)

into (2) $\Rightarrow$ $\therefore B_1 = \left[(E-V)e^{T/4} - (E+V)\cos\frac{\sqrt{3}T}{4}\right] / \sin\left(\frac{\sqrt{3}T}{4}\right)$ (4)

From (2) $\Rightarrow$ $\therefore A_2 = -E - V = -B_2$ (5)

into (1) $\Rightarrow$ $\therefore A_1 = \left[-(E-V)e^{T/4} + (E+V)\cos\frac{\sqrt{3}T}{4}\right] / \sin\left(\frac{\sqrt{3}T}{4}\right) = -B_1$ (6)

Hence, one half-cycle is enough and other could be obtained using symmetry.

And since $i_c(t) = \frac{C\,dv_c(t)}{dt} = \mathring{V}_c(t)$

$$\therefore \quad I = i_c\left(\tfrac{T}{2}\right) = \mathring{V}_c(T/2) = e^{-\frac{t}{2}}\left[\left(-A_1/2 - \sqrt{3}A_2/2\right)\sin(\sqrt{3}t/2) + \left(-A_2/2 + \sqrt{3}A_1/2\right)\cos\left(\tfrac{\sqrt{3}t}{2}\right)\right]_{T/2}$$

$$= e^{-T/4}\cdot\left[-\frac{A_1+\sqrt{3}A_2}{2}\cdot\sin\frac{\sqrt{3}T}{4} + \frac{\sqrt{3}A_1 - A_2}{2}\cdot\cos\frac{\sqrt{3}T}{4}\right] \qquad (7)$$

$$\& \quad -I = i_c(0) = \mathring{V}_c(0) = \frac{\sqrt{3}A_1 - A_2}{2} \qquad (8)$$

(5) & (6) into (7) + (8) gives: $\left(\text{Note}: T = \frac{60\,\text{sec}}{6} = 10\,\text{sec.}\right)$

$$0 = A_1\left[\left\{-\sin\left(\tfrac{\sqrt{3}T}{4}\right) + \sqrt{3}\cos\left(\tfrac{\sqrt{3}T}{4}\right)\right\}e^{-T/4} + \sqrt{3}\right] + A_2\left[\left\{-\sqrt{3}\sin\left(\tfrac{\sqrt{3}T}{4}\right) - \cos\left(\tfrac{\sqrt{3}T}{4}\right)\right\}e^{-T/4} - 1\right]$$

$$= A_1\left[2\cos\left(\tfrac{\sqrt{3}T}{4} + \tfrac{\pi}{6}\right)\cdot e^{-T/4} + \sqrt{3}\right] - A_2\left[2\sin\left(\tfrac{\sqrt{3}T}{4} + \tfrac{\pi}{6}\right)\cdot e^{-T/4} + 1\right]$$

$$= 1.7552\,A_1 - 0.83747\,A_2$$
$$= 1.7552 * [13.532E - 12.728V] - 0.83747 * [-E - V] =$$
$$= 24.589E - 21.503V$$

$$\therefore \quad V = \frac{24.589 * 100}{21.503} = 114.35 \text{ Volts} \qquad (9)$$

into (5) $\Rightarrow$ $\quad \therefore A_2 = -B_2 = -214.35$ Volts $\qquad$ (10)

& into (6) $\Rightarrow$ $\quad \therefore A_1 = -102.28$ Volts $\qquad$ (11)

$\therefore$ The equation of $V_c(t)$ in the first symmetrical half is given by:

$$V_c(t) = V_q(t) = 100 - e^{-t/2} * \left[102.28\sin\left(\tfrac{\sqrt{3}t}{2}\right) + 214.35\cos\left(\tfrac{\sqrt{3}t}{2}\right)\right];\ \text{Volts}, \quad t \in [0, \tfrac{T}{2})$$
(12)

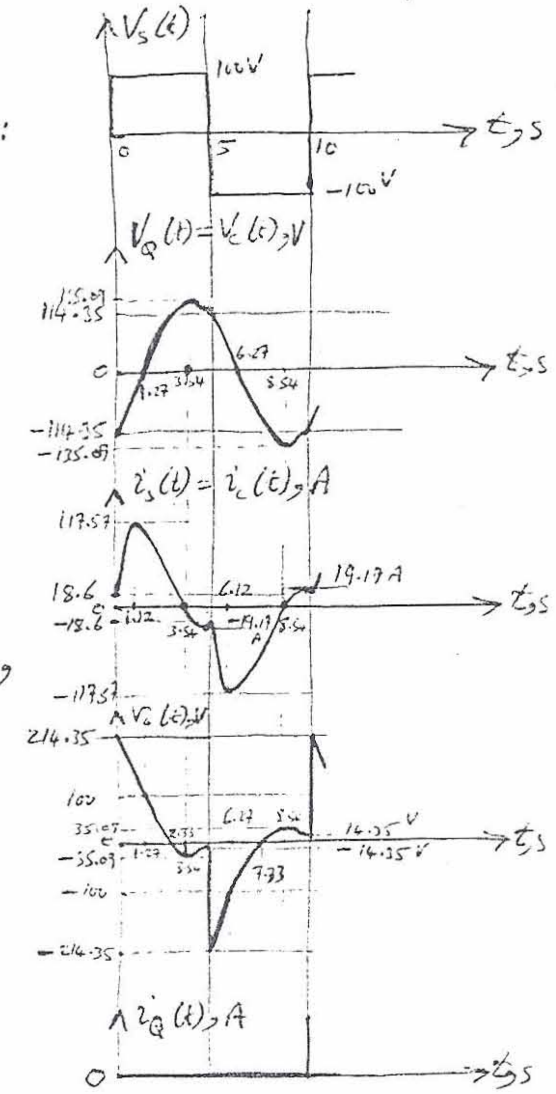
Using (8), (10) & (11):

$\therefore I = -18.602$ Amps. $\qquad$ (13)

The equation for $i_c(t)$ in the first symmetrical half is given by:

$$i_c(t) = i_s(t) = e^{-t/2} * \left[236.77\sin\left(\tfrac{\sqrt{3}t}{2}\right) + 18.602\cos\left(\tfrac{\sqrt{3}t}{2}\right)\right];\ \text{Amp}, \ t \in [0, \tfrac{T}{2})$$
(14)

These values modify the waveforms shown earlier to be as indicated beside. Note that i_Q is zero, and that:



$V_c(0) = -114.35$ Volts,

$\dot{V}_c(0) = i_c(0) = 18.602$ Amp or V/s,

$V_c(t)$ is zero at $t = 1.2686$ sec,

$\dot{V}_c(1.2686) = i_c(1.2686) = 116.31$ Amp or V/s,

$V_c(t)$ is max. at $t = 3.5371$ sec,

$\dot{i}_c(3.5371) = 0$,

$V_c(3.5371) = 135.09$ Volts,

$V_c(5) = -V_c(0) = 114.35$ Volts,

$\dot{V}_c(5) = i_c(5) = -\dot{V}_c(0) = -i_c(0) = -18.602$, Amp or V/s

$i_c(t)$ is max. at $t = 1.1187$ sec,

$i_c(1.1187) = 117.57$ Amp,

$\dot{i}_c(1.1187) = 0$ A/s

$V_c(1.1187) = -17.565$ Volts,

$i_c(t)$ is min. at $t = 4.7463$ sec,

$i_c(4.7463) = -19.167$ Amp,

$\dot{i}_c(4.7463) = 0$ A/s, and

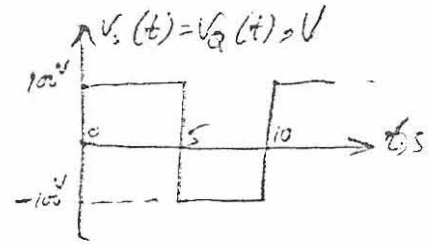
$V_c(4.7463) = 119.17$ volts.

Also note the corner points at $i_s(t)$ curve.

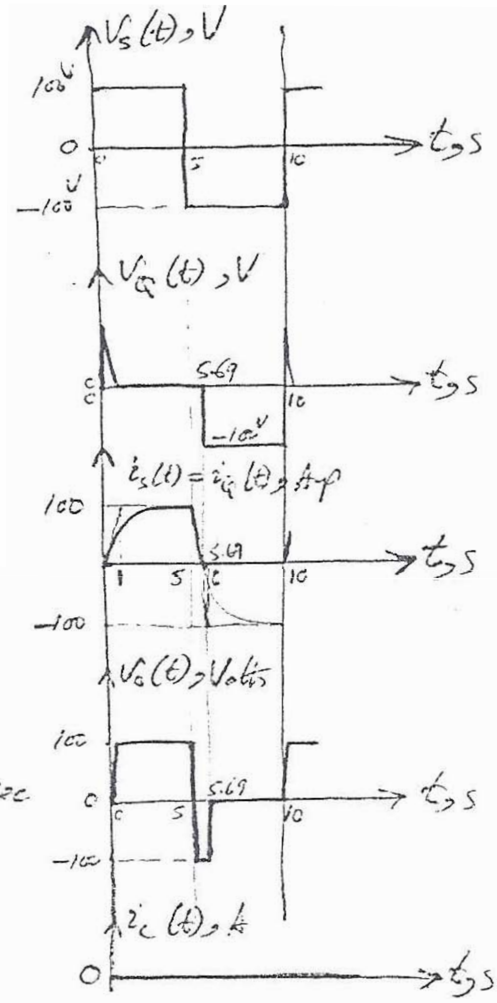
Zeros of $i_s(t)$ occures at $t = 2.3279$ sec & 7.3279 sec.

Hence, for Q, it is forward biased during $t \in [1.27, 6.27]$ sec, and in this interval the max. dv/dt is occurring at start when $t = 1.27$ sec with value of 116.31 V/sec

II) When Q is not gated and S is off then, $i_s(t) = i_c(t) = i_Q(t) = V_c(t) = 0$.
Hence, $V_Q(t) = V_s(t)$ as shown.



Hence, for Q, it is forward biased during $t \in [0, 5]$ sec and in this interval max. dV/dt is at start with value of infinity. This value causes Q to be on at the start of interval although gate is absent. Hence, current will flow into the load through Q. When the input reverses at $t = 5$ sec current will try to reverse with the result of commutating the thyristor. Waveforms are hence obtained as shown. Note that the load current is exponential with time constant of $L/R = 1$ sec and that almost it reaches its steady-state value of 100 Amp at the end of $t = 5$ sec, i.e:



$$i_Q(t) = 100\left(1 - e^{-t}\right), \text{Ap}, \; t \in [0, 5] \text{ sec}$$

$\&$ $i_Q(5) = 99.33$ Amps ≈ 100 Ap

After $t = 5$ sec, $i_Q(t)$ is given by

$$i_Q(t) = -100 + A_3 e^{-(t-5)}, \text{Ap}, \text{ where}$$

$$i_Q(5) = 99.33 = -100 + A_3 \quad \therefore A_3 = +199.33 \text{ Ap}$$

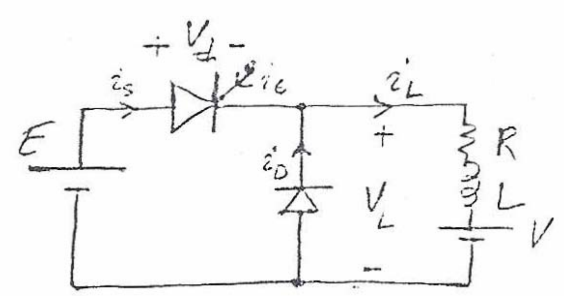
$$\therefore i_Q(t) = -100 + 199.33\, e^{-(t-5)}, \text{Ap}, \; t \in [5, t_c], \text{ where}$$

$i_Q(t_c) = 0 \qquad \therefore t_c = 5.6898$ sec after which $i_Q(t) = 0$.

$$\therefore i_Q(t) = \begin{cases} 100\left(1 - e^{-t}\right), \text{Ap}, & t \in [0, 5] \text{ sec} \\ -100 + 199.33\, e^{-(t-5)}, \text{Ap}, & t \in [5, 5.6898] \text{ sec} \\ 0, \text{Ap}, & t \in [5.6898, 10] \text{ sec} \end{cases}.$$

Hence, S introduces a snubber capacitor that protects against ∞ di_Q/dt rate at start of input positive step, with the result of Q being brought on only through gating current.

3-1 Since current mode is discontinuous, then current starts at steady state from zero when gate pulse is applied.



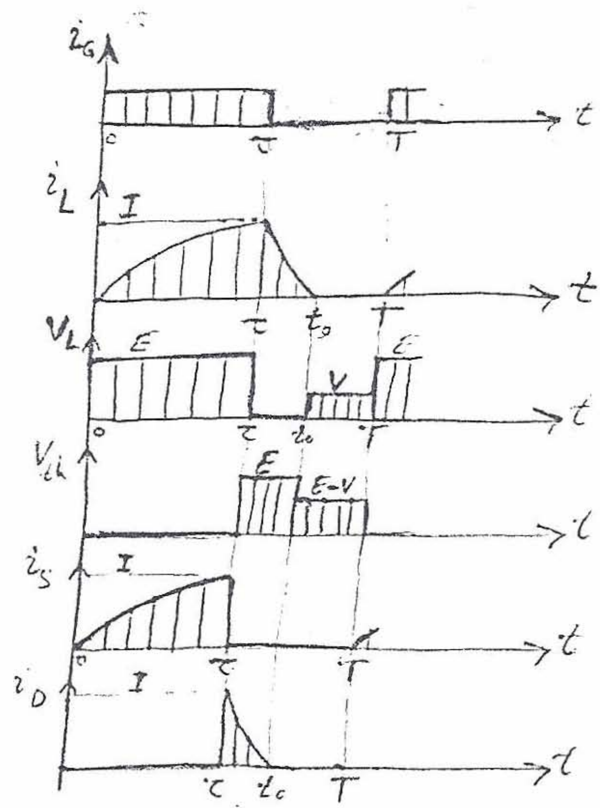
$$\therefore\ i_L(t) = \frac{E-V}{R} + Ae^{-Rt/L}$$

$$\&\ 0 = \frac{E-V}{R} + A$$

$$\therefore\ i_L(t) = \frac{E-V}{R}\left(1 - e^{-Rt/L}\right),\ t \in [0, \tau)$$

$$\therefore\ i_L(\tau) = I = \frac{E-V}{R}\cdot\left(1 - e^{-R\tau/L}\right) \quad (1)$$

$$\&\ \text{For } t \in (\tau, t_0]$$



$$\therefore\ i_L(t) = Be^{-R(t-\tau)/L} - \frac{V}{R}$$

$$\because\ i_L(\tau) = I \quad \text{due current continuity.}$$

$$\therefore\ i_L(\tau) = I = B - \frac{V}{R}$$

$$\therefore\ B = I + \frac{V}{R}$$

$$\therefore\ i_L(t) = \left(I + \frac{V}{R}\right)\cdot e^{-R(t-\tau)/L} - \frac{V}{R}, \quad t \in (\tau, t_0]$$

$$\therefore\ i_L(t_0) = 0 = \left(I + \frac{V}{R}\right)\cdot e^{-R(t_0-\tau)/L} - \frac{V}{R}$$

$$\therefore\ (V + IR) = Ve^{R(t_0-\tau)/L} \qquad \therefore\ \frac{t_0 - \tau}{L/R} = \ln\left(1 + \frac{IR}{V}\right)$$

$$\therefore\ t_0 = \tau + \frac{L}{R}\ln\left(1 + \frac{IR}{V}\right) \quad (2)$$

But, from (1):

$$1+\frac{IK}{V} = 1+\frac{E-V}{V}\cdot\left(1-e^{-K\tau/L}\right) = \left(\frac{E}{V}-1\right)\left(-e^{-K\tau/L}\right)+\frac{E}{V}\cdot$$

$$\therefore\ t_0 = \tau + \frac{L}{R}\ln\left[\frac{E}{V}\left(1-e^{-R\tau/L}\right)+e^{-R\tau/L}\right] \qquad (3)$$

Hence: to get discontinuous mode we must have:

$T \geq t_0$. where t_0 as given in (2) or (3)

The current $i_L(t)=0$ in $t\in[t_0, T]$.

$$i_{L_{av}} = \frac{1}{T}\int_0^T i_L(t)\,dt =$$

$$= \frac{1}{T}\left[\int_0^\tau i_L(t)\,dt + \int_\tau^{t_0} i_L(t)\,dt\right]$$

$$= \frac{1}{T}\left[\int_0^\tau \frac{E-V}{R}\left(1-e^{-Rt/L}\right)dt + \int_\tau^{t_0}\left[\left(I+\frac{V}{R}\right)e^{-K(t-\tau)/L}-\frac{V}{R}\right]dt\right]$$

$$= \frac{1}{T}\left[\frac{E-V}{R}\left(t+\frac{L}{R}e^{-Kt/L}\right)\Big|_0^\tau + \left[\left(I+\frac{V}{R}\right)\frac{e^{-K(t-\tau)/L}}{-K/L}-\frac{Vt}{R}\right]_\tau^{t_0}\right] =$$

$$= \frac{1}{T}\left[\frac{E-V}{R}\cdot\left(\tau+\frac{L}{R}e^{-K\tau/L}-\frac{L}{R}\right)+\left(I+\frac{V}{R}\right)\frac{L}{K}\cdot\left(1-e^{-K(t_0-\tau)/L}\right)-\frac{V}{R}(t_0-\tau)\right]$$

$$= \frac{1}{T}\left[\frac{E-V}{R}\cdot\tau-\frac{IL}{R}+\left(I+\frac{V}{K}\right)\cdot\frac{L}{K}\cdot\left(1-\frac{V}{V+IK}\right)-\frac{V}{R}\cdot\frac{L}{K}\ln\left(1+\frac{IK}{V}\right)\right]$$

$$= \frac{1}{T}\left[\frac{E-V}{R}\tau-\frac{IL}{R}+\frac{L}{R^2}\cdot IR-\frac{VL}{R^2}\ln\left(1+\frac{IK}{V}\right)\right].$$

$$\therefore\ i_{L_{av}} = \frac{E-V}{R}\cdot\frac{\tau}{T}-\frac{V}{R}\cdot\frac{L}{RT}\cdot\ln\left(1+\frac{IK}{V}\right) \qquad (4)$$

$\therefore$ power delivered to load battery, $P_0 = i_{L_{av}}\cdot V =$

$$= V\cdot\left(\frac{E-V}{R}\cdot\frac{\tau}{T}-\frac{V}{R}\cdot\frac{L}{RT}\cdot\ln\left(1+\frac{IK}{V}\right)\right) \qquad (5)$$

for $i_{L_{rms}}$

$$\therefore\ i_{L_{rms}}^2 = \frac{1}{T}\int_0^T i_L^2(t)\,dt = \frac{1}{T}\left[\int_0^{\tau} i_L^2(t)\,dt + \int_{\tau}^{t_0} i_L^2(t)\,dt\right] =$$

$$= \frac{1}{T}\left[\left(\frac{E-V}{R}\right)^2\left(t + \frac{2L}{R}e^{-Rt/L} - \frac{L}{2R}e^{-2Rt/L}\right)\Big|_0^{\tau} + \right.$$

$$\left. + \left(I+\frac{V}{R}\right)^2 \cdot \frac{e^{-2R(t-\tau)/L}}{-2R/L}\Big|_{\tau}^{t_0} + \left(\frac{V}{R}\right)^2 t\Big|_{\tau}^{t_0} - \frac{2V}{R}\left(I+\frac{V}{R}\right)\frac{e^{-R(t-\tau)/L}}{-R/L}\Big|_{\tau}^{t_0}\right]$$

$$= \frac{1}{T}\cdot\left[\left(\frac{E-V}{R}\right)^2\cdot\left(\tau + \frac{2L}{R}\left(e^{-R\tau/L}-1\right) - \frac{L}{2R}\left(e^{-2R\tau/L}-1\right)\right) + \left(\frac{V}{R}\right)^2(t_0-\tau) + \right.$$

$$\left. + \left(I+\frac{V}{R}\right)^2\cdot\frac{L}{2R}\cdot\left(1-e^{-2R(t_0-\tau)/L}\right) - \frac{2V}{R}\left(I+\frac{V}{R}\right)\cdot\frac{L}{R}\left(1-e^{-R(t_0-\tau)/L}\right)\right]$$

$$= \frac{1}{T}\cdot\left[\left(\frac{E-V}{R}\right)^2\cdot\left\{\tau + \frac{2L}{R}\cdot\left(\frac{-IR}{E-V}\right) - \frac{L}{2R}\left(\frac{IR}{E-V}-1\right)^2 + \frac{L}{2R}\right\} + \left(\frac{V}{R}\right)^2(t_0-\tau) + \right.$$

$$\left. + \left(I+\frac{V}{R}\right)^2\cdot\frac{L}{2R}\cdot\left(1-\frac{V^2}{(V+IR)^2}\right) - \frac{2V}{R^2}(IR+V)\cdot\frac{L}{R}\cdot\left(1-\frac{V}{V+IR}\right)\right] =$$

$$= \frac{1}{T}\cdot\left[\left(\frac{E-V}{R}\right)^2\cdot\left\{\tau + \frac{L}{2R} - \frac{2IL}{E-V} - \frac{I^2RL}{2(E-V)^2} + \frac{LI}{E-V} - \frac{L}{2R}\right\} + \left(\frac{V}{R}\right)^2(t_0-\tau) + \right.$$

$$\left. + \frac{(IR+V)^2L}{2R^3}\cdot\frac{IR(2V+IR)}{(V+IR)^2} - \frac{2VLIR}{R^3}\right] =$$

$$= \frac{1}{T}\left[\left(\frac{E-V}{R}\right)^2\cdot\left\{\tau - \frac{I^2RL + 2LI(E-V)}{2(E-V)^2}\right\} + \left(\frac{V}{R}\right)^2(t_0-\tau) + \right.$$

$$\left. + \frac{IL(2V+IR)}{2R^2} - \frac{2VIL}{R^2}\right] = \frac{1}{T}\left[\left(\frac{E-V}{R}\right)^2\tau + \left(\frac{V}{R}\right)^2(t_0-\tau) + \right.$$

$$\left. - \frac{I^2RL + 2LIE - 2LIV - 2ILV - I^2LR + 4VIL}{2R^2}\right] =$$

$$= \frac{1}{T}\cdot\left[\left(\frac{E-V}{R}\right)^2\tau + \left(\frac{V}{R}\right)^2(t_0-\tau) - \frac{LIE}{R^2}\right] =$$

$$= \left(\frac{E-V}{R}\right)^2\cdot\frac{\tau}{T} + \left(\frac{V}{R}\right)^2\left(\frac{t_0-\tau}{T}\right) - \frac{E}{R}\cdot I\cdot\frac{L}{R\cdot T}$$

$$\therefore\ i_{L_{rms}} = \sqrt{\left(\frac{E-V}{R}\right)^2\cdot\frac{\tau}{T} + \left(\frac{V}{R}\right)^2\cdot\frac{L}{RT}\ln\left(1+\frac{IR}{V}\right) - \left(\frac{EI}{R}\right)\cdot\frac{L}{RT}} \qquad (6)$$

—

$\boxed{3\text{-}2}$ a) $T_{critical} = t_o$ of $\boxed{3\text{-}1} = \tau + \frac{L}{R}\ln\left[\frac{E}{V}\left(1-e^{-R\tau/L}\right)+e^{-R\tau/L}\right] =$

$= 3.66794$ ms (see Eq.-3 in $\boxed{2-6}$).

$\therefore T_{critical} = 3.66794$ msec

b) $T = (T_{critical} + \tau)/2 = 3.334$ msec $\therefore$ Continuous Mode.

$$\therefore I_{min} = -\frac{V}{R} + \frac{E}{R}\cdot\frac{e^{R\tau/L}-1}{e^{RT/L}-1} = 0.20556 \text{ Amp}$$

$$\& \; I_{max} = -\frac{V}{R} + \frac{E}{R}\cdot\frac{1-e^{-R\tau/L}}{1-e^{-RT/L}} = 0.48534 \text{ Amp}$$

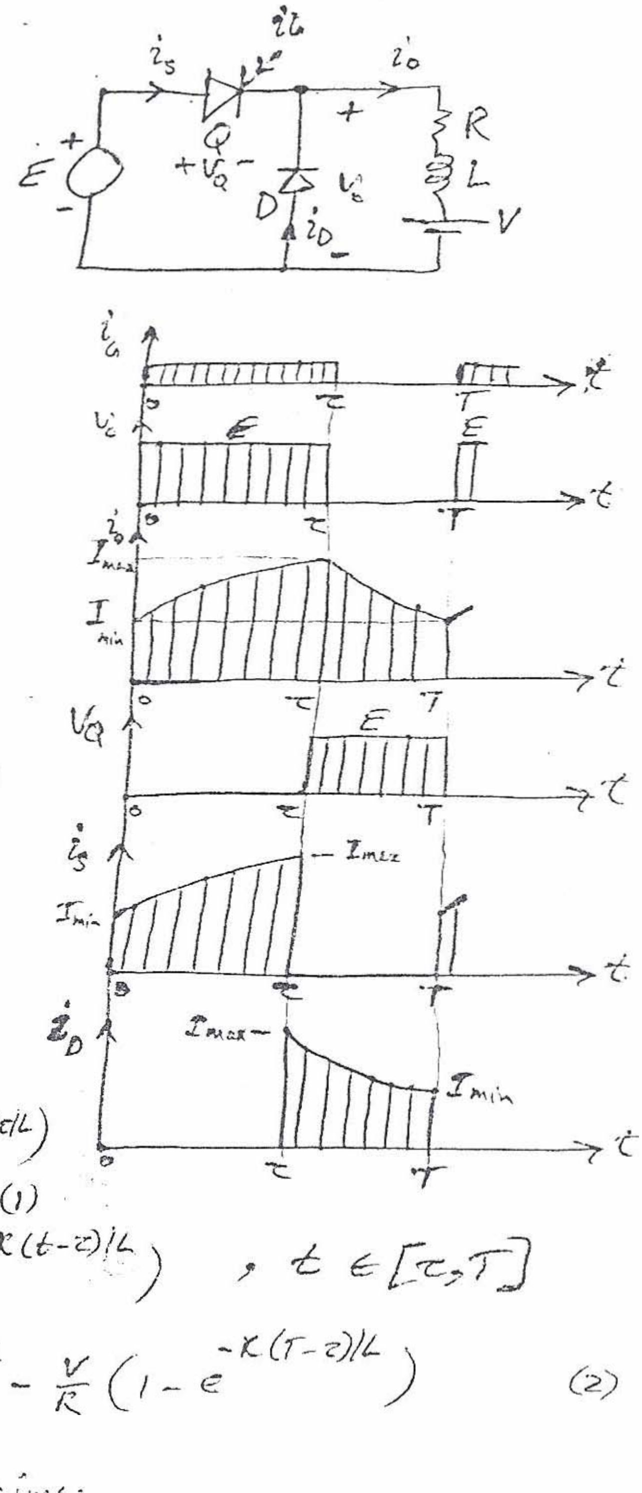
$$\therefore i_{L_{av}} \text{ (by linearization)} = \frac{I_{max}+I_{min}}{2} = 0.34545 \text{ Amp}$$

$$i_{L_{av}} \text{ (actual)} = \frac{E-V}{R}\cdot\frac{\tau}{T} - \frac{V}{R}\cdot\left(1-\frac{\tau}{T}\right) = 0.39982 \text{ Amps}$$

$$\therefore \% \text{ error in } i_{L_{av}} \text{ due to linearization} = \left|1-\frac{.34545}{.39982}\right| * 100 = 13.6\%$$

(Note: the actual value for $i_{L_{av}}$ is easy to calculate, without referring to the current expression, by using superposition. For dc, L looks short circuit and hence average voltage across R is found from the waveform of the load voltage, hence $i_{L_{av}}$ is obtained. You can also get it by integration, but this is long and avoidable).

$\boxed{3-3}$ A first quadrant chopper supplies positive load voltage and positive load current. It has three modes of operation, Discontinuous, Critical & Continuous, depending upon load currents.



1) The discontinuous mode is analized in $\boxed{3-1}$.
2) The critical mode is a limit case of $\boxed{3-1}$ and can be obtained when T is made $= t_o$ in Eq. 2 of $\boxed{3-1}$
3) The continuous mode is considered here for analysis.

$$i_o(t) = I_{min}\, e^{-Rt/L} + \frac{E-V}{R}\left(1-e^{-Rt/L}\right) \qquad t\in[0,\tau]$$

$$\therefore\ I_{max} = i_o(\tau) = I_{min}\, e^{-R\tau/L} + \frac{E-V}{R}\left(1-e^{-R\tau/L}\right) \tag{1}$$

$$\& \ i_o(t) = I_{max}\, e^{-R(t-\tau)/L} - \frac{V}{R}\left(1-e^{-R(t-\tau)/L}\right), \quad t\in[\tau, T]$$

$$\therefore\ I_{min} = i_o(T) = I_{max}\, e^{-R(T-\tau)/L} - \frac{V}{R}\left(1-e^{-R(T-\tau)/L}\right) \tag{2}$$

Solving (1) & (2) together gives:

$$I_{min} = -\frac{V}{R} + \frac{E}{R}\cdot\frac{e^{R\tau/L}-1}{e^{RT/L}-1} \tag{3}$$

$$I_{max} = -\frac{V}{R} + \frac{E}{R}\cdot\frac{e^{R\tau/L}-1}{e^{RT/L}-1}\cdot e^{R(T-\tau)/L} \tag{4}$$

The waveforms are as shown above.

Assuming a chopping period, $T, << L/R$, the current pattern $i_L(t)$ can be considered triangular.

$$\therefore \quad I_{o_{av}} = (I_{max} + I_{min})/2 \qquad (5)$$

$$\& \quad I_{o_{rms}} = \sqrt{I_{o_{av}}^2 + \left(\frac{I_{max} - I_{min}}{2\sqrt{3}}\right)^2} \qquad (6)$$

Other parameters can be obtained very easily.

(Note: the critical mode is also a limit case of continuous mode, and is obtained by letting $I_{min} = 0$, i.e

$$T_{critical} = \frac{L}{R} \ln\left[\frac{E}{V}\left(e^{R\tau/L} - 1\right) + 1\right]. \qquad (7)$$

The expression is just the same as the one obtained earlier:

$$T_{critical} = \tau + \frac{L}{R} \ln\left[1 + \frac{I_{max} \cdot R}{V}\right].\ ) \qquad (8)$$

$\boxed{3-4}$ $E = 600^V$, $V = 200^V$, $L = 1mh$, $R = 1.5\,\Omega$, $T = 4\,ms$
& $\tau = 2.5\,ms$

To judge mode of operation find $T_{critical}$:

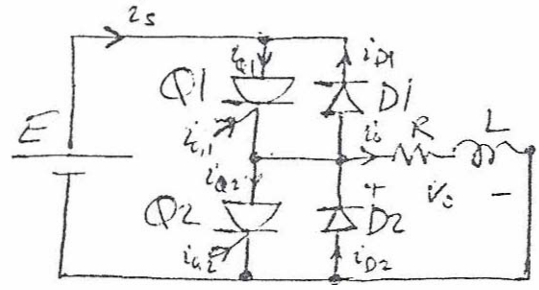
$$(7) \Rightarrow \quad T_{critical} = \frac{1m}{1.5} \ln\left[\frac{600}{200} \cdot \left(e^{2.5 \times 1.5} - 1\right) + 1\right] = 3.222 \text{ m sec}$$

Since $T = 4\,ms$ $\quad \therefore T > T_{critical}$ $\quad \therefore$ Discontinuous mode.

(Note: you can also judge it by I_{min} being negative:

$(3) \Rightarrow \quad I_{min} = -\frac{200}{1.5} + \frac{600}{1.5} \cdot \frac{e^{2.5 \times 1.5} - 1}{e^{4 \times 1.5} - 1} = -92.1$ Amp $\quad \therefore$ OK)

$\boxed{3\text{-}5}$ The circuit for bidirectional current chopper-converter with positive output is shown aside.

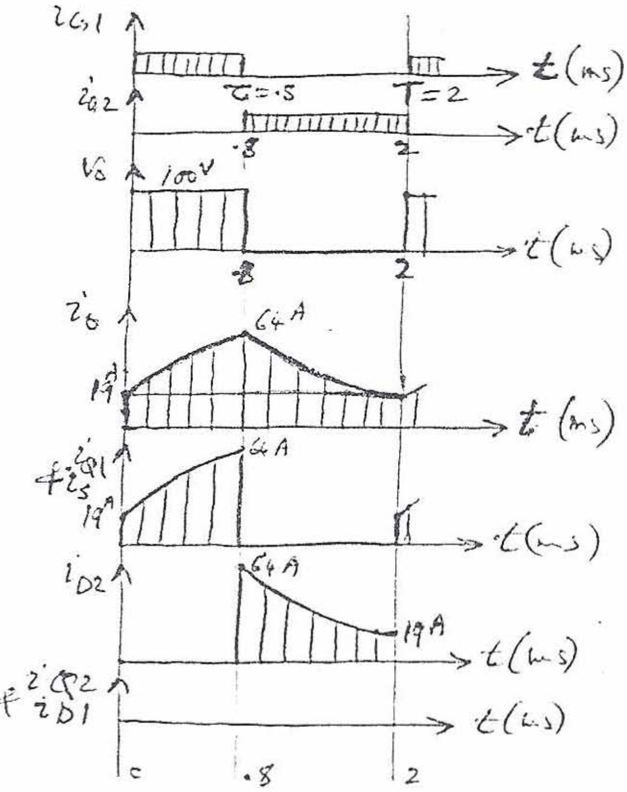


$\because$ $R = 1\Omega$, $L = 1mh$, $T = 2ms$
& $\tau = .8\,ms$ & $E = 100$ Volts.

$$\therefore I_{min} = \frac{E}{R}\cdot\frac{e^{R\tau/L}-1}{e^{RT/L}-1} = 19.182\,Ap$$

$$\& \; I_{max} = I_{min} * e^{R(T-\tau)/L} = 63.686\,Ap$$

Since I_{min} is positive then D1 & Q2 could be taken out of the circuit without harm.
The current mode is always continuous.
& Commutation type is Forced.
The waveforms are as shown.

