بسم الله الرحمن الرحيم

الحلول المختارة لطلاب الهندسة والعمارة

العلوم الاساسية للمعماريين ١

إعداد

سلطان محمد القاسمي

الاستاذ المساعد بكلية العلوم التطبيقية والهندسية بجامعة ام القرى

تمهيد

الحمد لله وحده والصلاة والسلام على من لا نبي بعده سيدنا محمد وعلى آله وصحبه

وبعد، فهذه مجموعة من المسائل المحلولة في مادة العلوم الأساسية للمعماريين ١ لطلبة الهندسة والعمارة اخترتها أثناء تدريسي لهذه المادة لجمع شتات المنهج المقرر وهي مأخوذة من كتاب "Calculus & Analytic Geometry" لمؤلفيه Thomas و Finny الطبعة الخامسة ١٩٨٢م

وقد قمت بتبويبها وفهرستها لتسهيل المراجعة فيها وترقيمها، واكتفيت عند ذكر المسألة بذكر رقمها والصفحة التي وردت فيها في الكتاب المذكور بما يلبي المقرر لمادة العلوم الأساسية للمعماريين ١ لطلاب العمارة الإسلامية في كلية العلوم التطبيقية والهندسية بجامعة أم القرى.

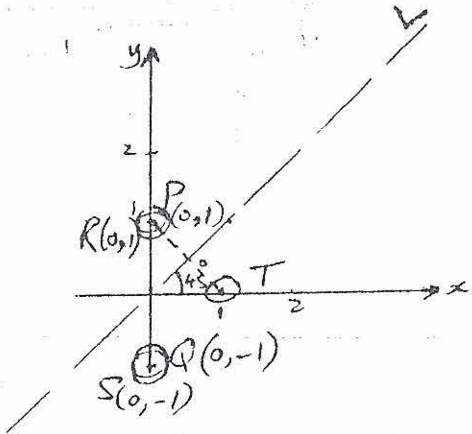
والله أسأل أن يجعل هذا العمل حافزاً للطلاب للسير قدماً في الدراسة والتحصيل لنفع العباد والبلاد، كما وأسأله أن لا يحرمني أجره في الآخرة والأولى.

٢٤/٦/١٤٠٨هـ

الفهرس

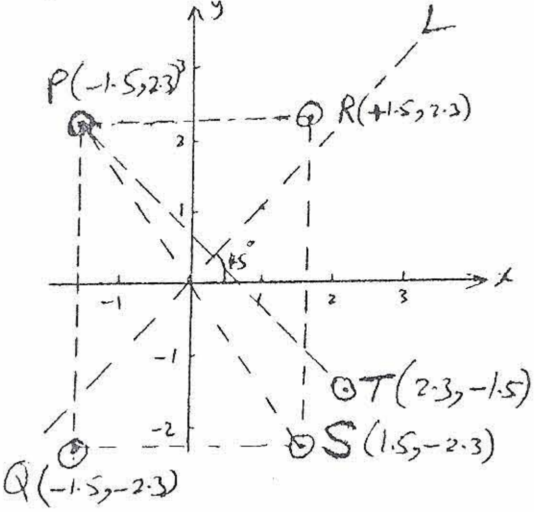
$\boxed{\frac{5}{4}}$

ⓐ Q(0, -1)
ⓑ R (0, 1)
ⓒ S (0, -1)
ⓓ T (1, 0)



$\boxed{\frac{12}{4}}$

ⓐ Q (-1.5, -2.3)
ⓑ R (+1.5, 2.3)
ⓒ S (+1.5, -2.3)
ⓓ T (2.3, -1.5)



$\boxed{\frac{17}{5}}$

$\Delta ABB'$ is similer to $A'BO$

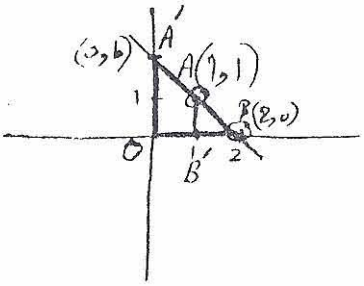
$\therefore \frac{AB'}{B'B} = \frac{A'O}{OB}$

$AB' = 1$
$B'B = 1$
$OB = 2$

$\therefore \frac{1}{1} = \frac{A'O}{2}$

$\therefore A'O = 2$

$\therefore \boxed{b=2}$



تابع

$\frac{3}{7}$

ⓐ $C(-1, 2)$

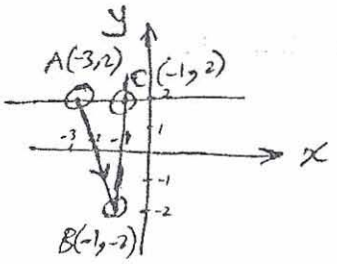
ⓑ $\Delta x = -1-(-3) = -1+3 = 2$

ⓒ $\Delta y = -2-2 = -4$

ⓓ $\overline{AB} = \sqrt{2^2+4^2}$

$= \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$



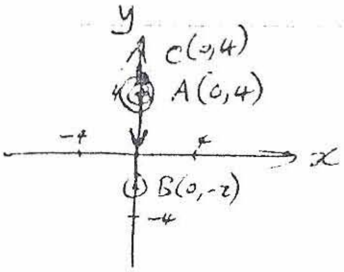
$\frac{6}{7}$

ⓐ $C(0, 4)$

ⓑ $\Delta x = 0-0 = 0$

ⓒ $\Delta y = -2-4 = -6$

ⓓ $\overline{AB} = \sqrt{0^2+6^2} = 6$



$\frac{9}{7}$

$P(x,y)$ is 5 units from $C(-3,4)$

$\therefore (y-4)^2 + [x-(-3)]^2 = 5^2$

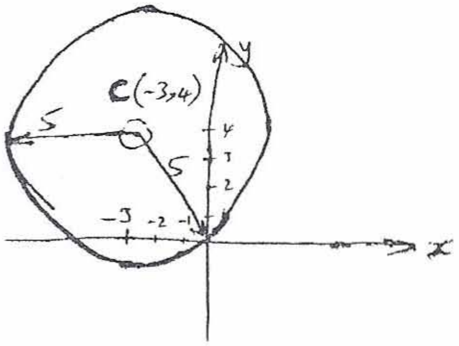
$\therefore y^2-8y+16+(x+3)^2 = 25$

$\therefore y^2-8y+16+x^2+6x+9 = 25$

$\therefore y^2-8y+x^2+6x = 25-25$

$\therefore$ The equation is:

$$\boxed{y^2-8y+x^2+6x=0}.$$



$\frac{12}{7}$

$\Delta x = 3 - x = 5 \Rightarrow \therefore x = 3-5 = -2$

$\Delta y = -3 - y = 6 \Rightarrow \therefore y = -3-6 = -9$

$\therefore A(x,y)$ is $(-2,-9)$

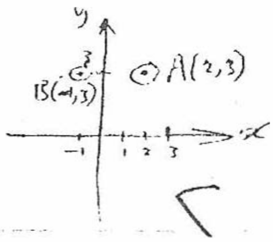
$\frac{14}{7}$

$A(-2,5) \longrightarrow B(0, 5+\Delta y)$ $\therefore -2+\Delta x = 0$ $\therefore \Delta x = 2$

$\Delta y = 3$ A x $= 3 * 2 = 6$ $\therefore$ New Coordinates are $(0, 11)$

$\frac{5}{10}$

$m_{AB} = \frac{3-5}{2+1} = 0$ $\therefore m'_{\perp AB} = \infty$ (// to y-axis)

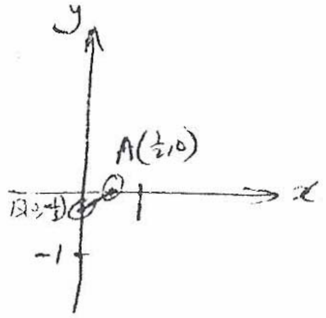


ح. ك

$\boxed{\frac{8}{10}}$ slope of AB $= \dfrac{0-(-\frac{1}{3})}{\frac{1}{2}-0}$

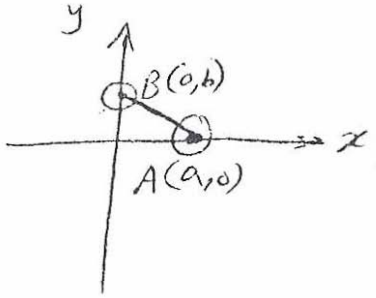
$= \dfrac{1/3}{1/2} = \dfrac{2}{3}$

slope of perpendicular $= -\dfrac{1}{2/3} = -\dfrac{3}{2}$



$\boxed{\frac{12}{10}}$ slope of A B $= \dfrac{b-0}{0-a} = -\dfrac{b}{a}$

slope of perpendicular $= \dfrac{-1}{-b/a} = \dfrac{a}{b}$

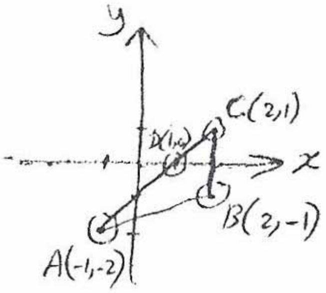


$\boxed{\frac{15}{10}}$ slope of $\overline{AB} = \dfrac{-1-(-2)}{2-(-1)} = \dfrac{-1+2}{2+1} = \dfrac{1}{3}$

slope of BC $= \dfrac{1-(-1)}{2-2} = \dfrac{1+1}{0}$ not defined

slope of CD $= \dfrac{0-1}{1-2} = \dfrac{-1}{-1} = 1$

slope of DA $= \dfrac{-2-0}{-1-1} = \dfrac{-2}{-2} = 1$



Since slope $\overline{AB} \neq$ slope CD & slope BC $\neq$ slope DA

Hence ABCD is not a parallelogram

$\frac{19}{10}$

$+2 = \frac{y-0}{x-0} = \frac{y}{x}$

$\therefore \boxed{y = 2x}$ (1)

$+1 = \frac{y-0}{x-(-1)} = \frac{y}{x+1}$

$\therefore \boxed{y = x+1}$ (2)

from (1) & (2) $\therefore 2x = x+1$

$\therefore 2x - x = 1$

$\therefore x = 1$

Using (2) $\therefore y = 2(1) = 2$

$\therefore P(x,y)$ is $(1, 2)$

$\frac{23}{10}$

slope of $\overline{AB} = \frac{1-1}{-1-(-2)} = \frac{0}{-1+2} = 0$

slope of $BC = \frac{5-1}{1-(-1)} = \frac{4}{1+1} = \frac{4}{2} = 2$

slope of $CD = \frac{7-5}{2-1} = \frac{2}{1} = 2$

$\therefore$ BC & CD have same slope and one common point

$\therefore$ BCD only are on a straight line.

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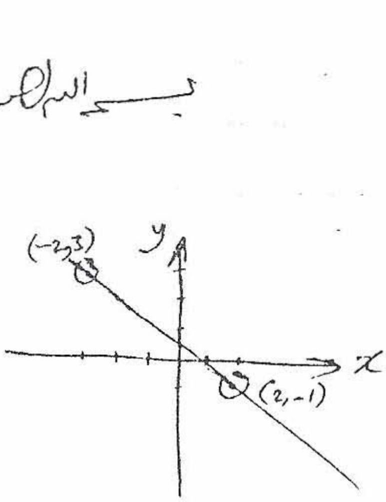
10/14

$\frac{y-3}{x+2} = \frac{3+1}{-2-2} = \frac{4}{-4} = -1$

$\therefore\ y-3 = -x-2$

$\therefore\ \boxed{y+x-1=0}$ is the equation.



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$3x+4y=12$

$\therefore\ 4y = 12-3x$

$\therefore\ y = 3 - \frac{3}{4}x$

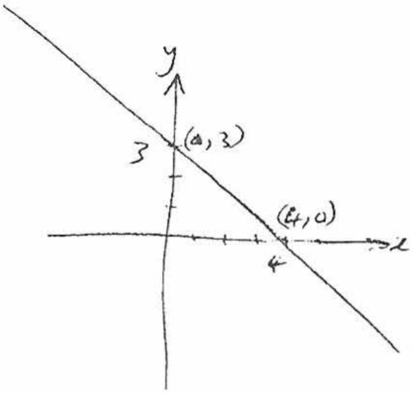
$y = -\frac{3}{4}x + 3$ cf $y = mx + c$

$\therefore$ slope, $m = -\frac{3}{4}$

y-intercept, $c = 3$

put $y=0$ $\therefore\ x = \frac{12}{3} = 4$

$\therefore$ x-intercept $= 4$



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ⓐ slope of $L' = -2$

$\therefore$ slope of $L = \frac{-1}{-2} = \frac{1}{2}$

$\therefore$ Equation of L is $\frac{y-2}{x+2} = \frac{1}{2} \Rightarrow 2y-4 = x+2$

or $\boxed{2y-x-6=0}$

ⓑ Solve:

$2y-x-6=0$ (1)

& $2x+y-4=0$ (2)

$2\times(1)+(2)$: $4y+y-12-4=0$

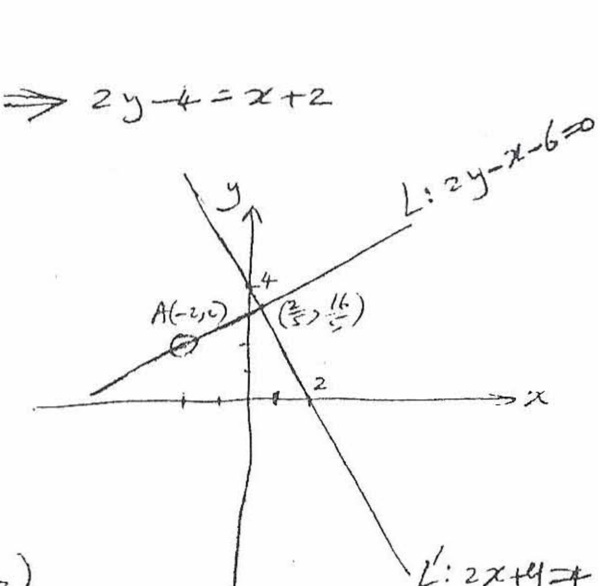
$\therefore\ 5y-16=0 \Rightarrow y = \frac{16}{5}$

from (1) $\therefore\ x = 2y-6 = \frac{32-30}{5} = \frac{2}{5}$

$\therefore$ point of intersection is $\left(\frac{2}{5}, \frac{16}{5}\right)$

ⓒ distance $= \sqrt{\left(2-\frac{16}{5}\right)^2 + \left(-2-\frac{2}{5}\right)^2} = \frac{1}{5}\sqrt{(10-16)^2 + (-10-2)^2}$

$= \frac{1}{5}\sqrt{36+144} = \frac{1}{5}\sqrt{180} = \frac{6}{5}\sqrt{5}$

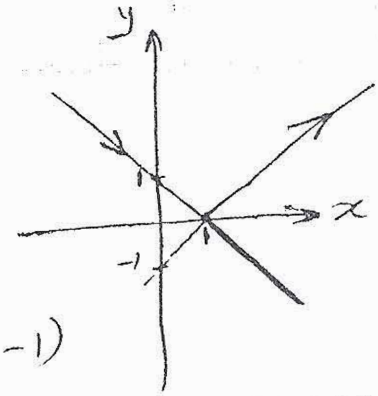


عط

25
14

The line $x+y=1$ has intercepts of $(1,0)$ & $(0,1)$

The line of reflection will have same x-intercept but image of y-intercept.



$\therefore$ Its intercepts will be $(1,0)$ & $(0,-1)$

$\therefore$ Equation of its new path will be:

$\frac{x}{1} + \frac{y}{-1} = 1$ OR $\boxed{x-y=1}$ & $\boxed{y \geqslant 0}$

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28
15

ⓐ slope of $L_1 : Ax + By + C = 0$ is $-\frac{A}{B}$

slope of $L_2 : Ax + By + C' = 0$ is $-\frac{A}{B}$

$\therefore L_1$ & L_2 are parallel unless $C = C'$ where they coincide.

ⓑ slope of $L_3 : Bx - Ay + C' = 0$ is $\frac{B}{A}$

$\therefore$ slope of $L_3 \times$ slope of $L_1 = \frac{B}{A} \cdot \left(-\frac{A}{B}\right) = -1$

$\therefore L_3$ & L_1 are $\perp$.

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1
27

$y = \sqrt{x+4}$ $\therefore x+4 \geqslant 0$ $\therefore x \geqslant -4$

$\therefore x \in [-4, \infty)$ $\therefore y \in [0, \infty)$

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2/27

Domain : $x \in [0,1]$
Range : $y \in [0,1]$

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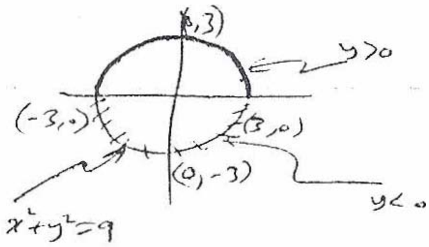
$y = \sqrt{9 - x^2}$
or $y^2 = 9 - x^2$ & $y > 0$
$y^2 + x^2 = 9$ & $y > 0$



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Graph d is for $y = (x-1)^2$
because its vertix is at $(1,0)$

17/27

$\left|\frac{x}{2} - 1\right| \leq 1$ means :

$\frac{x}{2} - 1 \leq 1$ and $-\left(\frac{x}{2} - 1\right) \leq 1$
i.e $\frac{x}{2} \leq 2$ and $1 - 1 \leq \frac{x}{2}$
i.e $x \leq 4$ and $0 \leq \frac{x}{2}$
i.e $x \in (-\infty, 4]$ and $x \in [0, \infty)$
$\therefore$ $x \in [0,4]$ is the domain of x or $0 \leq x \leq 4$

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$-5 < x < -1$ subtract $\frac{-5 + (-1)}{2} = -3$ from all sides
$\therefore$ $-2 < x + 3 < 2$
$\therefore$ $|x + 3| < 2$

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$f(-x) = \cos(-x) = \cos x = f(x)$ $\therefore$ $\cos x$ is even.

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$f(-x) = \tan(-x) = -\tan x = -f(x)$ $\therefore$ $\tan x$ is odd.