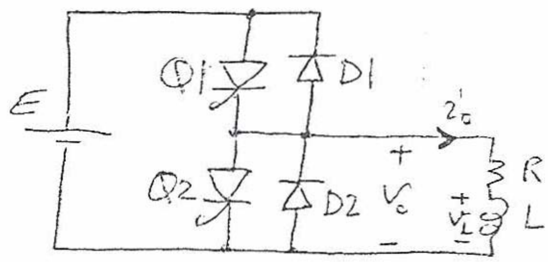


$$\therefore V_{o_{av}} = \frac{100 * .8}{2} = 40\,\text{Volts}$$

$$\therefore i_{o_{av}} = \frac{40V}{1\Omega} = 40\,\text{Amps}$$

(Note one could assume linear segments since $\frac{L}{R} = 1msec$ comparable to τ & $T-\tau$, $\therefore i_{o_{av}} \simeq \frac{I_{min}+I_{max}}{2} = 41.434\,Ap$, $\%e = 3.6\%$)

The circuit and waveforms of a half-bridge chopper with $R = 10\,\Omega$, $L = 100\ mH$, $\tau = 5\ ms$, $T = 7\ ms$ & $E = 100$ Volts; are as shown;



where:

$$I_n = \frac{100}{10} \cdot \frac{e^{5\cdot 10/100} - 1}{e^{7\cdot 10/100} - 1} = 6.40A$$

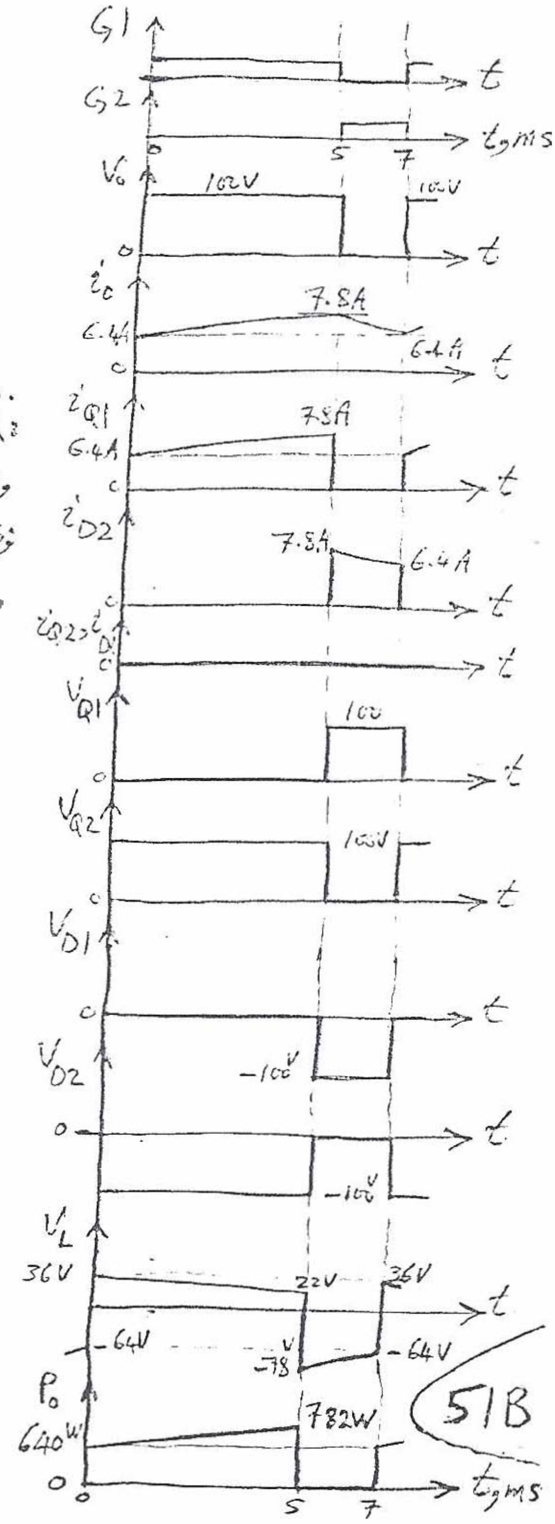
$$\therefore\ I_x = \frac{100}{10} + \left(6.4 - \frac{100}{10}\right) e^{-5\cdot 10/100} = 7.82A$$

$V_L = L\, di/dt \simeq$ level segments,
- $\cdot 1 * \frac{7.82 - 6.40}{.005} = 28.34V$ for $t \in (0, 5)\ ms$ &
- $-.1 * \frac{7.82 - 6.40}{.002} = -70.84V$ $(\simeq 28.34 - 100V)$ for $t \in (5, 7)\ ms$.

Or, exactly, exponential segments with:
$V_L(0^+ ms) = 100 - 10 * 6.40 = 36.01$ Volts,
and $V_L(5^- ms) = 100 - 10 * 7.82 = 21.84$ Volts;
$V_L(5^+ ms) = 0 - 10 * 7.82 = -78.16$ Volts,
and $V_L(7^- ms) = 0 - 10 * 6.40 = -63.99\ V$.
Line segments could be assumed since, $L/R = 10\ ms > T$.

$\therefore\ V_{o_{av}} = 100 * 5/7 = 71.43$ Volts,
$i_{o_{av}} = V_{o_{av}}/R = 7.14$ Amps
$P_{o_{av}} \simeq \frac{782 + 640}{2} \cdot \frac{5}{7} = 508\ W$
$(\text{or} \simeq i_{av}^2 R = 510\ W)$
$I_{x_{Q1}} = 7.8A$, $I_{av_{Q1}} = \frac{7.14 * 5}{7} = 5.10A$,
$V_{x_{Q1}} = .100\ V$, $V_{av_{Q1}} = 100 * 2/7 = 28.6V$,
$I_{x_{D2}} = 7.8\ A$, $I_{av_{D2}} = 7.14 * 2/7 = 2.04$,
$V_{x_{D2}} = 100\ V$, $V_{av_{D2}} = 100 * 5/7 = 71.4V$.
As for D1 & Q2, they are not in use.

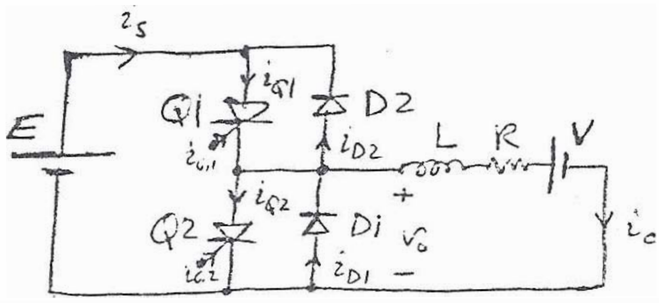
$$K_i = \frac{i_{ac}}{i_{dc}} = \frac{(7.82 - 6.40)/2\sqrt{3}}{7.14} = 5.73\%$$



51B

$\boxed{3\text{-}6}$

$E = 600$ volts
$V = 200$ volts
$L = 4$ mh
$R = 1.5\,\Omega$
$T = 4$ ms
$\tau = 2.5$ ms



$\therefore$ a) $i_{c_{av}} = I_o = \left(E\dfrac{\tau}{T} - V\right)/R$

$= \left(600 * \dfrac{2.5}{4} - 200\right)/1.5 = 116.7$ Amp

$\therefore V_{o_{av}} = V_o = \dfrac{E\tau}{T} = \dfrac{600*2.5}{4} = 375$ Volts.

b) $I_{max} = \dfrac{-V}{R} + \dfrac{E}{R} \cdot \dfrac{e^{R\tau/L} - 1}{e^{RT/L} - 1} \cdot e^{R(T-\tau)/L} =$

$= \dfrac{-200}{1.5} + \dfrac{600}{1.5} \cdot \dfrac{e^{1.5*2.5/4} - 1}{e^{1.5*4/4} - 1} \cdot e^{1.5*1.5/4}$

$= 179.92$ Amps

& $I_{min} = \dfrac{-V}{R} + \dfrac{E}{R} \cdot \dfrac{e^{R\tau/L} - 1}{e^{RT/L} - 1}$

$= \dfrac{-200}{1.5} + \dfrac{600}{1.5} \cdot \dfrac{e^{1.5*2.5/4} - 1}{e^{1.5*4/4} - 1} = 45.15$ Amps.

c) Since $L/R = 4/1.5 = 2.7$ msec $> \tau$

$\therefore$ we can assume linear pattern of current segments.

$\therefore\ i_{s_{av}} \simeq \dfrac{I_{max} + I_{min}}{2} \cdot \dfrac{\tau}{T} = \dfrac{179.92 + 45.15}{2} \cdot \dfrac{2.5}{4}$

$= 70.34$ Amps.

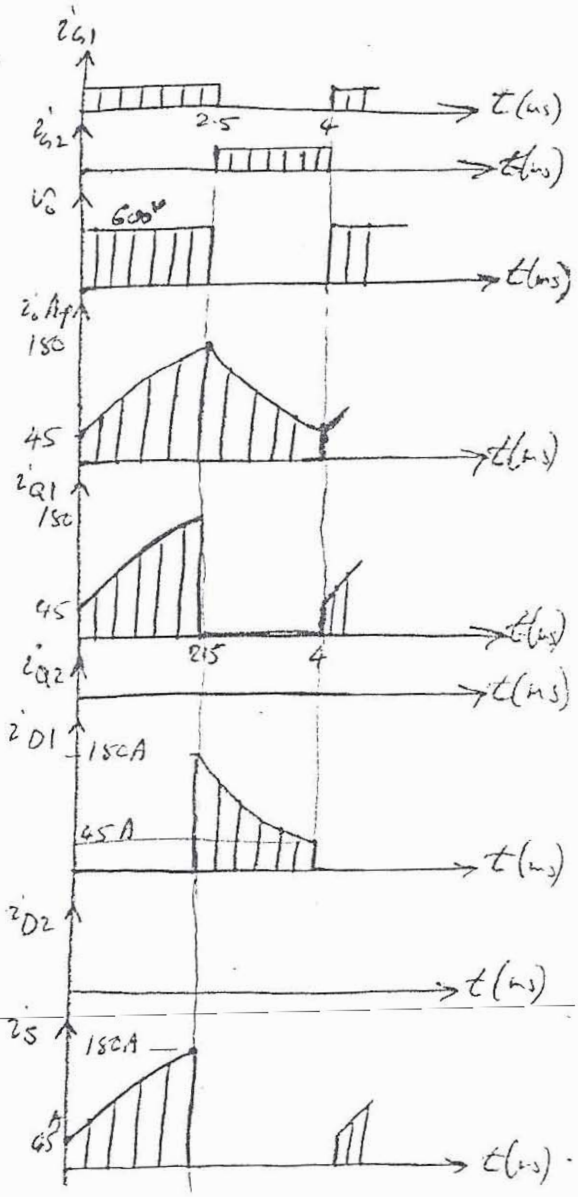
(Note: the accurate answer is

$i_{s_{av}} = \dfrac{1}{4}\displaystyle\int_0^{2.5}\left[45.15\,e^{-1.5t/4} + \dfrac{600-200}{1.5}\left(1 - e^{-1.5t/4}\right)\right]dt$

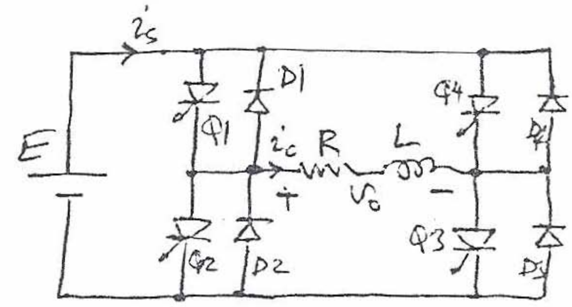
$= \left[-221.51 \cdot \dfrac{e^{-1.5t/4}}{-1.5/4} + 266.67\,t\right]_0^{2.5} / 4$

$= \left[590.70\left(e^{-1.5*2.5/4} - 1\right) + 266.67 * 2.5\right] / 4 = 76.82$ Amps.).

e) No difference, since Q2 & D2 are not in operation, for the given data

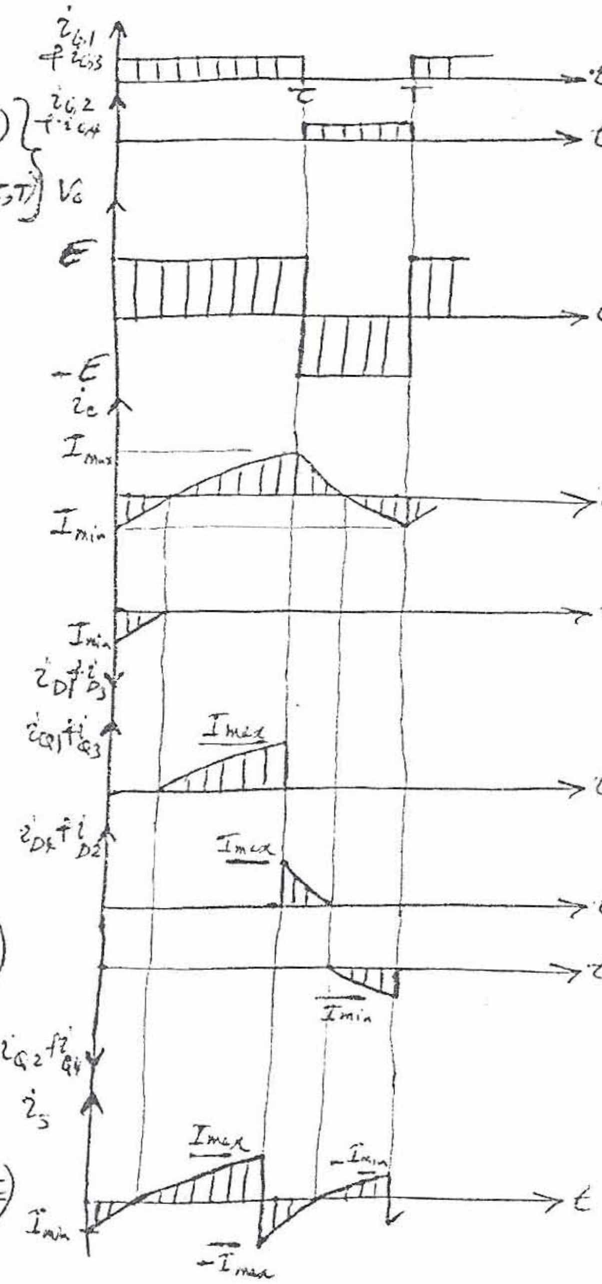


3-7 Consider the analysis of full bridge choppers feeding RL loads. The figs shows circuit and waveforms relevant.



Here:

$$v_o(t)=\begin{cases}E & t\in(0,\tau)\\ -E & t\in(\tau,T)\end{cases}$$



&

$$i_o=\begin{cases}I_{min}e^{-Rt/L}+\frac{E}{R}\left(1-e^{-Rt/L}\right), & t\in(0,\tau)\\ I_{max}e^{-R(t-\tau)/L}-\frac{E}{R}\left(1-e^{-R(t-\tau)/L}\right), & t\in(\tau,T)\end{cases}$$

$$\therefore I_{max}=i_o(\tau)=I_{min}e^{-R\tau/L}+\frac{E}{R}\left(1-e^{-R\tau/L}\right)$$

& $I_{min}=i_o(T)=I_{max}e^{-R(T-\tau)/L}-\frac{E}{R}\left(1-e^{-R(T-\tau)/L}\right)$

Solving together:

$$I_{max}=\frac{E}{R}\cdot\frac{e^{RT/L}-2e^{R(T-\tau)/L}+1}{e^{RT/L}-1}$$

& $$I_{min}=\frac{-E}{R}\cdot\frac{e^{RT/L}-2e^{R\tau/L}+1}{e^{RT/L}-1}$$

For large inductance, we can assume linear segments.

$$\therefore i_{c_{av}}\simeq\frac{I_{max}+I_{min}}{2}\quad\left(i_{c_{av}}=\frac{E(2\tau-T)}{R\cdot T}\right)$$

& $$i_{c_{rms}}\simeq\sqrt{i_{c_{av}}^2+\left(\frac{I_{max}-I_{min}}{2\sqrt{3}}\right)^2}$$

& $$i_{s_{av}}\simeq\left(\frac{I_{max}+I_{min}}{2}\right)\frac{\tau}{T}+\left(\frac{-I_{max}-I_{min}}{2}\right)\left(\frac{T-\tau}{T}\right)$$

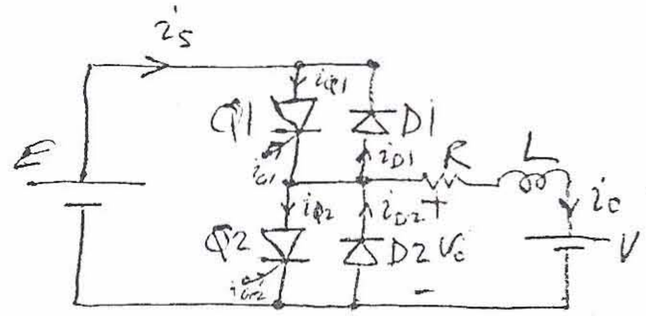
$$=\left(\frac{I_{max}+I_{min}}{2}\right)\left(\frac{\tau}{T}-\frac{T-\tau}{T}\right)$$

$$=\left(\frac{I_{max}+I_{min}}{2}\right)\left(\frac{2\tau}{T}-1\right)$$

& $$i_{s_{rms}}^2\simeq i_{s_{av}}^2+\left(\frac{I_{max}-I_{min}}{2\sqrt{3}}\right)^2\cdot\left(\frac{\tau}{T}+\frac{T-\tau}{T}\right)\Rightarrow i_{s_{rms}}\simeq\sqrt{i_{s_{av}}^2+\left(\frac{I_{max}-I_{min}}{2\sqrt{3}}\right)^2}$$

$\boxed{3\text{-}8}$ $I_{min} = -\frac{50}{20} + \frac{200}{20}\cdot\frac{e^{20*7/100}-1}{e^{20*10/100}-1}$

$= 2.28193$ Amps



$E = 200^V$, $V = 50^V$, $R = 20\Omega$, $L = .1h$, $\tau = 7ms$, $T = 10ms$

$I_{max} = \frac{-50}{20} + \frac{200}{20}\cdot\frac{e^{20*7/100}-1}{e^{20*10/100}-1}\cdot e^{20*3/100}$

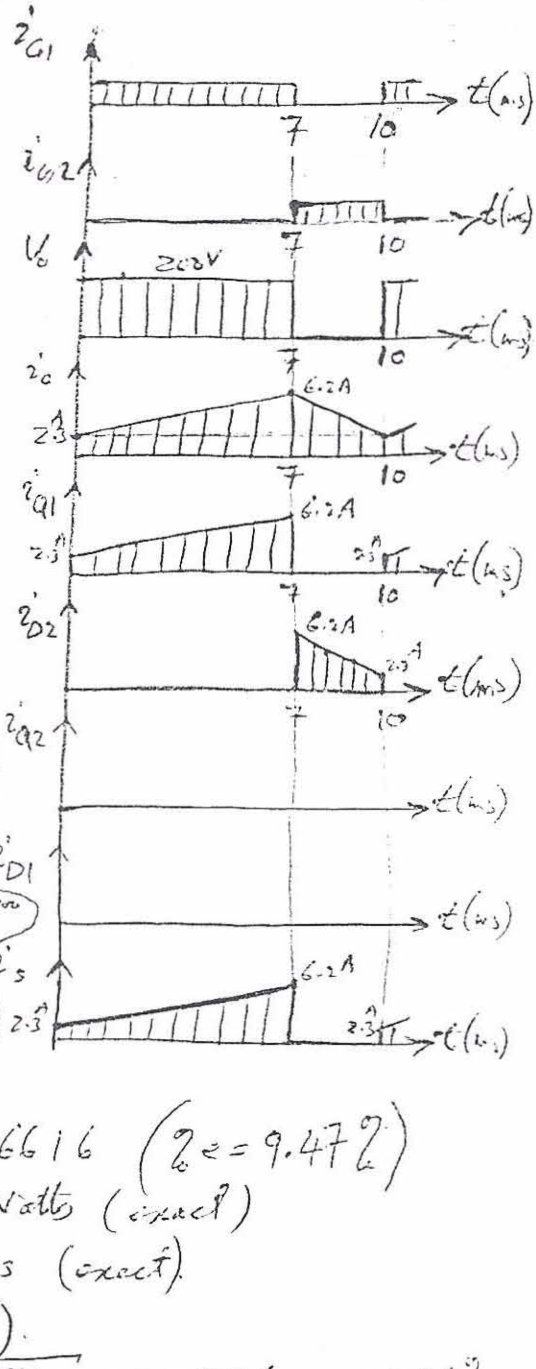
$= 6.21324$ Amps

$\because \frac{L}{R} = \frac{100\,mh}{20\Omega} = 5$ msec

is comparable to τ & $T-\tau$ (i.e. 7ms & 3ms)

$\therefore$ The exponential segment could be assumed linear.

The waveforms are as shown.



\# $i_{o_{av}} \simeq \frac{2.28193+6.21324}{2} = 4.24758A$

OR $i_{o_{av}} = \left(\frac{200*7}{10} - 50\right)/20 = 4.5$ Amp (exact)

(%e = 5.61% $\therefore$ assumption justified)

\# $i_{o_{rms}} \simeq \sqrt{4.5^2 + \left(\frac{6.21324-2.28193}{2\sqrt{3}}\right)^2} = 4.64090A$

OR $i^2_{o_{rms}} =$ (power supplied by source − power consumed by battery)/R (exact)

\# P_E = power supplied by $E = 200*\frac{1}{10}\int_0^7 \left(2.28193e^{-20t/100} + \frac{200-50}{20}\left(1-e^{-20t/100}\right)\right)dt = 20\left[2.28193\frac{e^{-.2t}}{-.2}\Big|_0^7 + 7.5\left(t - \frac{e^{-.2t}}{-.2}\right)\Big|_0^7\right] = 20\left[2.28193\left(\frac{1-e^{-1.4}}{.2}\right) + 7.5\left(7 - \frac{1-e^{-1.4}}{.2}\right)\right] = 656.8688$ Watt (exact)

OR $P_E \simeq 200 * \frac{2.28193+6.21324}{2} * \frac{7}{10} = 594.6616$ (%e = 9.47%)

\# P_V = power delivered to $V = 50 * 4.5 = 225$ Watts (exact)

$\therefore$ $i_{o_{rms}} = \sqrt{(656.8688-225)/20} = 4.64687$ Amps (exact)

(%e (of approximate value of 4.64090) = 0.13%).

\# $\therefore$ $K_i = \frac{i_{o_{ac}}}{i_{o_{av}}} = \sqrt{\frac{i^2_{o_{rms}} - i^2_{o_{av}}}{i^2_{o_{av}}}} = \sqrt{\left(\frac{i_{o_{rms}}}{i_{o_{av}}}\right)^2 - 1} = \sqrt{.06634} = 0.2576 = 25.76\%$

\# & $V_{o_{av}} = 200*7/10 = 140$ volts

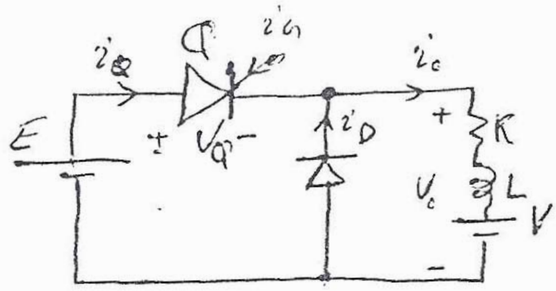
\# & $V_{o_{rms}} = 200\sqrt{\frac{7}{10}} = 167.3320$ volts

\# $\therefore$ $K_v = \sqrt{\left(\frac{V_{o_{rms}}}{V_{o_{av}}}\right)^2 - 1} = 0.6547 = 65.47\%$

\# D1 is not needed & for D2: $i_{F_{max}} \geq 6.21324$ Amp, $i_{F_{av}} \geq 4.24758 * \frac{3}{10} = 1.2743$ Amp

& $i_{F_{rms}} \geq \sqrt{\left[4.24758^2 + \left(\frac{6.21324-2.28193}{2\sqrt{3}}\right)^2\right] * .3} = 2.40810$ Amp. Current Mode is Continuous.

3-9 a) $E = 80$ Volts, $R = 10\,\Omega$, $L = 50$ mh, $\tau = 15$ ms, $T = 25$ ms & $V = 30$ Volts, with circuit as shown.



b) $\therefore I_{min} = \frac{-30}{10} + \frac{80}{10} \cdot \frac{e^{10*15/50} - 1}{e^{10*25/50} - 1} = -1.9642$ Amp

& since no path is available for negative current,
$\therefore$ Mode is discontinuous.

& The waveforms are as shown ($i_s = i_Q$)

$\therefore i_o(t) = \frac{80-30}{10}\left(1 - e^{-10t/50}\right)$, ($t$ in ms)

$= 5\left(1 - e^{-t/5}\right)$ Amp, $t \in [0, 15]$ ms

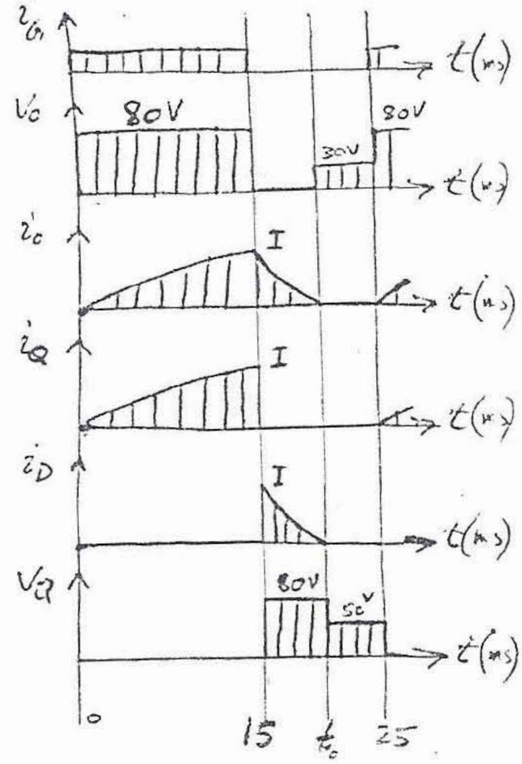
$\therefore I = i_o(15) = 5\left(1 - e^{-3}\right) = 4.7511$ Amp

& $i_o(t) = I e^{-10(t-15)/50} - \frac{30}{10}\left(1 - e^{-(t-15)/5}\right) =$

$= -3 + 7.7511\, e^{(15-t)/5}$ A, $t \in [0, t_o]$ ms.

$\therefore i_o(t_o) = 0 \Rightarrow t_o = 19.7461$ ms.

& $i_o(t) = 0$ for $t \in [t_o, 25]$ ms.



c) $\therefore V_{o_{av}} = \frac{1}{25} * \left[80 * 15 + 30 * (25 - 19.7461)\right] =$

$= 54.3047$ Volts.

& $i_{o_{av}} = \frac{54.3047 - 30}{10} = 2.4305$ Amp.

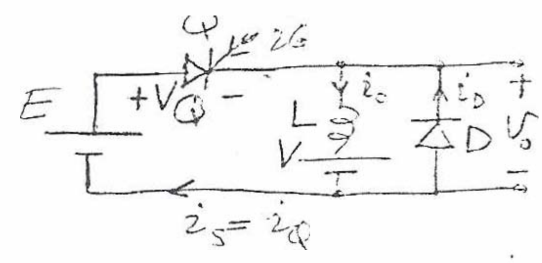
& $V_{o_{rms}}^2 = \frac{1}{25}\left[80^2 * 15 + 30^2 * (25 - 19.7461)\right] = 4024.141\ V^2$

$\therefore V_{o_{rms}} = 63.4755$ Volts

$\therefore V_{o_{ac}} = \sqrt{V_{o_{rms}}^2 - V_{o_{av}}^2} = 32.8655$ Volts

$\therefore K_v = \frac{V_{o_{ac}}}{V_{o_{av}}} = .6053 = 60.53\,\%$.

3-10 $E = 200$ Volts,
$\tau = 20$ ms,
$T = 60$ ms,
$L = 100$ mh &
$V = 80$ Volts.



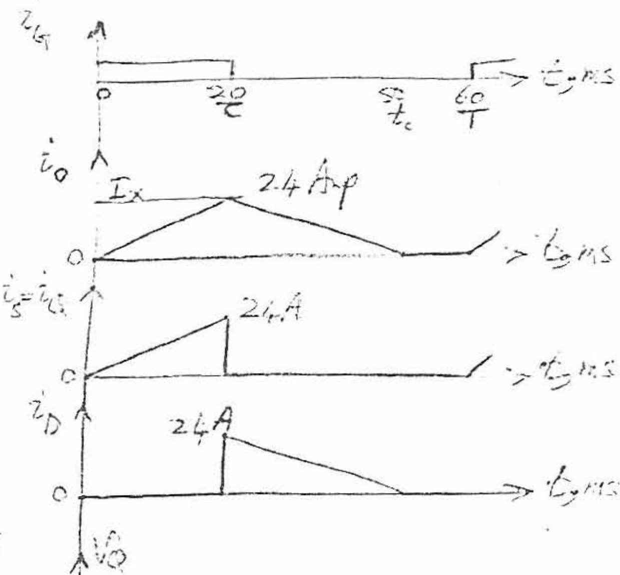
Assume $i_o(0) = I_o \geq 0$

∴ $i_o(\tau) = I_o + \frac{E-V}{L}\cdot\tau$

∴ $i_o(T) = I_o + \frac{E-V}{L}\tau - \frac{V}{L}(T-\tau)$

$= I_o + \frac{E\tau - VT}{L} = I_o + \Delta I$,

where $\Delta I = \frac{E\tau - VT}{L}$ (1)



For stable operation, ΔI must be ≤ 0; because if it was positive then the current at the end of a cycle would be more than his value at the beginning. This would cause the current to increase unstablly reaching after few cycles to very large values and hence the circuit becomes in-operational. You can equally argue it by saying that the energy pumped to L during the on-state 56 should be pumped out before (or just at) the end of the cycle.

If ΔI were equal to zero, continuous or critical mode occurs; whereas for $\Delta I < 0$, the current collapses to discontinuous mode.

For our case; $\Delta I = \frac{200 * 20 - 80 * 60}{100} = -8$ Amp $< 0 \Rightarrow I_o = 0$A.

∴ Current mode is discontinuous, with extinction at $t_o = \frac{E\tau}{V} = \frac{200}{80} * 20 = 50$ ms

This results in the indicated waveforms, with $I_x = \frac{E-V}{L}\tau = 24$ Amp.

$\therefore\ i_{c_{av}} = \frac{24 * 50}{2 * 60} = 10$ Amp.

& $i_{o_{rms}} = \sqrt{\frac{24^2 * 50}{3 * 60} = 160} = 4\sqrt{10} = 12.6491$ Amp

$\therefore\ K_i = \frac{i_{oac\,rms}}{i_{av}} = \sqrt{\left(\frac{i_{o\,rms}}{i_{o\,av}}\right)^2 - 1} = 0.7746 = 77.46\ \%$

& $V_{o_{av}} = \frac{200 * 20 + 80 * 10}{60} = 80$ Volts

& $V_{o_{rms}} = \sqrt{\frac{200^2 * 20 + 80^2 * 10}{60} = 14400} = 120$ Volts

$\therefore\ K_v = \sqrt{\left(\frac{V_{o\,rms}}{V_{o\,av}}\right)^2 - 1} = 1.12 = 112\ \%$

$P_v = V \cdot i_{o_{av}} = 80 * 10 = 800$ Watt $= P_E$ because $P_L = 0$

(check : $P_E = E \cdot i_{s_{av}} = 200 * \frac{24 * 20}{2 * 60} = 800\,W$ $\therefore$ OK)

& $i_{DF_{max}} = 24$ Amp.

& $i_{DF_{av}} = \frac{24 * 30}{2 * 60} = 6$ Amp.

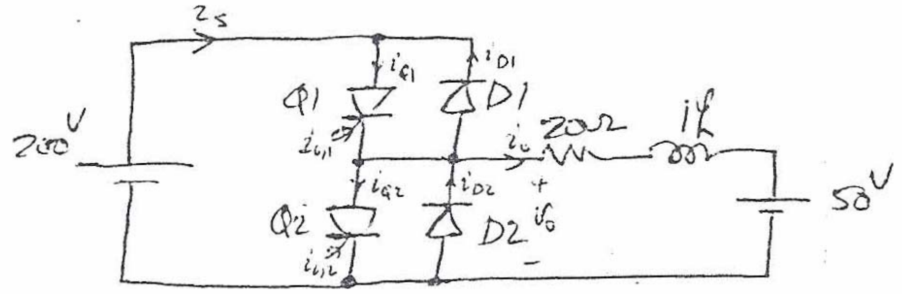
& $i_{DF_{rms}} = \sqrt{\frac{24^2 * 30}{3 * 60} = 96} = 4\sqrt{6} = 9.798$ Amp

Yes, τ is limited operationally by Eq (1) to give $\Delta I = \frac{E\tau - VT}{L} \leq 0 \Rightarrow E\tau \leq VT$

$\therefore\ \tau \in \left[0, \frac{VT}{E}\right] \equiv [0, 24]$ ms.

In case this limit is violated, current will rise gradually to ∞.

3-11 a)



b) $I_{min} = -\frac{50}{20} + \frac{200}{20} \cdot \frac{e^{.14} - 1}{e^{.2} - 1}$

$= 4.287$ Amp

$I_{max} = -\frac{50}{20} + \frac{200}{20} \cdot \frac{e^{.14} - 1}{e^{.2} - 1} \cdot e^{.06}$

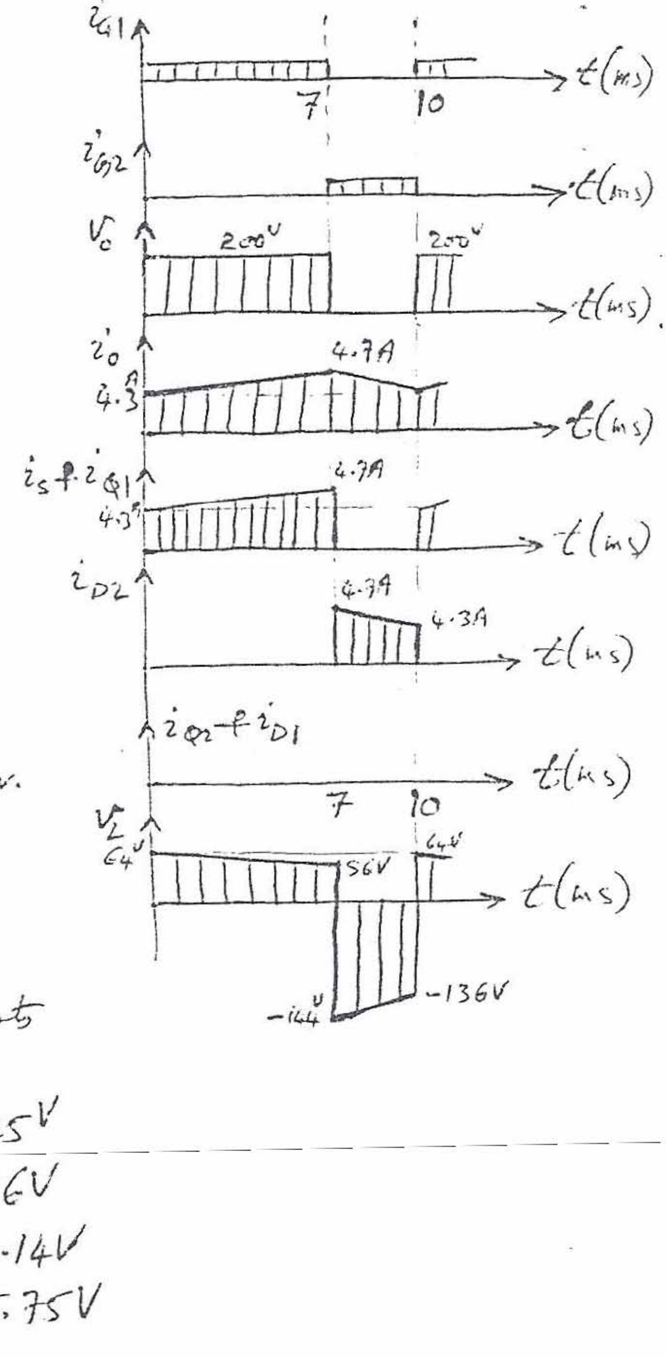
$= 4.707$ Amp

Time-Constant of current $= \frac{L}{R}$

$= \frac{1}{20} = 50\text{ ms} \gg \tau$ & $T - \tau$

$\therefore$ Segments could be assumed linear.

and waveforms are as shown.



$V_L = V_o - V - i_o R$, with linear segments

$V_L(0^+) = 200 - 50 - 4.287 * 20 = 64.25^V$

$V_L(7^-) = 200 - 50 - 4.707 * 20 = 55.86V$

$V_L(7^+) = 0 - 50 - 4.707 * 20 = -144.14V$

$V_L(10^-) = 0 - 50 - 4.287 * 20 = -135.75V$

c) $V_{o_{av}} = 140$ Volt, $V_{o_{rms}} = 167.33$ Volt & $K_v = 65.5\%$

$i_{o_{av}} = 4.50$ Amp, $i_{o_{rms}} = \sqrt{4.5^2 + \left(\frac{4.707 - 4.287}{2\sqrt{3}}\right)^2} = 4.502$ Amp & $K_i = 2.69\%$

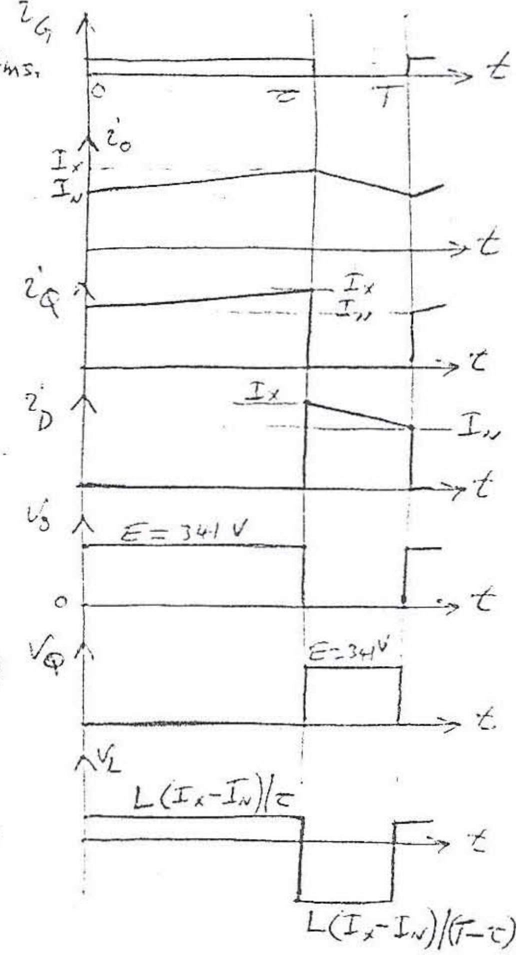
$P_V = 225$ Watt, $P_E = 225 + 4.502^2 * 20 = 630.3$ Watt, D1 is not required as Q2

$i_{F_{max}}$ for D2 ≥ 4.7 Amp & $i_{F_{rms}}$ for D2 $\geq 4.502 \times \sqrt{.3} = 2.466$ Amp & $i_{F_{av}}$ for D2 ≥ 1.35 Amp

d) Current Mode is contineous.

3-12 1. The circuit is a first quadrant chopper feeding RLV Load.

2. Waveforms are as shown, assuming linear segments for current due to $\frac{L}{R} = \frac{52}{2.5} = 20.8\,ms > \tau = 7\,ms$.



3. During $t \in [0, \tau)$:

$$\therefore\ i_o(t) = \frac{341 - 121}{2.5}\left(1 - e^{-2.5t/52m}\right) + 40e^{-2.5t/52m},\ A$$

$$\therefore\ I_N = i_o(0) = 40\ A$$

$$\& \ I_x = i_o(\tau) = i_o(7\,ms) = 53.72\ Ap$$

& During $t \in [\tau, T)$:

$$\therefore\ i_o(t) = \frac{-121}{2.5}\left(1 - e^{-\frac{2.5}{52m}(t-7m)}\right) + 53.72e^{-\frac{2.5}{52m}(t-7m)}$$

$$\therefore\ I_N = i_o(T) = 40\ A \Rightarrow T = 10.00\ msec$$

$$\therefore\ V_{o_{av}} = 341 * \frac{7}{10} = 238.7\ Volts$$

$$\therefore\ i_{o_{av}} = \frac{238.7 - 121}{2.5} = 47.08\ Ap$$

$$\& \ V_{o_{rms}} = 341\sqrt{\frac{7}{10}} = 285.3\ Volts$$

$$\& \ i_{o_{ac}} \simeq \frac{I_x - I_N}{2\sqrt{3}} = \frac{53.72 - 40.0}{2\sqrt{3}} = 3.961\ Ap$$

$$\therefore\ i_{o_{rms}} = \sqrt{i_{o_{av}}^2 + i_{o_{ac}}^2} = \sqrt{47.08^2 + 3.961^2} = 47.25\ Ap$$

$$\therefore\ K_v = \sqrt{\left(\frac{V_{o_{rms}}}{V_{o_{av}}}\right)^2 - 1} = \sqrt{\left(\frac{285.3}{238.7}\right)^2 - 1} = 0.6546 = 65.46\%$$

$$\& \ K_i = \frac{i_{o_{ac}}}{i_{o_{av}}} = \frac{3.961}{47.08} = .08413 \simeq 8.41\%.$$

$$P_R = i_{o_{rms}}^2 \cdot R = 47.25^2 * 2.5 = 5581.4\ W = 5.581\ KW$$

$$P_V = i_{o_{av}} \cdot V = 47.08 * 121 = 5696.68 = 5.697\ KW$$

$$\therefore\ P_S = P_R + P_V = 5.581 + 5.697 = 11.278\ KW$$

(OR equally: $P_S \simeq E \cdot i_{Q_{av}} = E * \frac{I_X + I_N}{2} \cdot \frac{\tau}{T} =$
$= 341 * \frac{53.72 + 40}{2} * \frac{7}{10} = 11185W = 11.185KW$)

4. Forced Commutation is used here.
5. For Q:

$$I_{av} = \frac{I_X + I_N}{2} \cdot \frac{\tau}{T} = \frac{53.72 + 40}{2} * \frac{7}{10} = 32.80\ Amp$$

$$I_{rms} = i_{o_{rms}} \sqrt{\frac{\tau}{T}} = 47.25 \sqrt{.7} = 39.53\ Amp$$

$$I_{max} = I_X = 53.72\ Amp$$

$$V_{FB} = 341\ Volts$$

$$V_{RB} = 0\ Volts$$

6. For D:

$$I_{av} = \frac{I_X + I_N}{2} \cdot \frac{T - \tau}{T} = \frac{53.72 + 40}{2} \cdot \frac{3}{10} = 14.06\ Amp$$

$$I_{max} = I_X = 53.72\ Amp$$

$$I_{rms} = i_{o_{rms}} \sqrt{\frac{T - \tau}{T}} = 47.25 \sqrt{.3} = 25.88\ Amp$$

$$V_{max} = 341\ Volts.$$

7. The current mode can be discontinuous by reducing the value of the inductance L

60

3-13 a) The circuit is a half bridge chopper that can supply the load with ± 50 DCV but with positive load currents only.

b) $i_o > 0$ & V_o can be either positive or negative.

c) It is very likely, due to $\tau : T$ ratio; that the current mode is discontinuous. To check this, assume $i_o(0) = 0$

$\therefore\ i_o(t) = \frac{50}{1}\left(1 - e^{-t}\right)$, Ap, $t \in [0, \tau)$

$\therefore\ i_o(\tau) = 50\left(1 - e^{-.01}\right)$, Ap $= 0.49751$ Ap $= 497.51$ mA

$\therefore\ i_o(t) = -50 + Ae^{-(t-.01)}$, with

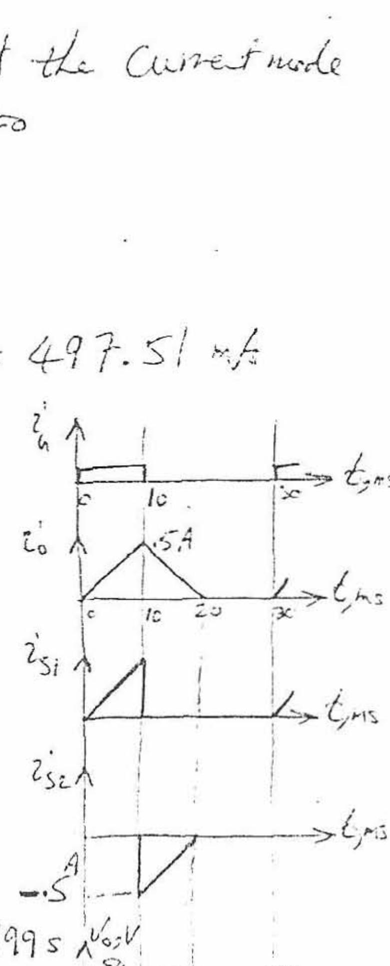
$i_o(\tau) = i_o(.01) = 497.51$ mA

$\therefore\ A = 50.49751$ Ap

$\therefore\ i_o(t) = -50 + 50.49751e^{-(t-.01)}$, Ap $t \in [\tau, t_x)$

where $i_o(t_x) = 0$ $\therefore\ t_x = \ln\left(\frac{50.49751}{50}\right) + .01 = .0199$ s

$\therefore\ t_x = 19.9$ msec, and hence, i_o is given by:



$$i_o(t) = \begin{cases} 50\left(1 - e^{-t}\right), A, & t \in [0, 10\text{ms}) \\ -50 + 50.49751e^{.01-t}, A, & t \in [10\text{ms}, 19.9\text{ms}) \\ 0, A, & t \in [19.9\text{ms}, 30\text{ms}) \end{cases}$$

Hence, current mode is discontinuous. This illustrates the waveforms above. Note that, because $L/R = 1$ sec $\ggg \tau, T$, then linear segments appears for the currents. Hence, $V_{oav} = 50\left(\frac{10 - 9.9 + 0}{30}\right) = 0.1650$ Volts

$\therefore\ i_{oav} = \frac{V_{oav}}{R} = \frac{.1650}{1} = 0.1650\ A = 165.0$ mA

& $V_{orms}^2 = 50^2\left(\frac{10 + 9.9 + 0}{30}\right) = 1658.42 \Rightarrow V_{orms} = 40.72$ Volts.

$\therefore\ K_v = \frac{V_{oac}}{V_{oav}} = \sqrt{\left(\frac{V_{orms}}{V_{oav}}\right)^2 - 1} = \sqrt{\left(\frac{40.72}{.165}\right)^2 - 1} = 246.8$ (Too High).

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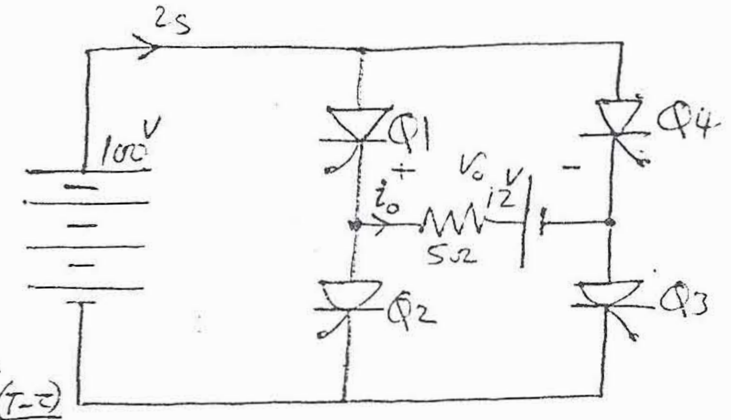
3-14 $f_{chopping} = 1\,KHz$



$\therefore T = 1\,msec$

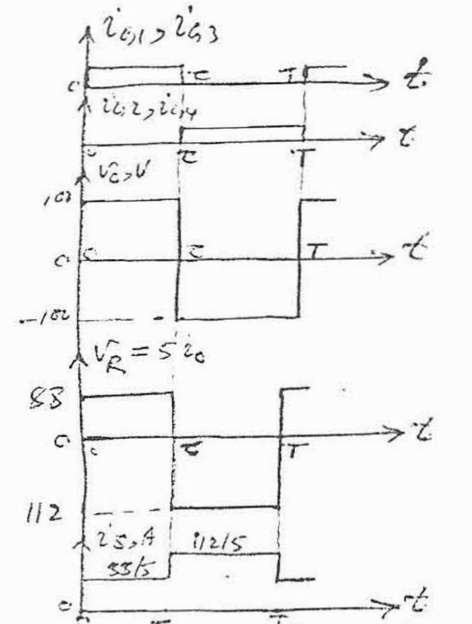
Let Q1 & Q3 be on for τ,

$\therefore$ The av. power delivered to 5Ω resistor is P_5 given by

$$P_5 = \frac{(100-12)^2}{5} * \frac{\tau}{T} + \frac{(100+12)^2}{5}\frac{(T-\tau)}{T}$$

$$= 2508.8 - 960\,\alpha \text{ , where}$$

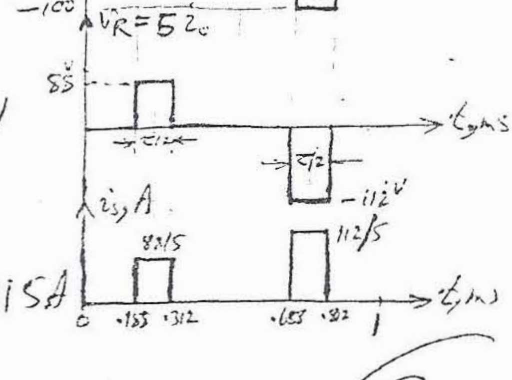
α is the duty ratio $\frac{\tau}{T}$



$\because P_5 = 500\,W$

$$\therefore \alpha = \frac{2508.8 - 500}{960} = 2.0925 > 1$$

$\therefore$ If Q1 & Q3 were on for τ & then Q2 & Q4 are brought on to end of T, it is impossible to supply power less than $2508.8 - 960 = 1548.8\,W$. To supply power less than this to the 5Ω load resistor, SPWM scheme is adopted with symmetrical on-duration of $\tau/2$ as shown



$$\therefore P_5 = \frac{88^2}{5}\cdot\frac{\tau/2}{T} + \frac{112^2}{5}\cdot\frac{\tau/2}{T} = 2028.8\,\alpha$$

$$\therefore \alpha = \frac{500}{2028.8} = 0.24645$$

$$\therefore P_{12} = 12 * \left[\frac{88-112}{5}\cdot\frac{\tau/2}{T}\right]$$

$$= 12 * \left(\frac{-24}{5}\right) * \frac{\alpha}{2} = -28.8 * .24645$$

$= -7.0978$ Watt $\therefore$ Battery supplies 7.1 W

$P_{source} = P_{100} = P_5 + P_{12} = 500 - 7.0978 = 492.9\,W$

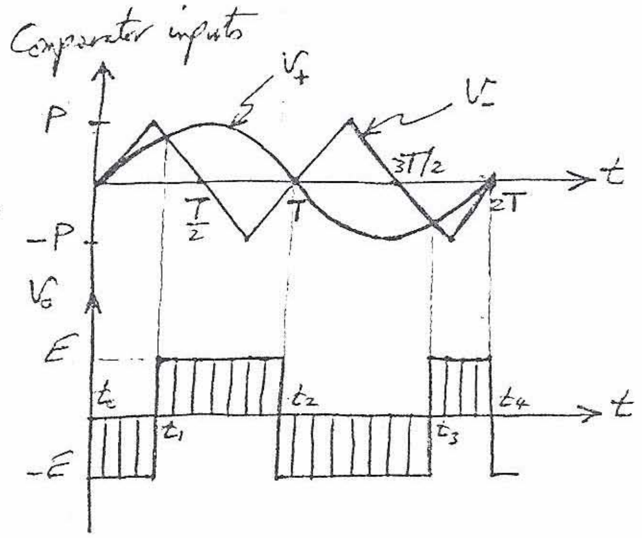
$V_{o_{rms}} = \sqrt{12^2(1-\alpha) + 100^2\alpha} = 50.725\,V$, $V_{o_{av}} = 12(1-\alpha) = 9.043\,V$

$i_{o_{rms}} = \sqrt{\frac{500}{5}} = 10\,Amp$, $i_{o_{av}} = \frac{7.0978}{12} = 0.5915\,A$

$\therefore K_v = \sqrt{(V_{orms}/V_{av})^2 - 1} = 5.520$ (very high ripple)

& $K_i = \sqrt{\left(\frac{i_{orms}}{i_{o_{av}}}\right)^2 - 1} = 16.88$ (very high ac components).

4-4 $V_+(t) = \begin{cases} \frac{4P}{T}.t \ ; & t\in[0,\frac{T}{4}] \\ -\frac{4Pt}{T}+2P, & t\in[\frac{T}{4},\frac{3T}{4}] \\ \frac{4P.t}{T}-4P, & t\in[\frac{3T}{4},\frac{5T}{4}] \\ -\frac{4Pt}{T}+6P, & t\in[\frac{5T}{4},\frac{7T}{4}] \\ \frac{4P.t}{T}-8P, & t\in[\frac{7T}{4},2T] \end{cases}$



& $V_-(t) = P\sin\left(\frac{2\pi}{2T}t\right) = P\sin(\pi t/T)$

$\therefore V_o(t) = \begin{cases} -E & t\in[t_0,t_1]\cup[t_2,t_3] \\ E & t\in[t_1,t_2]\cup[t_3,t_4] \end{cases}$

where t_0, t_1, t_2, t_3 & t_4 are the solutions of $V_+(t)=V_-(t)$ (Note that there are four solutions per fundamental period, i.e double the frequency ratio). Finding these solutions:

$\therefore \frac{4Pt_0}{T} = P\sin(\pi t_0/T) \Rightarrow f(t_0/T) = \sin(\pi t_0/T) - 4t_0/T = 0 \ \therefore \frac{t_0}{T}=0$

& $\frac{-4Pt_1}{T} + 2P = P\sin(\pi t_1/T) \Rightarrow f(t_1/T) = \sin(\pi t_1/T) + 4t_1/T - 2 = 0 \ \therefore \frac{t_1}{T} = .29846$

& $+\frac{4Pt_2}{T} - 4P = P\sin(\pi t_2/T) \Rightarrow f(t_2/T) = \sin(\pi t_2/T) - 4\frac{t_2}{T} + 4 = 0 \ \therefore \frac{t_2}{T} = 1.000$

& $-\frac{4Pt_3}{T} + 6P = P\sin(\pi t_3/T) \Rightarrow f(t_3/T) = \sin(\pi t_3/T) + 4\frac{t_3}{T} - 6 = 0 \ \therefore \frac{t_3}{T} = 1.70154$

& $t_4 = t_0 + 2T = 2T$ (Note: you can find only one iteration and use symmetry)

I) $\therefore$ Since $V_o(2T-t) = -V_o(t)$ $\therefore V_{o_{av}} = 0$ & $V_o(t) = \sum_{n=1}^{\infty} a_n \sin n\omega t + b_n \cos n\omega t$

where $\omega = \frac{2\pi}{2T} = \frac{\pi}{T}$, $\because \cos n\omega(2T-t) = \cos n\omega t \ \therefore b_n = 0$

$\because \sin n\omega(2T-t) = -\sin n\omega t \ \therefore a_n = \frac{2}{2T}\int_0^{2T} V_o(t)\sin n\omega t\, dt = \frac{1}{T} * 2\int_0^T V_o(t)\sin n\omega t\, dt$

$= 2 * \int_0^1 V_o\left(\frac{t}{T}\right)\sin\left(\frac{n\pi t}{T}\right) d\left(\frac{t}{T}\right) = 2\left(\int_0^{.29846} - \int_{.29846}^{1}\right) E\sin n\pi x\, dx =$

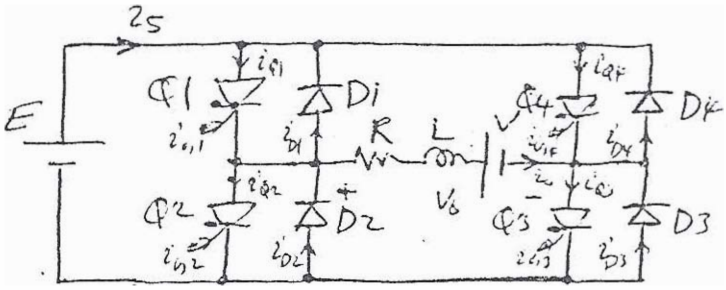
$= 2E\left[\frac{-\cos n\pi x}{n\pi}\Big|_0^{.29846} + \frac{\cos n\pi x}{n\pi}\Big|_{.29846}^{1}\right] = \frac{2E}{n\pi}\left[1 - 2\cos(n\pi * .29846) + \cos n\pi\right] = a_n$

II) $\therefore V_{o1_{rms}} = \frac{\sqrt{2}E}{\pi}\left|-2\cos .29846\pi\right| = 0.532712E$ & $\because V_{o_{tot_{rms}}} = E$

$\therefore C_v = \sqrt{1 - .532712^2} = 0.846297$

4-6

$E = 100$ Volts
$T = 1$ ms
$\tau = .75$ ms
$R = 1\Omega$
$L =$ very large
$V = 20$ Volts



$\because$ L is very large,

$\therefore$ exponential segments could be assumed linear (∵ ripple very small).

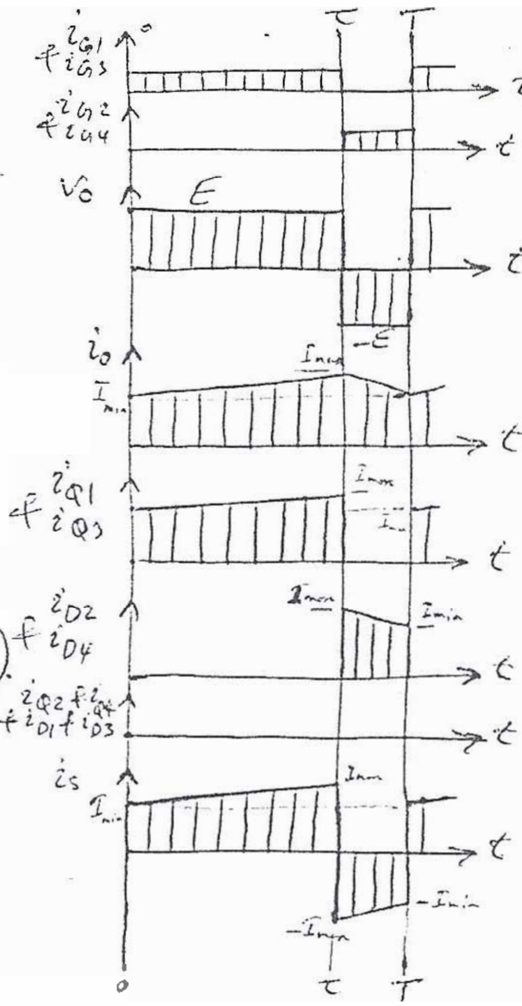
$\therefore V_{o_{av}} = [E\tau - E(T-\tau)]/T$

$= \frac{E}{T}(2\tau - T) = 50$ Volts

$\therefore i_{o_{av}} = \frac{V_{o_{av}} - V}{R} = 30$ Amps

$\therefore$ power delivered to $V = i_{o_{av}} \cdot V = 600W$

The waveforms are all shown here.



& $i_S = i_{Q1} + i_{Q4} - i_{D1} - i_{D4} =$

$= i_{Q1} - i_{D4}$ as shown.

(Note: D1, Q2, D3 & Q4 are not required).

As for K_v:

$K_v = \frac{V_{ac}}{V_{o_{av}}} = \sqrt{\frac{V_{o_{rms}}^2 - V_{o_{av}}^2}{V_{o_{av}}^2}} =$

$= \sqrt{\left(\frac{V_{o_{rms}}}{V_{o_{av}}}\right)^2 - 1}$.

& $\because V_{o_{rms}} = E = 100$ Volts & $V_{o_{av}} = 50$ Volts

$\therefore K_v = \sqrt{\left(\frac{100}{50}\right)^2 - 1} = \sqrt{3} = 1.732 = 173.2\%$ (very high ripple)

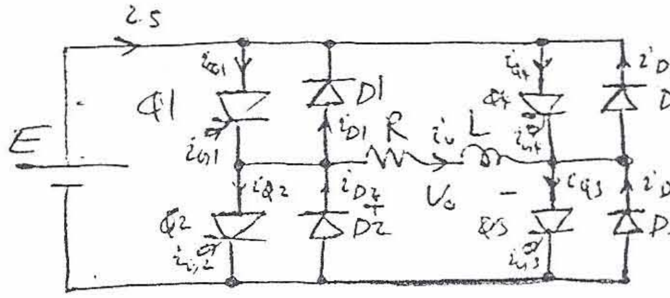
$\boxed{4\text{-}7}$ $E = 100$ Volts

$T = 1$ msec

$\tau = .75$ msec $\left(\frac{\tau}{T} = \frac{3}{4}\right)$

$R = 1\Omega$

$L =$ very large



$\because$ SPWM

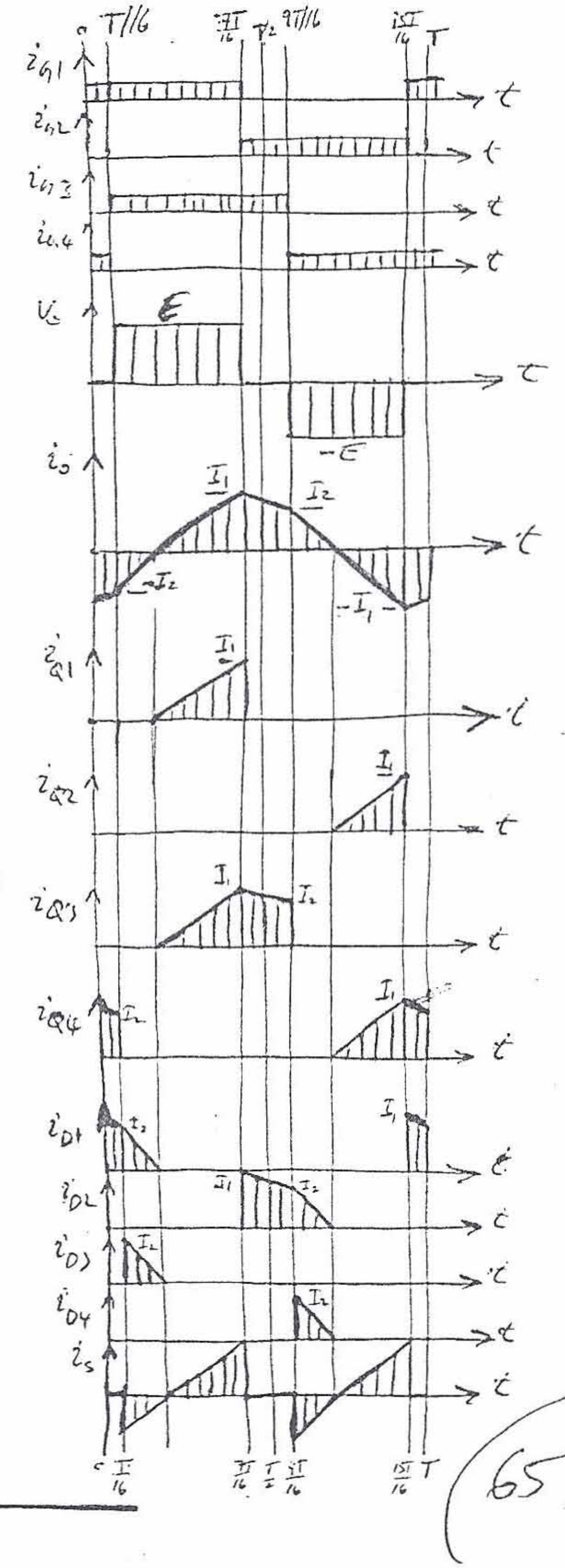
$\therefore$ Waveforms are as shown

$\because$ L is very large,

$\therefore$ i_o can be assumed as straight line segments, as shown.

As for i_s:

$i_s = i_{Q1} + i_{Q4} - i_{D1} - i_{D4}$, to give the shape shown.



$$V_{o\,\text{fundamental}\,rms} = \frac{4E}{\pi\sqrt{2}} \sin\frac{\pi\tau}{2T}$$

$$= 83.18 \text{ Volts}$$

$$\rho_v = \frac{V_{o\,\text{harmonics}}}{V_{o\,rms}} = \sqrt{1 - \left(\frac{V_{o\,\text{fundamental}}}{V_{o\,rms}}\right)^2},$$

$$\because V_{o\,rms} = E\sqrt{\frac{\tau}{T}} = 86.60 \text{ Volts}$$

$$\therefore \rho_v = .2784 = 27.84\%$$

4-8

$V = 500$ volts

$T = 2\,ms$

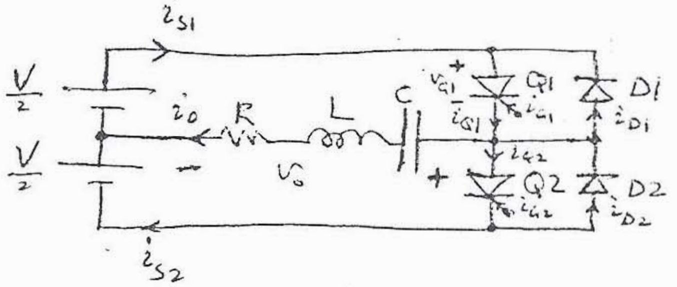
$R = 1.2\ \Omega$

$\omega = \frac{2\pi}{T}$

$\omega L = 10\Omega$

$\frac{1}{\omega C} = 10\ \Omega$

$t_q = 50\ \mu sec$



a) Since higher harmonics can be neglected,

$\therefore\ V_o \simeq V_m \sin \omega t = \overline{V_o}$, where:

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2m} = \pi\ K\,rad/sec,$

& $V_m = \frac{2}{T}\int_0^T V_o(t) \sin \omega t\, dt$

$= \frac{2}{2\pi/\omega} \cdot 2\int_0^{T/2} \frac{V}{2} \sin \omega t\, dt =$

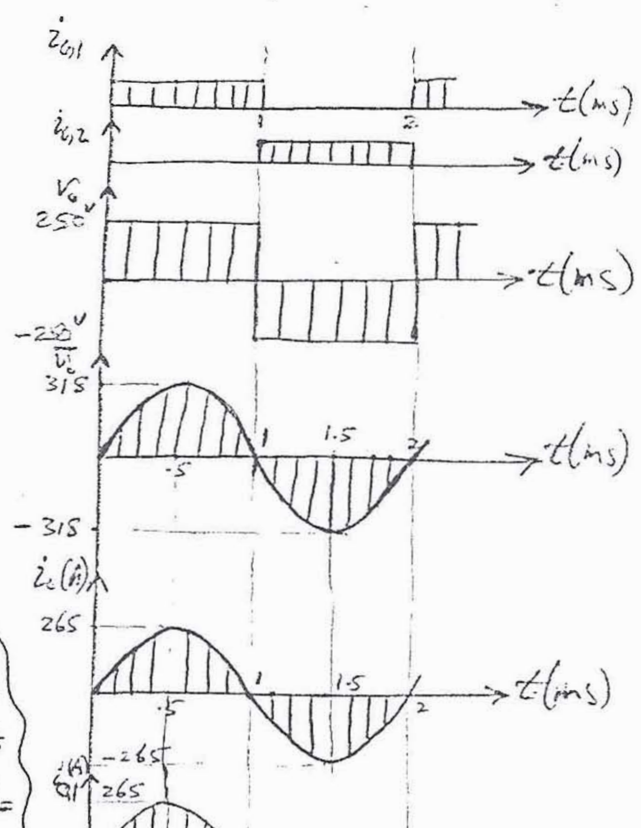
$= \frac{\omega}{\pi} \cdot \frac{2V}{2} \cdot \frac{-\cos \omega t}{\omega}\Big|_0^{T/2} = \frac{V}{\pi}\left(-\cos \frac{\omega T}{2} + 1\right)$

$= \frac{V}{\pi}\left(-\cos\left(\frac{2\pi}{T} \cdot \frac{T}{2}\right) + 1\right) = \frac{V}{\pi}\left(-\cos \pi + 1\right) = \frac{2V}{\pi}$

$\therefore\ \overline{V_o} = \frac{1000}{\pi} \sin \pi t(ms)$ volts $= 318.31 \sin \pi t$

$\therefore\ i_o \simeq \overline{i_o} = \frac{318.31}{\sqrt{1.2^2 + (10-10)^2}} \sin\left\{\pi t - \tan^{-1}\left(\frac{10-10}{1.2}\right)\right\} =$

$= 265.26\ Amp\ \sin \pi t(ms)$



b) $i_{Q1_{av}} = i_{Q2_{av}} = \frac{2}{\pi} * 1 * 265.26/2 = 84.43$ Amps

& $i_{Q1_{rms}} = i_{Q2_{rms}} = \frac{265.26}{\sqrt{2} \cdot \sqrt{2}} = 132.63$ Amps

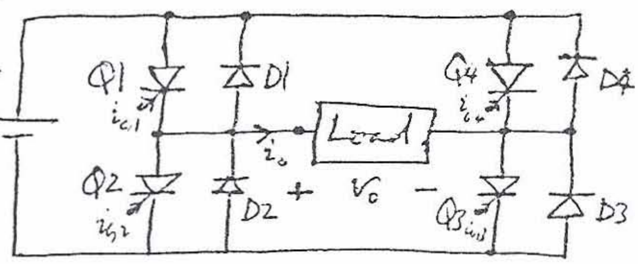
& $i_{D1\,av} = i_{D2\,av} = i_{D1\,rms} = i_{D2\,rms} = 0$ Amps

66

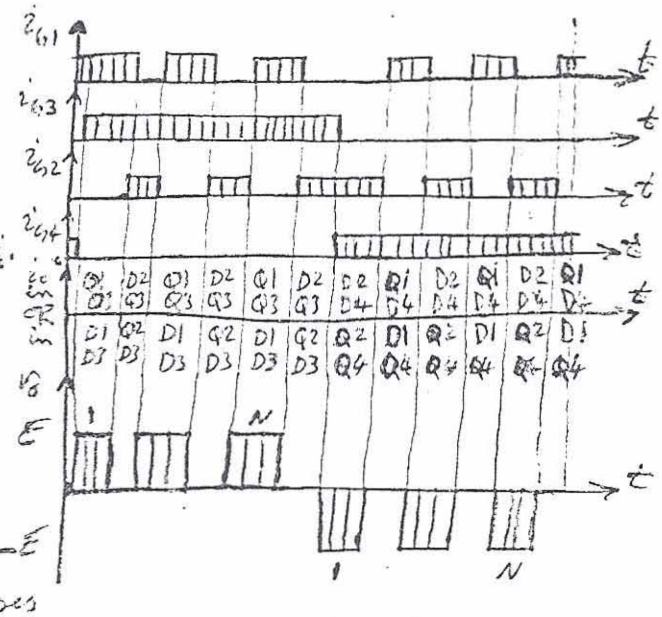
c). Since phase lead $= 0 \not> t_q = 50\mu sec$; then force Commutation is required to provide recovery, though commutation current is zero Amps.

4-9

Consider the inverter circuit shown, and let the gate signals of Q2 & Q4 be the complements of those of Q1 & Q3 respectively.



Now, if gate signals of Q1 & Q3 were as shown, the output voltage will be a multiple pulse width modulation of the input supply, E, providing bidirectional i_o.



The timing of all signals can be controlled to give the desired multiple pulse width modulated output. We will consider the case of equally-spaced N similar pulses per half cycle having centred symmetry to make life simpler.

$\therefore \quad V_o\left(t+\frac{T}{2}\right) = -V_o(t)$

$$\& \quad V_o(t) = \begin{cases} E & \text{for } t \in \left[\frac{(2k-1)T-\tau}{4N}, \frac{(2k-1)T+\tau}{4N}\right] \\ 0 & \text{for } t \in \left[\frac{(2k-3)T+\tau}{4N}, \frac{(2k-1)T-\tau}{4N}\right] \end{cases} \quad k = 1, \ldots, N.$$

where τ is the total energizing period in one whole cycle $\in [0, T]$.

$$\therefore \quad V_o(t) = V_{o_{av}} + \sum_{n=1}^{\infty} (a_n \sin n\omega t + b_n \cos n\omega t), \quad \text{where } \omega = \frac{2\pi}{T}$$

$\because$ Symmetrical $\therefore V_{o_{av}} = 0 \;\&\; b_n = 0$ for all n $\&$ $a_n = 0$ for even n

$$\therefore \quad V_o(t) = \sum_{\substack{n=1 \\ odd}}^{\infty} a_n \sin\left(\frac{2n\pi t}{T}\right)$$

$$\therefore \quad a_{n_{odd}} = \frac{2}{T}\int_0^T V_o(t) \sin\left(\frac{2n\pi t}{T}\right) dt = \frac{2}{T} * 2\int_0^{T/2} V_o \sin\left(\frac{2n\pi t}{T}\right) dt =$$

$$= \frac{4}{T}\sum_{k=1}^{N} \int_{\frac{(2k-1)T-\tau}{4N}}^{\frac{(2k-1)T+\tau}{4N}} E \sin\left(\frac{2n\pi t}{T}\right) dt = \frac{4E}{T}\sum_{k=1}^{N}\left[\frac{-\cos(2n\pi t/T)}{2n\pi/T}\right]_{\frac{(2k-1)T-\tau}{4N}}^{\frac{(2k-1)T+\tau}{4N}}$$

$$= \frac{2E}{n\pi}\sum_{k=1}^{N} -\cos\left\{\frac{2n\pi[(2k-1)T+\tau]}{4NT}\right\} + \cos\left\{\frac{2n\pi[(2k-1)T-\tau]}{4NT}\right\} =$$

$$= \frac{2E}{n\pi} \sum_{k=1}^{N} 2\sin\left\{\frac{n\pi(2k-1)}{2N}\right\} \sin\left(\frac{n\pi\tau}{2NT}\right)$$

$$= \frac{4E}{n\pi} \cdot \sin\left(\frac{n\pi\tau}{2NT}\right) \cdot \sum_{k=1}^{N} \sin\left\{\frac{n\pi}{2N}(2k-1)\right\}$$

(Let's revise Complex theory to find $\sum_{\substack{m=1\\ odd}}^{M} \sin m\theta = S$.