✓

$\frac{43}{28}$

$F(t) = 4t - 3$

$\therefore F(t+h) = 4(t+h) - 3 = 4t + 4h - 3$

$\therefore \frac{F(t+h) - F(t)}{h} = \frac{4t+4h-3-(4t-3)}{h} = \frac{4h}{h} = 4.$

$\frac{44}{28}$

$f(x) = x - 7 \quad , g(x) = \sqrt{x}$

$\therefore g(f) = \sqrt{f} = \sqrt{x-7}$

$\frac{45}{28}$

$f(x) = x + 2$

$g(x) = 3x$

$\therefore g(f(x)) = g(f) = 3f = 3(x+2)$

$\therefore g(f(x)) = 3(x+2)$

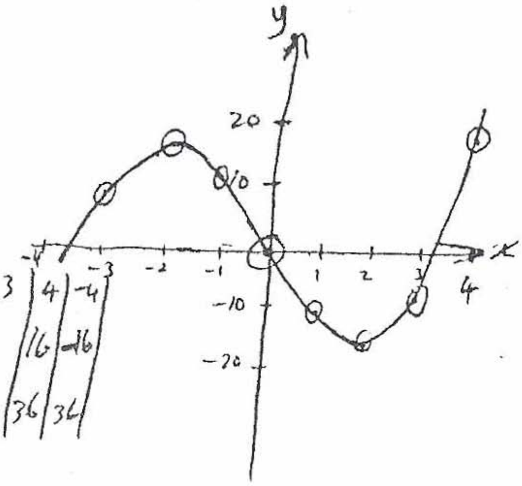
$\boxed{\frac{13}{30}}$

$$y' = \lim_{\Delta x\to 0} \frac{(x+\Delta x)^3-12(x+\Delta x)-\overline{(x^3-12x)}}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{(\cancel{x^3}+3x^2\Delta x+\cdots)-\cancel{12x}-12\Delta x-\cancel{x^3}+\cancel{12x}}{\Delta x}$$

$$= 3x^2-12$$

x	0	1	2	−1	−2	3	−3	4	−4
$y = x^3-12x$	0	−11	−16	11	16	−9	9	16	−16
$y' = 3(x^2-4)$	−12	−9	0	−9	0	15	15	36	36



$\boxed{\frac{14}{30}}$

slope of curve, $y' = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{(x+\Delta x)^2(4(x+\Delta x)+3)+1-[x^2(4x+3)+1]}{\Delta x}$

$$= \lim_{\Delta x\to 0}\frac{[x^2+2x\Delta x+(\Delta x)^2](4x+4\Delta x+3)+\cancel{1}-4x^3-3x^2-\cancel{1}}{\Delta x} =$$

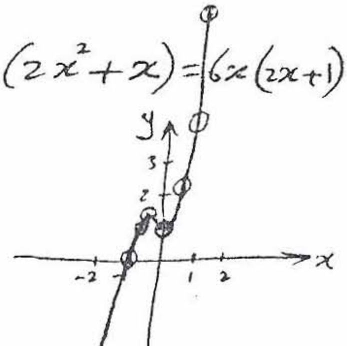
$$= \lim_{\Delta x\to 0}\frac{\cancel{4x^3}+8x^2\Delta x+4x(\Delta x)^2+4x^2\Delta x+8x(\Delta x)^2+4(\Delta x)^3+\cancel{3x^2}+6x\Delta x+3(\Delta x)^2-\cancel{4x^3}-\cancel{3x^2}}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{12x^2\Delta x+12x(\Delta x)^2+4(\Delta x)^3+6x\Delta x+3(\Delta x)^2}{\Delta x}$$

$$= 12X^2+6X$$

$\therefore$ slope at (x,y) is $y' = 12X^2+6x = 6(2x^2+x) = 6x(2x+1)$

X	0	1	−1/2	1/2	−1	−3/4	+3/4
$y = x^2(4x+3)+1$	1	8	5/4	9/4	0	1	35/8
$y' = 6x(2x+1)$	0	18	0	6	6	9/4	45/4



$\boxed{\frac{5}{37}}$ $f'(x) = \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to 0}\frac{-\cancel{h}}{\cancel{h}(x^2+xh)} = \lim_{h\to 0}\frac{-1}{x^2+xh} = -\frac{1}{x^3}$.

$\therefore$ slope (at $x=3$) $= f'(3) = -\frac{1}{3^2} = -\frac{1}{9}$, y (at $x=3$) $= \frac{1}{3}$

$\therefore \frac{y-1/3}{x-3} = -\frac{1}{9}$ $\quad\therefore\quad 9y-3=-x+3$ $\quad\therefore$ The tangent is $9y+x-6=0$

$\boxed{\frac{6}{37}}$ $f(x) = \frac{1}{x^2}$ $\quad {}^{\prime}(x)\ \lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{1,}$ $\quad \lim_{h\to 0}\frac{x^2-(x+h)^2}{x^2(x+}$ $=$

$= \lim_{h\to 0}\frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{hx^2(x+h)^2} = \lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$.

$\therefore f'(3) = \frac{-2}{3^3} = \frac{-2}{27}$ $\quad$ f $\quad f(3) = \frac{1}{3^2} = \frac{1}{9}$

$\therefore$ Equation of tangent is $\frac{y-\frac{1}{9}}{x-3} = \frac{-2}{27}$ $\quad$ or $\quad 27y+2x=9$

$\frac{10}{37}$

$f(x) = x^3 - 12x + 11$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 12(x+h) + 11 - (x^3 - 12x + 11)}{h} = \lim_{h \to 0} \frac{(\cancel{x^3} + 3x^2h + \dots) - 12\cancel{x} - 12h \; \overset{-x^3}{\underset{+11}{+12x}}}{h}$$

$= 3x^2 - 12$

$\therefore f'(x)\Big|_{x=3} = 3(3)^2 - 12 = 15$

$\therefore$ at $X = 3$ $\quad y = 3^3 - 12(3) + 11 = 2$ $\quad$ and slope $= 15$

$\therefore \frac{y-2}{x-3} = 15$ $\quad$ or $\quad y - 15X + 43 = 0$ is the equation for the tangent.

$\frac{13}{37}$

$f(x) = x - \frac{1}{\%}$ $\qquad \therefore f'(x) = \lim_{h \to 0} \frac{\cancel{x} + h - \frac{1}{x+h} - \cancel{x} + \frac{1}{x}}{h} =$

$$\lim_{h \to 0} \frac{h - \frac{x-(x+h)}{x\cdot(x+h)}}{h} = \lim_{h \to 0} \frac{-h + \frac{1}{x(x+h)}}{h} = 1 + \frac{1}{x^2}$$

$\therefore f'(3) = 1 + \frac{1}{9} = \frac{10}{9}$ $\quad \therefore$ at $X = 3$, $y = 3 - \frac{1}{3} = \frac{8}{3}$ and slope $= \frac{10}{9}$

$\therefore \frac{y - \frac{8}{3}}{x-3} = \frac{10}{9}$ $\quad$ or $\quad 9y - 10x + 6 = 0$ is the equation of tangent.

$\frac{17}{37}$

$$f(x) = \sqrt{2x+3} \qquad \therefore f'(x) = \lim_{h \to 0} \frac{\sqrt{2x+2h+3} - \sqrt{2x+3}}{h} \cdot \frac{\sqrt{2x+2h+3} + \sqrt{2x+3}}{\sqrt{2x+2h+3} + \sqrt{2x+3}} =$$

$$= \lim_{h \to 0} \frac{(\cancel{2x} + 2h + \cancel{3}) - (\cancel{2x} + \cancel{3})}{\cancel{h}(\sqrt{2x+2\cancel{h}+3} + \sqrt{2x+3})} = \frac{2}{2\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}} \qquad \therefore f'(3) = \frac{1}{3}$$

$\therefore$ at $x = 3$ $\quad y = 3$ $\quad$ and slope $= \frac{1}{3}$ $\quad \therefore \frac{y-3}{x-3} = \frac{1}{3}$ $\quad$ or $3y - x - 6 = 0$ is Eq.

$\frac{19}{37}$

$$f(x) = \frac{1}{\sqrt{2x+3}} \qquad \therefore f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{2(x+h)+3}} - \frac{1}{\sqrt{2x+3}}}{h} =$$

$$= \lim_{h \to 0} \frac{\sqrt{2x+3} - \sqrt{2(x+h)+3}}{h\sqrt{2(x+h)+3}\cdot\sqrt{2x+3}} \cdot \frac{\sqrt{2x+3} + \sqrt{2(x+h)+3}}{\sqrt{2x+3} + \sqrt{2(x+h)+3}} =$$

$$= \lim_{h \to 0} \frac{(\cancel{2x} + \cancel{3}) - (2(\cancel{x}+h) + \cancel{3})}{h\sqrt{[2(x+h)+3](2x+3)}\left(\sqrt{2x+3} + \sqrt{2(x+h)+3}\right)} = \frac{-2}{(2x+3)(2\sqrt{2x+3})} =$$

$= -(2x+3)^{-\frac{3}{2}} = f'(x)$, $\therefore \frac{y - \frac{1}{\sqrt{9}}}{x-3} = -(9)^{-\frac{3}{2}}$ $\quad$ or $\quad \frac{3y-1}{3x-9} = -\frac{1}{27}$ $\quad \therefore 27y + x = 12$

$\frac{20}{37}$

$$f(x) = \sqrt{x^2+1} \qquad \therefore f'(x) = \lim_{h \to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h} \cdot \frac{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}} =$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{2h\sqrt{x^2+1}} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{2h\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$

$\therefore f'(3) = \frac{3}{\sqrt{10}}$ $\quad \therefore$ at $x = 3$ $\quad y = \sqrt{10}$ $\quad$ and slope $= \frac{3}{\sqrt{10}}$ $\quad \therefore \frac{y - \sqrt{10}}{x-3} = \frac{3}{\sqrt{10}}$

$\therefore \sqrt{10}\,y - 3x - 1 = 0$ is the equation for the tangent at $x = 3$ $\quad$ ||

$\boxed{\frac{3}{41}}$ $\quad S = \frac{1}{2}\cdot g t^2 + V_i t + S_0$

$\therefore V = \frac{ds}{dt} = 2\cdot\frac{1}{2}\cdot g t + V_0 = g t + V_0$

$\boxed{\frac{4}{41}}$ $\quad S = 4 t + 3 \qquad \therefore \frac{ds}{dt} = 4$

$\boxed{\frac{8}{41}}$ $\quad S = (2 - t)^2 = 4 - 4t + t^2$

$\therefore V = \frac{ds}{dt} = -4 + 2t$

$\boxed{\frac{10}{41}}$ $\quad S = 64 t - 16 t^2 \qquad \therefore V = \frac{ds}{dt} = 64 - 32 t.$

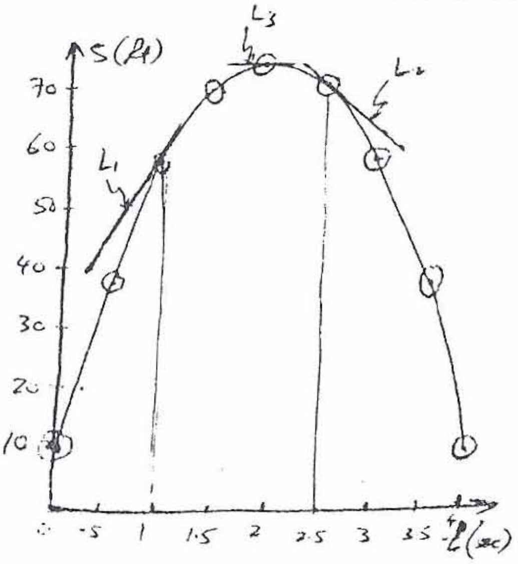
$\boxed{\frac{11}{41}}$

ⓐ Velocity at $t = 1.0$ is the slope of $L_1 = \frac{18}{0.75} = 24$ ft/sec

ⓑ Velocity at $t = 2.5$ is the slope of $L_2 = \frac{-10}{0.75} = -13$ ft/sec

ⓒ Velocity at $t = 2.0$ is the slope of $L_3 = 0$ because it is horizontal

$\therefore$ velocity at $t = 2$ is 0 ft/sec.



$\frac{13}{42}$ Count the number of divisions at $t = 19$ sec $\therefore$ speed $= 190$ ft/sec

$\frac{20}{42}$ $Q = 200(30 - t)^2$ gallons

$\therefore \frac{dQ}{dt} = -400(30 - t)$ gallons/minute

$\therefore \frac{dQ}{dt}$ when $t = 10$ min. is $-400(30 - 10) = -8000$ gal/min

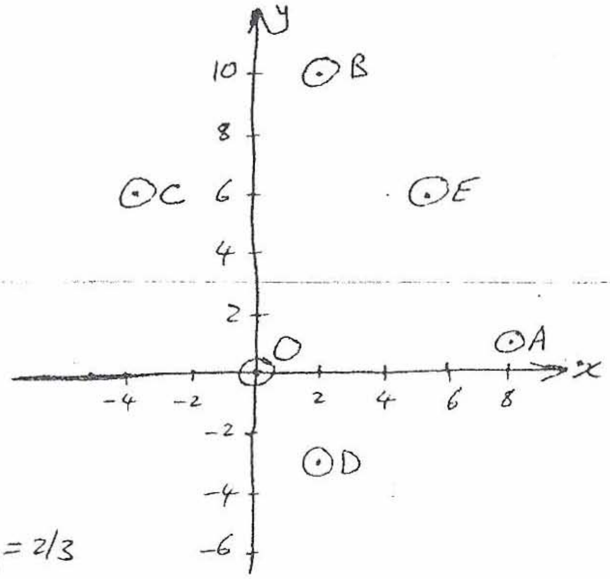
$\therefore$ Water is coming out at 8000 gallon/min at $t = 10$ min

The average rate $= \frac{\text{rate (at } t=0) + \text{rate (at } t=10)}{2}$

$$= -400 \cdot \frac{(30 - 0) + (30 - 10)}{2} = -400 \cdot \frac{30 + 20}{2} = -10{,}000 \ \frac{\text{gal}}{\text{min}}$$

$\therefore$ Average rate during the first 10 minutes is 10,000 gallons of water out per minute.

$\boxed{\frac{1}{56}}$



ⓐ See the plt.

ⓑ $m_{AB} = \frac{10-1}{2-8} = \frac{9}{-6} = -3/2$

$m_{BC} = \frac{6-10}{-4-2} = \frac{-4}{-6} = 2/3$

$m_{CD} = \frac{-3-6}{2+4} = \frac{-9}{6} = -3/2$

$m_{DA} = \frac{-3-1}{2-8} = \frac{-4}{-6} = 2/3$

$m_{CE} = \frac{6-6}{4\frac{2}{3}-4} = \frac{0}{2/3} = 0$

$m_{BD} = \frac{-3-10}{2-2} = \infty$

ⓒ Yes, ABCD is aparallelogram

because $m_{AB} = m_{CD} = -3/2$ & $m_{BC} = m_{DA} = 2/3$

ⓓ $m_{AE} = \frac{6-1}{4\frac{2}{3}-8} = \frac{5}{-3\frac{1}{3}} = \frac{5}{-\frac{10}{3}} = -\frac{15}{10} = -3/2$

$\therefore m_{AB} = m_{AE} = -3/2$ with A common $\therefore$ AEB isastraight segment.

ⓔ $m_{OD} = \frac{-3-0}{2-0} = -3/2$

$\therefore m_{CD} = m_{OD} = -3/2$ with D common $\therefore$ DOC is. astraight segment.

$\therefore$ O (0,0) lies on the straight line through D & C.

ⓕ Equation of AB is : $\frac{y-1}{x-8} = m_{AB} = -\frac{3}{2}$ $\therefore$ $2y+3x-26=0$

Equation of CD is : $\frac{y-6}{x+4} = m_{CD} = -3/2$ $\therefore$ $2y+3x=0$

Equation of AD is : $\frac{y-1}{x-8} = m_{DA} = \frac{2}{3}$ $\therefore$ $3y-2x+13=0$

Equation of CE is : $\frac{y-6}{x+4} = m_{CE} = 0$ $\therefore$ $y=6$

Equation of BD is : $\frac{y-10}{x-2} = m_{BD} = \infty$ $\therefore$ $x=2$

ⓖ AB intercepts are $(0,13)$ & $(\frac{26}{3},0)$

CD intercepts are $(0,0)$

AD intercepts are $(0,-\frac{13}{3})$ & $(\frac{13}{2},0)$

CE intercepts are $(0,6)$

BD intercepts are $(2,0)$

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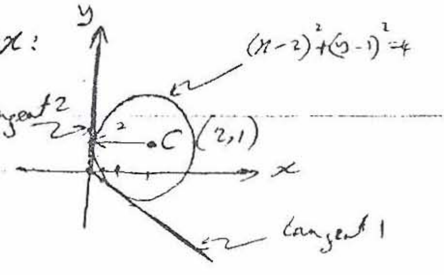
بسم الله الرحمن الرحيم

$\frac{4}{56}$

Equation of circle centre C(2,1) radius 2 is $(x-2)^2+(y-1)^2=4$
Equation of line through origin with slope m
is $y=mx$. Assume intersection & solve for x:



$\therefore (x-2)^2+(mx-1)^2=4$

$\therefore x^2-4x+4+m^2x^2-2mx+1=4$

$\therefore (1+m^2)x^2-(4+2m)x+1=0$

$\therefore x=\frac{(4+2m)\pm\sqrt{(4+2m)^2-4(1+m^2)}}{2(1+m^2)}$

$\therefore$ For tangency, $\sqrt{(4+2m)^2-4(1+m^2)}=0$,
or $(4+2m)^2-4(1+m^2)=0$

$\therefore 16+\cancel{4m^2}+16m-4-\cancel{4m^2}=0 \Rightarrow 16m=-12, \therefore \boxed{m=\frac{-3}{4}}$

Now assume intersection & solve for y

$\therefore \left(\frac{y}{m}-2\right)^2+(y-1)^2=4$

$\therefore \frac{y^2}{m^2}-4\frac{y}{m}+4+y^2-2y+1=4 \quad \therefore y^2\left(1+\frac{1}{m^2}\right)-\left(2+\frac{4}{m}\right)y+1=0$

$\therefore y=\frac{\left(2+\frac{4}{m}\right)\pm\sqrt{\left(2+\frac{4}{m}\right)^2-4\left(1+\frac{1}{m^2}\right)}}{2\left(1+\frac{1}{m^2}\right)}$ $\quad\therefore$ For tangency, $\sqrt{\left(2+\frac{4}{m}\right)^2-4\left(1+\frac{1}{m^2}\right)}=0$,

or $\left(2+\frac{4}{m}\right)^2-4\left(1+\frac{1}{m^2}\right)=0$, $\therefore \cancel{4}+\frac{16}{m}+\frac{16}{m^2}\cancel{-4}-\frac{4}{m^2}=0$

$\therefore \frac{16}{m}+\frac{12}{m^2}=0 \quad \therefore \frac{4}{m}\left(4+\frac{3}{m}\right)=0 \quad \therefore$ either $\frac{1}{m}=0$ or $m=\frac{-3}{4}$

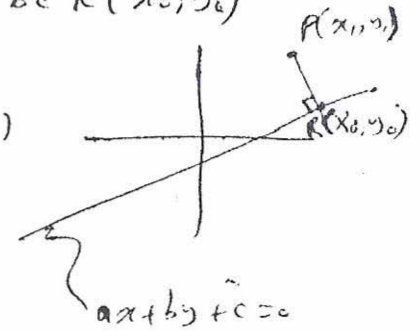
If $\frac{1}{m}=0 \quad \therefore y=mx$ or $\frac{y}{m}=x$ or $y(0)=0 \therefore \boxed{x=0}$
If $m=\frac{-3}{4} \quad \therefore y=\frac{-3}{4}x$ or $\boxed{4y+3x=0}$

$\therefore$ Equations of the straight lines required are $x=0$ & $4y+3x=0$

$\boxed{\frac{8}{57}}$

تابع

Let the projection of P on $ax+by+c=0$ be $R(x_0, y_0)$



$\therefore$ Equation of PR is $\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$ (1)

But PR is $\perp$ $ax+by+c=0$

$\therefore$ slope of PR $= \frac{-1}{-a/b} = \frac{b}{a}$

$\therefore$ Equation of PR is $\frac{y-y_1}{x-x_1} = \frac{b}{a}$ (2)

From (1) & (2) $\therefore \frac{y_1-y_0}{x_1-x_0} = \frac{b}{a}$ $\therefore y_1 - y_0 = \frac{b}{a}(x_1 - x_0)$ (3)

Since $R(x_0, y_0)$ is on $ax+by+c=0$ $\therefore ax_0 + by_0 + c = 0$ (4)

Solving (3) & (4) for x_0, we get:

$$ax_0 + b\left(y_1 - \frac{b}{a}(x_1 - x_0)\right) + c = 0 \quad \text{OR} \quad ax_0 + by_1 + \frac{b^2}{a}x_0 - \frac{b^2}{a}x_1 + c = 0$$

$\therefore$ $\left(a + \frac{b^2}{a}\right)x_0 = \frac{b^2}{a}x_1 - by_1 - c$ multiply all by a :

$\therefore$ $(a^2+b^2)x_0 = b^2x_1 - bay_1 - ca$ (5)

Now the required distance is $|PR| = \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2}$.