Let $z = e^{j\theta}$ $\therefore z^m = e^{jm\theta} = \cos m\theta + j\sin m\theta$

$$\therefore S = \sum_{\substack{m=1\\ odd}}^{M} Im\, z^m = Im \sum_{\substack{m=1\\ odd}}^{M} z^m = Im\left(z^1 + z^3 + \dots + z^M\right).$$

Now, let $\bar{S} = z + z^3 + \dots + z^M = \sum_{\substack{m=1\\ odd}}^{M} z^m$

$$\therefore z^2\bar{S} - \bar{S} = z^3 + z^5 + \dots + z^{M+2} - z - z^3 - z^5 - \dots - z^M =$$

$$= z^{M+2} - z$$

$$\therefore \bar{S}(z^2-1) = z^{M+2} - z \Rightarrow \bar{S} = \frac{z^{M+2} - z}{z^2 - 1}$$

$$\therefore S = Im\,\bar{S} = Im\left(\frac{z^{M+2} - z}{z^2-1}\right) = Im\left[\frac{\cos(M+2)\theta + j\sin(M+2)\theta - \cos\theta - j\sin\theta}{\cos 2\theta + j\sin 2\theta - 1}\right]$$

$$= Im\left[\frac{\{\cos(M+2)\theta - \cos\theta\} + j\{\sin(M+2)\theta - \sin\theta\}}{(\cos 2\theta - 1)^2 + (\sin 2\theta)^2} \cdot \{(\cos 2\theta - 1) - j\sin 2\theta\}\right]$$

$$= \frac{(\cos 2\theta - 1)\{\sin(M+2)\theta - \sin\theta\} - \sin 2\theta\{\cos(M+2)\theta - \cos\theta\}}{\cos^2 2\theta - 2\cos 2\theta + 1 + \sin^2 2\theta} =$$

$$= \frac{\cos 2\theta \sin(M+2)\theta - \sin 2\theta \cos(M+2)\theta - \cos 2\theta \sin\theta + \sin 2\theta \cos\theta - \sin(M+2)\theta + \sin\theta}{2 - 2\cos 2\theta} =$$

$$= \frac{\sin(M+2-2)\theta + \sin(2\theta - \theta) - \sin(M+2)\theta + \sin\theta}{2(1 - \cos 2\theta)} =$$

$$= \frac{\sin(M+1-1)\theta - \sin(M+1+1)\theta + 2\sin\theta}{2 * 2\sin^2\theta} = \frac{-2\cos(M+1)\theta \sin\theta + 2\sin\theta}{4\sin^2\theta} =$$

$$= \frac{1 - \cos(M+1)\theta}{2\sin\theta} \quad \therefore S = \sum_{\substack{m=1\\ odd}}^{M} \sin m\theta = \frac{1 - \cos(M+1)\theta}{2\sin\theta}\Bigg).$$

$$\therefore a_{\substack{n\\ odd}} = \frac{4E}{n\pi} \cdot \sin\left(\frac{n\pi\tau}{2NT}\right) \cdot \left[\sum_{k=1}^{N} \sin\left\{(2k-1)\left(\frac{n\pi}{2N}\right)\right\}\right] =$$

$$= \frac{4E}{n\pi} \cdot \sin\left(\frac{n\pi\tau}{2NT}\right) \left[\sum_{\substack{m=2k-1=1\\ m\ odd}}^{M=2N-1} \sin m\left(\frac{n\pi}{2N}\right)\right] = \frac{4E \sin\left(\frac{n\pi\tau}{2NT}\right) * [1 - \cos n\pi]}{n\pi * 2\sin\left(\frac{n\pi}{2N}\right)}$$

$$= \frac{4E}{n\pi} \cdot \sin\left(\frac{n\pi\tau}{2NT}\right) \Big/ \sin\left(\frac{n\pi}{2N}\right).$$

GS

$\therefore$ The fundamental output voltage, $V_{o1_{rms}} = \frac{4E}{\pi\sqrt{2}} \cdot \sin\left(\frac{\pi\tau}{2NT}\right) \Big/ \sin\left(\frac{\pi}{2N}\right)$

& The total output voltage, $V_{o\,tot_{rms}} = E \cdot \sqrt{\frac{\tau}{T}}$,

giving a distortion factor, $\mu_v = \frac{V_{o\,harmonics\,rms}}{V_{o\,tot\,rms}} = \sqrt{\frac{V^2_{o\,tot\,rms} - V^2_{o1\,rms}}{V^2_{o\,tot\,rms}}}$

$$= \sqrt{1 - \left(\frac{V_{o1\,rms}}{V_{o\,tot\,rms}}\right)^2} = \sqrt{1 - \left(\frac{2\sqrt{2}}{\pi} \cdot \sqrt{\frac{T}{\tau}} \cdot \frac{\sin(\pi\tau/2NT)}{\sin(\pi/2N)}\right)^2}.$$

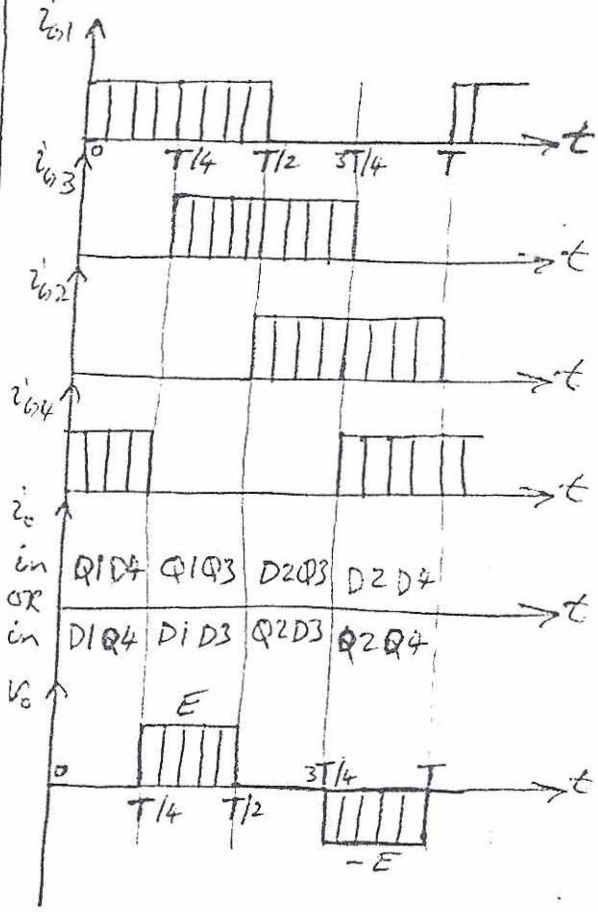
Hence, the two parameters τ & N facilitate control over μ_v & $V_{o1_{rms}}$.

If $N=1$, then single pulse modulation is used. Hence,

$$a_{n_{odd}} = \frac{4E}{n\pi} \cdot \sin\left(\frac{n\pi\tau}{2T}\right) \Big/ \sin\left(\frac{n\pi}{2}\right) = (-1)^{\frac{n-1}{2}} * \frac{4E}{n\pi} * \sin\left(\frac{n\pi\tau}{2T}\right)$$

& $V_{o1_{rms}} = \frac{4E}{\pi\sqrt{2}} \cdot \sin\left(\frac{\pi\tau}{2T}\right)$

& $\mu_v = \sqrt{1 - \frac{8 \cdot T}{\pi^2 \cdot \tau} \cdot \sin^2\left(\frac{\pi\tau}{2T}\right)}$



$\boxed{4\text{-}10}$ a) for $\delta = 90° \Rightarrow \tau = \frac{T}{2}$

$\therefore a_{n_{odd}} = (-1)^{\frac{n-1}{2}} * \frac{4E}{n\pi} \cdot \sin\left(\frac{n\pi}{4}\right)$

$\therefore V_{o1_{rms}} = \frac{4E}{\pi\sqrt{2}} \cdot \sin\left(\frac{\pi}{4}\right) = 0.63662E$

& $V_{o3_{rms}} = \frac{4E}{3\pi\sqrt{2}} \cdot \sin\left(\frac{3\pi}{4}\right) = 0.21221E$

& $V_{o5_{rms}} = \frac{4E}{5\pi\sqrt{2}} \left|\sin\left(\frac{5\pi}{4}\right)\right| = 0.12732E$

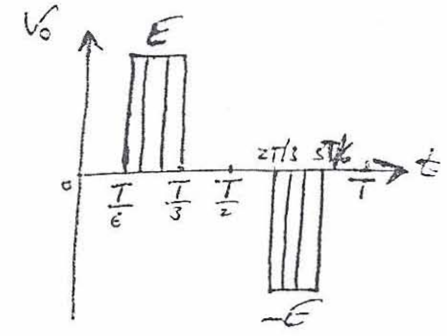
& $V_{o7_{rms}} = \frac{4E}{7\pi\sqrt{2}} \left|\sin\left(\frac{7\pi}{4}\right)\right| = 0.09095E$

& b) The gating waveforms indicated earlier will be for this case as shown above.

$\boxed{4-11}$ a) Single pulse modulation:

$$\therefore V_{o1_{rms}} = \frac{4E}{\pi\sqrt{2}} \cdot \sin\left(\frac{\pi\tau}{2T}\right) = 0.45E$$

$$\therefore \sin\left(\frac{\pi\tau}{2T}\right) = \frac{.45\sqrt{2}\,\pi}{4} = 0.49982$$



$$\therefore \frac{\tau}{T} = \frac{2}{\pi} \cdot \sin^{-1} .49982 = 0.33320 \approx \frac{1}{3}$$

$$\therefore \tau \approx \frac{T}{3} \Rightarrow \delta \approx 60^\circ\text{, width of a pulse.}$$

$$\& \quad V_{o_{tot_{rms}}} = E\sqrt{\frac{\tau}{T}} \left(\approx \frac{E}{\sqrt{3}}\right) = 0.57724E$$

$$\therefore \mu_v = \sqrt{1 - \left(\frac{.45}{.57724}\right)^2} = 0.62631$$

$$\& \quad K_v = \sqrt{\left(\frac{.57724}{.45}\right)^2 - 1} = 0.80340$$

b) Multiple pulse width modulation; $N = 10$:

$$\therefore V_{o1_{rms}} = \frac{4E}{\pi\sqrt{2}} \sin\left(\frac{\pi\tau}{2NT}\right) \Big/ \sin\left(\frac{\pi}{2N}\right) = .45E$$

$$\therefore \sin\left(\frac{\pi\tau}{20T}\right) = \frac{\pi\sqrt{2} * .45}{4} * \sin\left(\frac{\pi}{20}\right) = .07819$$

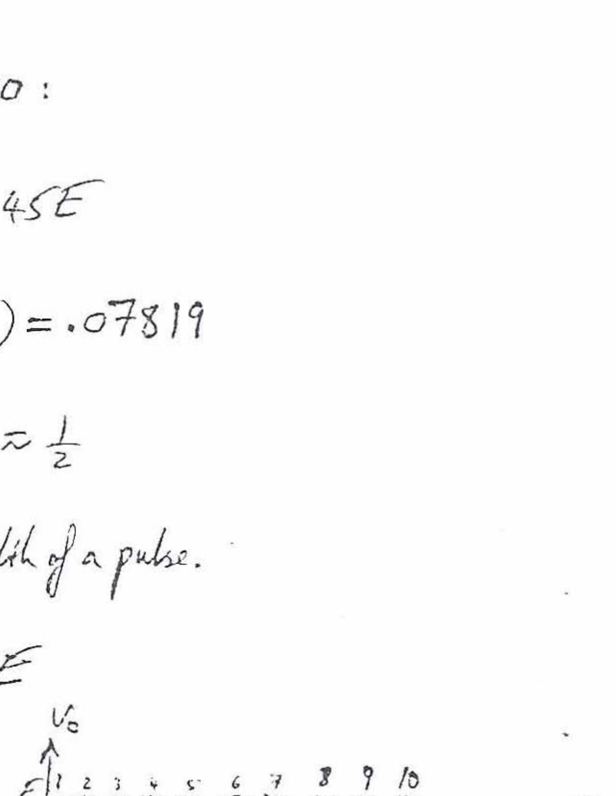
$$\therefore \frac{\tau}{T} = \frac{20}{\pi} \sin^{-1} .07819 = 0.49828 \approx \frac{1}{2}$$

$$\therefore \tau \simeq \frac{T}{2} \Rightarrow \delta \simeq \frac{90^\circ}{10} = 9^\circ\text{; width of a pulse.}$$

$$\& \quad V_{o_{tot_{rms}}} = E\sqrt{\frac{\tau}{T}} \left(\approx \frac{E}{\sqrt{2}}\right) = 0.70589E$$

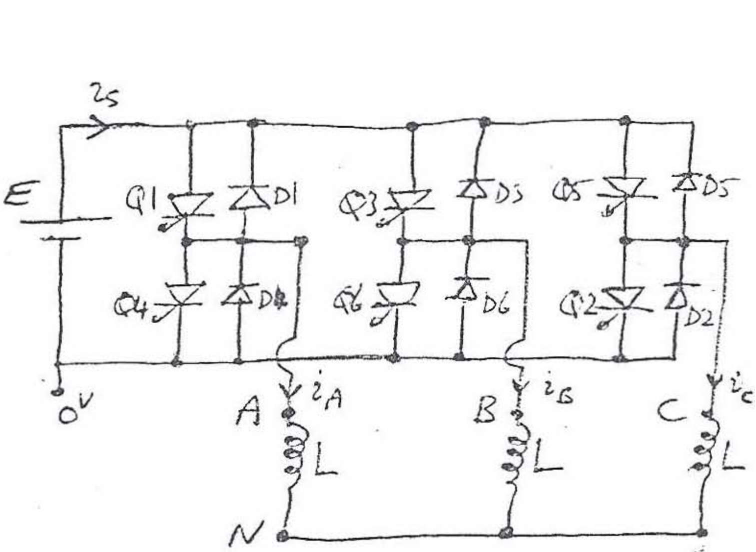
$$\therefore \mu_v = \sqrt{1 - \left(\frac{.45}{.70589}\right)^2} = 0.77046$$

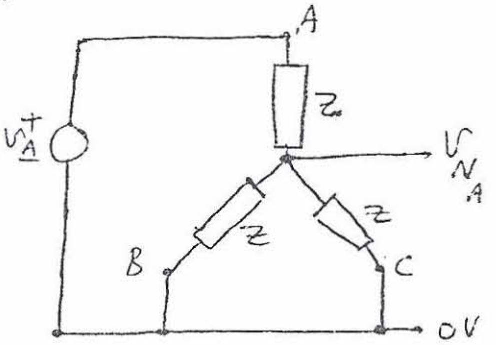
$$\& \quad K_r = \sqrt{\left(\frac{.70589}{.45}\right)^2 - 1} = 1.2086$$



4-12

Assuming symmetrical star loads, the waveforms are as shown. The potential of N relative to the negative battery terminal could be found by superposition and phase A analysis is as follows:





$$\frac{V_{NA}}{V_A} = \frac{(Z//Z)}{(Z//Z)+Z} = \frac{Z/2}{Z/2+Z} = \frac{1}{3}$$

Likewise $\frac{V_{NB}}{V_B} = \frac{1}{3} = \frac{V_{NC}}{V_C}$

$$V_N = V_{N_A} + V_{N_B} + V_{N_C} = \frac{1}{3}\left(V_A + V_B + V_C\right)$$

Hence, $V_{AN} = V_A - V_N$ as shown, other phase voltages has $\pm T/3$ phase shift

To calculate i_A, we assume that current initially at L is providing symmetry, i.e. $i_A(0) = -\frac{ET}{9L}$, slopes $= \frac{V_{AN}}{L}$.

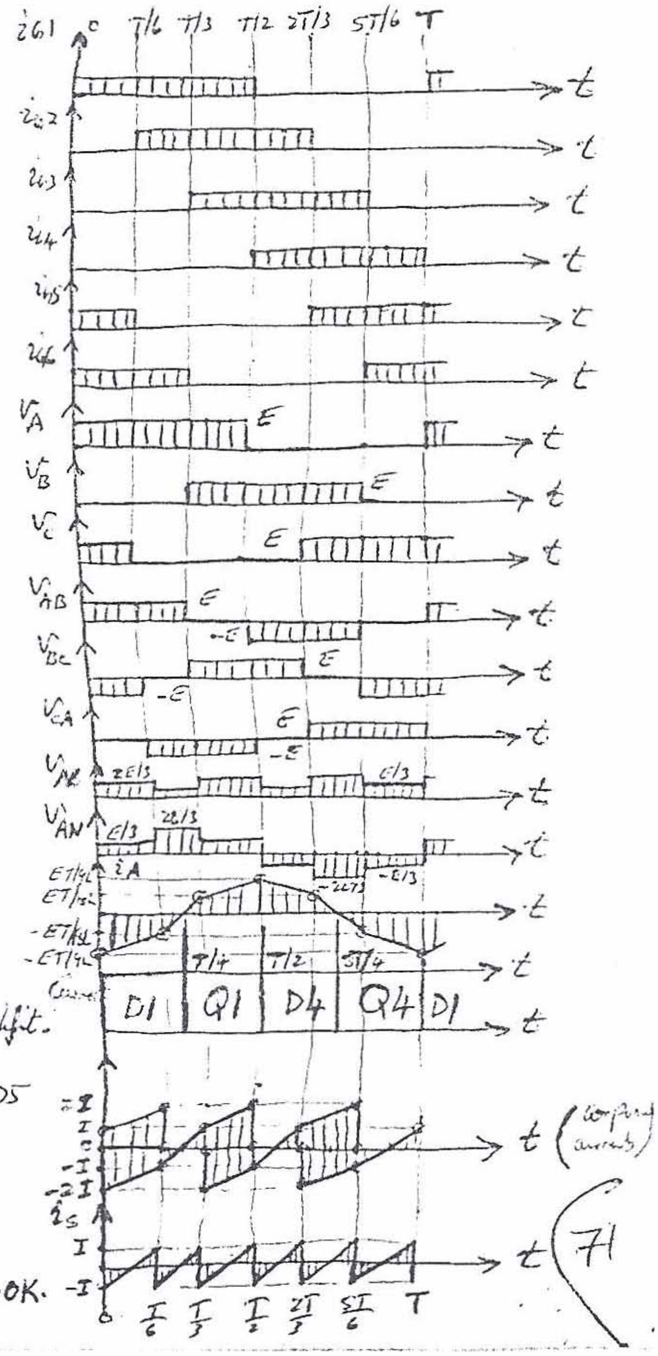
other i_B & i_C currents have $\pm T/3$ shift.

As for i_S: $i_S = i_{Q1} + i_{Q3} + i_{Q5} - i_{D1} - i_{D3} - i_{D5}$

as shown, where $I = ET/18L$

$P_A = \frac{1}{T}\int_0^T V_{AN}\, i_A\, dt = \frac{1}{T}\int_0^T \text{odd} \times \text{even} = 0$

$P_{tot} = P_A + P_B + P_C = 0$ (Consistant with the fact that L loads dissapite no power). OK.



4-13

Assuming symmetrical star loads, with circuit as in $\boxed{4\text{-}12}$ above, the waveforms are as shown. The shape at the indeterminate interval is decided by the load. Due to load being inductive, the load currents at instants of commutating Q1, Q3 & Q5 will have to freewheel through D4, D6 & D2 respectively. Likewise, at commutation of Q4, Q6 & Q2 through D1, D3 & D5 respectively. This gives the shapes shown of V_A, V_B & V_C. The potential of V_N is obtained as before as:

$V_N = \frac{V_A + V_B + V_C}{3}$, and is as shown.

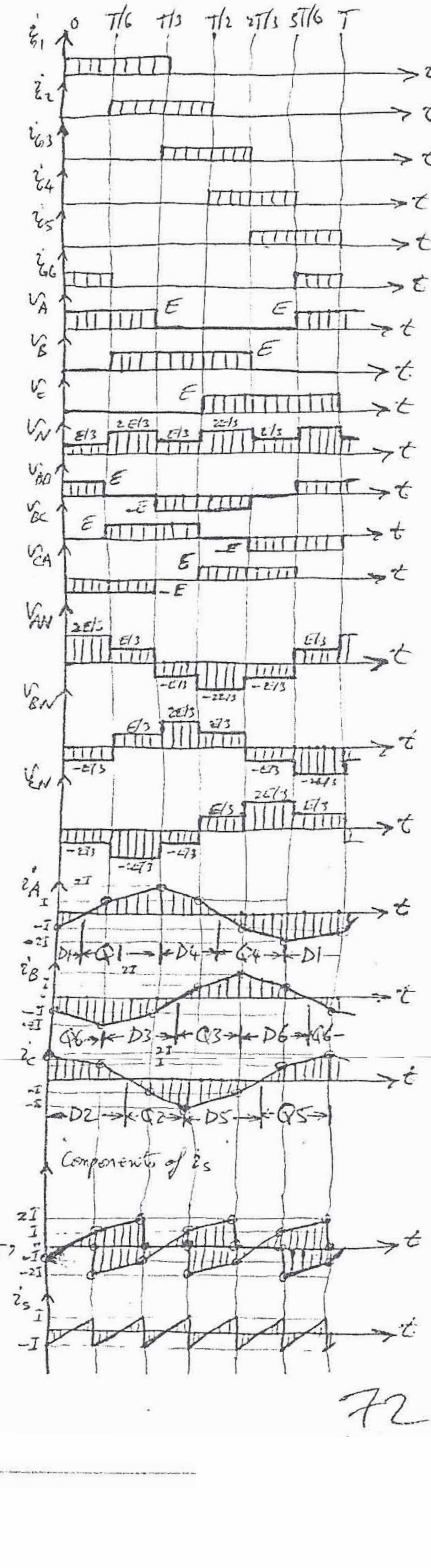
Hence, V_{AN}, V_{BN} & $V_{CN} = V_C - V_N$ are as shown. Note the $\frac{T}{3}$ shifts.

Also note that the presence of freewheeling diodes makes the conduction effectively for half cycle, so that the previous case waveforms show up again with T/6 phase shift.

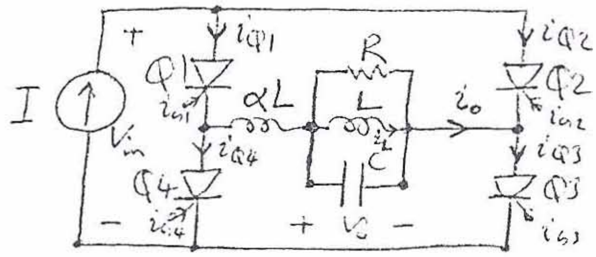
Again initial state currents are assumed to preserve symmetry hence, i_A, i_B & i_C are as shown, where $I = ET/18L$.

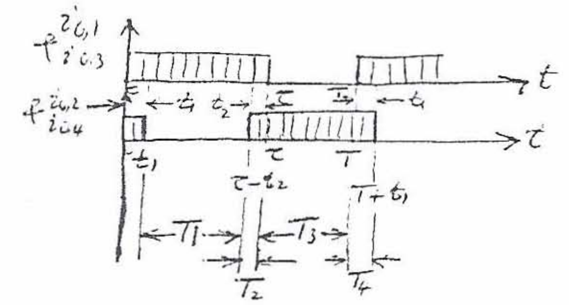
Again, $i_s = i_{Q1} + i_{Q3} + i_{Q5} - i_{D1} - i_{D3} - i_{D5}$, and is obtained from components as shown. Note the $\frac{T}{6}$ phase shift compared to the one obtained earlier.

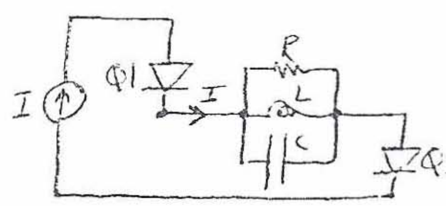
As before power consumed is zero.



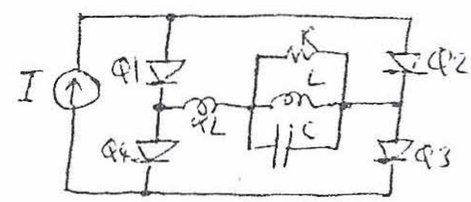
4-14 The load Commutation in the current inverter shown, is caused by the capacitor. When current flows into the capacitor the voltage accross it will build up, hence making a forward bias to the thyristors not in Conduction. When these thyristors are gated, the current will divert its flow and a short circuit will occure to the load combination. This short circuit current will force the commutation of the thyristors at conduction and hence half a cycle is complete. The same would happen in the other half cycle. If the gating of the thyristors not in conduction was delayed, a voltage reversal (due to oscilation) may occure and Commutation is not possible because these thyristor have not been turned on to short the load combination.
The importance of lead inductances is during the Commutation interval whereby, the current rate of change is so high that these lead inductances cause a substantial voltage drop.
Assuming ideal components, the circuit will look like the following at various instants of time, with shown rectifiers fully on.



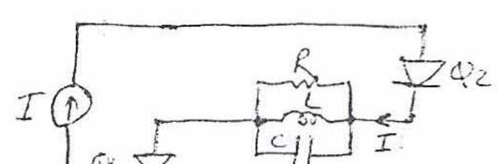




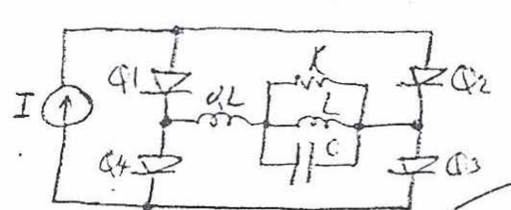
$t \in T_1 = (t_1, \tau - t_2)$



$t \in T_2 = [\tau - t_2, \tau]$



$t \in T_3 = (\tau, T)$



$t \in T_4 = [T, T + t_1]$

Now, assuming $\alpha <<< 1$ which is a practically justified assumption, then the commutation time becomes too small that the load inductance during this commutation interval could be considered as a current source passing the same value of pre-commutation current. Let us analize the circuit for each time interval.

$\therefore$ at $t \in T_1$, the initial i_L & V_o are assumed to be I_1 & V_1

$$\therefore I(s) = V_o \cdot \left(\frac{1}{R} + \frac{1}{sL} + \frac{1}{\frac{1}{sC}} \right) = V_o(s) \cdot \frac{s^2LCR + sL + R}{sLR}$$

$$\therefore V_o(s) = \frac{sLR}{s^2LCR + sL + R} \cdot I(s)$$

$$\therefore s = \frac{-L \pm \sqrt{L^2 - 4LCR^2}}{2LCR} \qquad \because \text{load mode is resonating}$$

$$\therefore s = \frac{-L \pm j\sqrt{4LCR^2 - L^2}}{2LCR} = -\frac{1}{2CR} \pm j\sqrt{\frac{1}{LC} - \left(\frac{1}{2CR}\right)^2} = -\sigma \pm j\omega$$

$$\therefore \sigma = \frac{1}{2CR} \quad \& \quad \omega = \sqrt{\frac{1}{LC} - \left(\frac{1}{2CR}\right)^2} = \sqrt{\frac{1}{LC} - \sigma^2}$$

$$\therefore V_o(t) = \frac{sLR}{s^2LCR + sL + R} \cdot I = \left[A_1 \sin\omega(t - t_1) + B_1 \cos\omega(t - t_1)\right] e^{-\sigma(t - t_1)}$$

$$\because V_o(t_1) = V_1 = B_1$$

$$\therefore V_o(t) = \left[A_1 \sin\omega(t - t_1) + V_1 \cos\omega(t - t_1)\right] \cdot e^{-\sigma(t - t_1)} \qquad t \in T_1 \qquad (1)$$

$$\& \quad i_o(t) = I \qquad t \in T_1 \qquad (2)$$

$$\& \quad i_L(t_1) = I - \frac{V_o(t_1)}{R} - C\dot{V}_o(t_1) = I - \frac{V_1}{R} - C(-\sigma V_1 + \omega A_1) = I_1$$

$$\therefore A_1 = \frac{I - I_1 - \sigma C V_1}{\omega C} \qquad (3)$$

$$\therefore V_o(\tau - t_2) = V_2 = \left[A_1 \sin\omega(\tau - t_1 - t_2) + V_1 \cos\omega(\tau - t_1 - t_2)\right] e^{-\sigma(\tau - t_1 - t_2)} \qquad (4)$$

$$\& \quad i_L(\tau - t_2) = I_2 = I - \sigma C V_2 - \omega C e^{-\sigma(\tau - t_1 - t_2)} \cdot \left[A_1 \cos\omega(\tau - t_1 - t_2) - V_1 \sin\omega(\tau - t_1 - t_2)\right] \qquad (5)$$

& At $t \in T_2$, with $\alpha <<< 1$ then commutation time is very short, during which, the load inductance current, i_L, could be assumed constant at I_2, hence appearing as a current source. The capacitor voltage could also be assumed constant, with value $V_2 > 0$ to enable Q2 & Q4 to fire.

$$\therefore \quad i_L'(t) = I_2 \qquad t \in T_2 \qquad (6)$$

$$\& \quad V_c(t) = V_2 \qquad t \in T_2 \qquad (7)$$

$$\& \quad -\dot{i}_0 \cdot \alpha L = V_c$$

$$\therefore \quad i_0 = \int \frac{-V_c}{\alpha L}\, dt = \frac{-V_2}{\alpha L} \cdot t + A_3$$

$$\because \quad i_c(\tau - t_2) = I \qquad \therefore \quad I = \frac{-V_2}{\alpha L} \cdot (\tau - t_2) + A_3$$

$$\therefore \quad A_3 = I + \frac{V_2}{\alpha L}(\tau - t_2)$$

$$\therefore \quad i_c(t) = \frac{V_2}{\alpha L}(\tau - t_2 - t) + I \qquad t \in T_2 \qquad (8)$$

$\therefore$ The condition for t_2 is that $i_c(\tau) = -I$

$$\therefore \quad -I = \frac{V_2}{\alpha L}(\tau - t_2 - \tau) + I \implies -2I = -\frac{V_2 t_2}{\alpha L}$$

$$\therefore \quad t_2 = \frac{2\alpha L I}{V_2} \qquad (9)$$

& At $t \in T_3$, the initial i_L & V_c are respectively I_2 & V_2.

$\therefore$ Like the analysis of T_1:

$$\therefore \quad i_c(t) = -I \qquad t \in T_3 \qquad (10)$$

$$\& \quad V_c(t) = \left[A_2 \sin\omega(t-\tau) + V_2 \cos\omega(t-\tau)\right] \cdot e^{-\sigma(t-\tau)}, \quad t \in T_3 \qquad (11)$$

where: $$A_2 = -(I + I_2 + \sigma C V_2)/\omega C \qquad (12)$$

$$\therefore \quad V_o(T) = V_1 = \left[A_2 \sin\omega(T-\tau) + V_2 \cos\omega(T-\tau)\right] \cdot e^{-\sigma(T-\tau)} \qquad (13)$$

$$\& \quad i_L(T) = I_1 = -I - \sigma C V_1 - \omega C e^{-\sigma(T-\tau)} \cdot \left[A_2 \cos\omega(T-\tau) - V_2 \sin\omega(T-\tau)\right] \qquad (14)$$

$\therefore$ At $t \in T_4$, like for $t \in T_2$; but $V_1 < 0$ for Q1 & Q3 to fire.

$$\therefore \quad i_L(t) = I_1 \qquad t \in T_4 \qquad (15)$$

$$\& \quad V_o(t) = V_1 \qquad t \in T_4 \qquad (16)$$

$$\& \quad i_o(t) = \frac{V_1}{\alpha L} \cdot (T - t) - I \qquad t \in T_4 \qquad (17)$$

Hence, the condition for t_1 is that $i_o(T+t_1) = I$

$$\therefore \quad t_1 = \frac{-2\alpha L I}{V_1} \qquad (V_1 < 0) \qquad (18)$$

This completes one cycle at steady state.

Now, we want to find I_1, V_1, I_2 & V_2 in terms of given parameters. This is done by solving (3), (4), (5), (12), (13) & (14) together.

(3) & (4) gives:

$$V_2 e^{\sigma(\tau - t_1 - t_2)} \cdot \omega C = I \sin\omega(\tau - t_1 - t_2) - I_1 \sin\omega(\tau - t_1 - t_2) + V_1 \left[\omega C \cos\omega(\tau - t_1 - t_2) - \sigma C \sin\omega(\tau - t_1 - t_2)\right]$$

Let $t_+ = \tau - t_1 - t_2$

$$\therefore \quad V_2 e^{\sigma t_+} \cdot \omega C = I \sin\omega t_+ - I_1 \sin\omega t_+ + C V_1 \left(\omega \cos\omega t_+ - \sigma \sin\omega t_+\right) \qquad (19)$$

& (3) & (5) gives:

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$$I_2 + \sigma C V_2 = I\left(1 - e^{-\sigma t_+} \cos\omega t_+\right) + I_1 e^{-\sigma t_+} \cos\omega t_+ + C V_1 e^{-\sigma t_+} \left(\omega \sin\omega t_+ + \sigma \cos\omega t_+\right) \qquad (20)$$

Let $t_- = T - \tau$

∴ (12) & (13) gives:

$$V_1 e^{\sigma t_-} \omega C = -I \sin\omega t_- - I_2 \sin\omega t_- + CV_2(\omega\cos\omega t_- - \sigma\sin\omega t_-) \qquad (21)$$

& (12) & (14) gives

$$I_1 + \sigma C V_1 = I(e^{-\sigma t_-}\cos\omega t_- - 1) + I_2 e^{-\sigma t_-}\cos\omega t_- + CV_2 e^{-\sigma t_-}(\omega\sin\omega t_- + \sigma\cos\omega t_-) \qquad (22)$$

For maximum power dilievered to R, then the following conditions must be met:

I) $t_+ = t_- = t_0$ i.e $\tau - t_1 - t_2 = T - \tau \Rightarrow \tau = \frac{T+t_1+t_2}{2}$

II) $t_+ + t_- \simeq \frac{2\pi}{\omega}$ i.e $T - t_1 - t_2 \simeq \frac{2\pi}{\omega} \Rightarrow T \simeq \frac{2\pi}{\omega} + t_1 + t_2$

∴ $2t_0 \simeq \frac{2\pi}{\omega} \Rightarrow t_0 \simeq \pi/\omega$. Let $t_0 = \frac{\pi - \delta}{\omega}$.

The above conditions implies that the behaviour of the circuit during T_1 is just same as during T_3 & T_2 as T_4 with opposite sense, i.e symmetrical.

∴ $V_1 = -V_2$ & $I_1 = -I_2$

∴ (19) & (21) become the same as:

$$-V_1 e^{\sigma t_0}\omega C \simeq I \overset{(+\delta)}{\cancel{\sin}}\omega t_0 - I_1 \overset{(+\delta)}{\cancel{\sin}}\omega t_0 + CV_1(\omega \overset{-1}{\cancel{\cos}}\omega t_0 - \sigma \overset{(+\delta)}{\cancel{\sin}}\omega t_0)$$

∴ $-V_1 e^{\sigma t_0}\omega C \simeq -CV_1\omega + \delta(I - I_1)$

or $\omega C V_1(1 - e^{\sigma\pi/\omega}) \simeq +\delta(I - I_1) \Rightarrow V_1 \simeq \frac{-\delta(I-I_1)}{\omega C(e^{\sigma\pi/\omega} - 1)}$ (23)

& (20) & (22) become the same as:

$$-I_1 - \sigma C V_1 = I(1 - e^{-\sigma t_0}\overset{-1}{\cancel{\cos}}\omega t_0) + I_1 e^{-\sigma t_0}\overset{-1}{\cancel{\cos}}\omega t_0 + CV_1 e^{-\sigma t_0}(\omega \overset{(+\delta)}{\cancel{\sin}}\omega t_0 + \sigma \overset{-1}{\cancel{\cos}}\omega t_0)$$

$$\simeq I(1 + e^{-\sigma t_0}) - I_1 e^{-\sigma t_0} - CV_1 e^{-\sigma t_0}\cdot\sigma$$

∴ $(I_1 + \sigma C V_1)(e^{-\sigma t_0} - 1) \simeq I(1 + e^{-\sigma t_0}) \Rightarrow I_1 + \sigma C V_1 = \frac{e^{\sigma t_0}+1}{1 - e^{\sigma t_0}}\cdot I$ (24)

Solving (23) & (24):

$$\therefore\ I_1+\sigma C\left(\frac{-\delta(I-I_1)}{\omega C\left(e^{\sigma t_o}-1\right)}\right)=\frac{e^{\sigma t_o}+1}{1-e^{\sigma t_o}}\cdot I$$

$$\therefore\ I_1\left[1+\frac{\sigma\delta}{\omega\left(e^{\sigma t_o}-1\right)}\right]=\frac{-\sigma C\delta+\omega C+\omega Ce^{\sigma t_o}}{\left(1-e^{\sigma t_o}\right)\omega C}\cdot I$$

$$\therefore\ I_1\approx\frac{1+e^{\sigma t_o}}{1-e^{\sigma t_o}}\,I=\frac{e^{\sigma t_o}+1}{e^{\sigma t_o}-1}\cdot(-I)$$

$$\therefore\ I_1=-I\cdot\frac{e^{\sigma\pi/\omega}+1}{e^{\sigma\pi/\omega}-1} \qquad (25)$$

$$(23)\Rightarrow\ \therefore\ V_1=\frac{-\delta}{\omega C}\cdot\left(\frac{I+I\cdot\frac{e^{\sigma\pi/\omega}+1}{e^{\sigma\pi/\omega}-1}}{e^{\sigma\pi/\omega}-1}\right)=\frac{-\delta I\left(e^{\sigma\pi/\omega}-1+e^{\sigma\pi/\omega}+1\right)}{\omega C\left(e^{\sigma\pi/\omega}-1\right)^2}$$

$$\therefore\ V_1=\frac{-2\delta I e^{\sigma\pi/\omega}}{\omega C\left(e^{\sigma\pi/\omega}-1\right)^2}=\frac{-\delta I}{2\omega C\sinh^2\left(\frac{\sigma\pi}{2\omega}\right)}\ ,\ \delta>0 \qquad (26)$$

(9) & (18) both become:

$$t_2=t_1=t_c=\frac{2\alpha LI}{V_2}=\frac{2\alpha LI}{-V_1}=\frac{2\alpha LI}{+\delta I}\left(2\omega C\sinh^2\left(\frac{\sigma\pi}{2\omega}\right)\right)$$

$$\therefore\ t_c=+4\alpha\omega LC\sinh^2\left(\frac{\sigma\pi}{2\omega}\right)/\delta\ ,\ \delta>0 \qquad (27)$$

$$\therefore\ \tau=\frac{T+t_1+t_2}{2}=\frac{T}{2}+t_c\approx T/2\ ,\quad \text{since } t_c<<<T$$

$$\&\ T=\frac{2\pi}{\omega}+2t_c\approx\frac{2\pi}{\omega}\ ,\quad \text{since } t_c<<<T$$

$$\therefore\ \omega=\frac{2\pi}{T}=\sqrt{\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2}\Rightarrow\frac{4\pi^2}{T^2}=\frac{1}{LC}-\frac{1}{4R^2C^2}$$

$$\therefore\ \frac{1}{C}=\frac{-\frac{1}{L}\pm\sqrt{\frac{1}{L^2}+4\cdot\frac{1}{4R^2}\cdot\frac{-4\pi^2}{T^2}}}{-2/4R^2}=\frac{2R^2}{L}\pm\sqrt{\left(\frac{2R^2}{L}\right)^2-16\pi^2R^2/T^2}$$

$$=\frac{2R}{TL}\cdot\left[RT\pm\sqrt{T^2R^2-4\pi^2L^2}\right]$$

$$\therefore\ C=\frac{TL}{2R}\cdot\frac{1}{RT\pm\sqrt{R^2T^2-4\pi^2L^2}}=\frac{TL}{2R}\cdot\frac{RT\mp\sqrt{R^2T^2-4\pi^2L^2}}{4\pi^2L^2}$$

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$\therefore$ C, the commutating capacitor $=\frac{T}{8\pi^2LR}\cdot\left(RT\pm\sqrt{R^2T^2-4\pi^2L^2}\right)$ (28)

The power maximized, P, is given by using (1):

$$P = \frac{1}{T}\int_0^T V_o^2(t)/R\,dt = \frac{2}{RT}\int_0^{T/2} V_o^2(t)\,dt \simeq \frac{2}{RT}\int_{t_c}^{T/2}\Big[A_1 \sin\omega(t-t_c) + V_1\cos\omega(t-t_c)\Big]^2$$

$$* \; e^{-2\sigma(t-t_c)}\,dt = \frac{2}{RT}\int_{t_c}^{T/2}\Big[A_1^2\Big(\frac{1-\cos 2\omega(t-t_c)}{2}\Big) + V_1^2\Big(\frac{1+\cos 2\omega(t-t_c)}{2}\Big) +$$

$$+ 2A_1V_1\frac{\sin 2\omega(t-t_c)}{2}\Big]\cdot e^{-2\sigma(t-t_c)}\,dt =$$

$$= \frac{2}{RT}\cdot\Bigg[\frac{A_1^2+V_1^2}{2}\cdot\frac{e^{-2\sigma(t-t_c)}}{-2\sigma} + e^{-2\sigma(t-t_c)}\cdot\frac{(A_1^2/2)\big(2\omega\sin 2\omega(t-t_c) - 2\sigma\cos 2\omega(t-t_c)\big) + (V_1^2/2)\big(2\sigma\cos 2\omega(t-t_c) - 2\omega\sin 2\omega(t-t_c)\big) + A_1V_1\big(2\omega\cos 2\omega(t-t_c) + 2\sigma\sin 2\omega(t-t_c)\big)}{-4\omega^2-4\sigma^2}\Bigg]_{t_c}^{T/2}$$

$$\simeq \frac{2}{RT}\cdot\Bigg[\frac{A_1^2+V_1^2}{4\sigma}\big(1-e^{-2\sigma\pi/\omega}\big) + \frac{(-A_1^2\sigma + V_1^2\sigma + 2A_1V_1\omega)(-e^{-2\sigma\pi/\omega}+1)}{4(\omega^2+\sigma^2)}\Bigg]$$

$$= \frac{(1-e^{-2\sigma\pi/\omega})}{2RT}\cdot\Bigg[\frac{A_1^2+V_1^2}{\sigma} + \frac{(-A_1^2+V_1^2)\sigma + 2A_1V_1\omega}{\omega^2+\sigma^2}\Bigg]$$

but:

$$A_1 = \frac{I - I_1 - \sigma C V_1}{\omega C} \simeq \frac{I - I_1}{\omega C} = \frac{I + \frac{I(e^{\sigma\pi/\omega}+1)}{(e^{\sigma\pi/\omega}-1)}}{\omega C} = \frac{2e^{\sigma\pi/\omega}\cdot I}{(e^{\sigma\pi/\omega}-1)\omega C} = \frac{e^{\sigma\pi/2\omega} I}{\omega C \sinh\frac{\sigma\pi}{2\omega}}$$

& $A_1^2 + V_1^2 \simeq A_1^2$ & $-A_1^2 + V_1^2 \simeq -A_1^2$

$$\therefore\; P \simeq \frac{1-e^{-2\sigma\pi/\omega}}{2RT}\cdot\Big[\frac{1}{\sigma} - \frac{\sigma}{\omega^2+\sigma^2}\Big]A_1^2 = \frac{\sinh(\sigma\pi/\omega)}{RTe^{\sigma\pi/\omega}}\cdot\Big[2RC - \frac{LC}{2RC}\Big]A_1^2$$

$$= \frac{\sinh(\sigma\pi/\omega)}{2R^2CTe^{\sigma\pi/\omega}}\cdot(4R^2C^2 - LC)\cdot\frac{4e^{2\sigma\pi/\omega}I^2}{\omega^2C^2(e^{\sigma\pi/\omega}-1)^2}$$

$\therefore$ Max. power, $P = (4R^2C - L)I^2\sinh(\sigma\pi/\omega)/2R^2T\omega^2C^2\sinh^2(\frac{\sigma\pi}{2\omega})$

or $P \simeq \dfrac{4R^2C - L}{R^2\omega^2C^2T}\cdot\coth\big(\frac{\sigma\pi}{2\omega}\big)\cdot I^2 = \dfrac{4R^2C-L}{R^2C^2T\cdot\frac{4LCR^2-L^2}{4L^2C^2R^2}}\cdot\coth\big(\frac{\sigma\pi}{2\omega}\big)\cdot I^2$

\# $\therefore\; P = 4LI^2\coth\big(\frac{\sigma\pi}{2\omega}\big)/T$ (29)

\# As for waveforms:

$i_{Q1} = i_{Q3}$ for all t

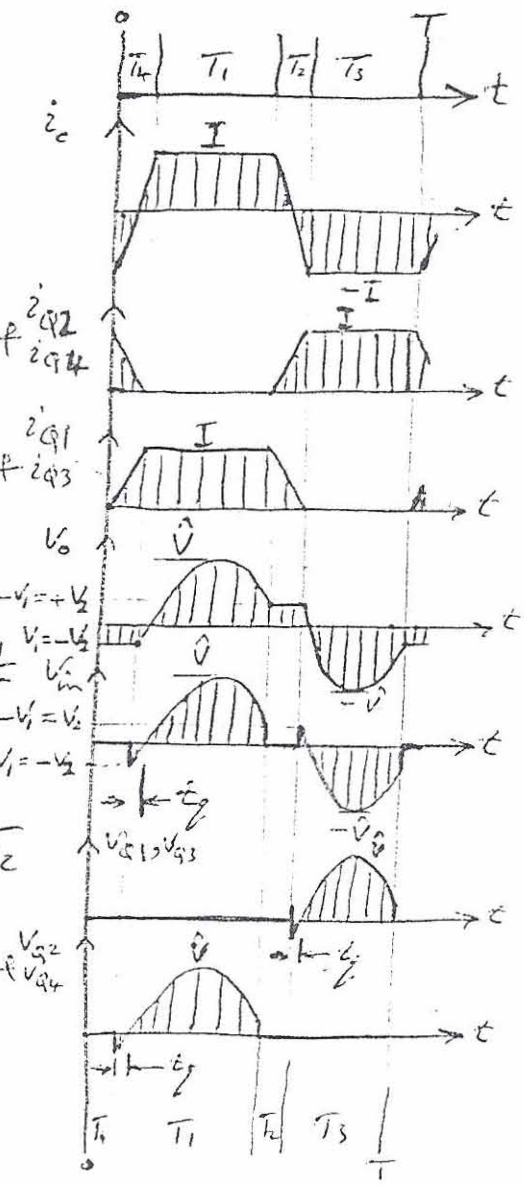
& $i_{Q2} = i_{Q4}$ for all t

during $t \in T_2$

$\therefore I = i_{Q1} + i_{Q2}$

& $i_{Q1} = i_{Q4} + i_o = i_{Q2} + i_o$

$\therefore I = i_{Q2} + i_o + i_{Q2} = 2i_{Q2} + i_o$



$$\therefore i_{Q2} = \frac{I - i_c}{2} = \frac{I - I - (\tau - t_2 - t)V_2/\alpha L}{2}$$
$$= +\left(\frac{T}{2} - t\right)V_1/2\alpha L$$

$$\therefore i_{Q2}(t) = \frac{\delta\,(t - T/2)}{4\omega L C\alpha \sinh^2\left(\frac{\sigma\pi}{2\omega}\right)} \cdot I \;, \; t \in T_2$$

$V_L = 0$ for $t \in T_2$

& $V_L(t) = V_c(t)$ for $t \in T_1$

& $i_{Q1} = i_{Q3} = i_c = I$ for $t \in T_1$

$$\hat{V} = V_c(t)_{max} \simeq A_1 e^{-\sigma\pi/2\omega} = \frac{2e^{\sigma\pi/\omega} \cdot I}{(e^{\sigma\pi/\omega} - 1)\omega C} e^{-\sigma\pi/2\omega} = \frac{I}{\omega C \sinh(\sigma\pi/2\omega)} \quad (30)$$

from (1) $\therefore V_c(t_1 + t_q) = 0$

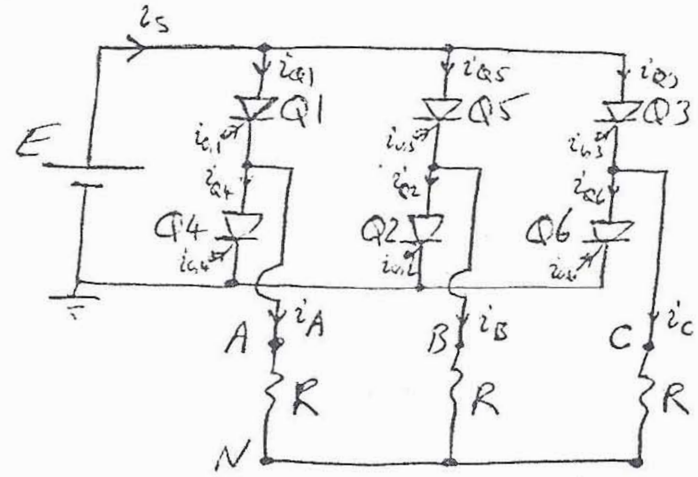
$\therefore A_1 \sin\omega t_q + V_1 \cos\omega t_q = 0 \quad \therefore \tan\omega t_q = \frac{-V_1}{A_1}$ = (from (26) & A_1 exp.)

$$= \frac{\delta I}{2\omega C \sinh^2(\sigma\pi/2\omega)} \cdot \frac{\omega C \sinh(\sigma\pi/2\omega)}{I e^{\sigma\pi/2\omega}} = \frac{\delta e^{-\sigma\pi/2\omega}}{2\sinh(\sigma\pi/2\omega)} = \frac{\delta}{e^{\sigma\pi/\omega} - 1}$$

$\therefore \delta = (e^{\sigma\pi/\omega} - 1) \cdot \tan\omega t_q \ll \pi \quad (31)$

This closes all the expressions for the problem.

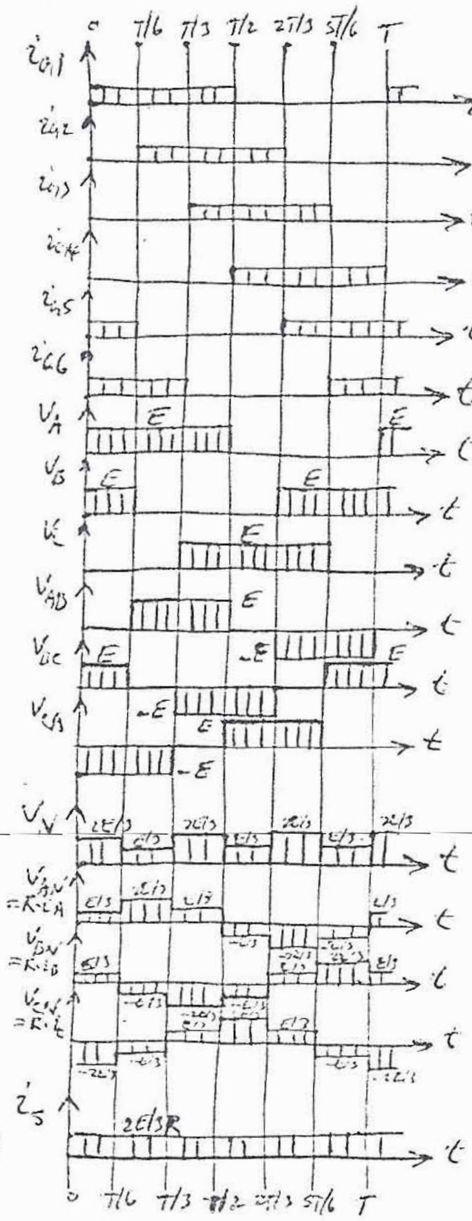
4-15 Since load is resistive, then no need for the six freewheeling diodes and the bridge becomes as shown here.



The neutral voltage V_N is again, due to symmetry, given by:

$$V_N = \frac{1}{3}(V_A + V_B + V_C)$$

$\therefore$ The waveforms are as shown.



The source current i_s is:

$$i_s = i_{Q1} + i_{Q5} + i_{Q3}.$$

where i_{Q1}, i_{Q5} & i_{Q3} are the positive parts of i_A, i_B & i_C respectively (i_{Q4}, i_{Q2} & i_{Q6} are respectively the negative)

$\therefore$ i_s is obtained as shown (constant)

$\therefore$ $V_{AN}(t) = a_n \sin n\omega t + b_n \cos n\omega t$

$b_n = 0$ due to symmetry.

&

$$a_n = \frac{2}{T}\int_0^T V_{AN}(t)\sin n\omega t\,dt =$$

$$= \frac{8}{T}\int_0^{T/4} V_{AN}(t)\sin n\omega t\,dt =$$

$$= \frac{8}{T}\left[\int_0^{T/6}(E/3)\sin n\omega t\,dt + \int_{T/6}^{T/4}(2E/3)\sin n\omega t\,dt\right] =$$

$$= \frac{E\cdot 8}{3T}\left[-\frac{\cos n\omega t}{n\omega}\Big|_0^{T/6} - 2\frac{\cos n\omega t}{n\omega}\Big|_{T/6}^{T/4}\right] =$$

$$= \frac{8E}{3\omega T n}\left[1 - \cos(n\omega T/6) - 2\cos n\omega T/4 + 2\cos n\omega T/6\right]$$

$$= \frac{8E}{3\times 2\pi n}\left[1 + \cos(n\pi/3) - 2\cos\left(\frac{n\pi}{2}\right)\right]$$

$$\therefore\ a_1 = \frac{4E}{3\pi}\left(1 + \cos\pi/3 + 0\right) = \frac{4E}{3\pi}\left(1 + \frac{1}{2}\right) = \frac{2E}{\pi}$$

$\therefore$ phase fundamental $= \frac{\sqrt{2}E}{\pi} = \frac{\sqrt{2}\times 100}{\pi} = 45.0$ Volts

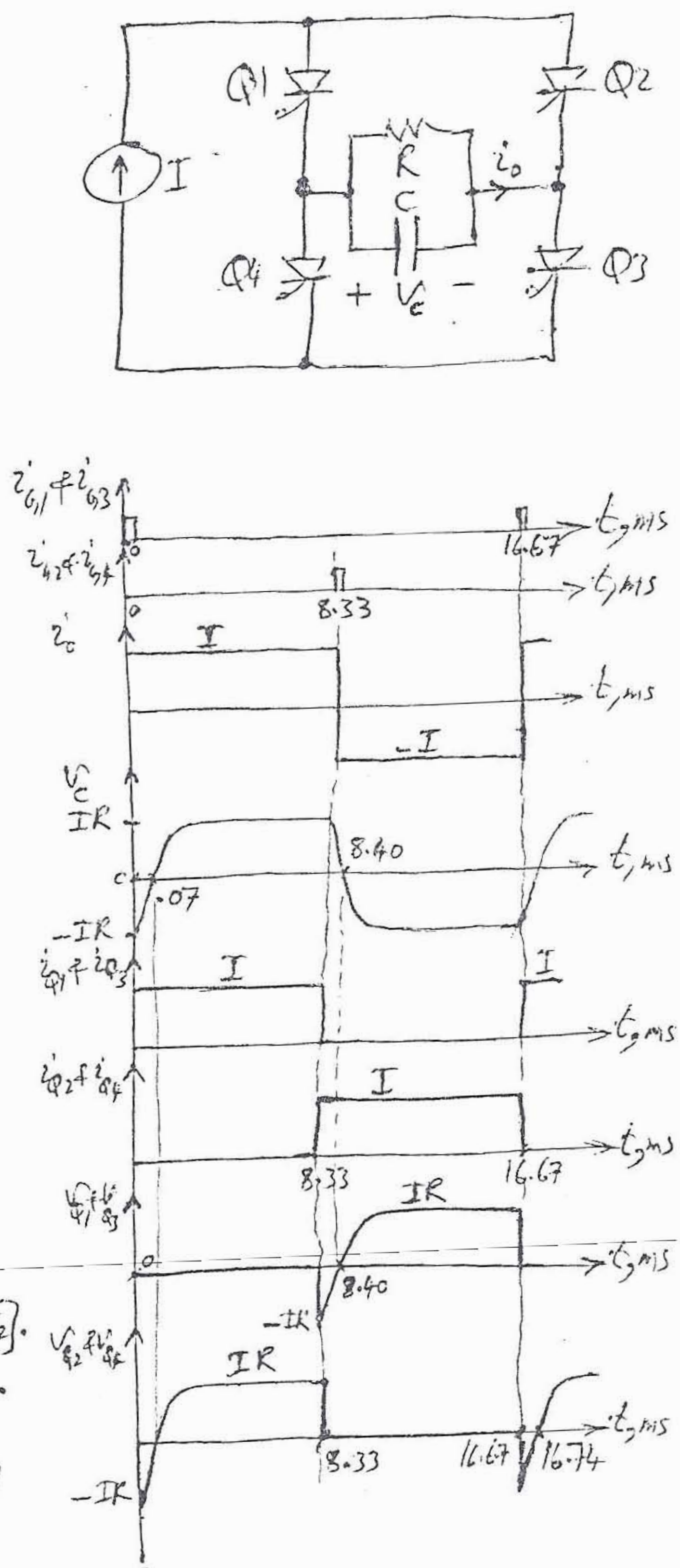
$$\text{phase rms} = \sqrt{\frac{4}{T}\left[\left(\frac{E}{3}\right)^2\frac{T}{6} + \left(\frac{2E}{3}\right)^2\frac{T}{12}\right]} = \frac{\sqrt{2}E}{3}$$

$\therefore$ phase distortion $= \frac{V_{harmonics}}{V_{rms}} = \sqrt{1 - \left(\frac{\sqrt{2}E/\pi}{\sqrt{2}E/3}\right)^2} = \sqrt{1 - \frac{9}{\pi^2}} = .297 \simeq 30\%$.

4-16 $f = 60\ Hz \Rightarrow T = 16.67\ ms$

$R = 100\ \Omega$ & $C = 1\mu F$.

Circuit shown gives the waveforms indicated. When Q1 & Q3 are turned on at $t=0$, the source charges C up so that as soon as Q2 & Q4 are gated, C discharges through Q4 Q3 causing Q3 to be off and through Q1 Q2 causing Q1 to be off. This makes I to recharge C with opposite sense till Q1 & Q3 are gated again at $t=T$ whereby a similar procedure ensures the turn off of Q2 & Q4. Hence, no force commutation is required. Since $RC = 100\mu s$ and $T \gg RC$; then waveform of V_c looks rectangular as shown with equation:



$$V_c(t) = IR\left(1 - 2e^{-t/RC}\right), \; t \in [0, T/2].$$

V_c is zero at $t = RC \ln 2 = 69.3\ \mu s$.

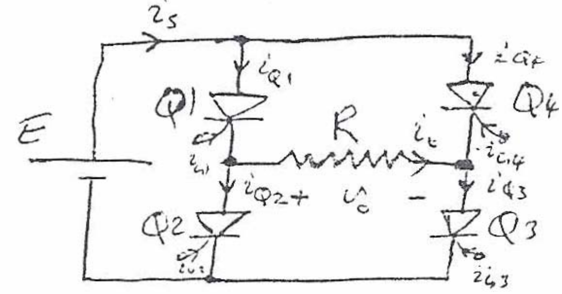
This is also the time during which thyristors are reversed biased after being put off.

$\therefore$ t_q must be $\leq$ 69.3 μs.

81B

4-17-a Chopper inverter:

The circuit is as shown aside where no diodes are required since no inductance is present.



The output fundamental is given by:

$$v_{o\,\text{fundamental}}(t) = a\sin\left(\frac{2\pi t}{T}\right) + b\cos\left(\frac{2\pi t}{T}\right)$$

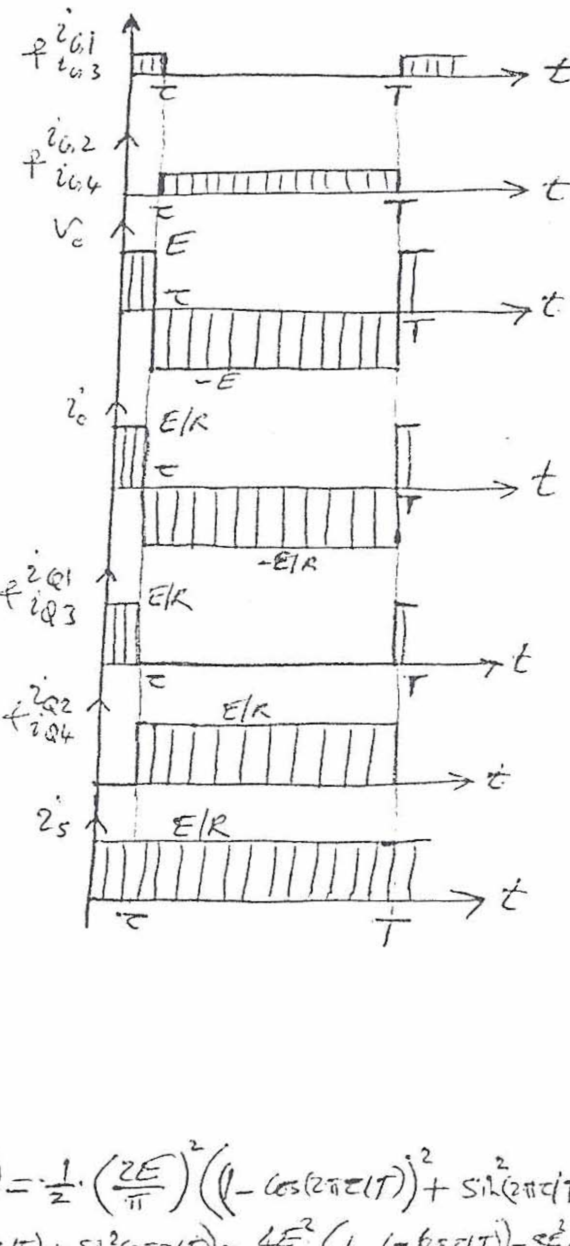
$$\therefore\ a = \frac{2}{T}\int_0^T v_o(t)\sin\left(\frac{2\pi t}{T}\right)dt =$$

$$= \frac{2}{T}\left[\int_0^{\tau} E\sin\left(\frac{2\pi t}{T}\right)dt - \int_{\tau}^{T} E\sin\left(\frac{2\pi t}{T}\right)dt\right] =$$

$$= \frac{2E}{T}\left[\frac{-\cos(2\pi t/T)\Big|_0^{\tau} + \cos(2\pi t/T)\Big|_{\tau}^{T}}{2\pi/T}\right]$$

$$= \frac{E}{\pi}\left[1 - \cos(2\pi\tau/T) + 1 - \cos(2\pi\tau/T)\right]$$

$$= \frac{2E}{\pi}\left(1 - \cos\left(\frac{2\pi\tau}{T}\right)\right).$$



$$\& \quad b = \frac{2}{T}\int_0^T v_o(t)\cos\left(\frac{2\pi t}{T}\right)dt =$$

$$= \frac{2}{T}\left[\int_0^{\tau} E\cos\left(\frac{2\pi t}{T}\right)dt - \int_{\tau}^{T} E\cos\left(\frac{2\pi t}{T}\right)dt\right] =$$

$$= \frac{2E}{T}\left[\frac{\sin(2\pi t/T)\Big|_0^{\tau} - \sin(2\pi t/T)\Big|_{\tau}^{T}}{2\pi/T}\right] =$$

$$= \frac{E}{\pi}\left[\sin(2\pi\tau/T) - 0 - 0 + \sin(2\pi\tau/T)\right] =$$

$$= \frac{2E}{\pi}\cdot\sin(2\pi\tau/T).$$

$$\therefore\ V^2_{o\,\text{fundamental}\,rms} = \left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{b}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(a^2 + b^2\right) = \frac{1}{2}\cdot\left(\frac{2E}{\pi}\right)^2\left(\left(1 - \cos(2\pi\tau/T)\right)^2 + \sin^2(2\pi\tau/T)\right) =$$

$$= \frac{2E^2}{\pi^2}\cdot\left(1 - 2\cos(2\pi\tau/T) + \cos^2(2\pi\tau/T) + \sin^2(2\pi\tau/T)\right) = \frac{4E^2}{\pi^2}\left(1 - \cos(2\pi\tau/T)\right) = \frac{8E^2}{\pi^2}\sin^2\left(\frac{\pi\tau}{T}\right)$$

$$\therefore\ V_{o\,\text{fundamental}\,rms} = \frac{2\sqrt{2}E}{\pi}\sin\left(\frac{\pi\tau}{T}\right) \Rightarrow \therefore\ \frac{2\sqrt{2} * 300}{\pi}\sin\left(\frac{\pi\tau}{T}\right) = 110 \Rightarrow \frac{\pi\tau}{T} = .41946 \text{ OR } 2.72214$$

$$\# \quad \therefore\ \frac{\tau}{T} = 0.13352 \quad \text{OR} \quad 0.86648$$

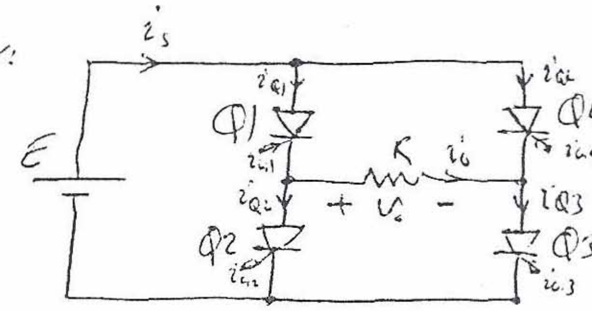
$$V^2_{o\,rms} = \left(E^2\tau + E^2(T-\tau)\right)/T = E^2 \quad \therefore\ V_{o\,rms} = E \quad \& \quad V_{o\,dc} = \left(E\tau - E(T-\tau)\right)/T = E\left(\frac{2\tau}{T} - 1\right)$$

$$= \pm E \times .73297$$

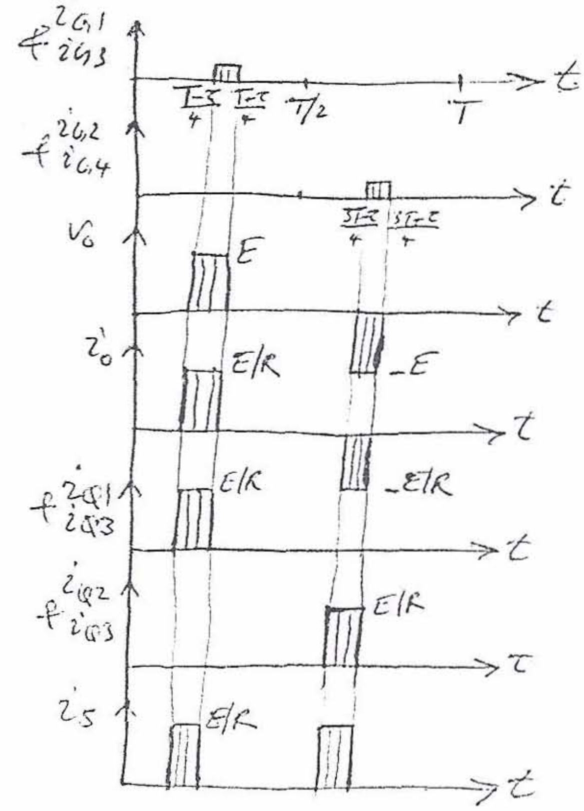
$$\# \quad \therefore\ \mu_v = \frac{V_{o\,\text{harmonics}\,rms}}{V_{o\,rms\,ac}} = \sqrt{\frac{V^2_{o\,rms} - V^2_{o\,dc} - V^2_{o\,\text{fundamental}}}{V^2_{o\,rms} - V^2_{o\,dc}}} = \sqrt{1 - \left(\frac{110}{300}\right)^2\cdot\frac{1}{(1 - .73297^2)}} = 0.84230 = 84.23\%.$$

$\therefore$ The distortion factor is 84.23 %. (Note: A filter is required at the output to get rid of the dc).

4-17-b Single pulse width-modulated inverter:



The circuit is as shown with no diodes as before. The gating needs not to be as complex as for RL load, but rather simpler as shown.



The output fundamental is given by:

$$v_{o_{fundamental}}(t) = a \sin\left(\frac{2\pi t}{T}\right)$$

where $a = \frac{4E}{\pi} \sin\left(\frac{\pi \tau}{2T}\right)$

$$\therefore V_{o_{fundamental_{rms}}} = \frac{2\sqrt{2}\,E}{\pi} \sin\left(\frac{\pi\tau}{2T}\right)$$

$$\therefore \frac{2\sqrt{2} * 300}{\pi} \sin\left(\frac{\pi\tau}{2T}\right) = 110$$

$\therefore \pi\tau/2T = .41946$ OR 2.72214

$\therefore \frac{\tau}{T} (\leq 1) = 0.26703$, the other answer $= 1.73297 > 1$ is rejected since $\frac{\tau}{T} \in [0, 1]$.

\# $\therefore \frac{\tau}{T} = 0.26703$ (i.e. $.13352T$ duration at each half cycle symmetrically).

(Note: no need for filter at the output since no dc component is there).

$V_{o_{rms}}^2 = \left(E^2\tau/2 + E^2\tau/2\right)/T = \left(\frac{E^2\tau}{T}\right)$ $\quad \therefore V_{o_{rms}} = E\sqrt{\frac{\tau}{T}} = 300\sqrt{.26703} = 155.026^V$

\# $\therefore \mu_v = \frac{V_{o_{harmonics_{rms}}}}{V_{o_{rms}}} = \sqrt{1 - \left(\frac{V_{o_{fundamental_{rms}}}}{V_{o_{rms}}}\right)^2} = \sqrt{1 - \left(\frac{110}{155.026}\right)^2} = .70465 \simeq 70.5\%$

$\therefore$ The distortion factor is 70.5%. (less distorted than part (a)).

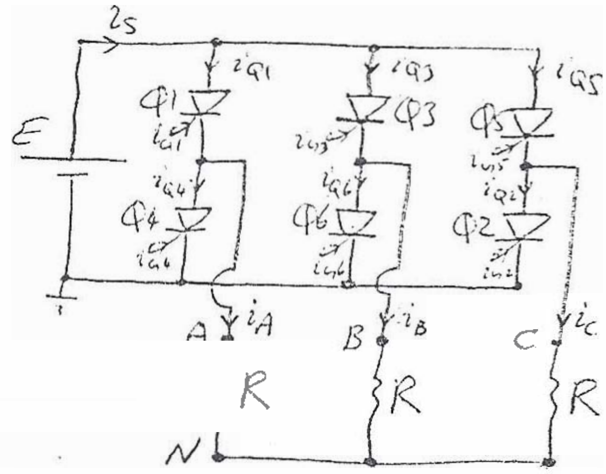
\# f Since resistive load; then waveform of i_o is same as that of v_o

$\therefore \mu_I = \mu_V = 70.5\%$.