Using (3), $\therefore |PR| = \sqrt{(x_1-x_0)^2 + \frac{b^2}{a^2}(x_1-x_0)^2} = |(x_1-x_0)| \sqrt{1+\frac{b^2}{a^2}} =$

$$= |x_1 - x_0| \cdot \sqrt{\frac{a^2+b^2}{a^2}} = \frac{a^2+b^2}{a\sqrt{a^2+b^2}} \cdot |x_1 - x_0| = \frac{|(a^2+b^2)(x_1-x_0)|}{a\sqrt{a^2+b^2}}$$

Using (5),

$\therefore$ $|PR| = \frac{|(a^2+\cancel{b^2})x_1 - (\cancel{b^2}x_1 - bay_1 - ca)|}{a\sqrt{a^2+b^2}} = \frac{|a^2x_1 + bay_1 + ca|}{a\sqrt{a^2+b^2}} =$

$$= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

$\therefore$ The distance between $P(x_1,y_1)$ and $ax+by+c=0$ is equal to $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

9
57

Q: 6

Assume that the centre is $C(h, k)$ and radius is R.

∴ According to L_1: ∴ $R = \frac{|h+k-1|}{\sqrt{1^2+1^2}}$ or $|h+k-1| = \sqrt{2}\,R$ (1)

& " " L_2: ∴ $R = \frac{|h-k+1|}{\sqrt{1^2+(-1)^2}}$ or $|h-k+1| = \sqrt{2}\,R$ (2)

& " " L_3: ∴ $R = \frac{|h-3k-1|}{\sqrt{1^2+(-3)^2}}$ or $|h-3k-1| = \sqrt{10}\,R$ (3)

Subtracting (1) from (2) ∴ $|h-k+1| - |h+k-1| = 0 \Rightarrow |h-k+1| = |h+k-1|$

∴ $(h-k+1)^2 = (h+k-1)^2$ ∴ $\cancel{h^2}+\cancel{k^2}+\cancel{1}-\cancel{2k}+2h-2kh = \cancel{h^2}+\cancel{k^2}+\cancel{1}-\cancel{2k}-2h+2kh$

∴ $-4kh = 0$ ∴ $h(1-k) = 0$ ∴ $h = 0$ or $k = 1$

If $h = 0$ ∴ (1) becomes $|k-1| = \sqrt{2}\,R$ ∴ $(k-1)^2 = 2R^2$ ∴ $k = 1 \pm \sqrt{2}\,R$

If $k = 1$. (1) becomes $|h| = \sqrt{2}\,R$ ∴ $h = \pm\sqrt{2}\,R$

∴ There are four possible circle centres $(0, 1+\sqrt{2}R)$, $(0, 1-\sqrt{2}R)$, $(\sqrt{2}R, 1)$ & $(-\sqrt{2}R, 1)$.

To 4 the radii use (3),

∴ for $(0, 1+\sqrt{2}R)$ ∴ $|0-3(1+\sqrt{2}R)-1| = \sqrt{10}\,R \Rightarrow \begin{cases} 3+3\sqrt{2}R+1 = \sqrt{10}\,R \\ -3-3\sqrt{2}R-1 = \sqrt{10}\,R \end{cases}$ ∴ $\overset{\times}{R} = -3.7$ ∴ ✗$R = -.54$

& for $(0, 1-\sqrt{2}R)$ ∴ $|0-3(1-\sqrt{2}R)-1| = \sqrt{10}\,R \Rightarrow \begin{cases} -3+3\sqrt{2}\text{ if}-1 = \sqrt{10}\,R \\ 3-3\sqrt{2}R+1 = \sqrt{10}\,R \end{cases}$ ∴ ✓$R = 3.7$ ∴ ✓$R = 0.54$

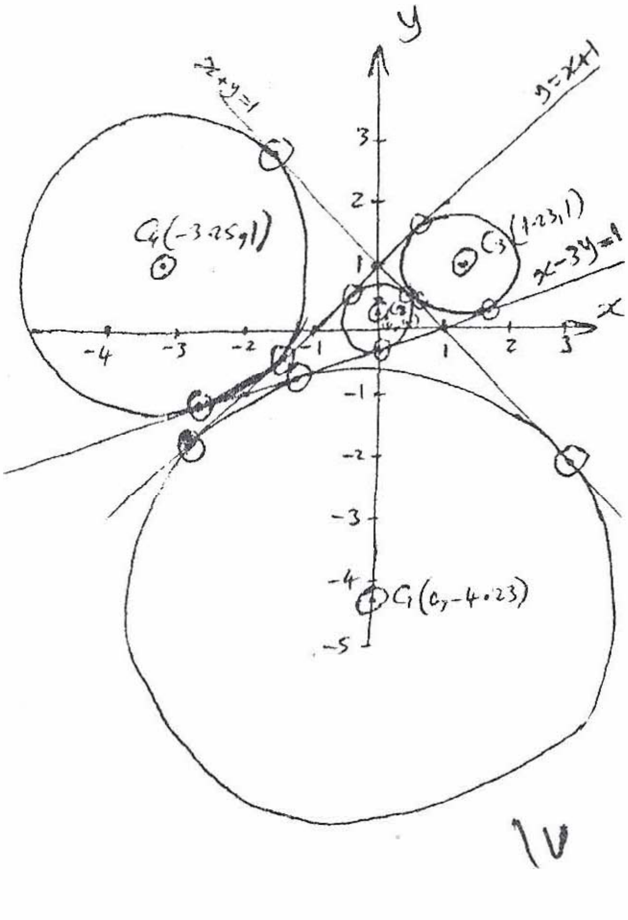
& for $(\sqrt{2}R, 1)$ ∴ $|\sqrt{2}R-3-1| = \sqrt{10}\,R \Rightarrow \begin{cases} \sqrt{2}R-3-1 = \sqrt{10}\,R \\ -\sqrt{2}R\ 3 = \sqrt{10}\,R \end{cases}$ ∴ ✗$R = -2.3$ ∴ ✓$R = 0.87$

& for $(-\sqrt{2}R, 1)$ ∴ $|-\sqrt{2}R-3-1| = \sqrt{10}\,R \Rightarrow \begin{cases} -\sqrt{2}R-3-1 = \sqrt{10}\,R \\ \sqrt{2}R+3+1 = \sqrt{10}\,R \end{cases}$ ∴ ✗$R = -.87$ ∴ ✓$R = 2.3$

∴ The four circles are

1. centre, $C_1(0, -4.23)$ radius 3.7
2. centre, $C_2(0, 0.24)$ radius 0.54
3. centre, $C_3(1.23, 1)$ 0.87
4. centre, $C_4(-3.25, 1)$ radius 2.3

graph shows an illustration.



تابع

$\boxed{\frac{14}{57}}$

$$y = \frac{x^2+2}{x^2-1}$$

$$\therefore (x^2-1)y = x^2+2 \qquad \therefore x^2(y-1) = 2+y \qquad \therefore x^2 = \frac{2+y}{y-1}$$

$$\therefore x = \pm\sqrt{\frac{y+2}{y-1}} \qquad x \text{ is real when } \frac{y+2}{y-1} \geqslant 0$$

$y = -\infty$ $\quad$ $y = -2$ $\quad$ $y = 1$ $\quad$ $y = +\infty$

$y+2$ $\quad$ $- - - - - - 0 + + + + + + + +$

$y-1$ $\quad$ $- - - - - - - - 0 + + + + + +$

$\frac{y+2}{y-1}$ $\quad$ $+ + + + + 0 - - - -\infty + + + + + + + + +$

$$\therefore x \text{ is real at } y \in (-\infty, -2] \cup (1, \infty)$$

$\boxed{\frac{15}{57}}$

$$A = \pi r^2$$

$$C = 2\pi r$$

$$\therefore A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi} \qquad \therefore A = \frac{C^2}{4\pi}$$

$\boxed{\frac{30}{57}}$

ⓐ
$$f'(x) = \lim_{h\to 0} \frac{\frac{x+h-1}{x+h+1} - \frac{x-1}{x+1}}{h} = \lim_{h\to 0} \frac{(x+h-1)(x+1) - (x-1)(x+h+1)}{(x+1)(x+h+1)h} =$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+\cancel{x}+\cancel{xh}+h-\cancel{x}-\cancel{1}-\cancel{x^2}-\cancel{xh}-\cancel{x}+\cancel{x}+h+\cancel{1}}{(x+1)(x+h+1)h} = \frac{2}{(x+1)^2}$$

ⓑ
$$f'(x) = \lim_{h\to 0} \frac{(x+h)^{3/2} - x^{3/2}}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h[(x+h)^{3/2} + x^{3/2}]} = \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h[(x+h)^{3/2}+x^{3/2}]}$$

$$= \lim_{h\to 0} \frac{3x^2+3xh+h^2}{(x+h)^{3/2}+x^{3/2}} = \frac{3x^2}{2x^{3/2}} = \frac{3}{2}x^{2-\frac{3}{2}} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}.$$

ⓒ
$$f'(x) = \lim_{h\to 0} \frac{(x+h)^{1/3} - x^{1/3}}{h} = \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h[(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{4/3}]} = \lim_{h\to 0} \cdot \frac{1}{(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}}$$

$$= \frac{1}{x^{2/3}+x^{1/3}\cdot x^{1/3}+x^{4/3}} = \frac{1}{3x^{2/3}} = \frac{1}{3}x^{-2/3}$$

٤٢

$\frac{36}{58}$

$S = 32t - 16t^2$

The ball will be at its highest point when its velocity is zero

$\therefore \frac{ds}{dt} = 32 - 32t = 0 \quad \therefore t = 1 \text{ sec} \quad \therefore S = 32 - 16 = 16 \text{ ft}$

$\therefore$ After 1sec the ball will be at the highest level 16 ft up.

—

$\frac{38}{58}$

$V = 2000 - 40t + 0.2t^2$

$\therefore dV/dt = -40 + 0.4\,t \Big|_{\text{at } t=30} = -40 + 0.4(30) = -40 + 12 = -28 \frac{\text{in}^3}{\text{sec}}$

$\therefore$ The volume is decreasing at 28 in³/sec.

—

$\frac{40}{58}$

$3x + 5y = 1$ & $(2+c)x + 5c^2y = 1$ intersect at (x_1, y_1) obtained by solving together both equations

$$\therefore \begin{pmatrix} 3 & 5 \\ 2+c & 5c^2 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \therefore x_1 = \frac{5c^2 - 5}{15c^2 - 5(2+c)}, \; y_1 = \frac{3 - (2+c)}{15c^2 - 5(2+c)},$$

Or, the point is $\left(\frac{c^2 - 1}{3c^2 - c - 2}, \frac{1 - c}{5(3c^2 - c - 2)} \right)$.

This point at $c = 1$ becomes $\left(\frac{0}{0}, \frac{0}{0} \right)$ which means that the solution does ha? exist.

However, when C approaches 1 ; the point will be:

$$\left(\lim_{c \to 1} \frac{c^2 - 1}{3c^2 - c - 2}, \lim_{c \to 1} \frac{1 - c}{5(3c^2 - c - 2)} \right) = \left(\lim_{c \to 1} \frac{\cancel{(c-1)}(c+1)}{\cancel{(c-1)}(3c+2)}, \lim_{c \to 1} \frac{-\cancel{(c-1)}}{5\cancel{(c-1)}(3c+2)} \right) =$$

$$= \left(\lim_{c \to 1} \frac{c+1}{3c+2}, \lim_{c \to 1} \frac{-1}{5(3c+2)} \right) = \left(\frac{2}{5}, \frac{-1}{25} \right)$$

—

بسم الله الرحمن الرحيم

$\boxed{\frac{2}{67}}$

$s = 2t^3 - 5t^2 + 4t - 3$

$v = \frac{ds}{dt} = 6t^2 - 10t + 4$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t - 10$

$\boxed{\frac{4}{67}}$ $s = 3 + 4t - t^2 \quad \therefore v = \frac{ds}{dt} = 4 - 2t \quad \& \quad a = \frac{dv}{dt} = -2$

$\boxed{\frac{9}{67}}$

$y = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x + 3$

$\therefore y' = x^3 - x^2 + x - 1 \quad \& \quad y'' = 3x^2 - 2x + 1.$

$\boxed{\frac{11}{67}}$

$12y = 6x^4 - 18x^2 - 12x \qquad \therefore y = \frac{x^4}{2} - \frac{3x^2}{2} - x$

$\therefore y' = \frac{dy}{dx} = 2x^3 - 3x - 1$

$\& \; y'' = \frac{dy'}{dx} = 6x^2 - 3$

$\boxed{\frac{16}{67}}$

At maximum hight, $v = 0 \quad \therefore v = \frac{ds}{dt} = 160 - 32t = 0$

$\therefore t = \frac{160}{32} = 5$ sec

$\therefore s(5) = 160(5) - 16(5)^2 = 800 - 400 = 400$ ft

$\therefore$ ⓐ It rise 400 ft.

$s = 256 = 160t - 16t^2 \qquad \therefore t^2 - 10t + 16 = 0$

$\therefore (t-8)(t-2) = 0 \qquad \therefore t = 2$ sec & $t = 8$ sec

$\therefore$ After 2sec it is 256 ft high going up & after 8sec down

$\therefore v = 160 - 32t = 160 - 32 \times 2 = 160 - 64 = 96$ ft/sec

or $v = 160 - 32 \times 8 = 160 - 256 = -96$ ft/sec

$\therefore$ ⓑ When it reaches 256 ft hight it is travelling at 96 ft/sec up or down.

$\boxed{\frac{22}{67}}$

$y' = 3x^2$, at $A(-2, -8) \quad \therefore y' = 3(-2)^2 = 3 \times 4 = 12 \quad \therefore$ Eq. of tangent $\frac{y+8}{x+2} = 12$

If $x = 0 \quad \therefore y = 2 * 12 - 8 = 16$. If $y = 0 \quad \therefore x = \frac{8}{12} - 2 = -\frac{4}{3} \quad \therefore$ Intercepts are $(0,16)$ & $(-\frac{4}{3}, 0)$

$\boxed{\frac{24}{67}}$ $y = x^2 \quad \therefore y' = 2x$ & $y'' = 2$

$\therefore$ Curvature, $K = \frac{2}{(1+(2x)^2)^{3/2}} = \frac{2}{(1+4x^2)^{3/2}} \therefore ⓐ K_{(x=0)} = 2$ & ⓑ $K_{(x=1)} = \frac{2}{5\sqrt{5}}$, ⓒ $K \to 0$ at $x \to \infty$

$\boxed{\frac{25}{67}}$ $y = ax^2 + bx + c$

The point $(1,2)$ satisfy the above equation $\therefore 2 = a + b + c$ (1)

The point $(0,0)$ also satisfy the above equation $\therefore 0 = c$ (2)

The slope at $(0,0) = 1 = y' = 2ax + b$ $\therefore 1 = b$ (3)

$\therefore a = 1$ & $b = 1$ & $c = 0$.

$\boxed{\frac{27}{67}}$ Slope of tangent $= 1 = 2x \quad \therefore x = \frac{1}{2} \quad \therefore$ point of tangency is $(\frac{1}{2}, \frac{1}{2})$

$\therefore \frac{1}{2} = (\frac{1}{2})^2 + c \quad \therefore c = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\therefore c$ is $\frac{1}{4}$.

$\boxed{\frac{2}{75}}$ $y = (x-1)^3 (x+2)^4$

$\therefore y' = 3(x-1)^2(x+2)^4 + 4(x+2)^3(x-1)^3 = (x-1)^2(x+2)^3[3x+6+4x-4]$

$\therefore y' = (7x+2)(x-1)^2(x+2)^3$.

$\boxed{\frac{9}{75}}$ $s = \frac{t}{t^2+1} \quad \therefore \frac{ds}{dt} = \frac{t^2+1-2t.t}{(t^2+1)^2} = \frac{1-t^2}{(t^2+1)^2}$.

$\boxed{\frac{12}{75}}$ $s = t^2(t+1)^{-1} = \frac{t^2}{t+1}$

$\therefore \frac{ds}{dt} = \frac{2t(t+1) - t^2}{(t+1)^2} = \frac{t^2+2t}{(t+1)^2} = \frac{t(t+2)}{(t+1)^2}$

$\boxed{\frac{14}{75}}$ $s = (t + t^{-1})^2 \quad \therefore \frac{ds}{dt} = 2(t+t^{-1}) \cdot (1 - t^{-2}) = 2\left(\frac{t^2+1}{t}\right) \cdot \left(\frac{t^2-1}{t^2}\right) = \frac{2(t^4-1)}{t^3}$

$\boxed{\frac{15}{75}}$ $s = (t^2 + 3t)^3$

$\therefore \frac{ds}{dt} = 3(t^2+3t)^2 \cdot (2t+3)$

<1

$\boxed{\frac{5}{79}}$

$y = x^{\frac{1}{2}}$

$\therefore y' = \frac{1}{2}x^{-\frac{1}{2}}$, $\therefore$ At $x = 4$, $y' = \frac{1}{2}(4)^{-\frac{1}{2}} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$\therefore$ At $x=4$, $y=2$ & slope $= \frac{1}{4}$

$\therefore$ Equation of tangent is $\frac{y-2}{x-4} = \frac{1}{4}$

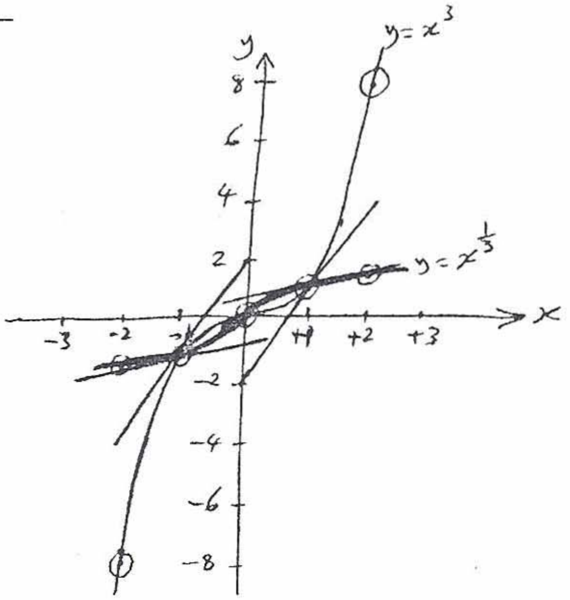
At $x=0$ $\therefore y = \frac{-4}{4} + 2 = 1$

At $y=0$ $\therefore x = -2*4+4 = -4$

$\therefore$ Intercepts are: $(0,1)$ & $(-4,0)$

$\boxed{\frac{7}{79}}$ ⓐ

x	$y = x^3$	$y = x^{1/3}$
-2	-8	-1.26
-1	-1	-1
0	0	0
1	1	1
2	8	1.26
slope	$y' = 3x^2$	$y' = \frac{1}{3}x^{-\frac{2}{3}}$
slope at $(1,1)$	$y' = 3$	$y' = \frac{1}{3}$
slope at $(-1,-1)$	$y' = 3$	$y' = \frac{1}{3}$



ⓑ $y = x^{1/3}$ fails to have derivative at $x=0$ because $y' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$.

Whereas $y = x^3$ has derivative of $y' = 3x^2$ defined in all the interval and at $x=0$ this $y' = 3(0)^2 = 0$.

This means that at $x=0$ the x-axis is tangent to $y = x^3$ and the y-axis is tangent to $y = x^{1/3}$

CC

$\boxed{\frac{9}{83}}$ $x^2 = \frac{x-y}{x+y}$ $\therefore\ 2x = \frac{(1-y')(x+y)-(1+y')(x-y)}{(x+y)^2}$

$\therefore\ 2x(x+y)^2 = \cancel{x}+y-\cancel{x}+y-y'(x\cancel{+y}+x\cancel{-y})$

$\therefore\ 2x(x+y)^2-2y = -2xy'$

$\therefore\ y' = \frac{y-x(x+y)^2}{x}$

$\boxed{\frac{16}{83}}$ $(x+y)^3+(x-y)^3 = x^4+y^4$ $\therefore\ 3(x+y)^2(1+y')+3(x-y)^2(1-y') =$

$= 4x^3+4y^3y'$ $\therefore\ 3\left[(x+y)^2+(x-y)^2\right]+3y'\left[(x+y)^2-(x-y)^2\right] = 4(x^3+y^3y')$

$\therefore\ 3(2x^2+2y^2)+3y'(4xy) = 4(x^3+y^3y')$

$\therefore\ 3x^2+3y^2-2x^3 = y'(2y^3-6xy) \Rightarrow \therefore\ y' = \frac{3x^2+3y^2-2x^3}{2y(y^2-3x)}$.

$\boxed{\frac{17}{83}}$ $(3x+7)^5 = 2y^3$ $\therefore\ \frac{dy}{dx} = \frac{5(3x+7)^4\cdot(3)}{6y^2} = \frac{5}{2}\cdot\frac{(3x+7)^4}{y^2}$

$\boxed{\frac{24}{83}}$ ⓐ $x^2-y^2=1$ $\therefore\ 2x-2yy'=0$ $\therefore\ y' = \frac{x}{y}$

ⓑ $\therefore\ y'' = \frac{y-y'x}{y^2} = \frac{y-\frac{x^2}{y}}{y^2} = \frac{y^2-x^2}{y^3} = \frac{-1}{y^3}$

$\boxed{\frac{22}{83}}$ $x^2y^2 = x^2+y^2$ $\therefore\ \cancel{2}xy^2+x^2\cdot\cancel{2}yy' = \cancel{2}x+\cancel{2}yy'$

$\therefore\ y'(x^2y-y) = x-xy^2$ $\therefore\ y' = \frac{-x(1-y^2)}{y(1-x^2)}$.

$\boxed{\frac{27}{83}}$ ⓐ $x^{2/3}+y^{2/3}=1$ $\therefore\ \frac{\cancel{2}}{3}x^{-1/3}+\frac{\cancel{2}}{3}y^{-\frac{1}{3}}y' = 0$ $\therefore\ y' = -\left(\frac{y}{x}\right)^{1/3} = -\frac{y^{1/3}}{x^{1/3}}$

ⓑ $\therefore\ y'' = -\frac{(1/3)y^{-2/3}y'\cdot x^{1/3}-(1/3)x^{-2/3}\cdot y^{1/3}}{x^{2/3}} = -\frac{(1/3)y^{-2/3}(-y^{1/3})-(1/3)x^{-2/3}y^{1/3}}{x^{2/3}} = \frac{1}{3}\cdot\frac{y^{-1/3}+x^{-2/3}y^{1/3}}{x^{2/3}} =$

$= \frac{1}{3}\cdot\frac{x^{2/3}+\frac{x^{2/3}}{y^{2/3}}}{x^{4/3}y^{1/3}} = \frac{1}{3}\cdot\frac{1}{x^{4/3}y^{1/3}} = \frac{1}{3}\left(\frac{1}{x^4y}\right)^{1/3}$.

$\boxed{\frac{33}{83}}$ $\frac{x-y}{x-2y} = 2$ $\therefore\ \frac{(1-y')(x-2y)-(1-2y')(x-y)}{(x-2y)^2} = 0$, at $(3,1)$ $\therefore\ \frac{(1-y')-(1-2y')(2)}{(1)^2} = 0$

$\therefore\ y' = \frac{-1}{-3} = 1/3$ $\therefore$ Tangent is $\frac{y-1}{x-3} = 1/3$, or $3y-x=0$

& Normal is $\frac{y-1}{x-3} = -3$ or $y+3x-10=0$

$\boxed{\frac{34}{83}}$

$(y-x)^2 = 2x+4 \quad \therefore \; 2(y-x)(y'-1) = 2 \Big|_{at\,(6,2)} \quad \therefore 2(-4)(y'-1)=2$

$\therefore y' = +1 + \frac{2}{-8} = +1 - \frac{1}{4} = \frac{3}{4}$

$\therefore$ slope at $(6,2)$ is $\frac{3}{4}$ $\quad \therefore$ Equation of tangent is $\frac{y-2}{x-6} = \frac{3}{4}$

or $4y - 3x + 10 = 0$.

& Equation of normal is $\frac{y-2}{x-6} = -\frac{4}{3}$, or $3y + 4x - 30 = 0$.

$\boxed{\frac{35}{83}}$

$x^2 + xy + y^2 = 7$ crosses the x-axis at $y=0 \; \therefore x^2 = 7$

$\therefore$ The two cross points are $(\sqrt{7}, 0)$ & $(-\sqrt{7}, 0)$

$\therefore 2x + xy' + y + 2yy' = 0$

$\therefore y' = -\frac{(2x+y)}{(x+2y)}$ $\quad$ when $y=0 \; \therefore y' = -\frac{2x}{x} = -2$

$\therefore$ The slope at both points is equal to -2 $\therefore$ The tangents are parallel.

$\boxed{\frac{3}{85}}$

ⓐ $y = x^3 - x \quad \therefore \Delta y = \left[(x+\Delta x)^3 - (x+\Delta x)\right] - \left[x^3 - x\right] =$

$= \left[\cancel{x^3} + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - \cancel{x} - \Delta x\right] - \left[\cancel{x^3} - \cancel{x}\right]$

$= (3x^2 - 1)\Delta x + \left[3x\Delta x + (\Delta x)^2\right]\Delta x$

ⓑ $\Delta y_{tan} = y'\Delta x = (3x^2 - 1)\Delta x$

ⓒ $\therefore \Delta y - \Delta y_{tan} = \left[3x\Delta x + (\Delta x)^2\right]\Delta x$

$\boxed{\frac{7}{85}}$

$y + \Delta y = x^{\frac{1}{3}} + \frac{1}{3}x^{-\frac{2}{3}}dx = 8^{\frac{1}{3}} + \frac{1}{3}\cdot 8^{-\frac{2}{3}}\cdot(-.5) = 2 + \frac{1}{3}\cdot\frac{1}{4}\cdot\left(\frac{-1}{2}\right) = 2 - \frac{1}{24} = \frac{47}{24} = 1.958$

$\boxed{\frac{8}{85}}$

$y = x^{-1}, \; x = 2, \; \Delta x = 0.1$

at $x = 2 \quad \therefore y = \frac{1}{2}$

at $x = 2.1 \quad \therefore y = \frac{1}{2} + \Delta y = \frac{1}{2} - x^{-2}\Delta x = \frac{1}{2} - (2)^{-2}(0.1) = \frac{20-1}{40} = \frac{19}{40} = .475$

$\frac{9}{85}$

$y = \sqrt{x^2+9}$, $x = -4$ & $\Delta x = -0.2$

at $x = -4$ $\therefore y = \sqrt{16+9} = 5$

at $x = -4 + \Delta x = -4.2$ $\therefore y = 5 + \Delta y$

$\Delta y = \frac{1}{2\sqrt{x^2+9}} \cdot (2x)\,\Delta x = \frac{1}{2(5)}(2)(-4)(-0.2) = 0.16$

$\therefore y$ (at $x = -4.2$) is $5 + 0.16 = 5.16$

$\frac{10}{85}$

$y = \frac{x}{x+1}$, $x = 1$, $\Delta x = 0.3$

Value of y at $x = 1$ is $\frac{1}{1+1} = 1/2$

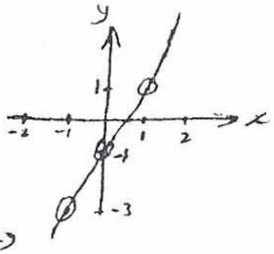
Value of y at $x = 1+.3$ is $\frac{1}{2} + \Delta y = \frac{1}{2} + \frac{x+1-x}{(x+1)^2}\Delta x$

$= \frac{1}{2} + \frac{\Delta x}{(x+1)^2} = \frac{1}{2} + \frac{0.3}{2^2} = \frac{1}{2} + \frac{.3}{4} = \frac{1}{2} + \frac{3}{40} = \frac{23}{40} = 0.575$

$\frac{2}{88}$

$f(x) = x^3 + x - 1$, $a = 0$, $b = 1$

$f(0) = -1$ & $f(1) = 1$ $\therefore f(a)$ & $f(b)$ have opposite signs.



$\therefore x_{n+1} = x_n - \frac{x_n^3 + x_n - 1}{3x_n^2 + 1}$

start with $x_0 = \frac{1}{2}$ $\therefore x_1 = 0.714$ $\therefore x_2 = 0.683$, $x_3 = 0.682$,

$x_4 = 0.682$ $\therefore$ The root is 0.682

$\frac{3}{88}$

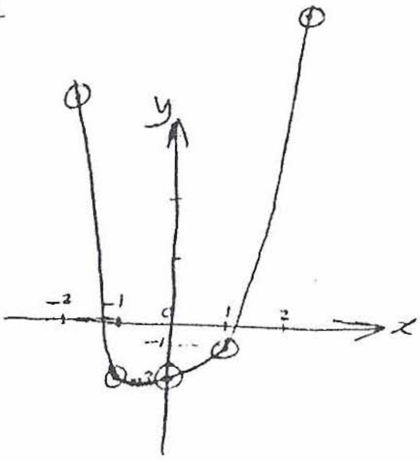
$y = x^4 + x - 3 = f(x)$, $a = 1$, $b = 2$

$f(a) = f(1) = -1$

$f(b) = f(2) = +15$

$\therefore f$ has a zero between a & b.

$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 + x_n - 3}{4x_n^3 + 1}$



Start with $\frac{a+b}{2} = \frac{1+2}{2} = 1.5$

n	0	1	2	3	4
x_n	1.5	1.254	1.172	1.164	1.164

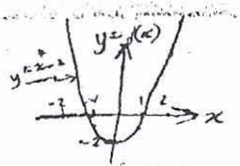
$\therefore$ The root of $x^4 + x - 3 = 0$ between 1 & 2 is $x = 1.164$

Co

$\frac{4}{88}$

$f(x)=x^4-2$, $f(1)=-1$, $f(2)=14$ $\therefore x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{x_n^4-2}{4x_n^3}$, $\therefore$ If $x_0=\frac{1+2}{2}=1.5$

$\therefore x_1=1.273$, $x_2=1.197$, $x_3=1.189$, $\underline{x_4=1.189}$ $\therefore$ The root is 1.189



$\frac{5}{88}$

$f(x)=2-x^4$

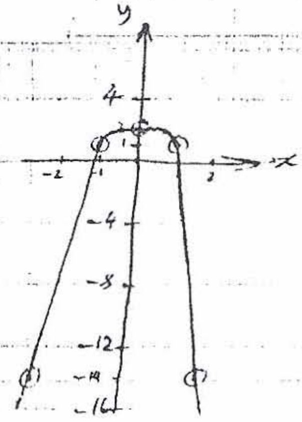
$f(-1)=2-1=1$ & $f(2)=2-16=-14$

$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{2-x_n^4}{-4x_n^3}$

If $x_0=\frac{-1-2}{2}=-1.5$ $\therefore x_1=-1.273$

$\therefore x_2=-1.197$, $x_3=-1.189$, $x_4=-1.189$

$\therefore$ The root is -1.189



$\frac{6}{88}$

$y=\sqrt{2x+1}-\sqrt{x+4}=f(x)$, $a=2$ & $b=4$

$f(a)=f(2)=\sqrt{5}-\sqrt{6}=-0.213$

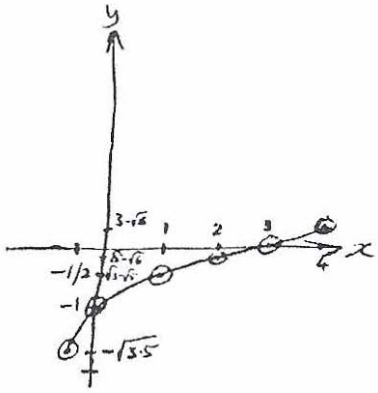
$f(b)=f(4)=\sqrt{9}-\sqrt{8}=+0.172$

$\therefore f$ has a zero between a & b.

$$\therefore x_{n+1}=x_n-\frac{\sqrt{2x_n+1}-\sqrt{x_n+4}}{\frac{2}{2\sqrt{2x_n+1}}-\frac{1}{2\sqrt{x_n+4}}}$$

Start with $\frac{a+b}{2}=\frac{2+4}{2}=3$

n	0	1
x_n	3	3



$\therefore$ The root of $\sqrt{2x+1}-\sqrt{x+4}=0$ between 2 & 4 is $x=3$.

$\frac{1}{93}$

$x=3t+1$, $y=t^2$, $\therefore t=\frac{x-1}{3}$ $\therefore y=\left(\frac{x-1}{3}\right)^2$

$\frac{dy}{dt}=2t$, $\frac{dy}{dx}=2\left(\frac{x-1}{3}\right)\cdot\frac{1}{3}=\frac{2}{9}(x-1)$, $\frac{dx}{dt}=3$

$\therefore \frac{dy/dt}{dx/dt}=\frac{2t}{3}=\frac{2}{3}t=\frac{2}{3}\left(\frac{x-1}{3}\right)=\frac{2}{9}(x-1)=\frac{dy}{dx}$.

$\boxed{\frac{3}{93}}$

$x = \frac{t}{1-t}$ ①, $y = t^2$ ② $\therefore\ x = \frac{y^{\frac{1}{2}}}{1-y^{\frac{1}{2}}} = \frac{\sqrt{y}\,(1+\sqrt{y})}{1-y}$

$\therefore\ x(1-y) = \sqrt{y} + y \quad \therefore\ (x - xy - y)^2 = y$

$\therefore\ x^2 + y^2(1+x)^2 - 2xy(1+x) = y, \therefore\ x^2 + y^2(1+x)^2 - y(1+2x+2x^2) = 0$

$\therefore\ y = \frac{1+2x+2x^2 \pm \sqrt{(1+2x+2x^2)^2 - 4(1+x)^2x^2}}{2(1+x)^2} = \frac{1+2x+2x^2 \pm \sqrt{1+4x^2(1+x)^2 + 4x(1+x) - 4x^2(1+x)^2}}{2(1+x)^2}$

$\therefore\ y = \frac{1+2x+2x^2 \pm (1+2x)}{2(1+x)^2} = \left< \begin{array}{l} \frac{1+2x+2x^2-1-2x}{2(1+x)^2} = \frac{x^2}{(1+x)^2} \\ \frac{1+2x+2x^2+1+2x}{2(1+x)^2} = 1 \text{ ✗} \end{array} \right\} = \frac{x^2}{(1+x)^2}.$

OR:

$x(1-t) = t \quad \therefore\ t = \frac{x}{(1+x)} \quad \therefore\ y = \frac{x^2}{(1+x)^2}$ ③

$\therefore\ dy/dt$ (from ②) $= 2t$

& dx/dt (from ①) $= \frac{1-t+t}{(1-t)^2} = \frac{1}{(1-t)^2}$

$\therefore\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{1/(1-t)^2} = 2t(1-t)^2$ ④

But dy/dx (from ③) $= \frac{2x(1+x)^2 - 2(1+x)x^2}{(1+x)^4} = \frac{2x+2x^2-2x^2}{(1+x)^3} = \frac{2x}{(1+x)^3}$

$=$ (using ①) $\frac{\frac{2t}{1-t}}{\left(1+\frac{t}{1-t}\right)^3} = \frac{2t}{1-t} \cdot \frac{(1-t)^3}{(1-t+t)^3} = 2t(1-t)^2 =$ ④.

$\boxed{\frac{10}{-93}}$

$y = 2v^3 + \frac{2}{v^3}$ ①, $v = (3x+2)^{2/3}$ ②

(a) $\therefore\ \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx} = (6v^2 - 6v^{-4}) \frac{2}{3} \cdot (3x+2)^{-1/3} \cdot (3) = 12(v^2 - v^{-4})(3x+2)^{-1/3} =$

$= 12\left((3x+2)^{4/3} - (3x+2)^{-8/3}\right)(3x+2)^{-1/3} = 12\left(3x+2 - (3x+2)^{-3}\right) =$

$= 12\left(3x+2 - \frac{1}{(3x+2)^3}\right)$ ③

(b) $y = 2(3x+2)^2 + \frac{2}{(3x+2)^2} \quad \therefore\ \frac{dy}{dx} = 4(3x+2)(3) - 4(3x+2)^{-3}(3)$

$= 12\left(3x+2 - \frac{1}{(3x+2)^3}\right) =$ ③

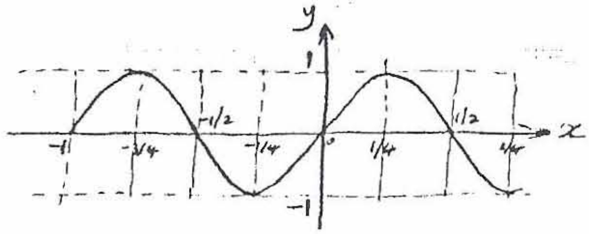
CV

3/100

$y = \sin 2\pi x.$

No shifts ; Peak = 1, period = 1

x-intercepts = 0, .., y-intercept = 0.