4-17-C-I 3ϕ Inverter with 180° scheme:

The circuit is as shown with no diodes as before. Waveforms are also as shown, where:

V_N (by symmetry) $= \frac{1}{3}(V_A + V_B + V_C)$



$E = 300V$

& $i_s = i_{Q1} + i_{Q3} + i_{Q5}$.

The fundamental phase voltage, $V_{AN_{fundamental}}(t)$, is given by:

$$V_{AN_{fundamental}}(t) = a\sin\left(\frac{2\pi t}{T}\right) + b\cos\left(\frac{2\pi t}{T}\right)$$

where: $a = \frac{2}{T}\int_0^T V_{AN}(t)\sin\left(\frac{2\pi t}{T}\right)dt$

$$= \frac{2}{T}\times 2\int_0^{T/2} V_{AN}(t)\cdot\sin\left(\frac{2\pi t}{T}\right)dt =$$

$$\frac{4}{T}\times 2\int_0^{T/4} V_{AN}(t)\sin\left(\frac{2\pi t}{T}\right)dt =$$

$$= \frac{8}{T}\left[\int_0^{T/6} V_{AN}(t)\sin\left(\frac{2\pi t}{T}\right)dt + \int_{T/6}^{T/4} V_{AN}(t)\sin\left(\frac{2\pi t}{T}\right)dt\right] =$$

$$= \frac{8}{T}\left[\int_0^{T/6}\frac{E}{3}\sin\left(\frac{2\pi t}{T}\right)dt + \int_{T/6}^{T/4}\frac{2E}{3}\sin\left(\frac{2\pi t}{T}\right)dt\right] =$$

$$= \frac{8E}{3T}\left[\frac{-\cos(2\pi t/T)\Big|_0^{T/6} - 2\cos(2\pi t/T)\Big|_{T/6}^{T/4}}{(2\pi/T)}\right] =$$

$$= \frac{4E}{3\pi}\left[1 - \cos(\pi/3) + 2\cos\left(\frac{\pi}{3}\right)\right]$$

$$= \frac{4E}{3\pi}\left(1 + \cos\frac{\pi}{3}\right) = \frac{4E}{3\pi}\left(1 + \frac{1}{2}\right) = \frac{2E}{\pi}$$

& $b =$ (by symmetry) 0

$\therefore V_{AN_{fundamental}}(t) = \frac{2E}{\pi}\sin\left(\frac{2\pi t}{T}\right)$

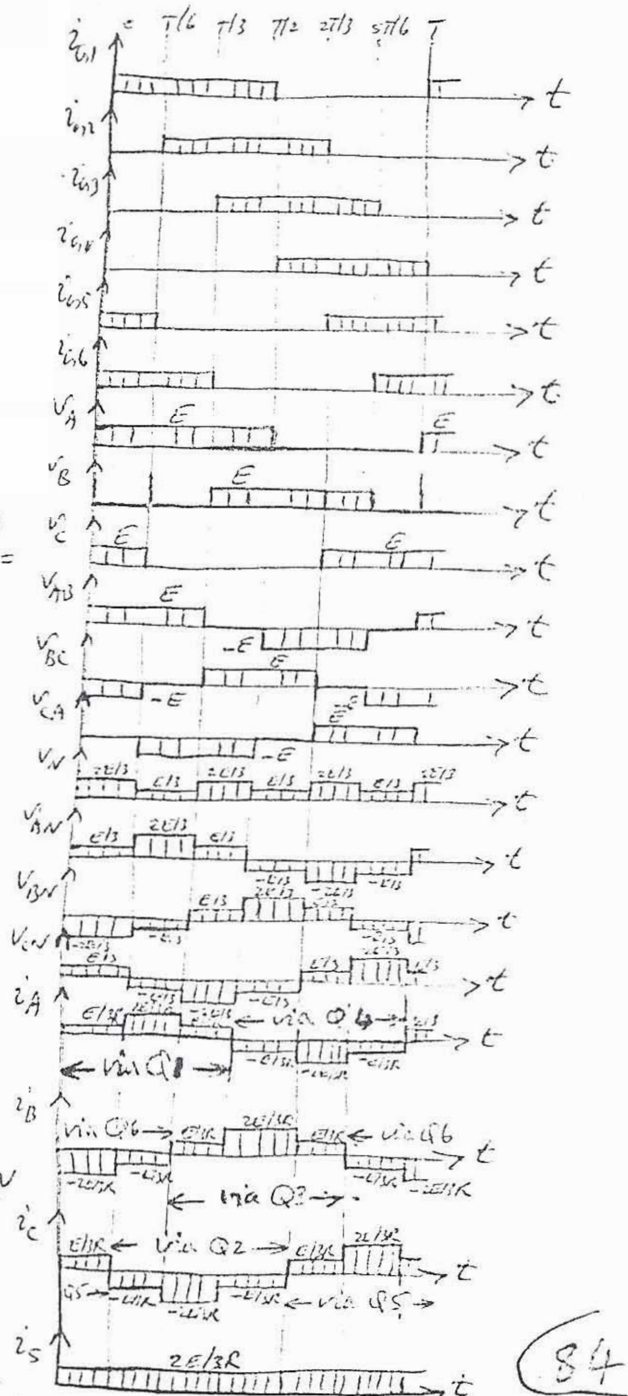
Other phases fundamentals are same but with $\pm 120°$ shifts & $V_{phase_{fundamental\ rms}} = \frac{\sqrt{2}E}{\pi} = 135.047^V$

$\therefore$ % e in achievement $= +22.77\%$

& $V_{phase_{rms}} = \sqrt{\left(\frac{E}{3}\right)^2 \cdot \frac{4}{6} + \left(\frac{2E}{3}\right)^2 \cdot \frac{2}{6}} = \frac{E}{3}\sqrt{\frac{2}{3} + \frac{4}{3}} = \frac{\sqrt{2}E}{3}$

$\therefore \mu_v$ (of phase voltage) $= \sqrt{1 - \left(\frac{\sqrt{2}/\pi}{\sqrt{2}/3}\right)^2} = .2968 = 29.68\%$.

$\therefore$ The phase voltage distortion factor is about 30%. (less distorted than both part ⓐ & part ⓑ).



4-17-C-II 3ϕ - Inverter with 120° scheme:

The circuit is as in part (I) but with waveforms as shown here. The voltage during the indeterminate interval of one terminal is obtained by the average of the other two terminal voltages due to the resistive symmetry of the star load. This voltage is $E/2$.

Again, $V_N =$ (by symmetry) $\frac{1}{3}(V_A + V_B + V_C)$

$= E/2$ all times.

& $i_S = i_{Q1} + i_{Q3} + i_{Q5} = \frac{E}{2R}$ all times

The fundamental phase voltage is:

$$V_{BN_{fundamental}}(t) = a \sin\left(\frac{2\pi t}{T}\right) + b \cos\left(\frac{2\pi t}{T}\right)$$

$a = 0$ by symmetry.

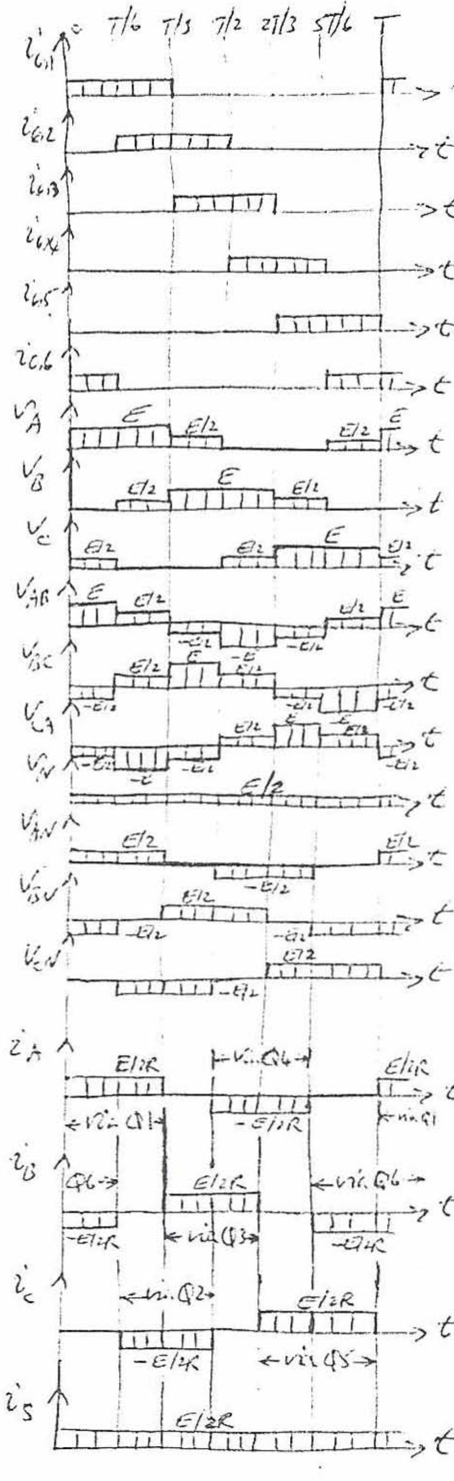
& $b = \frac{2}{T}\int_0^T V_{BN}(t) \cos\left(\frac{2\pi t}{T}\right) dt =$

$$= \frac{4 \times 2}{T}\int_0^{T/4} V_{BN}(t) \cos\left(\frac{2\pi t}{T}\right) dt =$$

$$= \frac{8}{T}\int_0^{T/6}\left(-\frac{E}{2}\right)\cos\left(\frac{2\pi t}{T}\right) dt =$$

$$= -\frac{4E}{T}\left[\frac{\sin(2\pi t/T)}{2\pi/T}\right]_0^{T/6} =$$

$$= \frac{-2E}{\pi}\cdot\left(\sin\left(\frac{\pi}{3}\right) - 0\right) = \frac{-E\sqrt{3}}{\pi}$$



$\therefore V_{BN_{fundamental}}(t) = \frac{-\sqrt{3}E}{\pi}\cdot\cos\left(\frac{2\pi t}{T}\right)$ & other phases fundamentals are same but with shifts of $\pm 120°$. Hence, $V_{phase_{fundamental\ rms}} = \sqrt{\frac{3}{2}}\cdot\frac{E}{\pi} = 116.95^V$

\# $\therefore$ %e in achievement $= +6.322\%$. (better than (I)).

& $V_{phase_{rms}} = \sqrt{\left(\frac{E}{2}\right)^2 \times \frac{4}{6}} = \frac{E}{\sqrt{6}}$

\# $\therefore C_v$ (of phase voltage) $= \sqrt{1 - \left(\frac{\sqrt{3/2}/\pi}{1/\sqrt{6}}\right)^2} = \sqrt{1 - \left(\frac{3}{\pi}\right)^2} = .2968 = 29.68\%$.

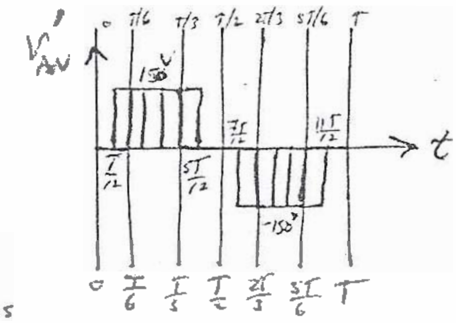
$\therefore$ The phase voltage distortion factor is about 30% (same as (I)).

(Note you can also calculate $V_{\phi_{rms_{fundamental}}}$ using shifting & formula $a_1 = \frac{4E}{\pi} \sin\left(\frac{\pi\tau}{2T}\right)$ of SPWM, by shifting to the right, through $\frac{T}{12}$, $V_{AN}(t)$ into the waveform shown in the figure as V'_{AN}.

$\therefore$ SPWM formula applies with $E = 150$, $\tau = \frac{2T}{3}$:

$\therefore$ $a_1\big|_{V'_{AN}} = \frac{600}{\pi} \sin\left(\frac{\pi}{3}\right) = \frac{300\sqrt{3}}{\pi}$ Volts.

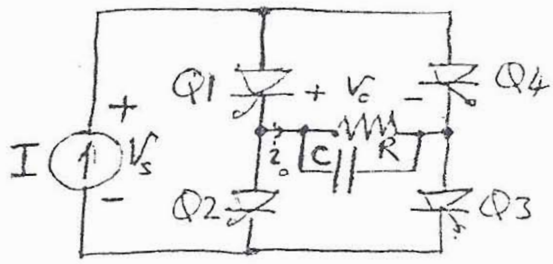
& Since shifting does not change peak.



$\therefore$ $a_1\big|_{V'_{AN}} = \hat{V}_{AN_{fundamental}} = \frac{300\sqrt{3}}{\pi}$ Volts $= V_{AN_{fundamental_{rms}}} \sqrt{2}$.

$\therefore$ $V_{\phi_{rms_{fundamental}}} = a_1\big|_{V'_{AN}} \div \sqrt{2} = \frac{300}{\pi} \cdot \sqrt{\frac{3}{2}} = \underline{116.95 \text{ Volts}}$ $\therefore$ OK).

4-18 $I = 50$ Amp
$R = 10\,\Omega$
$C = 400\,\mu F$
$f = 200$ Hz
$\therefore$ $T = 5$ msec
& $\omega = 2\pi f = 400\pi$ rad/sec



$\therefore$ Circuit is as shown
Since $RC = 4\,ms > \frac{T}{2} = 2.5\,ms$
$\therefore$ Exponential variations could be assumed linear and hence waveforms are as shown, where:

$$V(t) = -E e^{-t/RC} + IR\left(1 - e^{-t/RC}\right),\ t \in (0, T/2)$$

& $v(t) = -v(t - T/2)$

$$\therefore\ E = V(T/2) = -E e^{-T/2RC} + IR\left(1 - e^{-T/2RC}\right)$$

$$\therefore\ E = IR \cdot \frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} = 151.35\,V$$

& $\theta_c = 2\pi f \cdot RC \cdot \ln\left(\frac{E + IR}{IR}\right) = .4231\pi$

$$\therefore\ P_o = P_R \cong \frac{E^2}{3R} = 763.61 \text{ Watts}$$

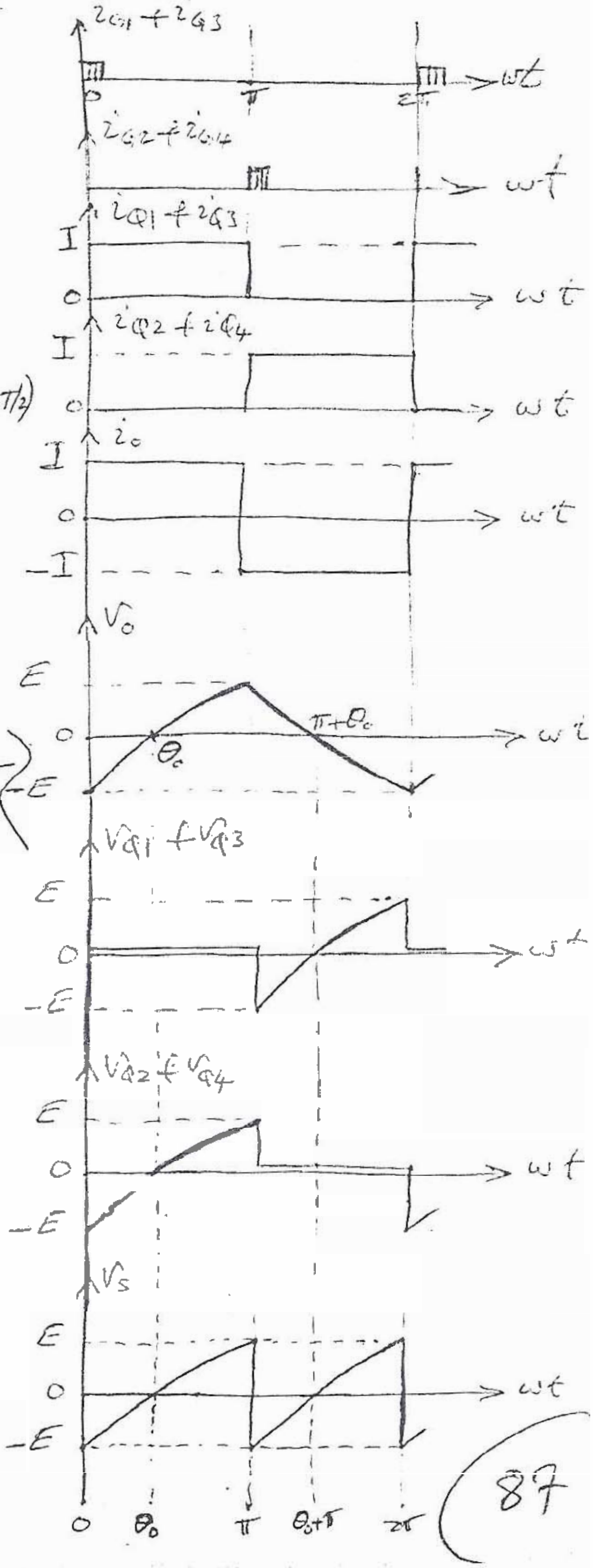
(Note: Exact value of P_R is:

$$P_R = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt = \frac{1}{\pi}\int_0^\pi \frac{v^2(\omega t)\,d\omega t}{R} =$$

$$= \frac{1}{\pi R}\cdot\left[I^2R^2\theta + (E+IR)^2\frac{e^{-2\theta/\omega CR}}{-2/\omega CR} + \right.$$

$$\left. - \frac{2IR(E+IR)}{-1/\omega CR}\cdot e^{-\theta/\omega CR}\right]_0^\pi$$

$= 783.22$ Watts & % e $= 2.5\%$)



f $P_I = P_o \simeq 763.61$ Watts

∴ $V_{s_{av}} = \frac{P_I}{I} \cong \frac{763.61}{50} = 15.272$ Volts

(Note: The exact value of $V_{s_{av}}$ can be obtained in two ways:

I) $V_{s_{av}} = \frac{P_I}{I} = \frac{P_R}{I} = \frac{783.22}{50} = 15.664$ Volts

II) $V_{s_{av}} = \frac{1}{\pi}\int_0^{\pi} v_s(\omega t)\,d\omega t = \frac{1}{\pi}\left[IR\theta + (IR+E)\,e^{-\theta/\omega CR}\cdot\omega CR\right]_0^{\pi} = 15.664V$

both ways give an error of 2.5% due to non-linearity)

f As for $V_{o_1 rms}$, this could be found from: $V_{o_1}(t) \simeq a\cos\omega t$,

where: $a = \frac{2}{2\pi}\int_0^{2\pi} V_o(\omega t)\cos\omega t\,d\omega t = \frac{2}{\pi}\int_0^{\pi}\left(-E + \frac{2E}{\pi}\cdot\theta\right)\cos\theta\,d\theta =$

$= \frac{2E}{\pi}\left[-\sin\theta + \frac{2}{\pi}(\theta\sin\theta + \cos\theta)\right]_0^{\pi} = \frac{2E}{\pi}\cdot\frac{2}{\pi}(-1-1) = \frac{-8E}{\pi^2}$

∴ $V_{o_1 rms} \simeq \frac{a}{\sqrt{2}} = 4\sqrt{2}E/\pi^2 = 86.75$ Volts (Exact $= \frac{2\sqrt{2}IR/\pi}{\sqrt{1+(\omega cR)^2}} = 87.83$ V, %e $= 1.2\%$)

f $V_{o_{av}} = 0$ since $V_o(t)$ is symmetrical.

f $V_{o_{rms}} = \sqrt{RP_o} \simeq \sqrt{7636.1} = 87.38$ Volts (exact $= 88.50$ V, %e $= 1.3\%$)

f $C_v = \frac{V_{oh\,rms}}{V_{o\,rms}} = \sqrt{1-\left(\frac{V_{o_1 rms}}{V_{o\,rms}}\right)^2} \cong 0.120 = 12.0\%$

f t_q of thyristor must be $< \frac{\theta_c}{\omega} = \frac{.4231\pi}{400\pi} = 1.058$ msec

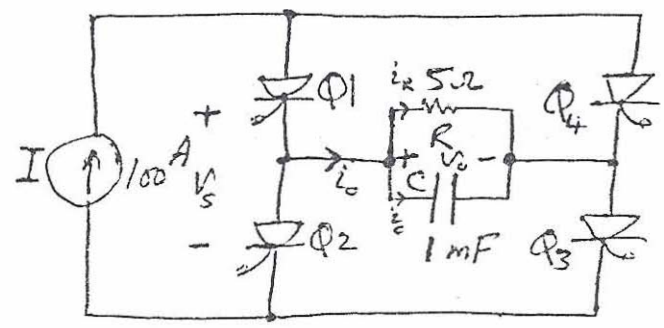
f $V_{QFB} = V_{QRB} = E = 151.35$ Volts. This is min. required rating

f $\frac{dV_Q}{dt} = \frac{dV_o}{dt}\left(\frac{\theta_c}{\omega}\right) = (E+IR)\cdot e^{-\theta_c/\omega CR}/RC = 125000\,V/s = 0.125\,\frac{V}{\mu s}$

∴ $\frac{dV}{dt}$ rating of thyristor must be $> .125\,V/\mu s$ $\left(\simeq \frac{2E}{T/2} = \frac{4E}{T} = .121\,V/\mu s\right)$

$\boxed{4\text{–}19}$ $f = 50\ Hz$

$\therefore\ T = 20\ msec$



Waveforms are as shown, assuming negligible lead inductances and commutation times.

i_c is seen to be a square wave with amplitude of 100 Amp.

$\therefore\ V_o(t) = 500 + A\,e^{-t/5m},\ V,\ t \in [0, 10]\ ms.$

Due to symmetry:

$\therefore\ V_o(0) = -V_o(T/2)$

$\therefore\ 500 + A = -\left[500 + A\,e^{-10m/5m}\right]$

$\therefore\ 1000 = -A\left(1 + e^{-2}\right)$

$\therefore\ A = -880.80$ Volts

$\therefore\ V_o(t) = 500 - 880.80\,e^{-200t},\ V,\ t \in [0, 10]\ ms$

$\therefore\ V_o(0) = -380.80$ Volts

$V_o = 0$ at $t = 2.8311\ ms$

$i_R(t) = \dfrac{V_o}{R} = 100 - 176.16\,e^{-200t},\ A,\ t \in [0, 10]\ ms$

$\therefore\ i_R(0) = -76.16$ Amp.

$\therefore\ i_c(t) = 100 - i_R(t) = 176.16\,e^{-200t},\ A,\ t \in [0, 10]\ ms$

$\therefore\ i_c(0) = 176.16\ A,\ i_c(2.8311\ ms) = 100\ A,\ i_c(10\ ms) = 23.84\ A.$

Note that V_s is running at double frequency, i.e. 100 Hz.

From waveforms, then time available for thyristors recovery is 2.8311 ms,

To find C_I of load:

$\therefore\ i_{o_{rms}} = I = 100$ Amp

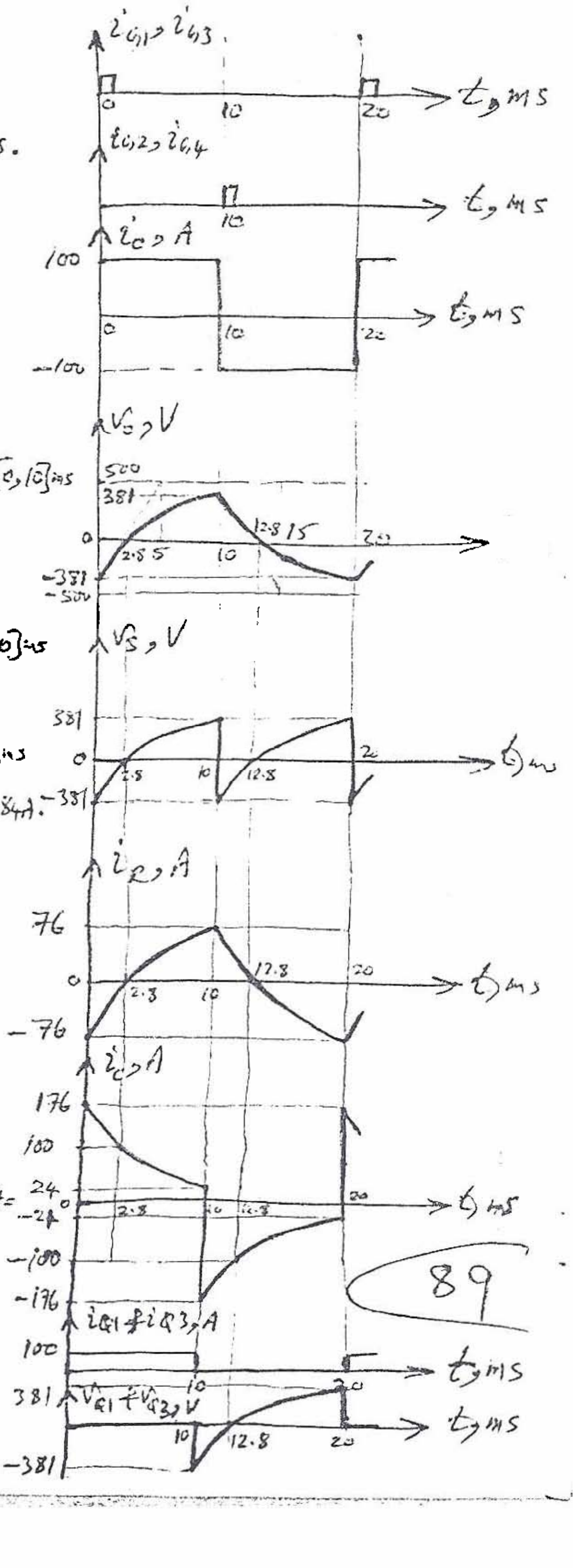
& $i_{o_{fundamental_{rms}}} = \dfrac{2}{20\sqrt{2}}\displaystyle\int_0^{20} i_o \sin\omega t\,dt = \dfrac{4}{20\sqrt{2}}\int_0^{10} 100 \sin\frac{2\pi t}{T}\,dt =$

$= \dfrac{400}{20\sqrt{2}}\left[-\cos(2\pi t/T)\right]_0^{10} \Big/ (2\pi/T) =$

$= \dfrac{400}{20\sqrt{2}} \cdot \dfrac{20}{2\pi} \cdot \left[-\cos\pi + \cos 0\right] = \dfrac{400}{\pi\sqrt{2}}$

$\therefore\ C_I = \dfrac{i_{o\,harmonics_{rms}}}{i_{o_{rms}}} = \sqrt{1 - \left(\dfrac{i_{o\,fundamental_{rms}}}{i_{o_{rms}}}\right)^2} = \sqrt{1 - \left(\dfrac{4}{\pi\sqrt{2}}\right)^2} =$

$= \sqrt{1 - \dfrac{8}{\pi^2}} = .4352 = 43.52\%.$



5-1-a $V \in [0, \sqrt{2}E]$

$\therefore$ Current mode is always discontinuous, and the control range of α is given by:

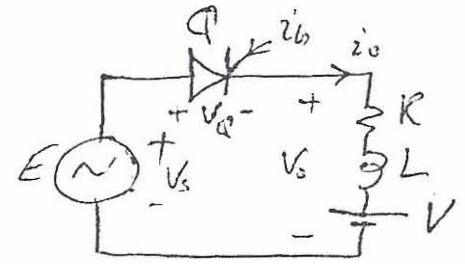
$V_Q(\alpha) = \sqrt{2}E\sin\alpha - V \geq 0$

$\therefore \alpha \in \left[\sin^{-1}\left(\frac{V}{\sqrt{2}E}\right), \pi - \sin^{-1}\left(\frac{V}{\sqrt{2}E}\right)\right]$

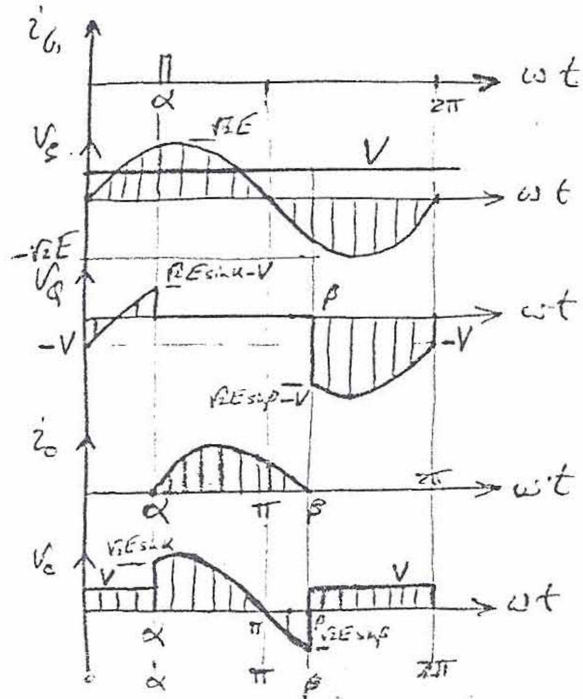
If α is outside the above range, the thyristor will not fire since V_Q is negative.

The output current is of pulsating nature and is given by:

$$i_o(\omega t) = \frac{\sqrt{2}E}{Z}\sin(\omega t - \phi) - \frac{V}{R} + Ae^{-(\omega t-\alpha)/\tan\phi}, \quad t \in [\alpha, \beta]$$



HWCR for RLV load



with $Z = \sqrt{R^2 + \omega^2 L^2}$ & $\tan\phi = \omega L / R$

$\because i_o(\alpha) = 0 = \frac{\sqrt{2}E}{Z}\sin(\alpha - \phi) - \frac{V}{R} + A \quad \therefore A = \frac{V}{R} - \frac{\sqrt{2}E}{Z}\sin(\alpha - \phi)$

\# $\therefore i_o(\omega t) = \frac{\sqrt{2}E}{Z}\left\{\sin(\omega t - \phi) - \sin(\alpha - \phi)\cdot e^{-(\omega t - \alpha)/\tan\phi}\right\} + \frac{V}{R}\left\{e^{-(\omega t - \alpha)/\tan\phi} - 1\right\}$ (1).

$\because i_o(\beta) = 0 \quad \therefore \beta$ could be obtained from the following expression:

\# $\frac{\sqrt{2}E}{Z}\left\{\sin(\beta - \phi) - \sin(\alpha - \phi)e^{-(\beta - \alpha)/\tan\phi}\right\} = \frac{V}{R}\cdot\left\{1 - e^{-(\beta - \alpha)/\tan\phi}\right\}$. (2)

\# & $v_o(\omega t) = \begin{cases} \sqrt{2}E\sin\omega t & \omega t \in [\alpha, \beta] \\ V & \text{otherwise} \end{cases}$ during one cycle $[0, 2\pi]$ (3)

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$$\therefore\; V_{o_{av}} = \frac{1}{2\pi}\int_0^{2\pi} v_o(\omega t)\,d\omega t = \frac{1}{2\pi}\left[V\left(2\pi-(\beta-\alpha)\right)+\int_\alpha^\beta \sqrt{2}E\sin\omega t\,d\omega t\right] =$$

$$= \frac{1}{2\pi}\left[V(2\pi+\alpha-\beta)-\frac{\sqrt{2}E}{1}\cos\omega t\Big|_\alpha^\beta\right] = \frac{1}{2\pi}\left[V(2\pi+\alpha-\beta)+\sqrt{2}E(\cos\alpha-\cos\beta)\right]$$

$$\#\;\therefore\; V_{o_{av}} = V\cdot\left(1-\frac{\beta-\alpha}{2\pi}\right)+\frac{E}{\pi\sqrt{2}}\left(\cos\alpha-\cos\beta\right). \qquad (4)$$

$$\#\;\&\; i_{o_{av}} = \frac{V_{o_{av}}-V}{R} = \left[\frac{E}{\pi\sqrt{2}}\left(\cos\alpha-\cos\beta\right)-V\left(\frac{\beta-\alpha}{2\pi}\right)\right]\Big/R \qquad (5)$$

As for $i_{o_{rms}}$: $\therefore 2\pi i_{o_{rms}}^2 = \int_0^{2\pi} i_o^2(\omega)\,d\omega t$

$$\therefore\; 2\pi\, i_{o_{rms}}^2 = \int_\alpha^\beta\left[\frac{2E^2}{Z^2}\left\{\sin^2(\omega t-\phi)-2\sin(\alpha-\phi)\sin(\omega t-\phi)\cdot e^{-(\omega t-\alpha)/\tan\phi}+\right.\right.$$

$$\left.+\sin^2(\alpha-\phi)\cdot e^{-2(\omega t-\alpha)/\tan\phi}\right\}+\frac{V^2}{R^2}\left\{e^{-2(\omega t-\alpha)/\tan\phi}-2e^{-(\omega t-\alpha)/\tan\phi}+1\right\}+$$

$$\left.+\frac{\sqrt{2}EV}{ZR}\left\{\sin(\omega t-\phi)e^{-(\omega t-\alpha)/\tan\phi}-\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{-2(\omega t-\alpha)/\tan\phi}+\sin(\alpha-\phi)e^{-(\omega t-\alpha)/\tan\phi}\right\}\right]d\omega t$$

$$= \frac{2E^2}{Z^2}\left\{\frac{\omega t}{2}-\frac{\sin 2(\omega t-\phi)}{4}+2\sin(\alpha-\phi)\cdot e^{-(\omega t-\alpha)/\tan\phi}\cdot\frac{\cos(\omega t-\phi)+\cot\phi\cdot\sin(\omega t-\phi)}{1+\cot^2\phi}\right.$$

$$\left.+\sin^2(\alpha-\phi)\cdot\frac{e^{-2(\omega t-\alpha)/\tan\phi}}{-2\cot\phi}\right\}_\alpha^\beta+\frac{V^2}{R^2}\cdot\left\{\frac{e^{-2(\omega t-\alpha)/\tan\phi}}{-2\cot\phi}-2\cdot\frac{e^{-(\omega t-\alpha)/\tan\phi}}{-\cot\phi}+\omega t\right\}_\alpha^\beta$$

$$+\frac{\sqrt{2}EV}{ZR}\left\{-e^{-(\omega t-\alpha)/\tan\phi}\cdot\frac{\cos(\omega t-\phi)+\cot\phi\sin(\omega t-\phi)}{1+\cot^2\phi}+\cos(\omega t-\phi)-\frac{\sin(\alpha-\phi)\cdot e^{-2(\omega t-\alpha)/\tan\phi}}{-2\cot\phi}+\frac{\sin(\alpha-\phi)\cdot e^{-(\omega t-\alpha)/\tan\phi}}{-\cot\phi}\right\}_\alpha^\beta =$$

$$= \frac{2E^2}{Z^2}\left\{\frac{\beta-\alpha}{2}-\frac{\sin 2(\beta-\phi)-\sin 2(\alpha-\phi)}{4}+2\sin^2\phi\sin(\alpha-\phi)\left[e^{-(\beta-\alpha)/\tan\phi}\left(\cos(\beta-\phi)+\cot\phi\sin(\beta-\phi)\right)-\cos(\alpha-\phi)-\cot\phi\sin(\alpha-\phi)\right]\right.$$

$$\left.+\sin^2(\alpha-\phi)\tan\phi\cdot\frac{1-e^{-2(\beta-\alpha)/\tan\phi}}{2}\right\}+\frac{V^2}{R^2}\cdot\left\{\tan\phi\cdot\left(\frac{1-e^{-2(\beta-\alpha)/\tan\phi}}{2}\right)+2\tan\phi\left(e^{-(\beta-\alpha)/\tan\phi}-1\right)\right.$$

$$\left.+\beta-\alpha\right\}+\frac{\sqrt{2}EV}{ZR}\cdot\left\{\sin^2\phi\left[\cos(\alpha-\phi)+\cot\phi\sin(\alpha-\phi)-e^{-(\beta-\alpha)/\tan\phi}\cdot\left(\cos(\beta-\phi)+\cot\phi\sin(\beta-\phi)\right)\right]+\right.$$

$$\left.+\cos(\beta-\phi)-\cos(\alpha-\phi)+\tan\phi\cdot\left(\frac{e^{-2(\beta-\alpha)/\tan\phi}-1}{2}\right)\cdot\sin(\alpha-\phi)+\tan\phi\sin(\alpha-\phi)\left(1-e^{-\frac{\beta-\alpha}{\tan\phi}}\right)\right\}=$$

$$= \frac{2E^2}{Z^2}\left\{\frac{\beta-\alpha}{2}+\frac{\sin 2(\alpha-\phi)-\sin 2(\beta-\phi)}{4}+2\sin\phi\sin(\alpha-\phi)\left[e^{-(\beta-\alpha)/\tan\phi}\sin\beta-\sin\alpha\right]+\right.$$

$$\left.+.5\sin^2(\alpha-\phi)\tan\phi\cdot\left(1-e^{-2(\beta-\alpha)/\tan\phi}\right)\right\}+\frac{V^2}{R^2}\cdot\left\{\beta-\alpha+.5\tan\phi\left(4e^{-(\beta-\alpha)/\tan\phi}-3-e^{-2(\beta-\alpha)/\tan\phi}\right)\right\}+$$

$$+\frac{\sqrt{2}EV}{2R}\cdot\left\{\sin\phi\left(\sin\alpha - e^{-(\beta-\alpha)/\tan\phi}\sin\beta\right) + \cos(\beta-\phi) - \cos(\alpha-\phi) + \right.$$

$$\left. + .5\tan\phi\,\sin(\alpha-\phi)\cdot\left(e^{-2(\beta-\alpha)/\tan\phi} + 1 - 2e^{-(\beta-\alpha)/\tan\phi}\right)\right\} \qquad (6)$$

Hence:

$$\# \quad i_{o_{rms}} = \sqrt{\left(\text{RHS of } (6)\right)/2\pi} \qquad (7)$$

As for $V_{o_{rms}}$, it is much simpler:

$$\therefore V_{o_{rms}}^2 * 2\pi = \int_0^{2\pi} v_o^2(\omega t)\,d\omega t = V^2(2\pi+\alpha-\beta) + \int_\alpha^\beta (\sqrt{2}E\sin\omega t)^2\,d\omega t =$$

$$= V^2(2\pi+\alpha-\beta) + 2E^2\left(\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right)_\alpha^\beta =$$

$$= V^2(2\pi+\alpha-\beta) + 2E^2\left(\frac{\beta-\alpha}{2} - \frac{\sin 2\beta - \sin 2\alpha}{4}\right)$$

$$\# \quad \therefore V_{o_{rms}} = \sqrt{V^2\left(1-\frac{\beta-\alpha}{2\pi}\right) + E^2\left(\frac{\beta-\alpha}{2\pi} - \frac{\sin 2\beta - \sin 2\alpha}{4\pi}\right)} \qquad (8)$$

$$\# \ \& \ K_i = \frac{i_{o_{ac}}}{i_{o_{av}}} = \sqrt{\frac{i_{o_{rms}}^2 - i_{o_{av}}^2}{i_{o_{av}}^2}} = \sqrt{\left(\frac{i_{o_{rms}}}{i_{o_{av}}}\right)^2 - 1} \qquad (9)$$

$$\# \ \& \ K_v = \frac{V_{o_{ac}}}{V_{o_{av}}} = \sqrt{\left(\frac{V_{o_{rms}}}{V_{o_{av}}}\right)^2 - 1} \qquad (10)$$

$$\# \ \& \ P_{o_{av}} = R * i_{o_{rms}}^2 + V * i_{o_{av}} \qquad (11)$$

$\boxed{5\text{-}1\text{-}b}$ This case is easily obtained by letting $V=0$ in all the expressions of $\boxed{5\text{-}1\text{-}a}$.