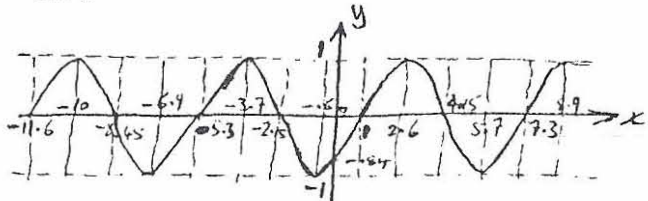
8/100

$y = \sin(x-1).$

x-shift = 1, no y-shift,

peak = 1, period = 2π = 6.3

x-intercepts = 1, .., y-intercept = sin(−1) = −0.84.



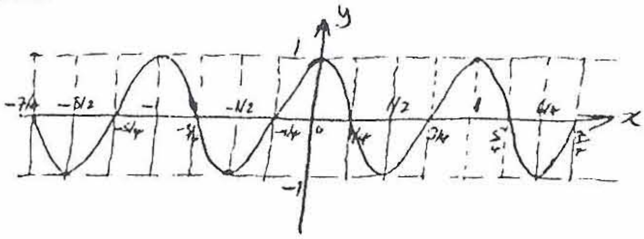
9/100

$y = \cos[2\pi(x+1)]$

x-shift = −1, no y-shift,

peak = 1, period = 1,

x-intercepts = $\frac{1}{4}$, .., y-intercept = 1.



10/100

$f(x) = 37 \sin\left[\frac{2\pi}{365}(x-101)\right] + 25$

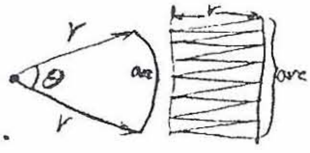
ⓐ Amplitude = peak = 37

ⓑ Period = $\frac{2\pi}{2\pi/365}$ = 365 , ⓒ Horizontal shift = 101, ⓓ Vertical = 25.

12/101

We can open the sector up to the triangles shown aside

∴ Area = $\frac{(arc) * r}{2} = \frac{(r*\theta)*r}{2} = \frac{r^2\theta}{2}$, θ in radian.



16/101

$\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B} = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$

17/101

$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B - \sin A \sin B} = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$

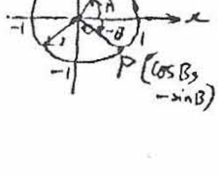
21/101

Law of cosines ∴ $\overline{PQ}^2 = \overline{OP}^2 + \overline{OQ}^2 - 2 . \overline{OP} . \overline{OQ} . \cos(A+B)$

∴ $(\cos A - \cos B)^2 + (\sin A + \sin B)^2 = 1 + 1 - 2 \cdot 1 \cdot 1 \cdot \cos(A+B)$

∴ $\cancel{\cos^2 A} - 2\cos A \cos B + \cancel{\cos^2 B} + \cancel{\sin^2 A} + 2\sin A \sin B + \cancel{\sin^2 B} = \cancel{2} - 2\cos(A+B)$

∴ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

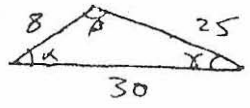


CA

بسم الله الرحمن الرحيم

$\boxed{1}$ Law of cosines, $\therefore 8^2 = 30^2 + 25^2 - 2*30*25*\cos\gamma \Rightarrow \cos\gamma = 0.974$

$\therefore \gamma = 13^\circ$



& Law of sines, $\therefore \frac{8}{\sin 13} = \frac{25}{\sin\alpha} = \frac{30}{\sin\beta}$

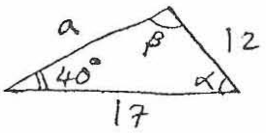
$\therefore \sin\alpha = \frac{25}{8} . \sin 13 = 0.703$

$\therefore \alpha = 45^\circ$.

& $\sin\beta = \frac{30}{8} . \sin 13 = 0.844$

$\therefore \beta = 122^\circ$.

$\boxed{2}$ Law of sines, $\therefore \frac{12}{\sin 40} = \frac{17}{\sin\beta}$



$\therefore \sin\beta = \frac{17}{12} . \sin 40 = 0.911$

$\therefore \beta = 66^\circ$ or $\beta = 114^\circ$

$\therefore \alpha = 74^\circ$ } $\therefore \alpha = 26^\circ$

$\therefore a^2 = 12^2 + 17^2 - 2(12)(17)\cos 74$ } $\therefore a^2 = 12^2 + 17^2 - 2(12)(17)\cos 26$

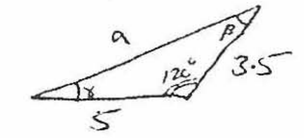
$= 321$ } $= 66$

$\therefore a = 18$ } $\therefore a = 8.1$

$\boxed{3}$ $a^2 = 3.5^2 + 5^2 - 2*5*3.5*\cos 120 = 54.8$

$\therefore a = 7.4$

$\therefore \frac{7.4}{\sin 120} = \frac{5}{\sin\beta} \quad \therefore \sin\beta = \frac{5}{7.4} . \sin 120 = .585$



$\therefore \beta = 36^\circ \quad \therefore \gamma = 24^\circ$.

$\frac{2}{107}$ $\lim_{\theta \to \pi} \frac{\sin \theta}{\pi - \theta} = \lim_{x \to 0} \frac{\sin(\pi - x)}{x} = \lim_{x \to 0} \frac{\sin x}{x} = 1 .$

$\frac{6}{107}$ $\lim_{x \to 0} \frac{x}{\sin 3x} = \lim_{x \to 0} \frac{\cancel{x}}{3\cancel{x}} = \frac{1}{3} .$

$\frac{7}{107}$ $\lim_{x \to 0} \frac{\sin 5x}{\sin 3x} = \lim_{x \to 0} \frac{5\cancel{x}}{3\cancel{x}} = 5/3 .$

$\frac{8}{107}$ $\lim_{x \to 0} \tan 2x \cdot \csc 4x = \lim_{x \to 0} \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{\sin 4x} = \frac{2\cancel{x}}{1 \cdot 4\cancel{x}} = \frac{1}{2} .$

$\frac{10}{107}$ $\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta} = \lim_{\theta \to 0} \frac{(1-\cos\theta)(1+\cos\theta)}{\theta(1+\cos\theta)} = \lim_{\theta \to 0} \frac{1-\cos^2\theta}{\theta(1+\cos\theta)} = \lim_{\theta \to 0} \frac{\sin^2\theta}{\theta(1+\cos\theta)} =$

$= \lim_{\theta \to 0} \frac{\theta^2}{\cancel{\theta}(1+\cos\theta)} = \lim_{\theta \to 0} \frac{\theta}{1+\cos\theta} = \frac{0}{1} = 0$

$\frac{14}{107}$ $\lim_{x \to \infty} x \sin \frac{1}{x} = \lim_{u \to 0} \frac{1}{u} \cdot \sin u = 1$

$\frac{18}{107}$ $\lim_{x \to 0} \frac{x^2 + 2x}{\sin 2x} = \lim_{x \to 0} \frac{x^2 + 2x}{2x} = \lim_{x \to 0} \frac{x+2}{2} = 1$

$\frac{26}{107}$ $y = \cos 5x \quad \therefore \frac{dy}{dx} = -5 \sin 5x$

$\frac{27}{107}$ $y = x^2 \sin 3x \quad \therefore y' = 2x \sin 3x + x^2 \cdot \cos 3x \cdot (3) = 2x \sin 3x + 3x^2 \cos 3x$

$\frac{30}{107}$ $y = \frac{2}{\cos 3x} \quad \therefore y' = \frac{0 - (-\sin 3x) \cdot (3) \cdot 2}{\cos^2 3x} = \frac{6 \sin 3x}{\cos^2 3x} .$

$\frac{37}{107}$ $y = \sin^2 x^2 \quad \therefore y' = 2 \sin x^2 \cdot \cos x^2 \cdot 2x = 4x \sin x^2 \cos x^2 .$

$\frac{38}{107}$ $y = \cos\left(\sin \sqrt{x^2+1}\right) \quad \therefore y' = -\sin\left(\sin\sqrt{x^2+1}\right) \cdot \cos\sqrt{x^2+1} \cdot \frac{1}{2} \cdot \frac{2x}{\sqrt{x^2+1}} =$
$= -\frac{x}{\sqrt{x^2+1}} \cdot \left(\cos \sqrt{x^2+1}\right) \cdot \sin\left(\sin\sqrt{x^2+1}\right) .$

$\frac{39}{107}$ $x \sin 2y = y \cos 2x \quad \therefore \sin 2y + x(\cos 2y) \cdot 2y' = y' \cos 2x - 2(\sin 2x) \cdot y$
$\therefore y' = \frac{\sin 2y + 2y \sin 2x}{\cos 2x - 2x \cos 2y} .$

$\frac{40}{107}$ $y^2 = \sin^4 2x + \cos^4 2x \Rightarrow y' = \frac{4 \sin^3 2x (\cos 2x) \cancel{2} + 4 \cos^3 2x (-\sin 2x) \cdot \cancel{2}}{2y} =$
$= \frac{4 \sin 2x \cos 2x}{y} \cdot \left(\sin^2 2x - \cos^2 2x\right) = -\frac{2}{y} \sin 4x \cdot \cos 4x = -\frac{\sin 8x}{2 \cdot y} .$

$\frac{42}{107}$

$v_x = \frac{dx}{dt} = -a\omega \sin\omega t \quad \therefore a_x = \frac{dv_x}{dt} = -a\omega^2 \cos\omega t = -\omega^2 x.$

$v_y = \frac{dy}{dt} = b\omega \cos\omega t \quad \therefore a_y = \frac{dv_y}{dt} = -b\omega^2 \sin\omega t = -\omega^2 y.$

$\frac{2}{117}$

ⓐ $f(x) = \frac{x}{x+1}$, $\therefore f$ is discontineous at $x=-1$, because

$\lim_{x\to -1^+} = \frac{-1}{0^+} = -\infty$ f $\lim_{x\to -1^-} = \frac{-1}{0^-} = +\infty.$

ⓑ $f(x) = \frac{x+1}{x^2-4x+3} = \frac{x+1}{(x-1)(x-3)}$, $\therefore f$ is discontineous at $x = 1$ f 3

because at $x=1$ $\lim_{x\to 1^+} f(x) = \frac{2}{0^+\cdot(-2)} = -\infty$ f $\lim_{x\to 1^-} f(x) = \frac{2}{0^-(-2)} = +\infty$

f at $x=3$ $\lim_{x\to 3^+} f(x) = \frac{4}{2\cdot 0^+} = +\infty$ f $\lim_{x\to 3^-} f(x) = \frac{4}{2\cdot 0^-} = -\infty$

$\frac{3}{117}$

$f(x) = \frac{x^2-1}{x-1}$ (f $x \neq 1$) $= \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x+1$

$\therefore f(x) = \begin{cases} x+1 & \text{when } x \neq 1 \\ 2 & \text{when } x=1 \end{cases}$

$\therefore \lim_{x\to 1^+} = 1+1 = 2$ f $\lim_{x\to 1^-} = 1+1 = 2$ f $f(1) = 2$

$\therefore f$ is contineous because $f(1) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$.

$\frac{10}{117}$

For $x \neq 2$, $f(x) = \frac{x^2+3x-10}{x-2} = \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}} = x+5$

$\lim_{x\to 2^+} f(x) = 7$ f $\lim_{x\to 2^-} f(x) = 7$

$\therefore$ for $f(x)$ to be contineous at $x=2$ $f(2)$ must be assigned the value of 7.

$\frac{2}{121}$

$y^2 = (3x^2+1)^{3/2} \quad \therefore dy = \frac{\frac{3}{2}(3x^2+1)^{\frac{1}{2}}\cdot 6x\,dx}{2y} = \frac{9x}{2y}\cdot(3x^2+1)^{1/2}\,dx.$

$\frac{9}{121}$

Let $y = \sqrt{x}$, $x = 144$, $\Delta x = 1 \quad \therefore y\,(\text{at } x=144) = 12$

f $y\,(\text{at } x = 145) = 12 + \Delta y = 12 + \frac{\Delta x}{2\sqrt{x}} = 12 + \frac{1}{2\times 12} = \frac{288+1}{24} = \frac{289}{24} = 12.04$

$\boxed{\frac{11}{121}}$

let $y = x^{1/4}$ and $x = 16$ and $\Delta x = 1$

$\therefore y(\text{at } x=16)$ is $16^{1/4} = 2$, and y at $(x=16+1=17)$ is $2+\Delta y =$

$2 + \frac{1}{4}x^{-3/4}\cdot\Delta x = 2 + \frac{1}{4}(16)^{-3/4}\cdot(1) = 2 + \frac{1}{4}\left(\frac{1}{8}\right)(1) = \frac{64+1}{32} = \frac{65}{32} = 2.03$

$\therefore (17)^{1/4} \cong 2.03$

$\boxed{\frac{12}{121}}$

Let $x = 0.125$ & $\Delta x = 0.001$ and $\sqrt[3]{x} = f(x)$

$\therefore \sqrt[3]{0.126} = \sqrt[3]{0.125+0.001} = f(x+\Delta x) \simeq f(x) + \Delta f(x) = f(x) + f'(x)\cdot\Delta x =$

$= \sqrt[3]{0.125} + \frac{1}{3}x^{\frac{-2}{3}}\Delta x = .5 + \frac{1}{3}(.125)^{\frac{-2}{3}}\cdot(.001) = .5 + \frac{1}{3}\cdot(.5)^{-2}\cdot(.001) =$

$= .5 + \frac{1}{3}\cdot(4)\cdot(.001) = .5 + .00133 = .50133$

$\therefore \sqrt[3]{0.126} \simeq 0.50133$

$\boxed{\frac{13}{121}}$

let $y = x^{4/3} + x^2 - x^{-1/3}$ at $x = 8$ $\therefore y = 16 + 64 - \frac{1}{2} = 79.5$

If x become 8.01 $\therefore dx = 0.01$

$\therefore dy = \left(\frac{4}{3}x^{1/3} + 2x + \frac{1}{3}x^{-4/3}\right)dx = \left[\frac{4}{3}(2) + 16 + \frac{1}{3}\left(\frac{1}{16}\right)\right] * 0.01 = \frac{897}{4800} = .187$

$\therefore y\ (\text{at } x = 8.01) = 79.5 + 0.187 = 79.687.$

$\boxed{\frac{17}{121}}$

$x = \frac{t-1}{t+1}$, $y = \frac{t+1}{t-1}$

ⓐ $\therefore dx = \frac{t+1-(t-1)}{(t+1)^2}dt = \frac{2}{(t+1)^2}\cdot dt$ & $dy = \frac{t-1-(t+1)}{(t-1)^2}dt = \frac{-2}{(t-1)^2}\cdot dt$

ⓑ $\frac{dy}{dx}\ (\text{at } t=2) = \frac{-2/(t-1)^2}{2/(t+1)^2} = -\frac{(t+1)^2}{(t-1)^2} = -\frac{3^2}{1^2} = -9.$

ⓒ at $t=2$ $\therefore x = \frac{1}{3}$, $y = \frac{3}{1} = 3$ $\therefore$ The point is $\left(\frac{1}{3}, 3\right)$

$\therefore$ Equation of tangent is $\frac{y-3}{x-\frac{1}{3}} = -9$ or $y + 9x - 6 = 0.$

$\boxed{\frac{20}{121}}$

$x = \frac{t-1}{t+1}$, $y = \frac{t+1}{t-1}$, $y' = \frac{dy}{dx} = -\left(\frac{t+1}{t-1}\right)^2$

$\therefore y'' = \frac{dy'}{dx}\cdot\frac{dt}{dt} = \frac{dy'/dt}{dx/dt} = \frac{-2\left(\frac{t+1}{t-1}\right)\left(\frac{t-1-(t+1)}{(t-1)^2}\right)}{\frac{2}{(t+1)^2}} = \frac{(t+1)^2}{2}\cdot\frac{(-2)(t+1)(-2)}{(t-1)^3}$

$\therefore y'' = \frac{d^2y}{dx^2} = 2\cdot\left(\frac{t+1}{t-1}\right)^3 = 2y^3 = \frac{2}{x^3}.$

$\boxed{\frac{21}{121}}$

$x = f(t)$, $y = g(t)$ $\therefore y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'}{f'}$

$\therefore \frac{d^2y}{dx^2} = \frac{dy'}{dx}\cdot\frac{dt}{dt} = \frac{dy'/dt}{dx/dt} = \frac{\frac{g''f'-f''g'}{f'^2}}{f'} = \frac{g''f'-f''g'}{f'^3}$

$\therefore \frac{d^2y}{dx^2} = \frac{\left(\frac{dx}{dt}\right)\cdot\frac{d^2y}{dt^2} - \frac{d^2x}{dt^2}\cdot\frac{dy}{dt}}{(dx/dt)^3}.$

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$\boxed{\frac{6}{123}}$ $y = (x+1)^2 (x^2+2x)^{-2}$ $\therefore$ $y' = 2(x+1)(x^2+2x)^{-2} - 2(x^2+2x)^{-3}(2x+2)(x+1)^2$

$= (x+1)(x^2+2x)^{-3} \cdot \left[2(x^2+2x) - 2(2x+2)(x+1)\right] =$

$= 2(x+1)(x^2+2x)^{-3} \cdot (x^2+2x-2x^2-4x-2) = -2(x+1)(x^2+2x+2)(x^2+2x)^{-3}.$

$\boxed{\frac{11}{123}}$ $y = x^2\sqrt{x^2-a^2}$ $\therefore$ $y' = 2x\sqrt{x^2-a^2} + x^2 \cdot \frac{1}{2} \cdot \frac{2x}{\sqrt{x^2-a^2}} = \frac{4x(x^2-a^2)+2x^3}{2\sqrt{x^2-a^2}}$

$\therefore$ $y' = \frac{3x^3-2a^2x}{\sqrt{x^2-a^2}}$.

$\boxed{\frac{16}{123}}$ $y^3 = \sin^3 x + \cos^3 x$ $\therefore$ $\cancel{3}y^2y' = \cancel{3}\sin^2 x \cdot \cos x - \cancel{3}\cos^2 x \sin x$

$\therefore$ $y' = \frac{\sin x \cdot \cos x(\sin x - \cos x)}{y^2}$.

$\boxed{\frac{21}{123}}$ $x^{2/3} + y^{2/3} = a^{2/3}$ $\therefore$ $\cancel{\frac{2}{3}}x^{-1/3} + \cancel{\frac{2}{3}}y^{-1/3} \cdot y' = 0$ $\therefore$ $y' = -\sqrt[3]{\frac{y}{x}} = -\left(\frac{y}{x}\right)^{1/3}$

$\boxed{\frac{32}{123}}$ $x = \frac{t}{1+t^2}$, $y = 1+t^2$ $\therefore$ $y' = \frac{dy/dt}{dx/dt} = \frac{2t}{\frac{1+t^2-2t\cdot t}{(1+t^2)^2}} = \frac{2t(1+t^2)^2}{1-t^2}$.

$\boxed{\frac{34}{123}}$ $y = \frac{x}{x^2+1}$ $\therefore$ $y' = \frac{x^2+1-2x\cdot x}{(x^2+1)^2} = \Big|_{\text{at the origin}} \frac{1}{1^2} = 1$

$\therefore$ Equation of tangent is $y = x$.

$\boxed{\frac{35}{123}}$ $x^2 - 2xy + y^2 + 2x + y - 6 = 0$. $\therefore$ $2x - 2y - 2xy' + 2yy' + 2 + y' = 0$

at $(2,2)$ $\therefore$ $\cancel{4} - \cancel{4} - \cancel{4y'} + \cancel{4y'} + 2 + y' = 0$ $\therefore$ $y' = -2$

$\therefore$ $\frac{y-2}{x-2} = -2$ or, $y + 2x - 6 = 0$ is the equation of the tangent at $(2,2)$.

$\boxed{\frac{36}{123}}$ Equation of the straight line is $\frac{y-3}{x} = \frac{-2-3}{5-0} = -1$ = slope of tangent

$\because$ point of tangency satisfy both equations $\therefore$ $y(x+1) = c \Rightarrow y \cdot \left(\frac{y-3}{-1}+1\right) = c$ (1)

& $y'(x+1) + y(1) = 0$ $\therefore$ $-1\left(\frac{y-3}{-1}+1\right) + y = 0$ $\Rightarrow$ $y - 3 - 1 + y = 0$ $\therefore$ $y = 2$

$\therefore$ $x = \frac{2-3}{-1} = 1$ $\therefore$ point of tangency is $(1,2)$ and c (from (1)) is 4.

$\boxed{\frac{39}{123}}$

ⓐ $y = (x^2+2x)^5 \quad \therefore y' = 5(x^2+2x)^4 \cdot (2x+2) = 10(x+1)(x^2+2x)^4.$

ⓑ $f = (3t^2-2t)^{1/2} \quad \therefore f' = \frac{1}{2}(3t^2-2t)^{-1/2}(6t-2) = \frac{3t-1}{\sqrt{3t^2-2t}}.$

ⓒ $f = \sqrt{r^2+5} + \sqrt{r^2-5} \quad \therefore f' = \frac{\cancel{2}r}{\cancel{2}\sqrt{r^2+5}} + \frac{\cancel{2}r}{\cancel{2}\sqrt{r^2-5}} = r \cdot \left(\frac{1}{\sqrt{r^2+5}} + \frac{1}{\sqrt{r^2-5}}\right).$

ⓓ $f = \frac{x^2-1}{x^2+1}$

$\therefore f' = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}.$

$\boxed{\frac{52}{124}}$

$$\frac{d\sqrt{x^2+16}}{d\left(\frac{x}{x-1}\right)} = \frac{\frac{\cancel{2}x\,\cancel{dx}}{\cancel{2}\sqrt{x^2+16}}}{\frac{x-1-x}{(x-1)^2}\cancel{dx}} = \frac{x}{\sqrt{x^2+16}} \cdot \frac{(x-1)^2}{(-1)}\Bigg|_{\text{at } x=3} = \frac{3}{\sqrt{9+16}} \cdot \frac{2^2}{(-1)} = -\frac{12}{5}.$$

$\boxed{\frac{55}{124}}$

$x = y^2+y, \quad u = (x^2+x)^{3/2}$

$\therefore \frac{du}{dy} = \frac{du}{dx} \cdot \frac{dx}{dy} = \frac{3}{2}(x^2+x)^{1/2} \cdot (2x+1) \cdot (2y+1), \quad \therefore \frac{dy}{du} = \frac{2/3}{(2x+1)(2y+1)\sqrt{x^2+x}}.$

$\boxed{\frac{57}{124}}$

$f'(x) = \sin x^2, \quad y = f\left(\frac{2x-1}{x+1}\right) \quad \therefore \frac{dy}{dx} = f'\left(\frac{2x-1}{x+1}\right) \cdot \frac{2(x+1)-(2x-1)}{(x+1)^2} =$

$= \left[\sin\left(\frac{2x-1}{x+1}\right)^2\right] \cdot \frac{3}{(x+1)^2} = \frac{3\sin[(2x-1)/(x+1)]^2}{(x+1)^2}.$

$\boxed{\frac{62}{124}}$

$x = t-t^2, \quad y = t-t^3 \quad \therefore y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-3t^2}{1-2t}\Bigg|_{\text{at } t=1} = \frac{1-3}{1-2} = \frac{-2}{-1} = 2$

& $\frac{d^2y}{dx^2} = \frac{dy'}{dx} \cdot \frac{dt}{dt} = \frac{dy'/dt}{dx/dt} = \frac{\frac{-6t(1-2t)-(-2)(1-3t^2)}{(1-2t)^2}}{1-2t} =$

$= \frac{-6t+12t^2+2-6t^2}{(1-2t)^3} = \frac{6t^2-6t+2}{(1-2t)^3}\Bigg|_{\text{at } t=1} = \frac{6-6+2}{(1-2)^3} = \frac{2}{-1} = -2$

$\therefore y'(\text{at } t=1) = 2$ & $y''(\text{at } t=1) = -2$

$\boxed{\frac{64}{124}}$

ⓐ $y = (2x-1)^{1/2} \quad \therefore y' = \frac{1}{\cancel{2}}(2x-1)^{-1/2}(\cancel{2}), \quad \therefore y'' = -\frac{1}{\cancel{2}}(2x-1)^{-3/2}(\cancel{2}), \quad \therefore y''' = \frac{3}{\cancel{2}}(2x-1)^{-5/2}(\cancel{2})$

$\therefore d^3y/dx^3 = y''' = 3(2x-1)^{-5/2}.$

ⓑ $y = (3x+2)^{-1} \quad \therefore y' = -(3x+2)^{-2}(3) \quad \therefore y'' = 2(3x+2)^{-3}(9) \quad \therefore y''' = -6(3x+2)^{-4}(27)$

$\therefore d^3y/dx^3 = y''' = -162(3x+2)^{-4}.$

ⓒ $y = ax^3+bx^2+cx+d \quad \therefore y' = 3ax^2+2bx+c$

$\therefore y'' = 6ax+2b \quad \therefore y''' = 6a$

$\therefore d^3y/dx^3 = y''' = 6a.$

$\boxed{\frac{70}{124}}$

ⓐ $y = \frac{x^2}{1+x} \quad \therefore dy = \frac{2x(1+x)-x^2}{(1+x)^2} \cdot dx = \frac{2x+x^2}{(1+x)^2} \cdot dx.$

ⓑ $x^2-y^2 = 1 \quad \therefore \cancel{2}x\,dx - \cancel{2}y\,dy = 0 \quad \therefore dy = \frac{x\,dx}{y}.$

ⓒ $xy+y^2 = 1 \quad \therefore x\,dy + y\,dx + 2y\,dy = 0 \quad \therefore dy = -\frac{y\,dx}{2y+x}.$

$\boxed{\frac{5}{127}}$

$y = x^4 - 8x^2 + 16 = (x^2-4)^2$

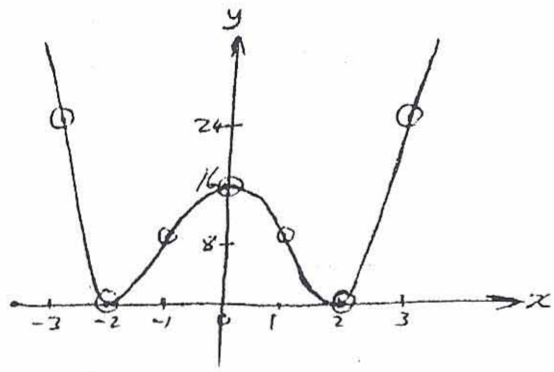
$\therefore\ y' = 2(x^2-4)(2x) = 4x(x^2-4)$

$= 4x(x-2)(x+2)$

x	$-\infty \quad -2 \quad 0 \quad 2 \quad \infty$
x	$- - - - - - - 0 + + + + + +$
$x-2$	$- - - - - - - - - - - 0 + + + +$
$x+2$	$- - - 0 + + + + + + + + + + +$
y'	$- - - 0 + + + 0 - - - 0 + + +$

∴ falling in $(-\infty,-2) \cup (0,2)$

Rising in $(-2,0) \cup (2,\infty)$

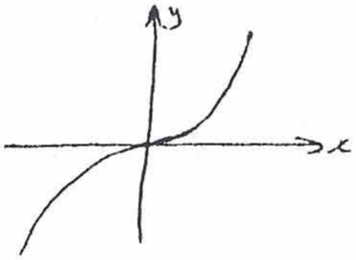


$\boxed{\frac{9}{127}}$

$y = x|x| = x\sqrt{x^2}$

$\therefore\ y' = \sqrt{x^2} + \frac{(2x)}{2\sqrt{x^2}}\cdot x = \frac{x^2+x^2}{\sqrt{x^2}} = \frac{2x^2}{|x|}$

∴ $y' = 2|x|$ always positive i.e rising.

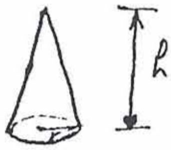


$\boxed{\frac{3}{131}}$

$dV/dt = 10\ ft^3/min \quad \& \ r = \frac{h}{2},\ h = 5ft$

Since $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$

$\therefore\ dV/dt = \frac{3\pi}{12} h^2\, dh/dt \quad \therefore\ \frac{dh}{dt} = \frac{10}{\frac{\pi}{4}5^2} = \frac{8}{5\pi} = 0.51\ ft/min.$

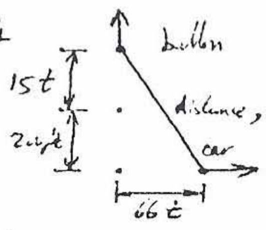


$\boxed{\frac{8}{131}}$

$D^2 = (200+15t)^2 + (66t)^2 \quad at\ t=1 \quad \therefore D = 224.9ft$

$\therefore\ \cancel{2}D\frac{dD}{dt} = \cancel{2}(200+15t)(15) + \cancel{2}(66t)(66)$

$\therefore\ \frac{dD}{dt} = \Big|_{at\ t=1sec} \frac{(200+15)(15)+66^2}{224.9} = 33.7\ ft/sec$



$\boxed{\frac{10}{131}}$

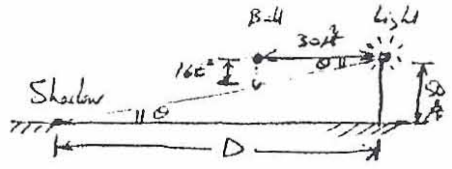
$x^2+y^2=1 \quad \therefore\ 2x+2y\frac{dy}{dx}=0 \quad \therefore\ \frac{dy}{dx} = -\frac{x}{y} \quad \therefore\ dy = -\frac{x}{y}dx$

$\therefore\ \frac{dy}{dt} = -\frac{x}{y}\cdot\frac{dx}{dt} = -\frac{x}{y}\cdot y = -x \quad \therefore\ \frac{dy}{dt} = -x$ ∴ The velocity is negative in the right half of the circle & positive in the left ∴ The particle moves with the clock.

$\frac{13}{132}$

By trigmometry, $\therefore \frac{D}{50} = \frac{30}{16t^2}$ $\therefore D = \frac{1500}{16t^2}$

$\therefore \frac{dD}{dt} = -2\left(\frac{1500}{16t^3}\right)\Big|_{at\ t=\frac{1}{2}} = -1500$ ft/sec.

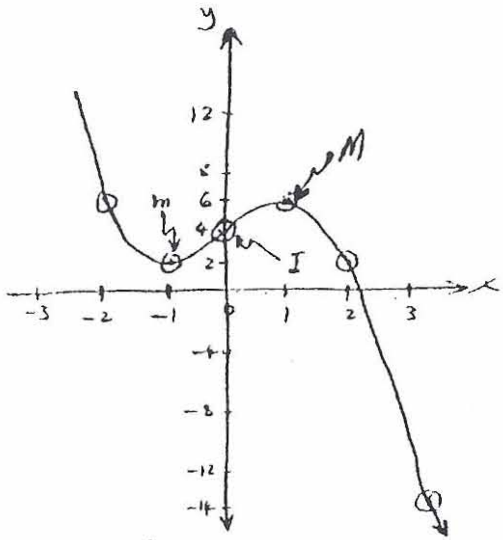


$\frac{3}{137}$

$y = 4 + 3x - x^3$

$y' = 3 - 3x^2 = 3(1-x^2) = 3(1-x)(1+x)$

x	$-\infty$	-1	1	∞
$(1-x)$	$+ + + + + + + 0 - - - -$			
$(1+x)$	$- - - 0 + + + + + + + +$			
y'	$- - - - 0 + + + 0 - - - -$			



ⓐ y is rising in $x \in (-1, 1)$

ⓑ y is falling in $x \in (-\infty, -1) \cup (1, \infty)$

$y'' = -6x$

$\therefore$ ⓒ y concaves up in $x \in (-\infty, 0)$

ⓓ y concaves down in $x \in (0, \infty)$

$\frac{14}{138}$

$y = x^4 - 32x + 48$

$\therefore y' = 4x^3 - 32$

$\& \ y'' = 12x^2$

$y' = 0$ at $x^3 = 8$ $\therefore x = 2$ $\therefore y = 2^4 - 32(2) + 48 = 0$

$y''(at\ x = 2) = 48$ (positive) $\therefore (2, 0)$ is a minimum

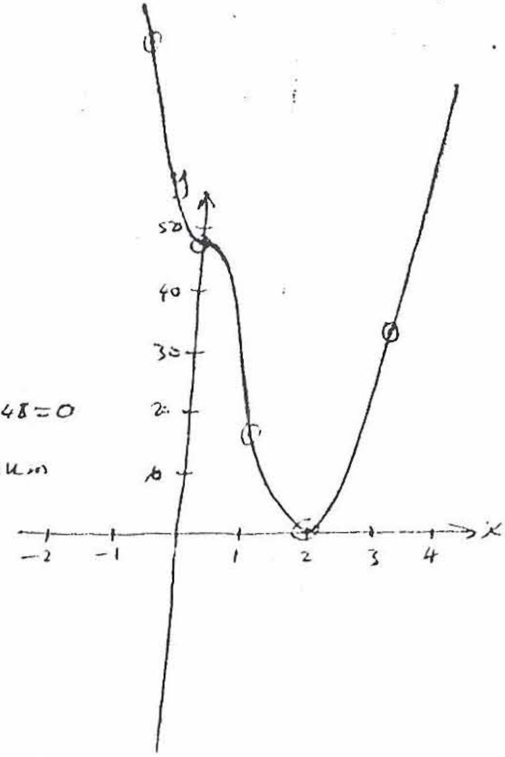
$\therefore$ Low turning points are just one at $(2, 0)$

$\&$ High turning points are none

Inflection at $y'' = 0$ $\therefore x = 0$ $\therefore y = 48$

$\therefore$ point of inflection is $(0, 48)$

x	-2	-1	0	1	2	3	4
y	128	81	48	17	0	33	176



$\frac{15}{138}$

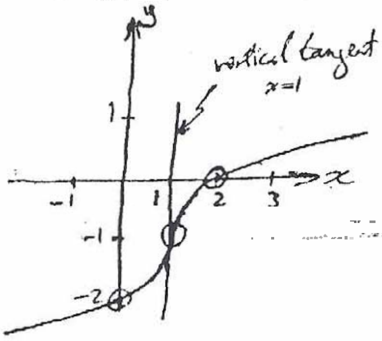
$x = y^3 + 3y^2 + 3y + 2$

At vertical tangents slope $= \frac{dy}{dx} = \infty$

$\therefore \frac{dx}{dy} = 0 \quad \therefore 3y^2 + 6y + 3 = 0$

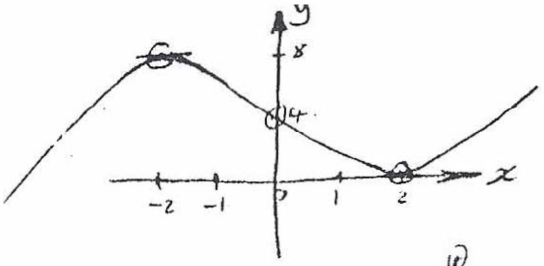
$\therefore 3(y+1)^2 = 0 \qquad \therefore x = -1 + 3 - 3 + 2 = 1$

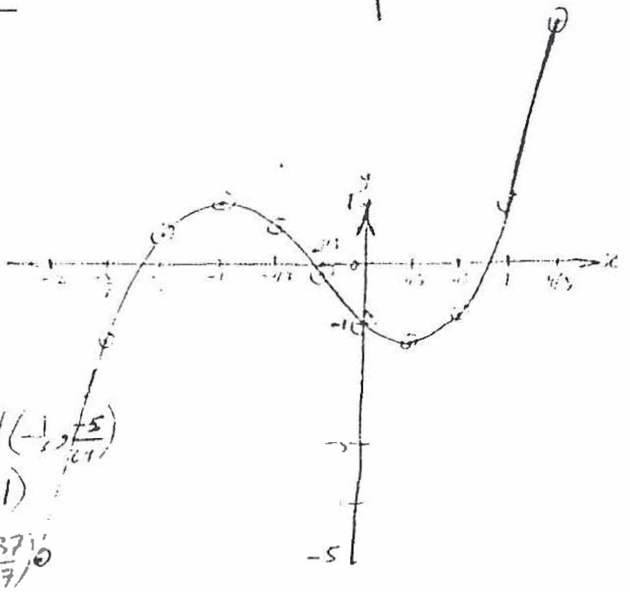
$\therefore$ The vertical tangent is $x = 1$, tangent at $(1, -1)$



$\frac{19}{138}$

The required curve is shown aside





$\frac{3}{138}$

$y = 2x^3 + 2x^2 - 2x - 1$

$y' = 2(3x^2 + 2x - 1) = 2(3x - 1)(x + 1)$

$\therefore$ Extrema at $x = -1, -\frac{5}{3}$

$\therefore y'' = 2(6x + 2) = 4(3x + 1) \therefore$ Inflection at $\left(-\frac{1}{3}, \pm\frac{5}{27}\right)$

$\therefore$ at $x = -1 \quad \therefore y'' = -8 \quad \therefore$ Max. is at $(-1, -1)$

$\therefore$ at $\frac{1}{3} \quad \therefore y'' = 8 \quad \therefore$ Min. is at $\left(\frac{1}{3}, \frac{-37}{27}\right)$

x	-2	$-5/3$	$-4/3$	-1	$-2/3$	$-1/3$		$1/3$	$2/3$	1	$4/3$
y	-5	$-\frac{37}{27}$	$\frac{13}{27}$	1	$\frac{17}{27}$	$-\frac{5}{27}$	-1	$-\frac{37}{27}$	$-\frac{23}{27}$	1	$\frac{135}{27}$

(a) The curve crosses the x-axis three times at $x \cong -1.47, -.40$ & 0.867

(b) The [illegible] +3, all y-values will now cross the x-axis once only at $x \cong -1.83$

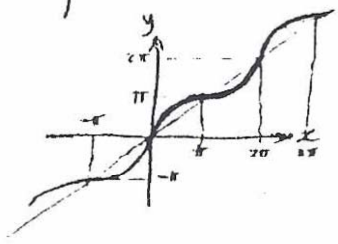
(c) Curve after adding -3 to all y-values will cross the x-axis once at $x = 1.17$

$\frac{24}{138}$

$y = x + \sin x$

$y' = 1 + \cos x \geq 0 \quad \therefore x = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$

$y'' = -\sin x = 0$ at the above $x \therefore$ no extreama.