$\therefore$ Control range of α is between $[0, \pi]$

$$\# \ \& \ i_o(\omega t) = \frac{\sqrt{2}E}{Z}\left\{\sin(\omega t-\phi) - \sin(\alpha-\phi)\cdot e^{-(\omega t-\alpha)/\tan\phi}\right\} \qquad (1)$$

$$\# \ \& \ \text{Condition for } \beta \text{ is:} \quad \sin(\beta-\phi) = \sin(\alpha-\phi)\cdot e^{-(\beta-\alpha)/\tan\phi} \qquad (2)$$

$$\# \ \& \ V_o(\omega t) = \sqrt{2}E\sin\omega t \quad , \ \omega t \in [\alpha, \beta] \quad (\text{zero otherwise}) \qquad (3)$$

\# & $V_{o_{av}} = \frac{E}{\pi\sqrt{2}} \cdot (\cos\alpha - \cos\beta)$ (4)

\# & $i_{o_{av}} = \frac{E}{\pi\sqrt{2} R} \cdot (\cos\alpha - \cos\beta)$ (5)

\# & $i_{o_{rms}} = \frac{E}{Z} \sqrt{\frac{\beta-\alpha}{2\pi} + \frac{\sin 2(\alpha-\phi) - \sin 2(\beta-\phi)}{4\pi} + \frac{2}{\pi} \cdot \sin\phi \cdot \sin(\alpha-\phi) \cdot \left[e^{-(\beta-\alpha)/\tan\phi} \sin\beta - \sin\alpha \right] + \frac{1}{2\pi} \cdot \sin^2(\alpha-\phi) \cdot \tan\phi \left(1 - e^{-2\frac{\beta-\alpha}{\tan\phi}}\right)}$ (6)

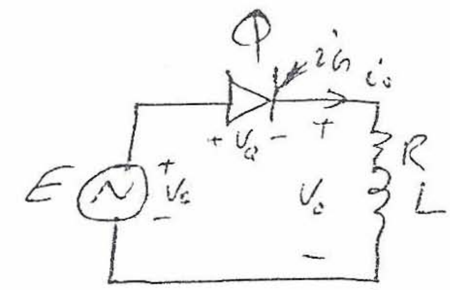
\# & $V_{o_{rms}} = E \cdot \sqrt{\frac{\beta-\alpha}{2\pi} - \frac{\sin 2\beta - \sin 2\alpha}{4\pi}}$ (7)

\# & $K_i = \sqrt{\left(\frac{i_{o_{rms}}}{i_{o_{av}}}\right)^2 - 1}$ (8)

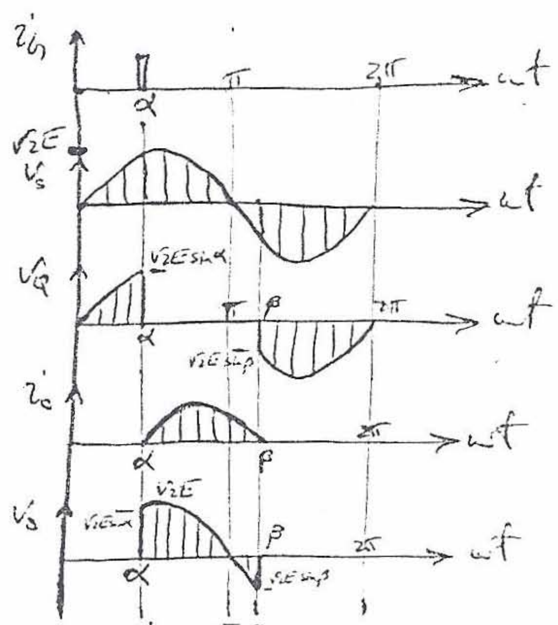
\# & $K_v = \sqrt{\left(\frac{V_{o_{rms}}}{V_{o_{av}}}\right)^2 - 1}$ (9)

\# & $P_{o_{av}} = R * i_{o_{rms}}^2$ (10)

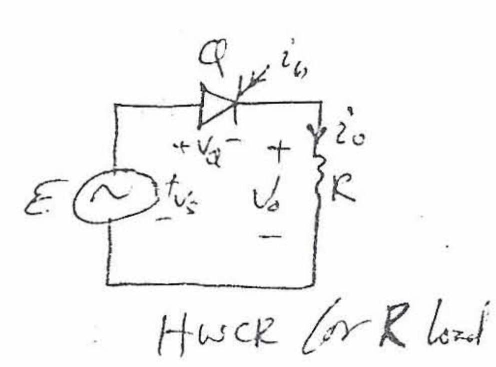
The circuit & waveforms are as shown here.



HWCR for RL-load



$\boxed{5\text{-}1\text{-}C}$ This case is further obtained from the expressions of $\boxed{5\text{-}1\text{-}b}$ by letting $L=0$; whereby, all load voltages are proportional to load currents, as follows:



\# $\beta = \pi$ always (1)

\# : $V_o(\omega t) = R\, i_o(\omega t) = \sqrt{2} E \sin \omega t$ (2)
$\omega t \in [\alpha, \pi]$

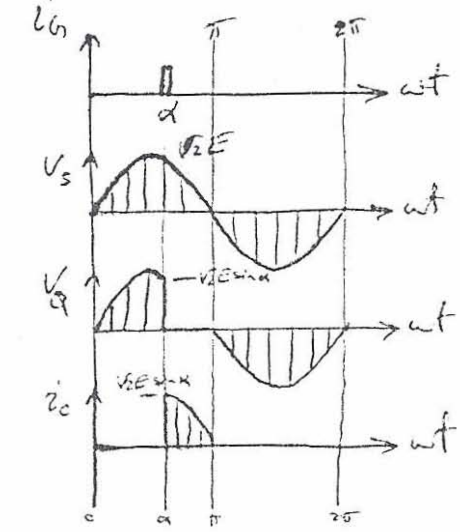
\# & $V_{o_{av}} = R * i_{o_{av}} = \frac{E}{\pi\sqrt{2}}(1+\cos\alpha)$ (3)

\# & $V_{o_{rms}} = R * i_{o_{rms}} = E \cdot \sqrt{\frac{\pi-\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}}$ (4)

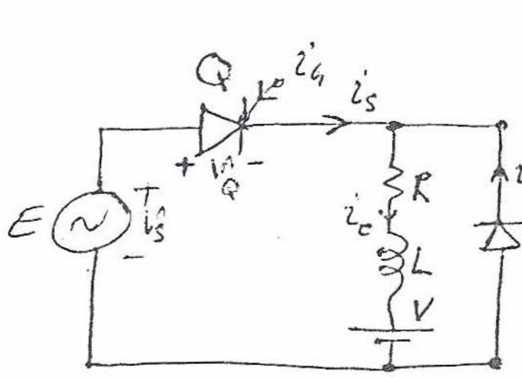
\# & $K_i = K_v = \sqrt{\frac{(i \text{ OR } V)^2_{rms}}{(i \text{ OR } V)^2_{av}} - 1}$ (5)

\# & $P_{o_{av}} = R * i^2_{o_{rms}} = \frac{E^2}{R}\left(\frac{\pi-\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right)$ (6)

The circuit & waveforms are as shown.



$\boxed{5-2-a}$ The HWCR when forced commutation is used will be modified by placing a free-wheeling diode accross the load circuit as shown.



Again, for circuit to perform, then the range of V is:

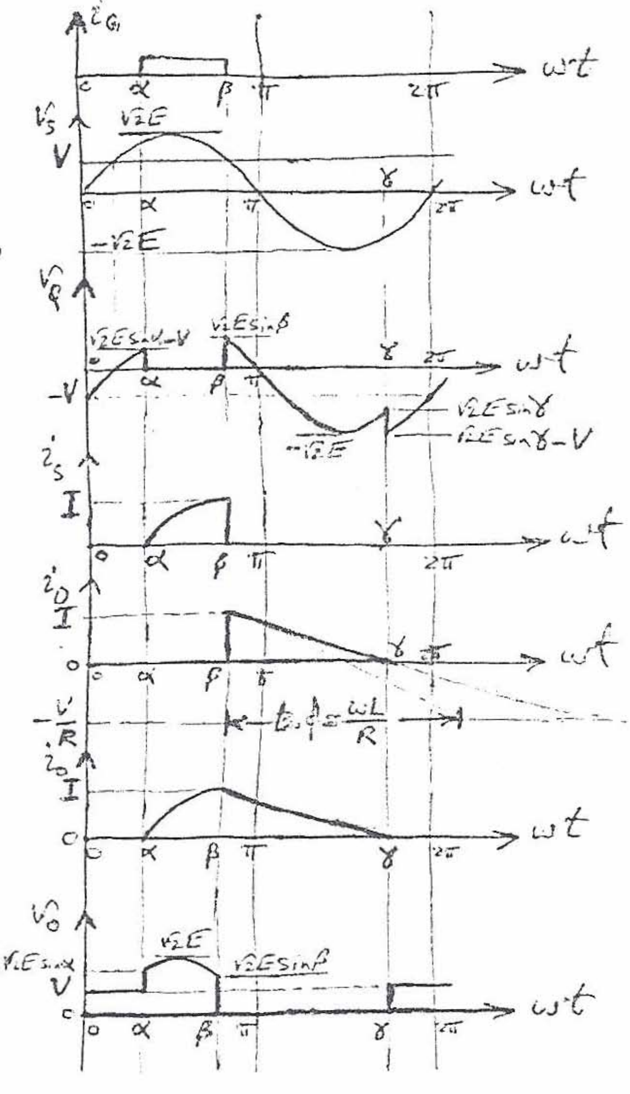
$V \in [0, \sqrt{2}E]$,

and the control range of α is:

$$\alpha \in \left[\sin^{-1}\left(\frac{V}{\sqrt{2}E}\right), \pi - \sin^{-1}\left(\frac{V}{\sqrt{2}E}\right)\right],$$

where $\sin^{-1}$ is defined over first quadrant. Current Mode could be continuous, discontinuous or critical depending upon circuit parameters. Each mode will be considered separately.

$\boxed{5-2-a-I}$ First, the discontinuous mode, where waveforms are as shown.



Here:

α is the firing angle,
β is the commutation angle,
γ is the extinction angle.

The control range of β is:

$\beta \in (\alpha, \pi]$, because after $\omega t = \pi$, v_s reverses and quits Q through D.

The value of I is given using Eq.1 of $\boxed{5-1-a}$ by:

$$I = i_o(\beta) = \frac{\sqrt{2}E}{Z}\left\{\sin(\beta-\phi) - \sin(\alpha-\phi)e^{(\alpha-\beta)/\tan\phi}\right\} - \frac{V}{R}\left\{1 - e^{(\alpha-\beta)/\tan\phi}\right\}, \quad (1)$$

where the above equation describes the current during $\omega t \in [\alpha, \beta]$.

For $\omega t \in [\beta, \gamma]$, $i_o(\omega t)$ is given by:

$$i_o(\omega t) = -\frac{V}{R} + \left(I + \frac{V}{R}\right) e^{-(\omega t - \beta)/\tan\phi} \qquad (2)$$

Hence, $i_o(\gamma) = 0$ gives γ as:

$$\# \quad \gamma = \beta + \tan\phi \cdot \ln\left(1 + \frac{IR}{V}\right) \qquad (3)$$

$i_o(\omega t)$ then keeps at zero till $2\pi + \alpha$ comes for a new cycle.

$$\# \therefore i_o(\omega t) = \begin{cases} \text{as in Eq. 1 of } \boxed{5-1-a} & \text{for } \omega t \in [\alpha, \beta] \\ \text{as in Eq. 2 above} & \text{for } \omega t \in [\beta, \gamma] \\ \text{Zero} & \text{for } \omega t \in [\gamma, 2\pi + \alpha] \end{cases} \qquad (4)$$

The output voltage, on the other hand, is given by:

$$\# \quad v_o(\omega t) = \begin{cases} \sqrt{2} E \sin \omega t & \text{for } \omega t \in (\alpha, \beta) \\ \text{Zero} & \text{for } \omega t \in (\beta, \gamma) \\ V & \text{for } \omega t \in (\gamma, 2\pi + \alpha) \end{cases} \qquad (5)$$

Hence,

$$V_{o_{av}} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} \sqrt{2} E \sin \omega t \, d\omega t + 0 + \int_{\gamma}^{2\pi+\alpha} V \, d\omega t\right] = \frac{1}{2\pi}\left[-\sqrt{2} E \cos \omega t \Big|_{\alpha}^{\beta} + V \cdot \omega t \Big|_{\gamma}^{2\pi+\alpha}\right]$$

$$\# \therefore V_{o_{av}} = \frac{1}{2\pi}\left[\sqrt{2} E (\cos\alpha - \cos\beta) + V \cdot (2\pi + \alpha - \gamma)\right] \qquad (6)$$

$$\# \therefore I_{o_{av}} = \frac{V_{o_{av}} - V}{R} = \frac{\sqrt{2} E \cdot (\cos\alpha - \cos\beta) - V(\gamma - \alpha)}{2\pi R} \qquad (7)$$

$$\& \quad V_{o_{rms}}^2 = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} 2E^2 \sin^2 \omega t \, d\omega t + 0 + \int_{\gamma}^{2\pi+\alpha} V^2 \, d\omega t\right] = \frac{1}{2\pi}\left[E^2 \int_{\alpha}^{\beta} (1 - \cos 2\omega t) \, d\omega t + V^2 \omega t \Big|_{\gamma}^{2\pi+\alpha}\right]$$

$$= \frac{1}{2\pi} \cdot \left[\frac{E^2}{2} \cdot (2\beta - 2\alpha - \sin 2\beta + \sin 2\alpha) + V^2 (2\pi + \alpha - \gamma)\right]$$

$$\# \therefore V_{o_{rms}} = \sqrt{V^2 \cdot \left(1 - \frac{\gamma - \alpha}{2\pi}\right) + E^2 \cdot \left(\frac{\beta - \alpha}{2\pi} - \frac{\sin 2\beta - \sin 2\alpha}{4\pi}\right)} \qquad (8)$$

As for $i_{o_{rms}}$ it is three components and is given as:

$$i_{o_{rms}}^2 = \frac{1}{2\pi}\left[2\pi i_{o_{rms}}^2 \text{ of (b) in } \boxed{5-1-a} + \int_\beta^\gamma \left[-\frac{V}{R} + \left(I + \frac{V}{R}\right)e^{-(\omega t-\beta)/\tan\phi}\right]^2 d\omega t + 0\right] =$$

$$= i_{o_{rms}}^2 \text{ of } \boxed{5-1-a} + \frac{1}{2\pi}\left[\frac{V^2}{R^2}\omega t + \left(I+\frac{V}{R}\right)^2 \cdot \frac{e^{-2(\omega t-\beta)/\tan\phi}}{-2/\tan\phi} - \frac{2V}{R}\left(I+\frac{V}{R}\right)\frac{e^{-(\omega t-\beta)/\tan\phi}}{-1/\tan\phi}\right]_\beta^\gamma$$

$$\# \therefore i_{o_{rms}} = \sqrt{i_{o_{rms}}^2 \text{ of } \boxed{5-1-a} + \frac{V^2}{R^2}\cdot\frac{\gamma-\beta}{2\pi} + \left(\frac{V+IR}{R}\right)^2 \cdot \tan\phi \cdot \left(\frac{1-e^{-2(\gamma-\beta)/\tan\phi}}{4\pi}\right) - \frac{V(V+IR)}{\pi R^2} \cdot \tan\phi \cdot \left(1-e^{-(\gamma-\beta)/\tan\phi}\right)} \quad (9)$$

$$\# \therefore K_i = \sqrt{\left(\frac{i_{o_{rms}}}{i_{o_{av}}}\right)^2 - 1} \quad (10)$$

$$\# \text{ \& } K_v = \sqrt{\left(\frac{v_{o_{rms}}}{V_{o_{av}}}\right)^2 - 1} \quad (11)$$

$$\# \text{ \& } P_{o_{av}} = i_{o_{rms}}^2 \cdot R + i_{o_{av}} \cdot V \quad (12)$$

$\boxed{5-2-a-II}$ Second, the critical mode, whereby γ in the above waveforms extends till it is just equal to $2\pi+\alpha$. Hence, this mode occures, using Eq. 3, when:

$$2\pi+\alpha = \beta + \tan\phi \cdot \ln\left(\frac{V+IR}{V}\right), \text{ i.e. when:}$$

$I = \frac{V}{R}\cdot\left[e^{(2\pi+\alpha-\beta)/\tan\phi} - 1\right]$. Using this in (1), then the critical mode occures when β satisfies the equation:

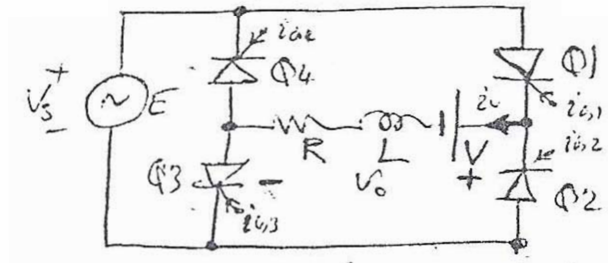
$$\frac{\sqrt{2}E}{Z}\left\{\sin(\beta-\phi) - \sin(\alpha-\phi)\cdot e^{(\alpha-\beta)\cot\phi}\right\} - \frac{V}{R}\cdot\left\{e^{2\pi\cot\phi} - 1\right\}\cdot e^{(\alpha-\beta)\cot\phi} = 0$$

OR:

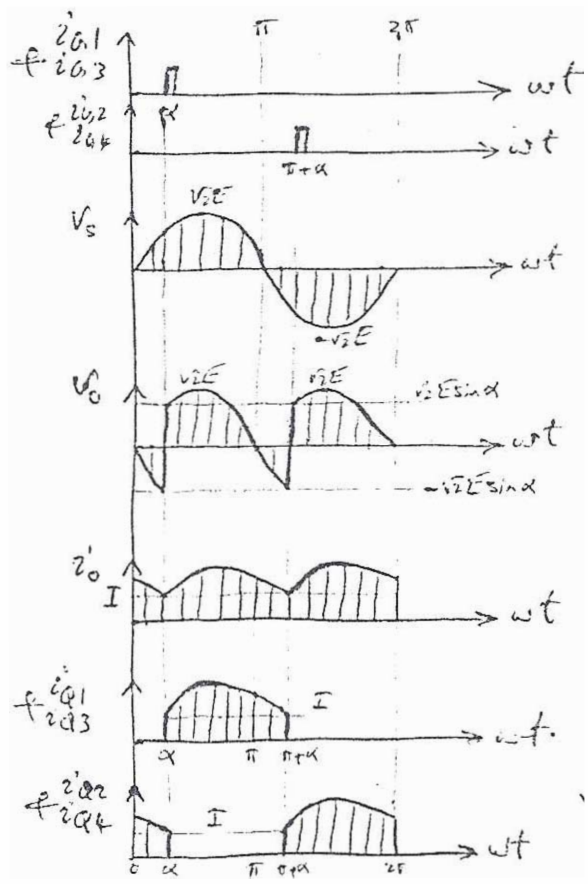
$$\sqrt{2}\cdot E\cdot\cos\phi\cdot\left\{e^{(\beta-\alpha)\cot\phi}\cdot\sin(\beta-\phi) - \sin(\alpha-\phi)\right\} - V\left(e^{2\pi\cot\phi} - 1\right) = 0 \quad (13)$$

As for the expressions, they are just as for discontinuous mode with $\beta = \beta_{crit}$.

$\boxed{5\text{-}5\text{-}b\text{-}I}$ Since Current mode is continuous, then V_o is a switched version of V_s as shown in the figure. The gating signals are symmetrical so that the output voltage and current is at twice the frequency.



FWCR for KLV load.



$$V_{o_{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}E \sin\omega t \, d\omega t =$$

$$= \frac{1}{\pi}\cdot\sqrt{2}E\cdot\frac{-\cos\omega t}{1}\Big|_{\alpha}^{\pi+\alpha} =$$

$$= \frac{\sqrt{2}E}{\pi}\left(\cos\alpha - \cos(\pi+\alpha)\right) =$$

$$= \frac{\sqrt{2}E}{\pi}\left(\cos\alpha + \cos\alpha\right) = \frac{2\sqrt{2}E}{\pi}\cos\alpha$$

$$\therefore i_{o_{av}} = \frac{V_{o_{av}} - V}{R} = \frac{2\sqrt{2}E\cos\alpha - \pi V}{\pi R}$$

$$\& \; V_{o_{rms}}^2 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} 2E^2 \sin^2\omega t \, d\omega t =$$

$$= \frac{2E^2}{2\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)_{\alpha}^{\pi+\alpha} =$$

$$= \frac{E^2}{\pi}\left(\pi + \alpha - \alpha - \frac{\sin(2\pi+2\alpha) - \sin 2\alpha}{2}\right)$$

$$= \frac{E^2}{\pi}\left(\pi - \frac{\sin 2\alpha - \sin 2\alpha}{2}\right) = E^2 \qquad \therefore V_{o_{rms}} = E$$

(This is expected, since squaring V_o is equal to squaring V_s)

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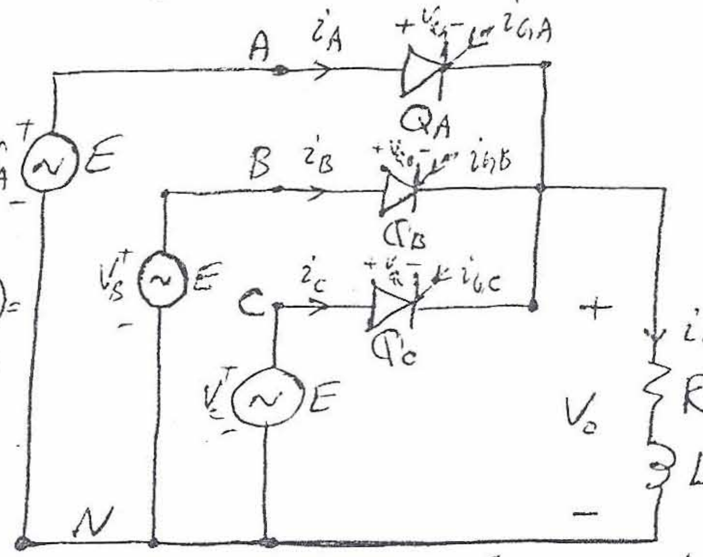
$$\therefore K_v = \frac{V_{o_{ac}}}{V_{o_{av}}} = \sqrt{\frac{V_{o_{rms}}^2 - V_{o_{av}}^2}{V_{o_{av}}^2}} = \sqrt{\left(\frac{V_{o_{rms}}}{V_{o_{av}}}\right)^2 - 1} = \sqrt{\frac{\pi^2}{8\cos^2\alpha} - 1}$$

5-10 a) Circuit is as shown (assuming symmetrical 3ϕ supply).

b) $V_{AN} = \sqrt{2}E \sin \omega t$

$V_{BN} = \sqrt{2}E \sin(\omega t - 120^\circ)$

$V_{CN} = \sqrt{2}E \sin(\omega t + 120^\circ)$



$\therefore V_{AB} = V_A - V_B =$

$= \sqrt{2}E(\sin \omega t - \sin(\omega t - 120^\circ)) =$

$= \sqrt{2}E[\sin \omega t (1 - \cos 120^\circ) + \cos \omega t \sin 120^\circ] =$

$= \sqrt{2}E[\sin \omega t (1 + \frac{1}{2}) + \cos \omega t \cdot \frac{\sqrt{3}}{2}] =$

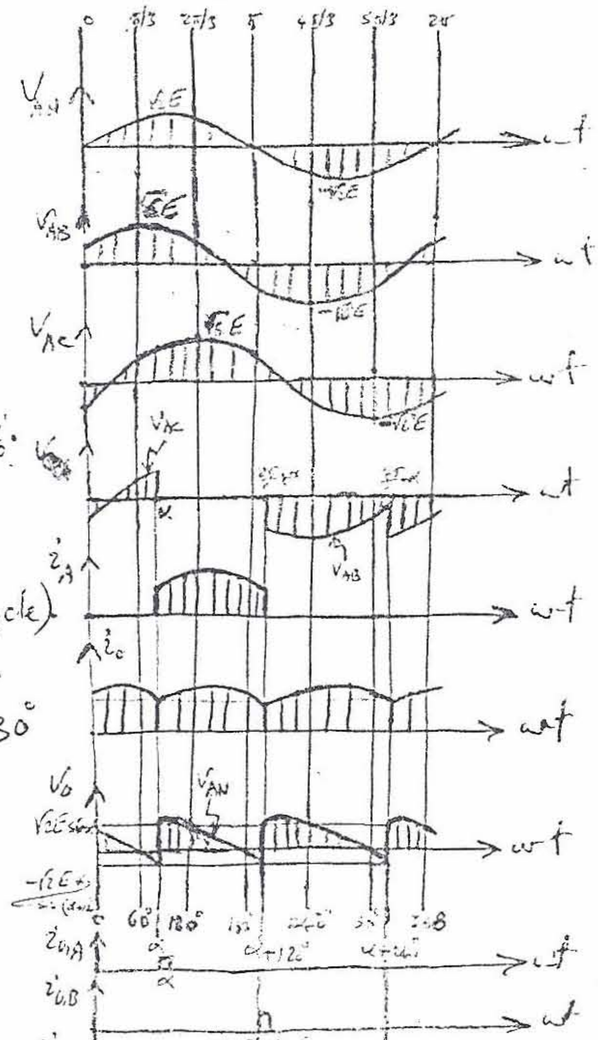
$= \sqrt{6}E[\frac{\sqrt{3}}{2} \sin \omega t + \frac{1}{2} \cos \omega t] =$

3ϕ HWCR for RL Load.

$= \sqrt{6}E[\sin \omega t \cos 30^\circ + \cos \omega t \sin 30^\circ]$

$= \sqrt{6}E \sin(\omega t + 30^\circ)$ &

Similarly:



$\therefore V_{BC} = \sqrt{6}E \sin(\omega t - 90^\circ)$

& $V_{CA} = \sqrt{6}E \sin(\omega t + 150^\circ)$

$\therefore V_{AC} = -\sqrt{6}E \sin(\omega t + 150^\circ) = \sqrt{6}E \sin(\omega t - 30^\circ)$

$\because$ Supply is AC $\therefore$ Three current modes for i_o.

$\therefore$ Current, i_o, is assumed continuous, first.

& $\because$ Gating is symmetrical.

$\therefore$ Every thyristor conducts for $\frac{2\pi}{3}$ ($\frac{1}{3}$ cycle).

& $\therefore$ Q_A conducts after Q_C & before Q_B.

$\therefore$ Before firing Q_A, $V_{QA} = V_{AC}$ $\therefore \alpha > 30^\circ$

$\therefore$ $\alpha \in (30^\circ, 30^\circ + 120^\circ) = (30^\circ, 150^\circ)$,

and $V_{QA} = 0$ for $\frac{1}{3}$ cycle after α.

Then when Q_B is fired at $\frac{2\pi}{3} + \alpha$,

$\therefore$ $V_{QA} = V_{AB}$ for $\frac{1}{3}$ cycle after $\frac{2\pi}{3} + \alpha$.

Then Q_C is fired at $\frac{4\pi}{3} + \alpha$ and

$V_{QA} = V_{AC}$ for $\frac{1}{3}$ cycle after $\frac{4\pi}{3} + \alpha$.

This completes one cycle, giving the waveforms shown above. The other phase waveforms are obtained by 120° shift for B & 240° shift for C.

During the conduction of Q_A $\therefore V_o = V_{AN}$ for $\omega t \in (\alpha, \alpha + \frac{2\pi}{3})$

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And for $\omega t \in \left[\alpha + \frac{2\pi}{3}, \alpha + \frac{4\pi}{3}\right]$ $\therefore V_o = V_{BN}$ since Q_B is on.
Finally for $\omega t \in \left[\alpha + \frac{4\pi}{3}, \alpha + 2\pi\right]$ Q_C is on & $V_o = V_{CN}$.
As for the output current, i_o:
$\therefore$ $i_o = i_A + i_B + i_C$ as shown above.
(Note that i_o & V_o are of period $= \frac{2\pi}{3}$).

c): Range of α is between $30°$ & $150°$ relative to each phase, [i.e within $\pm 60°$ off phase peak], for controlled operation.

d) $$V_{o_{av}} = \frac{1}{2\pi/3} \int_{\alpha}^{\alpha + 2\pi/3} \sqrt{2} E \sin\omega t \, d\omega t = \frac{3\sqrt{2}E}{2\pi} \left[-\cos\omega t \right]_{\alpha}^{\alpha + 2\pi/3} =$$

$$= \frac{3\sqrt{2}E}{2\pi} \left[\cos\alpha - \cos(120° + \alpha) \right] = \frac{3\sqrt{2}E}{2\pi} \left[\cos(\alpha + 60° - 60°) - \cos(\alpha + 60° + 60°) \right] =$$

$$= \frac{3\sqrt{2}E}{2\pi} \cdot 2 \sin(\alpha + 60°) \sin 60° = \frac{3\sqrt{2}E}{2\pi} \cdot 2 \cdot \frac{\sqrt{3}}{2} \cdot \sin(\alpha + 60°) =$$

$$\therefore V_{o_{av}} = \frac{3}{\pi} \cdot \sqrt{\frac{3}{2}} \cdot E \sin(\alpha + 60°)$$, $\alpha \in$ Contineous range.

Note: $V_{o_{av}} \in \left[\frac{-3}{2\pi} \sqrt{\frac{3}{2}} E, \frac{3}{\pi} \sqrt{\frac{3}{2}} E \right]$ for $\alpha \in [30°, 150°]$, i.e the output controlled voltage varies between $-.585$ & 1.170 of E.

Hence, $i_{o_{av}} \in [-.585, 1.170]$ of E/R, and since i_o can't be negative, then α must not reach $150°$ for contineous mode operation. In fact $i_{o_{av}}$ must be strictly > 0 for contineous current to flow. i.e $\sin(\alpha_{max} + 60°) > 0$ $\therefore \alpha_{max} < 120°$.
The smaller L/R the lower α_{max} is, and at $L = 0$, $\therefore \alpha_{max} = 60°$.
To get the exact value of α_{max} for any RL, then

Consider the Condition for β in the 1ϕ HWCR of $\boxed{5\text{-}1\text{-}b}$,

which is just the same as for this case.

$$\therefore \quad \sin(\beta-\phi) = \sin(\alpha-\phi)\, e^{(\alpha-\beta)/\tan\phi}$$

Now put $\beta = \alpha_{max} + 2\pi/3$ (just critical mode)

$$\therefore \quad \sin\left(\alpha_{max} + \tfrac{2\pi}{3} - \phi\right) = \sin(\alpha_{max} - \phi)\cdot e^{-\frac{2\pi}{3\tan\phi}}$$

$$\therefore \quad \sin(\alpha_{max}-\phi)\cos\tfrac{2\pi}{3} + \cos(\alpha_{max}-\phi)\sin\tfrac{2\pi}{3} - \sin(\alpha_{max}-\phi)\, e^{-2\pi/3\tan\phi} = 0$$

$$\therefore \quad -\sin(\alpha_{max}-\phi)\left[\tfrac{1}{2} + e^{-2\pi/3\tan\phi}\right] + \cos(\alpha_{max}-\phi)\cdot\tfrac{\sqrt{3}}{2} = 0$$

$$\therefore \quad \tan(\alpha_{max}-\phi) = \frac{\sqrt{3}}{1+2e^{-2\pi/3\tan\phi}}$$

$$\therefore \quad \alpha_{max} = \phi + \tan^{-1}\left[\frac{\sqrt{3}}{1+2e^{-2\pi/3\tan\phi}}\right]$$

Indeed, for resistive load, $L=0$ & $\phi = 0$ $\therefore \alpha_{max_R} = 60°$

& for inductive load, $R = 0$ & $\phi = \pi/2$ $\therefore \alpha_{max_L} = 120°$

Hence:

For continuous mode $\therefore$ $\alpha \in [30°, \alpha_{max}]$

& for discontinuous mode $\therefore$ $\alpha \in [\alpha_{max}, 150°]$.

Thus giving an overall control range of $\alpha \in [30°, 150°]$.