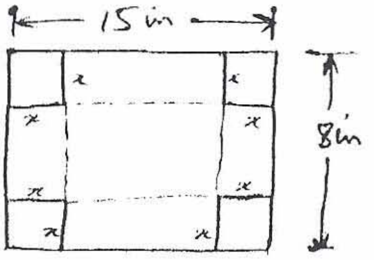
$\frac{4}{151}$ Let the side of the square be x

$\therefore$ Volume, $V = x\cdot(15-2x)\cdot(8-2x) = 4x^3-46x^2+120x$

$\therefore$ Max. volume at $\frac{dV}{dx}=0$

$\therefore$ $12x^2-92x+120=0$ $\therefore x=\frac{92\pm\sqrt{92^2-4(12)(120)}}{2(12)}$

$=\frac{92\pm52}{24} = <$ 6 X no width is left / 1.67 in



$\therefore$ Dimensions of box are 1.67 in, 4.7 1", & 11.7 in

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$\frac{8}{151}$ Equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

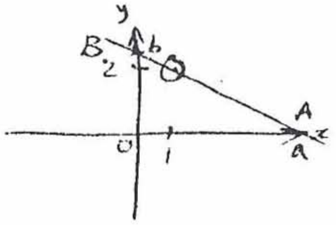
Since $(1,2)$ is on the line $\therefore \frac{1}{a}+\frac{2}{b}=1$

$\therefore$ $b=\frac{2}{1-\frac{1}{a}}=\frac{2a}{a-1}$

Area of $\Delta OAB, R = \frac{a\cdot b}{2} = \frac{a}{2}\cdot\frac{2a}{a-1} = \frac{a^2}{a-1}$

$\therefore$ $R'=0$ $\therefore \frac{2a(a-1)-a^2}{(a-1)^2}=0$ $\therefore a^2-2a=0$ $\therefore a=0$ or $a=2$

When $a=0$ no triangle is there $\therefore$ $a=2$ & area, $R=\frac{2^2}{2-1}=4$.



$\frac{12}{151}$ $48EIy = w(2x^4-5Lx^3+3L^2x^2)$ $\therefore y'=0=8x^3-15Lx^2+6L^2x$

$\therefore$ $\left(\frac{x}{L}\right)\left(8\left(\frac{x}{L}\right)^2-15\left(\frac{x}{L}\right)+6\right)=0$ $\therefore \frac{x}{L}=0$ X not accepted because its wall-end

or $\frac{x}{L}=\frac{15\pm\sqrt{15^2-4(8)(6)}}{2(8)}=\frac{15\pm\sqrt{33}}{16}= <$ 1.3 OR 0.58 $\therefore x = <$ 1.3L OR 0.58L

$y''=24x^2-30Lx+6L^2 = <$ x=1.3L +ve / x=.58L −ve

$\therefore$ At $x=0.58L$ the maximum deflection occurs.

$\frac{20}{152}$ By trigonometry, $\therefore \frac{R-r}{h}=\frac{R}{H}$ $\therefore h=H\left(1-\frac{r}{R}\right)$

Volume of cone, $V=\frac{1}{3}\pi R^2H$

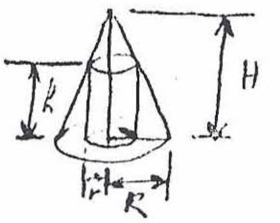
Volume of cylinder, $v=\pi r^2h=\pi r^2H\left(1-\frac{r}{R}\right)=\pi H\left(r^2-\frac{r^3}{R}\right)$.

$\therefore$ $\pi H(2r-3r^2/R)=0$ $\therefore r(2-3r/R)=0$

$\therefore$ $r=0$ X not accepted because the volume, v is then zero,

or $2-3r/R=0$ $\therefore r=\frac{2}{3}R$ $\therefore v=\pi H\left(\left(\frac{2}{3}R\right)^2-\left(\frac{2}{3}R\right)^3/R\right)=\pi HR^2\left(\frac{4}{9}-\frac{8}{27}\right)$

$\therefore$ $\frac{v}{V}=\frac{\pi R^2H\left(\frac{12-8}{27}\right)}{\pi R^2H/3}=\frac{3*4}{27}=\frac{12}{27}=\frac{4}{9}$, $\therefore v=\frac{4}{9}V$.



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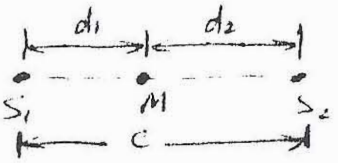
23/152

Intensity, $i \propto \frac{\text{strengths, s}}{(\text{distance, d})^2}$, $\therefore i = K\cdot\frac{s}{d^2}$



$\therefore i_1 = K\cdot\frac{S_1}{(d_1)^2}$ & $i_2 = K\cdot\frac{S_2}{d_2^2}$

$\because S_1 = a$ & $S_2 = b$ & $d_2 = c - d_1$

$\therefore i_1 = \frac{Ka}{d_1^2}$ & $i_2 = \frac{Kb}{(c-d_1)^2}$

$\therefore$ Intensity, i at point M is $i = i_1 + i_2 = K\left(\frac{a}{d_1^2} + \frac{b}{(c-d_1)^2}\right)$

We want i to be min. $\therefore i' = 0$

$\therefore \cancel{-2}\cdot\frac{a}{d_1^3} \cancel{-2}\cdot\frac{b}{(c-d_1)^3}(-1) = 0$ $\therefore \frac{a}{d_1^3} = \frac{b}{(c-d_1)^3}$

$\therefore \frac{b}{a} = \left(\frac{c-d_1}{d_1}\right)^3$ $\therefore d_1 = c\Big/\left[\left(\frac{b}{a}\right)^{1/3} + 1\right]$

27/152

Let the area of a cube be A_c & that of a sphere be A_s.

$\therefore A_c + A_s =$ constant, c $\therefore A_s = c - A_c$

$\therefore$ Volume of a cube, $V_c = \left(\frac{A_c}{6}\right)^{3/2}$, where its side $S = \sqrt{\frac{A_c}{6}}$.

& Volume of a sphere, $V_s = \frac{4\pi}{3}\left(\frac{A_s}{4\pi}\right)^{3/2}$, where its diameter $D = \sqrt{\frac{A_s}{\pi}}$.

$\therefore$ Sum of volumes, $V = V_c + V_s$ is extream when $V' = 0$

$\therefore V = \left(\frac{A_c}{6}\right)^{3/2} + \frac{4\pi}{3}\left(\frac{c-A_c}{4\pi}\right)^{3/2}$

$\therefore V' = 0 = \cancel{\frac{3}{2}}\left(\frac{A_c}{6}\right)^{1/2}\cdot\left(\frac{1}{6}\right) + \cancel{\frac{3}{2}}\cdot\frac{4\pi}{3}\left(\frac{c-A_c}{4\pi}\right)^{1/2}\cdot\left(\frac{-1}{4\pi}\right)$

$\therefore \frac{\cancel{1}}{\cancel{6}2}\cdot\sqrt{\frac{A_c}{6}} = \frac{\cancel{1}}{\cancel{3}}\sqrt{\frac{c-A_c}{4\pi}}$ $\therefore \frac{A_c}{24} = \frac{c-A_c}{4\pi}$ $\therefore A_c = c\Big/\left(\frac{4\pi}{24}+1\right)$

$\therefore A_c = \frac{6c}{\pi+6}$, to know max. or min. $\therefore V'' = \frac{1}{2}\left(\frac{A_c}{6}\right)^{-\frac{1}{2}}\frac{1}{36} + \frac{4\pi}{3}\left(\frac{c-A_c}{4\pi}\right)^{-1/2}\frac{1}{(4\pi)^2}$

$\therefore V''$ is always positive $\therefore$ At the above A_c, sum of volumes is min.

ⓐ $\therefore S/D = \sqrt{(A_c/6)/[(c-A_c)/\pi]} = \sqrt{\frac{\pi}{6}\cdot\frac{A_c}{c-A_c}} = \sqrt{\frac{\pi}{6}\cdot\frac{1}{\frac{c}{A_c}-1}} = \sqrt{\frac{\pi}{6}\cdot\frac{1}{\frac{\pi+6}{6}-1}}$

$= \sqrt{\frac{\pi}{6}\cdot\frac{1}{\frac{\pi+6-6}{6}}} = \sqrt{\frac{\pi}{6}\cdot\frac{6}{\pi}} = 1$ $\therefore S/D = 1$ or $S = D$ for min. sum of volumes.

ⓑ Since this is the only extreama $\therefore$ The other max. will be at either ends.

Let $A_s = 0$ $\therefore V_s = 0$ & $V = V_c = \left(\frac{c}{6}\right)^{3/2} = .068\,c^{3/2}$

Let $A_c = 0$ $\therefore V_c = 0$ & $V = V_s = \frac{4\pi}{3}\left(\frac{c}{4\pi}\right)^{3/2} = .094\,c^{3/2}$

By comparison, $\therefore$ Volume, V, is max, when $A_c = 0$ $\therefore \frac{S}{D} = \frac{0}{D} = 0$.

$\boxed{\frac{45}{143}}$

Let the base be b, perimeter p, other side x $\therefore$ The third is $p-b-x$

$\because A=\sqrt{\frac{p}{2}\cdot\left(\frac{p}{2}-b\right)\cdot\left(\frac{p}{2}-x\right)\cdot\left[\frac{p}{2}-(p-b-x)\right]}=\sqrt{\frac{p}{2}\left(\frac{p}{2}-b\right)}\cdot\left(\frac{p}{2}-x\right)^{1/2}\cdot\left(-\frac{p}{2}+b+x\right)^{1/2}$

$\therefore A'=\frac{dA}{dx}=0 \quad \therefore -\frac{1}{2}\left(\frac{p}{2}-x\right)^{-\frac{1}{2}}\left(b+x-\frac{p}{2}\right)^{1/2}+\left(\frac{p}{2}-x\right)^{1/2}\cdot\frac{1}{2}\left(b+x-\frac{p}{2}\right)^{-1/2}=0 \quad \therefore \frac{p}{2}-x=b+x-\frac{p}{2}$

$\therefore x=\frac{p-b}{2} \quad \therefore$ the two sides are $\frac{p-b}{2}$ & $p-b-\frac{p-b}{2}=\frac{p-b}{2}$ (2 Equal sides).

$\boxed{\frac{6}{181}}$

$\frac{dy}{dx}=2xy^2 \quad \therefore \frac{dy}{y^2}=2x\,dx \quad \therefore \int y^{-2}dy=\int 2x\,dx$

$\therefore \frac{y^{-1}}{-1}=x^2+c \quad \therefore -\frac{1}{y}=x^2+c \quad \therefore y=\frac{-1}{x^2+c}$.

$\boxed{\frac{10}{181}}$

$\frac{du}{dv}=2u^2\left(4v^3+4v^{-3}\right) \quad \therefore \frac{du}{2u^2}=4\left(v^3+v^{-3}\right)dv$

$\therefore \int\frac{du}{2u^2}=4\int\left(v^3+v^{-3}\right)dv \quad \therefore \frac{u^{-1}}{2(-1)}=4\left(\frac{v^4}{4}+\frac{v^{-2}}{-2}\right)+c$

$\therefore \frac{u^{-1}}{-2}=v^4-2v^{-2}+c$

$\therefore u=\frac{1}{-2\left(v^4-2v^{-2}+c\right)}$.

$\boxed{\frac{12}{181}}$

$\frac{dy}{dt}=\left(2t+t^{-1}\right)^2 \quad \therefore y=\int\left(4t^2+t^{-2}+4\right)dt=4\frac{t^3}{3}+\frac{t^{-1}}{-1}+4t+c$

$\therefore y=\frac{4}{3}t^3-\frac{1}{t}+4t+c$

$\boxed{\frac{19}{181}}$

$\int\frac{dx}{(3x+2)^2}=\frac{1}{3}\int(3x+2)^{-2}d(3x+2)=\frac{(3x+2)^{-1}}{-3}+c=c-\frac{1}{3(3x+2)}$.

$\boxed{\frac{20}{181}}$

$\int\frac{3r\,dr}{\sqrt{1-r^2}}=I \quad$ Let $1-r^2=u \quad \therefore \frac{du}{dr}=-2r \quad \therefore r\,dr=\frac{du}{-2}$

$\therefore I=\int\frac{3\left(\frac{du}{-2}\right)}{u^{\frac{1}{2}}}=-\frac{3}{2}\int u^{-\frac{1}{2}}du=-\frac{3}{2}\cdot\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+C=-3\sqrt{1-r^2}+C$

$\boxed{\frac{25}{181}}$

$\int\frac{(z+1)\,dz}{\sqrt[3]{z^2+2z+2}}=\frac{1}{2}\int\left(z^2+2z+2\right)^{-\frac{1}{3}}d\left(z^2+2z+2\right)=\frac{\left(z^2+2z+2\right)^{\frac{2}{3}}}{2\cdot\frac{2}{3}}+c$

$=\frac{3}{4}\left(z^2+2z+2\right)^{2/3}+C$.

$\boxed{\frac{1}{184}}$

$v=\frac{ds}{dt}=3t^2 \quad \therefore s=\int 3t^2dt=t^3+C \quad \therefore S_0=0+C=C$

$\therefore s=S_0+t^3$.

$\frac{5}{184}$

$v = (t+1)^{-2} \therefore S = \int v\,dt = \int (t+1)^{-2}\,dt = \frac{(t+1)^{-1}}{-1} + c$ ƒ $S_0 = -1 + c$

$\therefore c = 1 + S_0 \therefore S = 1 + S_0 - \frac{1}{t+1} = S_0 + \frac{t}{t+1}$.

$\frac{8}{184}$

$a = \frac{dv}{dt} = t \therefore v = \int t\,dt = \frac{t^2}{2} + c$ at $t=0$ $v = v_0 \therefore v_0 = 0 + c$

$\therefore v = v_0 + \frac{t^2}{2} \therefore \frac{ds}{dt} = v_0 + \frac{t^2}{2} \therefore s = \int \left(v_0 \cdot \frac{t^2}{2}\right) dt = v_0 t + \frac{t^3}{6} + c$

at $t=0, s = s_0 \therefore s_0 = 0 + 0 + c \therefore S = s_0 + v_0 t + \frac{t^3}{6}$.

$\frac{10}{184}$

$a = (2t+1)^{-3} \therefore V = \int a\,dt = \int (2t+1)^{-3}\,dt = \frac{(2t+1)^{-2}}{-2} \cdot \frac{1}{2} + c$ ƒ $V_0 = -\frac{1}{4} + c$

$\therefore V = V_0 + \frac{1}{4} - \frac{(2t+1)^{-2}}{4} \therefore s = \int v\,dt = \int \left[V_0 + \frac{1}{4} - \frac{(2t+1)^{-2}}{4}\right] dt = \left(V_0 + \frac{1}{4}\right)t - \frac{(2t+1)^{-1}}{4(-1)(2)} + c'$

ƒ $S_0 = \frac{1}{8} + c' \therefore S = S_0 - \frac{1}{8} + \left(V_0 + \frac{1}{4}\right)t + \frac{1}{8(2t+1)}$.

$\frac{16}{184}$

$\frac{dy}{dx} = \frac{x^2+1}{x^2} \therefore y = \int (1 + x^{-2})\,dx = x + \frac{x^{-1}}{-1} + c$ ƒ $1 = 1 - 1 + c \Rightarrow c = 1$

$\therefore y = x - \frac{1}{x} + 1$.

$\frac{17}{184}$

$\frac{dy}{dx} = x\sqrt{y} \therefore \frac{dy}{\sqrt{y}} = x\,dx \therefore \int y^{-\frac{1}{2}}\,dy = \int x\,dx \therefore \frac{y^{\frac{1}{2}}}{\frac{1}{2}} = \frac{x^2}{2} + c$

$\therefore 2\sqrt{y} = \frac{x^2}{2} + c$ at $x = 0, y = 1 \therefore 2 = 0 + c \therefore 2\sqrt{y} = \frac{x^2}{2} + 2$

$\therefore \sqrt{y} = 1 + \frac{x^2}{4} \therefore y = \left(1 + \frac{x^2}{4}\right)^2$.

$\frac{19}{184}$

$\frac{dy}{dx} = x\sqrt{1+x^2} \therefore y = \frac{1}{3}\int (1+x^2)^{\frac{1}{2}}\,d(1+x^2) = \frac{1}{2}\,\frac{(1+x^2)^{3/2}}{3/2} + c$ ƒ $-3 = \frac{1}{3} + c \Rightarrow c = -\frac{10}{3}$

$\therefore y = \frac{1}{3}\left[(1+x^2)^{3/2} - 10\right]$.