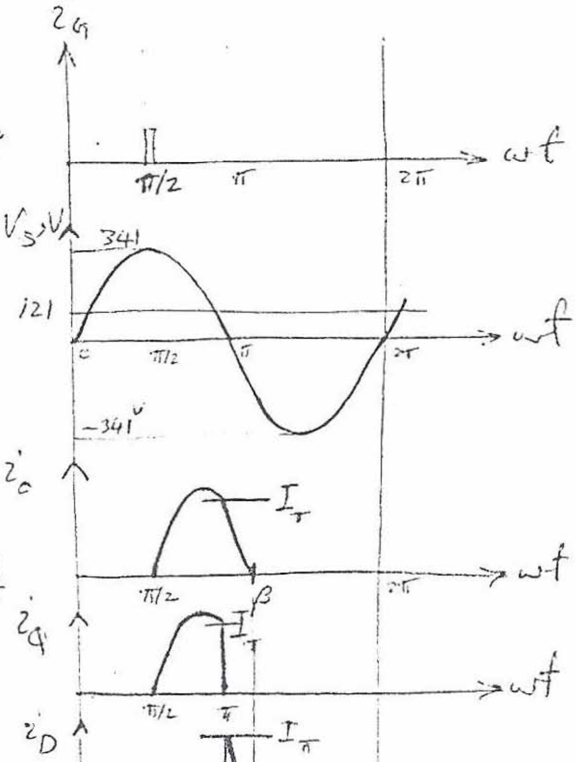
$\therefore$ For continuous mode, $V_{o_{av}} = \frac{3}{\pi}\cdot\sqrt{\frac{3}{2}}\cdot E\sin(\alpha+60°)$

e) & For continuous mode, $i_{o_{av}} = \frac{3}{\pi}\cdot\sqrt{\frac{3}{2}}\cdot\frac{E}{R}\cdot\sin(\alpha+60°)$.

(10)

$\boxed{5\text{-}12}$ 1. The circuit is HW HCR with RLV Load, with control range of: $\alpha \in \left[\sin^{-1}\frac{121}{341}, \pi - \sin^{-1}\frac{121}{341}\right]$, OR in degrees:

$$\alpha \in [20.8^\circ, 159.2^\circ]$$



2. Waveforms are as shown for $\alpha = 90^\circ$. Current mode is expected to be discontinuous, since, the charging angle is $\pi/2$ & the angle available for discharge is $3\pi/2$ with -121^V force. Let's find β to be absolutely sure.

For $\omega t \in [\alpha, \pi]$:

$$\therefore i_o(\omega t) = \frac{341 \sin(\omega t - \tan^{-1}(120\pi \times 52m/2.5))}{\sqrt{2.5^2 + (120\pi \times 52m)^2}} - \frac{121}{2.5} + A e^{-2.5\omega t/\omega \times 52m} =$$

$$= 17.255 \sin(\omega t - 82.73^\circ) - 48.4 + A e^{-.12753(\omega t - \pi/2)}, \text{ Amp}$$

$\because i_o(\frac{\pi}{2}) = 0 \quad \therefore A = 46.22$ Amp

$\therefore I_\pi = i_o(\pi) = 6.546$ Amp

$\therefore$ For $\omega t \in [\pi, \beta]$:

$\therefore i_o = (48.4 + 6.546)e^{-.12753(\omega t - \pi)} - 48.4$, A

$\therefore 0 = i_o(\beta) \Rightarrow \beta = 4.136$ rad $= 237^\circ$

$\because \beta = 237^\circ < 2\pi + \alpha = 360 + 90 = 450^\circ$

$\therefore$ Mode is actually discontinuous.

3. Q conducts in each cycle for 90° i.e. $\frac{\pi}{2\omega} = \frac{\pi}{240\pi} = \frac{1}{240}s = 4.17ms$ whereas D conducts for $237^\circ - 180^\circ = 57^\circ$ i.e. $\frac{57}{180} \cdot \frac{\pi}{120\pi} = 2.64$ ms. Q is commutated by the line voltage through D.

4. $V_{o_{av}} = \frac{121 \times \pi/2 + 341 \times \pi/2 \times \frac{2}{\pi} + 0 + 121 \times (2\pi - 4.136)}{2\pi} = 125.87$ Volts

$\therefore i_{o_{av}} = \frac{V_{o_{av}} - 121}{2.5} = \frac{125.87 - 121}{2.5} = 1.949$ Amp

f. $V_{o_{rms}}^2 = \frac{1}{2\pi}\left[121^2 * \frac{\pi}{2} + 341^2 * \frac{\pi}{2} * \frac{1}{2} + 0 + 121^2(2\pi - 4.136)\right] = 23199\,V^2$

$\therefore\ V_{o_{rms}} = 152.3$ Volts

$P_V = i_{o_{av}} \cdot V = 1.949 * 121 = 235.83$ Watts

5. For Q:

$t_q = \left[180 + \sin^{-1}\left(\frac{121}{341}\right)\right] / 180 * \pi / 120\pi = 9.296$ msec

$V_{FB} = 220$ Volts

$V_{RB} = 462$ Volts

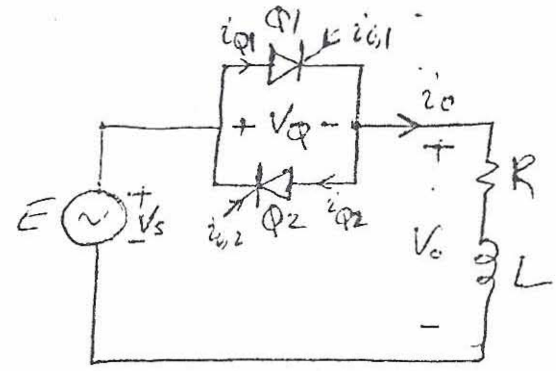
For D:

P_j Can be found assuming $V_F = 1$ Volt & straight line current segment (as $\frac{L}{R} = \frac{52m}{2.5} = 20.8\,ms \gg \frac{\beta - \pi}{\omega} = \frac{57}{180} \cdot \frac{\pi}{120\pi} = 2.64\,ms$) hold.

$P_j = I_\pi * 1 * (\beta - \pi)/2/2\pi = 6.546 * (4.136 - 3.142)/4\pi = 0.518\,W$

$T_j = 25 + .5 * .518 = 25.26\ ^\circ C$

$\boxed{6\text{-}1}$ The two back-to-back thyristors could be well replaced by a triac. The current mode for large (near π) α values is discontinuous and is given by:



(see $\boxed{6.1\text{-}b}$)

$$i_o(t) = \frac{\sqrt{2}E}{Z}\left\{\sin(\omega t-\phi) - \sin(\alpha-\phi)\,e^{\frac{\alpha-\omega t}{\tan\phi}}\right\} \quad (1)$$

for $\omega t \in [\alpha, \beta]$

& $i_o(t) = 0$ for $\omega t \in [\beta, \pi+\alpha]$

The other half of $i_o(t)$ is just symmetrical.

The condition for β is given by:
(see $\boxed{6.1\text{-}b}$)

$$\sin(\beta-\phi) - \sin(\alpha-\phi)\,e^{\frac{\alpha-\beta}{\tan\phi}} = 0 \quad (2)$$

As α decreases, the discontinuity gap will narrow till at $\beta-\pi=\alpha$ it's zero

$\therefore$ Controllability of α is such that $\beta = \pi+\alpha$. Putting this into (2):

$\therefore \sin(\pi+\alpha-\phi) - \sin(\alpha-\phi)\,e^{\frac{\alpha-\pi-\alpha}{\tan\phi}} = 0$

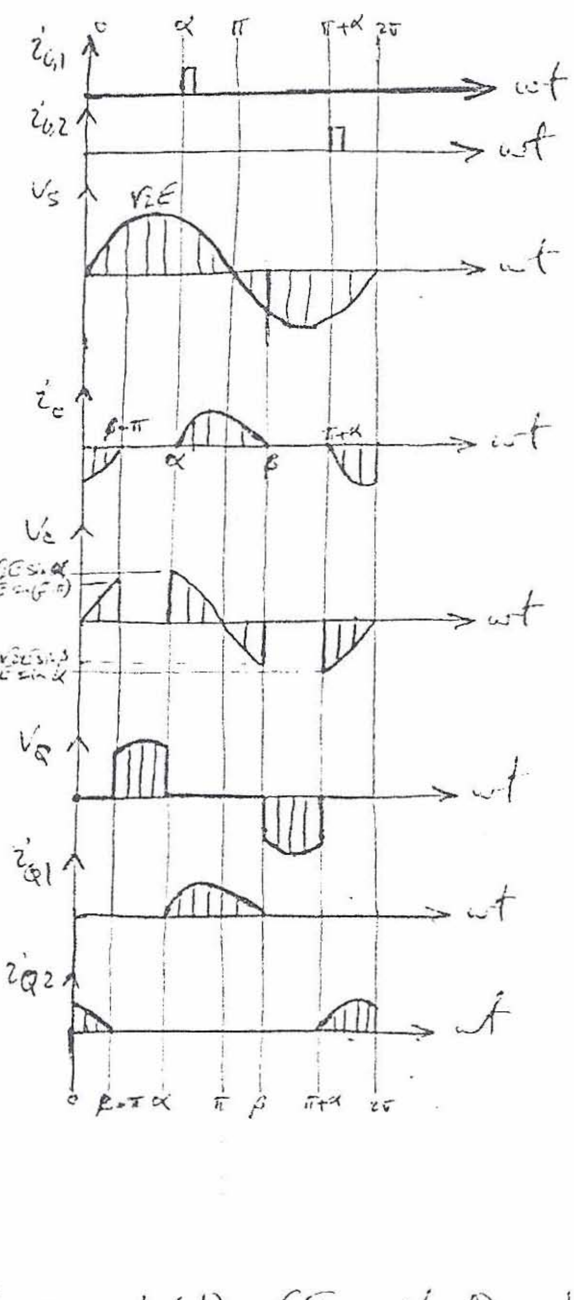
$\therefore -\sin(\alpha-\phi)\cdot\left[1+e^{-\pi/\tan\phi}\right] = 0$

$\because 1+e^{-\pi/\tan\phi} > 0$

$\therefore \sin(\alpha-\phi) = 0 \quad \therefore \alpha = \phi$

$\therefore$ Control range of α is

$$\alpha \in [\phi, \pi]$$



(Note: This is obvious since at critical case $i_o(\omega t) = \frac{\sqrt{2}E}{Z}\sin(\omega t-\phi)$ and has no exponential term in $i_o(t)$, hence α must be ϕ).

Hence; within α range, this problem is typical to $\boxed{6.1-b}$ with the difference that it has one more pulse of both current and voltage at the negative half, i.e averages are zero.

$$\therefore V_{o_{rms}}^2 = 2\left[V_{o_{rms}} \text{ of } \boxed{6.1-b}\right]^2$$

$$\therefore V_{o_{rms}} = \sqrt{2}\cdot E \cdot \sqrt{\frac{\beta-\alpha}{2\pi} - \frac{\sin 2\beta - \sin 2\alpha}{4\pi}} \qquad (3)$$

$$\& \quad i_{o_{rms}} = \sqrt{2}\left[i_{o_{rms}} \text{ of } \boxed{6.1-b}\right] \Rightarrow$$

$$\therefore i_{o_{rms}} = \frac{\sqrt{2}E}{Z}\sqrt{\frac{\beta-\alpha}{2\pi} + \frac{\sin 2(\alpha-\phi) - \sin 2(\beta-\phi)}{4\pi} + \frac{2}{\pi}\sin\phi\sin(\alpha-\phi)\left[e^{\frac{\alpha-\beta}{\tan\phi}}\sin\beta - \sin\alpha\right] + \frac{\sin^2(\alpha-\phi)\tan\phi\,(1-e^{\frac{2(\alpha-\beta)}{\tan\phi}})}{2\pi}}$$

$$(4)$$

$$\& \quad P_{o_{av}} = i_{o_{rms}}^2 * R \qquad (5)$$

$$\& \quad pf = \frac{P}{i_{o_{rms}} \cdot V_{o_{rms}}} = \frac{i_{o_{rms}}^2 * R}{i_{o_{rms}} * V_{o_{rms}}} = \frac{i_{o_{rms}} * R}{V_{o_{rms}}} \qquad (6)$$

When $\alpha = \phi$ the above parameters comes simply to:

$$i_o(t) = \frac{\sqrt{2}E}{Z}\sin(\omega t - \phi) \qquad (7)$$

$$\& \quad \beta - \phi = \pi \Rightarrow \beta = \pi + \phi \qquad (8)$$

$$\& \quad V_{o_{rms}} = E \qquad (9)$$

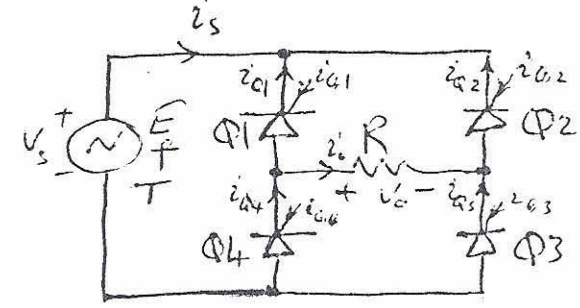
$$\& \quad i_{o_{rms}} = \frac{E}{Z} \qquad (10)$$

$$\& \quad pf = \frac{R}{Z} = fpf \qquad (11)$$

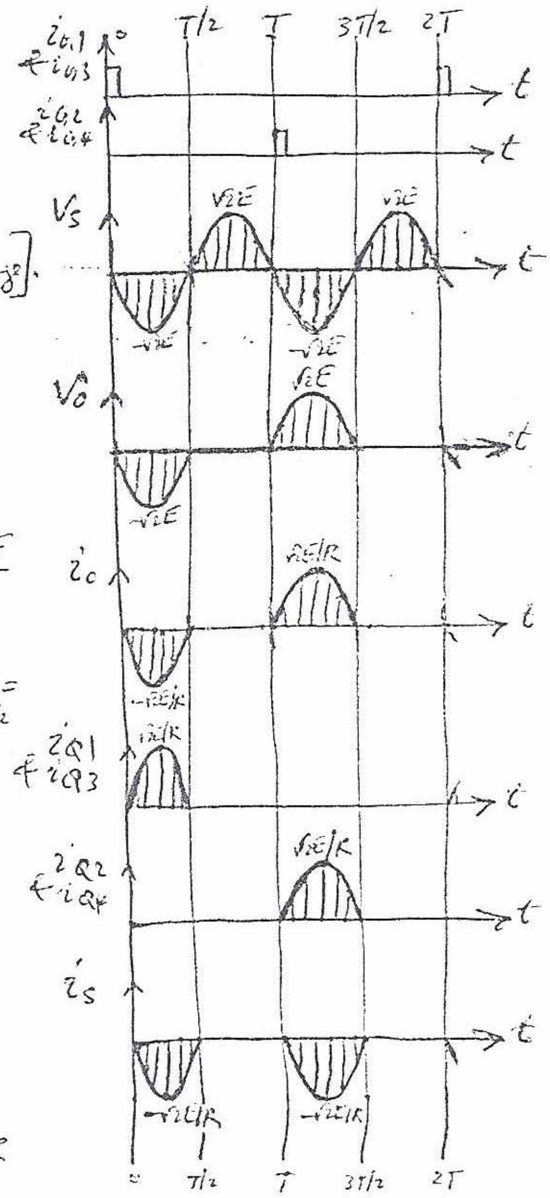
$$\& \quad \text{Finally } \rho_i = \frac{i_{harmonics}}{i_{rms}} = 0 \qquad (12)$$

(Note: this is simply because we lost control i.e. Q1 & Q2 appear as short-circuit)

$\boxed{7\text{-}1}$ a) The shown circuit is of a cyclo-converter that supplies ac to the load at half the frequency of the supply, since the period of the output is twice that of the input.



b) The waveforms are as shown.



c) $v_{o\,fundamental} = a\sin\left(\frac{2\pi t}{2T}\right) + b\cos\left(\frac{2\pi t}{2T}\right)$ [no average].

$$\therefore\ a = \frac{2}{2T}\int^{2T} v_o(t)\sin\left(\frac{\pi t}{T}\right)dt =$$
$$= \frac{1}{T} * 2\int_0^{T/2} -\sqrt{2}E\sin\left(\frac{2\pi t}{T}\right)\sin\left(\frac{\pi t}{T}\right)dt$$
$$= -\frac{2\sqrt{2}E}{T}\int_0^{T/2} 2\sin^2\left(\frac{\pi t}{T}\right)\cos\left(\frac{\pi t}{T}\right)dt$$
$$= \frac{-4\sqrt{2}E}{T}\cdot\frac{\sin^3(\pi t/T)}{3\pi/T}\Big|_0^{T/2} = \frac{-4\sqrt{2}E}{3\pi}\left[1-0\right] = \frac{-4\sqrt{2}E}{3\pi}$$

$$\&\ b = \frac{2}{2T}\int^{2T} v_o(t)\cos\left(\frac{\pi t}{T}\right)dt = \frac{1}{T} * 2\int_0^{T/2} -\sqrt{2}E\sin\frac{2\pi t}{T}\cos\frac{\pi t}{T}dt =$$
$$= \frac{-2\sqrt{2}E}{T}\int_0^{T/2} 2\sin\frac{\pi t}{T}\cos^2\frac{\pi t}{T}\,dt = \frac{-4\sqrt{2}E}{T}\left[-\frac{\cos^3(\pi t/T)}{3\pi/T}\right]_0^{T/2} =$$
$$= \frac{-4\sqrt{2}E}{3\pi}\left[1-0\right] = \frac{-4\sqrt{2}E}{3\pi}$$

$$\therefore\ v_{o\,fundamental\,rms} = \sqrt{(a^2+b^2)/2} = \frac{4\sqrt{2}E}{3\pi}.$$

$$\therefore\ i_{o\,fundamental\,rms} = 4\sqrt{2}E/3\pi R.$$

d) $P_{load} = \frac{V_{o\,rms}^2}{R} = \frac{1}{R}\cdot\frac{1}{2T}\left[E^2\cdot\frac{T}{2} + E^2\frac{T}{2}\right] = \frac{E^2}{2R} = i_{o\,rms}^2\cdot R$

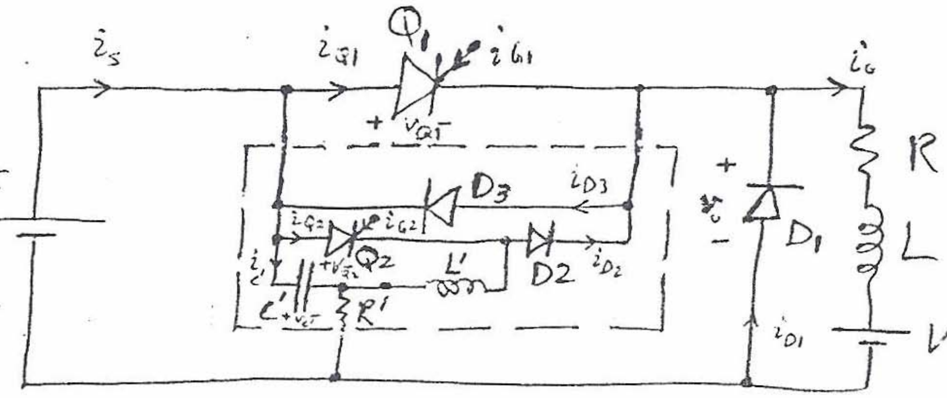
e) $pf = \frac{P_{ave}}{i_{s\,rms}\cdot V_{s\,rms}} = \frac{E^2/2R}{i_{s\,rms}\cdot E} = \frac{E/2R}{\sqrt{E^2/2R^2}} = \frac{1/2}{\sqrt{1/2}} = \sqrt{1/2} = \frac{1}{\sqrt{2}} = .7071$

f) $\because$ Resistive load, $\therefore \angle\frac{v_{o\,fundamental}}{i_{o\,fundamental}} = 0 \quad \therefore fpf = \cos 0 = 1$ 106

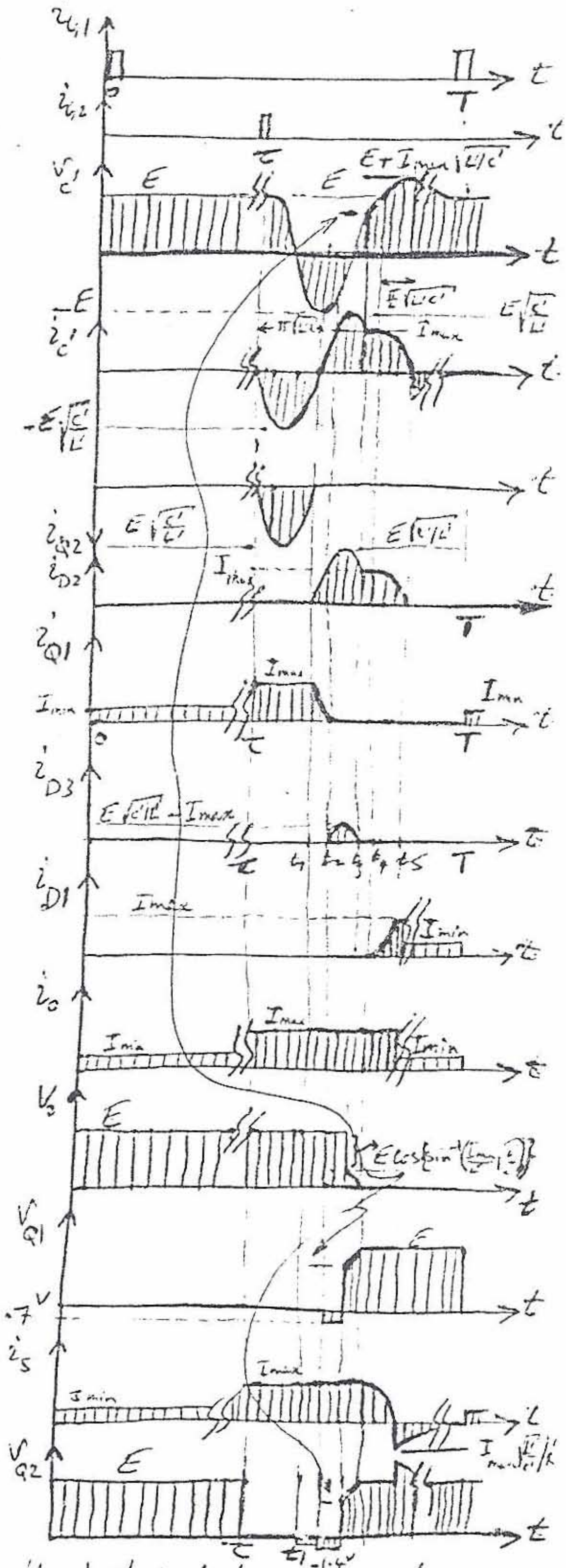
g) $\mu_{i_o} = \mu_{v_o}$ (resistive load) $= \frac{V_{harmonics}}{V_{rms}} = \sqrt{1-\left(\frac{V_{o\,fundamental\,rms}}{V_{rms}}\right)^2} = \sqrt{1-\left(\frac{4\sqrt{2}E/3\pi}{E/\sqrt{2}}\right)^2} = \sqrt{1-\frac{64}{9\pi^2}} = .5287 \simeq 53\%$

h) Disadvantage of circuit is that the supply current has a dc offset.

8-2



The dashed box is the circuit for force commutation. It is so designed that $L'C'$ will ring at frequency much higher than that of chopping, and the ringing period is very small compared to $R'C'$ time constant. At the start of a chopping cycle, C' is charged to E then Q_1 is gated, hence current i_o flows from supply through Q1 to the load. No other supply current is going to flow at the stage, since D3 is reversed biased by drop on Q1 & since Q2 is not yet gated and since C' is fully charged to E. When we want to commutate Q_1 at $t=\tau$ a gate pulse is provided to Q2. Hence, the loop consisting of $L'C'$ & Q_2 will ring for half a cycle then Q2 will be off. During this half cycle, current through Q1 is almost constant, D3 is still off, D2 is still off, current through R' is almost zero, and D1 is off. As Q2 becomes off, voltage across c' is almost $-E$ and it will force its ringing current through $C'L'$ D2 & Q1. As the ringing current increases, the



net Q1 current decreases, till it is shut off at the instant ringing current $= I_{max}$.

At this instant on, ringing current flows through $C'L'D2$ and the parallel paths of D3 and KLVE combination. After peaking, ringing current becomes again equal to I_{max} whereby, ringing stops, and its held there till C' charges back again to almost E. Now, D1 is on and current through $C'L'D2$ oscilates to a stop. During freewheeling period C' discharges to E ready for next chopping cycle. The waveforms are as shown above, and components are assumed ideal.

The circuit will look like the following, at various instants of time. All rectifiers in the illustrative figures are on and could be assumed short circuit, where:

$$t_1 = \tau + \pi\sqrt{L'C'}$$

$$\text{\&}\quad t_2 = \tau + \sqrt{L'C'}\left\{\pi + \sin^{-1}\left(\frac{I_{max}}{E}\cdot\sqrt{\frac{L'}{C'}}\right)\right\}$$
$$= t_1 + \sqrt{L'C'}\sin^{-1}\left(\frac{I_{max}}{E}\sqrt{\frac{L'}{C'}}\right)$$

$$\text{\&}\quad t_3 = \tau + \sqrt{L'C'}\left\{2\pi - \sin^{-1}\left(\frac{I_{max}}{E}\cdot\sqrt{\frac{L'}{C'}}\right)\right\}$$

$$\text{\&}\quad t_4 = t_3 + \frac{C'E}{I_{max}}\left[1 - \cos\left\{\sin^{-1}\left(\frac{I_{max}}{E}\cdot\sqrt{\frac{L'}{C'}}\right)\right\}\right]$$

$$\text{\&}\quad t_5 = t_4 + \frac{\pi}{2}\cdot\sqrt{L'C'}$$

The total commutation time is just below $\frac{5\pi}{2}\sqrt{L'C'}$ and

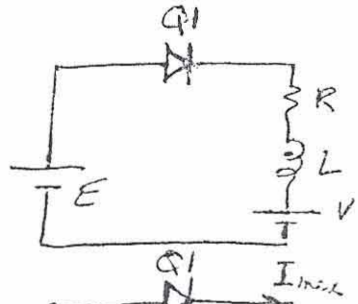
thyristor Q1 is reversed biased during:

$$t_{off} = t_3 - t_2 =$$
$$= \sqrt{L'C'}\cdot\left\{\pi - 2\sin^{-1}\left(\frac{I_{max}}{E}\cdot\sqrt{\frac{L'}{C'}}\right)\right\},$$

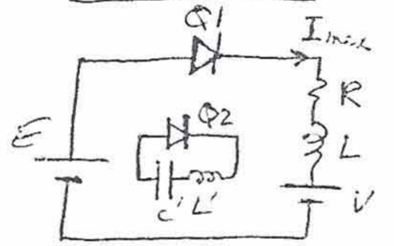
provided that $I_{max} < E\sqrt{\frac{C'}{L'}}$,

otherwise commutation fails.

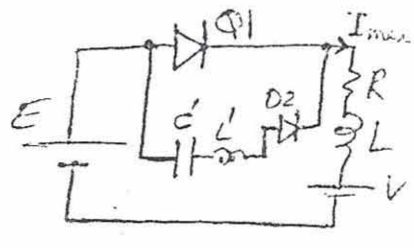
$t \in [0, \tau]$



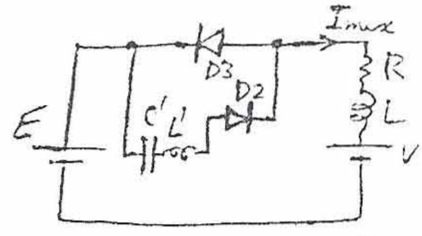
$t \in [\tau, t_1]$



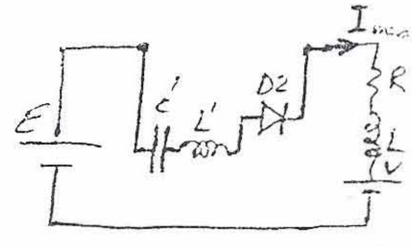
$t \in [t_1, t_2]$



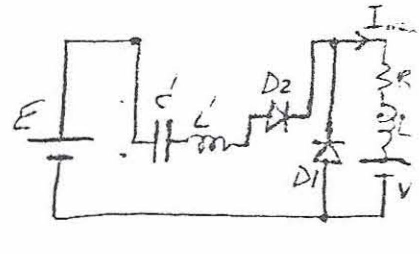
$t \in [t_2, t_3]$



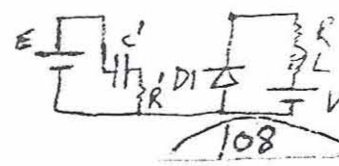
$t \in [t_3, t_4]$



$t \in [t_4, t_5]$



$t \in [t_5, T]$



$E = 600V$, $L' = 42\mu h$, $C' = 6\mu F$, $T = 2500\mu s$,

$I_{max} = 150\ A$

a) $t_{off_{Q1}} = \sqrt{42 \times 6}\left\{\pi - 2\sin^{-1}\left(\frac{150}{600}\cdot\sqrt{\frac{42}{6}}\right)\right\} = 26.93\ \mu sec$

$\therefore\ t_{q_1}$ must be $\leq 26.93\ \mu sec$

b) $t_c \approx \frac{5}{2}\pi\sqrt{L'C'} = \frac{5}{2}\pi\cdot\sqrt{42 \times 6} = 124.68\ \mu sec \ll T$ (ok)

(Note: the exact figure is slightly less $= t_5 - \tau =$

$= t_4 + \frac{\pi}{2}\sqrt{L'C'} - \tau = t_3 + \frac{C'E}{I_{max}}\left[1 - \cos\left\{\sin^{-1}\left(\frac{I_{max}}{E}\sqrt{\frac{L'}{C'}}\right)\right\}\right] + \frac{\pi}{2}\sqrt{L'C'} - \tau$

$= \tau + \sqrt{L'C'}\left\{2\pi - \sin^{-1}\left(\frac{I_{max}}{E}\sqrt{\frac{L'}{C'}}\right)\right\} + \frac{C'E}{I_{max}}\cdot\left[1 - \sqrt{1 - \frac{I_{max}^2 L'}{E^2 C'}}\right] + \frac{\pi}{2}\sqrt{L'C'} - \tau$

$= \frac{5\pi}{2}\sqrt{L'C'} - \left[\sqrt{L'C'}\sin^{-1}\left(\frac{I_{max}}{E}\sqrt{\frac{L'}{C'}}\right) - \frac{C'E - \sqrt{C'^2E^2 - C'L'I_{max}^2}}{I_{max}}\right]$

$= 124.68 - 5.47 = 119.21\ \mu sec$).

d) $t_{off_{Q2}} = t_3 - t_1 = \sqrt{L'C'}\left\{\pi - \sin^{-1}\left(\frac{I_{max}}{E}\cdot\sqrt{\frac{L'}{C'}}\right)\right\} = 38.40\ \mu sec$

c) For sketch, consider start of commutation as reference:

$\therefore\ t_1 = \pi\sqrt{L'C'} = 49.9\ \mu sec$,

& $t_2 = 61.3\ \mu s$, $t_3 = 88.3\ \mu sec$, $t_4 = 94.3\ \mu sec$

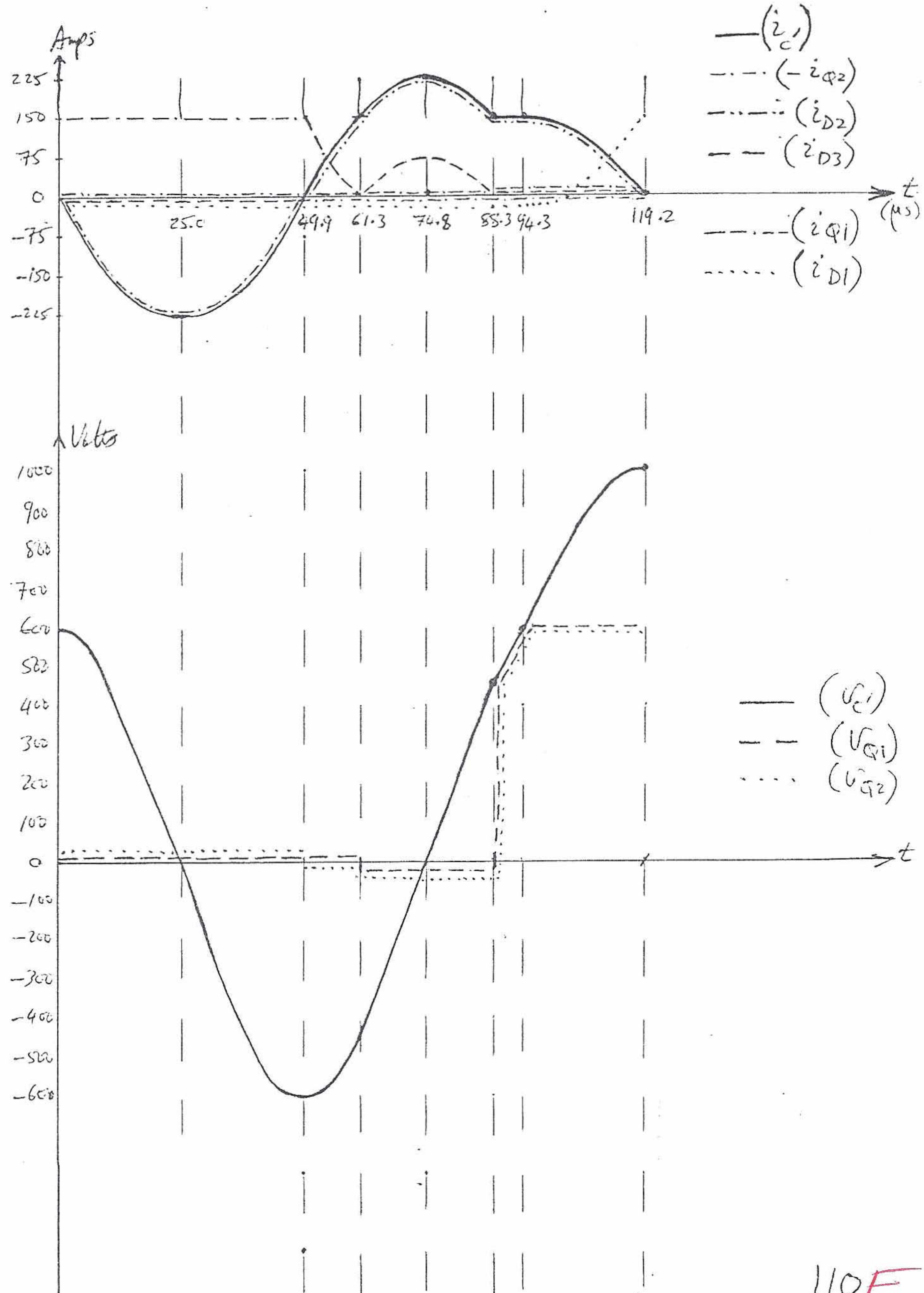
& $t_5 = 119.2\ \mu sec$

& $V_{c_{max}} = E + I_{max}\sqrt{\frac{L'}{C'}} = 996.9$ volts

& $i_{c_{max}} = E\sqrt{\frac{C'}{L'}} = 226.8$ Amps

& $V_{c_{precharge}} = E\cos\left\{\sin^{-1}\left(\frac{I_{max}}{E}\sqrt{\frac{L'}{C'}}\right)\right\} = 450$ volts

& $i_{D3_{max}} = E\sqrt{\frac{C'}{L'}} - I_{max} = 76.8$ Amp.



Amps
225
150
75
0
-75
-150
-225
25.0
49.9
61.3
74.8
88.3
94.3
119.2
t (μs)
(i_{c1})
($-i_{Q2}$)
(i_{D2})
(i_{D3})
(i_{Q1})
(i_{D1})
Volts
1000
900
800
700
600
500
400
300
200
100
0
-100
-200
-300
-400
-500
-600
t
(V_{c1})
(V_{Q1})
(V_{Q2})

110E