$\frac{1}{187}$

$\int \sin 3x\,dx = \frac{-\cos 3x}{3} + c$

$\frac{3}{187}$

$\int x \sin(2x^2)\,dx = I$, let $2x^2 = u \therefore 4x\,dx = du$

$\therefore I = \int \sin u \cdot \frac{du}{4} = -\frac{1}{4}\cos u + C = C - \frac{\cos 2x^2}{4}$

$\boxed{\frac{5}{187}}$ $\int \sin 2t\,dt = \frac{-\cos 2t}{2} + C$

$\boxed{\frac{16}{187}}$ $\int \sin^3 \frac{y}{2} \cos\frac{y}{2}\,dy = I$, Let $\sin\frac{y}{2} = u$ $\therefore \frac{du}{dy} = \frac{1}{2}\cos\frac{y}{2}$ $\therefore \cos\frac{y}{2}\cdot dy = 2du$

$\therefore I = \int u^3 \cdot 2du = 2\frac{u^4}{4} + C = \frac{u^4}{2} + C = \frac{\sin^4(y/2)}{2} + C$

$\boxed{\frac{17}{187}}$ $\int \frac{\sin[(z-1)/3]\,dz}{\cos^2[(z-1)/3]} = I$, let $\cos[(z-1)/3] = u$

$\therefore -\sin[(z-1)/3]\cdot\frac{1}{3}\,dz = du$ $\therefore I = \int \frac{-3du}{u^2} = -3\frac{u^{-1}}{-1} + C =$

$= \frac{3}{\cos[(z-1)/3]} + C\cdot$

$\boxed{\frac{21}{187}}$ $\int \sin t \cos t (\sin t + \cos t)\,dt = \int \sin^2 t \cos t\,dt + \int \sin t \cos^2 t\,dt =$

$= \int \sin^2 t\, d\sin t + \int \cos^2 t\, d\frac{\cos t}{-1} = \frac{\sin^3 t}{3} - \frac{\cos^3 t}{3} + C$

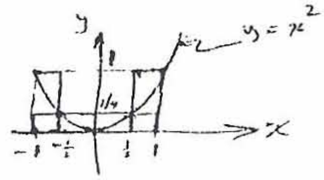
$\boxed{\frac{24}{187}}$ $\frac{dy}{dx} = \frac{\pi \cos \pi x}{\sqrt{y}}$ $\therefore \int \sqrt{y}\,dy = \int \pi \cos \pi x\,dx$ $\therefore \frac{y^{3/2}}{3/2} = \sin \pi x + C$

at $x = \frac{1}{2}$, $y = 1$ $\therefore \frac{1^{3/2}}{3/2} = \sin\frac{\pi}{2} + C$ $\therefore C = \frac{2}{3} - 1$

$\therefore y^{3/2} = \frac{3}{2}\left(\sin \pi x + \frac{2}{3} - 1\right) = \frac{3}{2}\sin \pi x - \frac{1}{2}$.

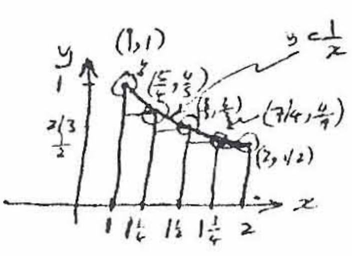
$\boxed{\frac{2}{191}}$ ⓐ $\therefore A_i = \frac{1}{2} * \frac{1}{4} + 0 + 0 + \frac{1}{2} * \frac{1}{4} = \frac{1}{4}$.

ⓑ $\therefore A_s = \frac{1}{2} * 1 + \frac{1}{2} * \frac{1}{4} + \frac{1}{2} * \frac{1}{4} + \frac{1}{2} * 1 = \frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \frac{1}{2} = \frac{5}{4}$.



$\boxed{\frac{4}{191}}$ ⓐ $A = \frac{1}{4} * \frac{4}{5} + \frac{1}{4} * \frac{2}{3} + \frac{1}{4} * \frac{4}{7} + \frac{1}{4} * \frac{1}{2}$

$= \frac{1}{4}\cdot\left[\frac{168 + 140 + 120 + 105}{210}\right] = \frac{533}{840} = 0.635$



ⓑ $A = \frac{1}{4} * 1 + \frac{1}{4} * \frac{4}{5} + \frac{1}{4} * \frac{2}{3} + \frac{1}{4} * \frac{4}{7} =$

$= \frac{1}{4}\cdot\left[\frac{105 + 84 + 70 + 60}{105}\right] = \frac{319}{420} = 0.760$

$\boxed{\frac{10}{191}}$ $\sum_{k=1}^{3} \frac{k-1}{k} = \frac{1-1}{1} + \frac{2-1}{2} + \frac{3-1}{3} = 0 + \frac{1}{2} + \frac{2}{3} = \frac{3+4}{6} = \frac{7}{6}$

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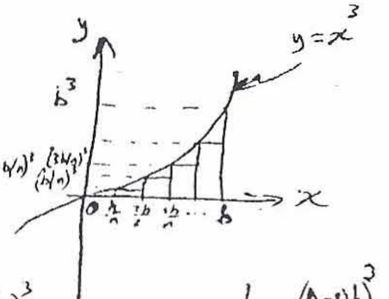
$\frac{2}{196}$

The width of any rectangle when the we divide the area into n rectangles is $\frac{b-0}{n} = \frac{b}{n}$



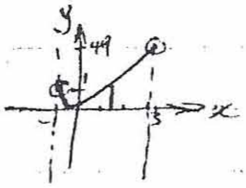
$$\therefore \text{Area}, A = \frac{b}{n} * 0 + \frac{b}{n} * \left(\frac{b}{n}\right)^3 + \frac{b}{n} * \left(\frac{2b}{n}\right)^3 + \frac{b}{n} * \left(\frac{3b}{n}\right)^3 + \cdots + \frac{b}{n} * \left(\frac{(n-1)b}{n}\right)^3$$

$$= \frac{b}{n} \cdot \sum_{k=1}^{k=n} \left[\frac{(k-1)b}{n}\right]^3 = \frac{b}{n} \cdot \sum_{k=1}^{n} (k-1)^3 \frac{b^3}{n^3} = \frac{b^4}{n^4} \cdot \sum_{k=1}^{n-1} k^3$$

$$= \frac{b^4}{n^4} \cdot \left(\frac{(n-1)n}{2}\right)^2 = \frac{b^4}{4} \cdot \left(\frac{n-1}{n}\right)^2 = \frac{b^4}{4} \cdot \left(1 - \frac{1}{n}\right)^2$$

$\therefore$ If n was very very large then $A = \frac{b^4}{4}(1-0)^2 = \frac{b^4}{4}$.

$\frac{6}{200}$



$$A = \int y\,dx = \int^{3} (2x+1)^2\,dx = \int_{-1}^{3} (4x^2 + 4x + 1)\,dx = \frac{4x^3}{3} + 2x^2 + x \Big|_{-1}^{3}$$

$$= \frac{4}{3}(27-(-1)) + 2(9-1) + (3-(-1)) = \frac{4}{3}\cdot(28) + 2\cdot(8) + 4 =$$

$$= \frac{112}{3} + 16 + 4 = \frac{112 + 20(3)}{3} = \frac{172}{3} = 57\frac{1}{3}$$

$\frac{7}{200}$

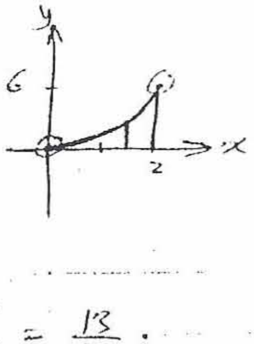
$$A = \int_0^2 y\,dx = \int_0^2 (x^3 + 2x + 1)\,dx = \frac{x^4}{4} + x^2 + x \Big|_0^2 = \frac{2^4 - 0^4}{4} + (2^2 - 0^2) + (2-0)$$

$$= 4 + 4 + 2 = 10$$

$\therefore$ $A = 10$.

$\frac{8}{200}$

$y = x\sqrt{2x^2+1}$



$\therefore$ $A = \int_0^2 y\,dx = \int_0^2 x\sqrt{2x^2+1}\,dx$,

Let $2x^2 + 1 = u$ $\therefore \frac{du}{dx} = 4x$ $\therefore x\,dx = \frac{du}{4}$

when $x = 0$ $\therefore u = 1$ & when $x = 2$ $\therefore u = 9$

$$\therefore A = \int_1^9 \sqrt{u} \cdot \frac{du}{4} = \frac{1}{4} \cdot \frac{u^{3/2}}{3/2} \Big|_1^9 = \frac{9^{3/2} - 1^{3/2}}{6} = \frac{27-}{6} = \frac{26}{6} = \frac{13}{3}.$$

9/202

$$A = \int_0^2 y\,dx = \int_0^2 \frac{x}{\sqrt{2x^2+1}}\,dx = \int_0^2 (2x^2+1)^{-1/2}\,d\,\frac{(2x^2+1)}{4} = \frac{(2x^2+1)^{1/2}}{(1/2)(4)}\Big|_0^2 =$$

$$= \frac{(2x^2+1)^{1/2}}{2} = \frac{(2\cdot 2^2+1)^{1/2} - (0+1)^{1/2}}{2} = \frac{9^{1/2} - 1^{1/2}}{2} = \frac{3-1}{2} = 1$$

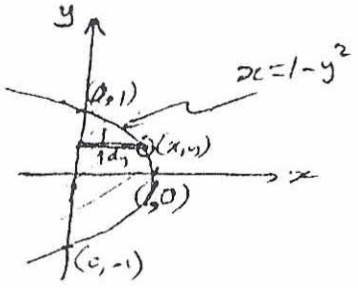
∴ $A = 1.$

15/202

Plotting $x = 1 - y^2$ ∴ $(1,0)$ & $(+1,0)$ & $(-1,0)$ are points on the curve. Let A be the dashed area ∴ $A = \int_{-1}^{1} x\,dy =$

$$= \int_{-1}^{1} (1-y^2)\,dy = y - \frac{y^3}{3}\Big|_{-1}^{1} = [1-(-1)] - \frac{1^3-(-1)^3}{3} = 2 - \frac{1+1}{3}$$

$$= 2 - \frac{2}{3} = \frac{6-2}{3} = 4/3 \quad ∴ \quad A = 4/3.$$



7/210

$$\int_0^2 \sqrt{4x+1}\,dx = \frac{(4x+1)^{3/2}}{(3/2)(4)}\Big|_0^2 = \frac{(9)^{3/2} - 1^{3/2}}{6} = \frac{27-1}{6} = \frac{26}{6} = \frac{13}{3}.$$

14/210

$$\int_0^{\pi} \sin^2 x\,dx = \int_0^{\pi} \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\Big[x - \frac{\sin 2x}{2}\Big]_0^{\pi} = \frac{1}{2}\Big[\pi - 0 - \frac{0-0}{2}\Big] = \frac{\pi}{2}$$

19/210

$$F(x) = \int_1^x \frac{dt}{t} \quad ∴ \quad F'(x) = \frac{d}{dx}\int_1^x \frac{dt}{t} = \frac{1}{x}.$$

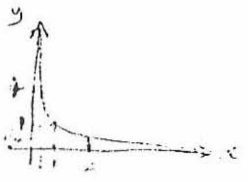
20/210

$$F(x) = \int_x^1 \sqrt{1-t^2}\,dt = -\int_1^x \sqrt{1-t^2}\,dt \quad ∴ \quad F'(x) = -\sqrt{1-x^2}.$$

5/213

ⓐ $\int_1^2 \frac{1}{x^2}\,dx = \frac{2-1}{4}\cdot\Big[\frac{\frac{1}{1^2}+\frac{1}{2^2}}{2} + \frac{1}{(5/4)^2} + \frac{1}{(6/4)^2} + \frac{1}{(7/4)^2}\Big] = 0.508994$

ⓑ $\int_1^2 \frac{1}{x^2}\,dx = \frac{(2-1)/4}{3}\cdot\Big[\frac{1}{1^2} + 4\cdot\frac{1}{(5/4)^2} + \frac{2}{(6/4)^2} + \frac{4}{(7/4)^2} + \frac{1}{2^2}\Big] = 0.500418$



ⓒ $\int_1^2 \frac{1}{x^2}\,dx = \frac{x^{-1}}{-1}\Big|_1^2 = \frac{2^{-1}-1^{-1}}{-1} = \frac{0.5-1}{-1} = 0.500000$ ∴ b is more accurate than a

ⓓ error in trapezoidal rule, $e = 10^{-5} = \frac{2-1}{12}\cdot\Big[\frac{6}{(1)^4}\Big]\cdot\Big(\frac{2-1}{n}\Big)^2$ ∴ $n = 223.6$ ∴ $n \geq 224$ subdivisions

ⓔ error in simpson rule, $e = 10^{-5} = \frac{2-1}{180}\cdot\Big[\frac{170}{(1)^6}\Big]\cdot\Big(\frac{2-1}{n}\Big)^4$ ∴ $n = 16.1$ ∴ $n \geq 18$ subdivisions

ΣΣ

$\frac{2}{224}$

$\frac{dy}{dx} = \sqrt{1+x+y+xy} = \sqrt{(1+x)+y(1+x)} = \sqrt{(1+x)(1+y)} = (1+x)^{1/2} \cdot (1+y)^{1/2}$

$\therefore \int \frac{dy}{(1+y)^{1/2}} = \int (1+x)^{1/2}\, dx \quad \therefore \frac{(1+y)^{1/2}}{1/2} = \frac{(1+x)^{3/2}}{3/2} + C_1 \quad \therefore 3(1+y)^{1/2} = (1+x)^{3/2} + C_2$

$\frac{4}{224}$

$\frac{dx}{dy} = \frac{y-\sqrt{y}}{x+\sqrt{x}} \quad \therefore \int (y-\sqrt{y})\,dy = \int (x+\sqrt{x})\,dx$

$\therefore \frac{y^2}{2} - \frac{y^{3/2}}{3/2} = \frac{x^2}{2} + \frac{x^{3/2}}{3/2} + C \quad \therefore \frac{y^2-x^2}{2} - \frac{y^{3/2}+x^{3/2}}{3/2} = c$

$\therefore 4(x^{3/2}+y^{3/2}) + 3(x^2-y^2) = c'$

$\frac{6}{224}$

(a) $\frac{dy}{dx} = x\sqrt{x^2-4} \quad \therefore y = \int x\sqrt{x^2-4}\,dx = \frac{(x^2-4)^{3/2}}{(3/2)(2)} + C = \frac{(x^2-4)^{3/2}}{3} + C$

at $x = 2$, $y = 3$ $\therefore C = 3 \quad \therefore y = \frac{(x^2-4)^{3/2}}{3} + 3$.

(b) $\frac{dy}{dx} = xy^3 \quad \therefore \int \frac{dy}{y^3} = \int x\,dx \quad \therefore \frac{y^{-2}}{-2} = \frac{x^2}{2} + C_1 \quad \therefore -y^{-2} = x^2 + C_2 \quad \therefore y = \frac{1}{\sqrt{-(C_2+x^2)}}$

at $x = 0$, $y = 1$ $\therefore C_2 = -1 \quad \therefore y = \frac{1}{\sqrt{1-x^2}}$

$\frac{8}{224}$

$y' = 3x^2+2 \quad \therefore \frac{dy}{dx} = 3x^2+2 \quad \therefore \int dy = \int (3x^2+2)\,dx$

$\therefore y = x^3+2x+C \quad$ at $x = 1, y = -1 \quad \therefore -1 = 1^3 + 2(1) + C \quad \therefore C = -4$

$\therefore$ The equation of the curve is $y = x^3+2x-4$

$\frac{9}{224}$

$a = \frac{dv}{dt} = -t^2 \quad \therefore v = \frac{ds}{dt} = \int -t^2\,dt = -\frac{t^3}{3} + C_1 \quad \therefore s = \int (-\frac{t^3}{3} + C_1)\,dt = -\frac{t^4}{12} + C_1 t + C_2$

$\therefore$ since $s = 0$ at $t = 0$ $\therefore C_2 = 0 \quad \therefore s = -\frac{t^4}{12} + C_1 t$

$\therefore$ since $s = h > 0$ and this is the maximum which occurs at $v = 0 = -\frac{t^3}{3} + C_1$

$\Rightarrow$ at $t = \sqrt[3]{3C_1} \quad \therefore h = -\frac{(3C_1)^{4/3}}{12} + C_1(3C_1)^{1/3} = C_1^{4/3}\left[-\frac{3^{4/3}}{12} + 3^{1/3}\right] = \frac{(3C_1)^{4/3}}{4}$

$\therefore C_1 = \frac{1}{3}(4h)^{3/4} \quad \therefore$ at $t = 0$ $v = C_1 = \frac{1}{3}\cdot(4h)^{3/4} = \frac{(4)^{3/4}}{3} \cdot h^{3/4}$ [illegible]

$\frac{17}{224}$

$a = \frac{dv}{dt} = -32 \quad \therefore v = \frac{ds}{dt} = \int -32\,dt = -32t + V_0 = -32t + 96$

$\therefore s = \int v\,dt = \int (96-32t)\,dt = 96t - 16t^2 + S_0 = 96t - 16t^2 + 0$

$\therefore s(t) = 16(6t-t^2) = 16t(6-t)$ ft above ground.

$\therefore$ At max. s, $\therefore v = 0 \quad \therefore t = \frac{96}{32} = 3$ sec $\therefore s(3) = 16(3)(6-3) = 144$ ft.

$\therefore$ max. height $= 144$ ft

20

$\frac{21}{225}$ $$\int \frac{x^3+1}{x^2}\,dx = \int\left(\frac{x^3}{x^2}+\frac{1}{x^2}\right)dx = \int\left(x+x^{-2}\right)dx = \frac{x^2}{2}+\frac{x^{-1}}{-1}+C$$

$$= \frac{x^2}{2}-\frac{1}{x}+C = \frac{x^3-2+C'x}{2x}$$

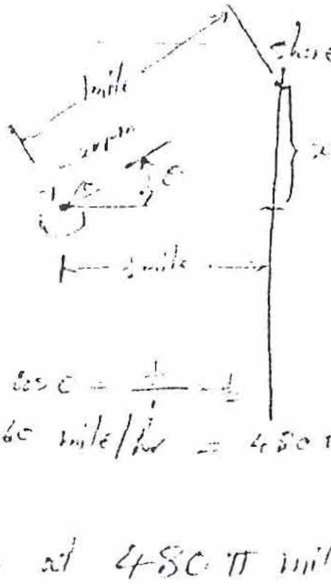
$\frac{23}{225}$ $$\int t^{1/3}\left(1+t^{4/3}\right)^{-7}dt = \frac{\left(1+t^{4/3}\right)^{-6}}{(-6)(4/3)}+C = -\frac{\left(1+t^{4/3}\right)^{-6}}{8}+C$$

$\frac{24}{225}$ $$\int \frac{\left(1+\sqrt{u}\right)^{1/2}}{\sqrt{u}}\,du = \int u^{-1/2}\left(1+u^{1/2}\right)^{1/2}du = \frac{\left(1+u^{1/2}\right)^{3/2}}{(3/2)(1/2)}+C = \frac{4}{3}\cdot\left(1+u^{1/2}\right)^{3/2}+C.$$

$\frac{39}{225}$ $$\frac{x}{(1/2)} = \tan\theta \qquad \therefore 2x = \tan\theta$$

$$2\frac{dx}{dt} = \sec^2\theta\cdot\frac{d\theta}{dt} = \sec^2\theta\cdot(2\times 2\pi)\frac{rad}{min}$$

$$\therefore \frac{dx}{dt} = \frac{\sec^2\theta\cdot 2\cdot 2\pi}{2} = 2\pi\sec^2\theta = \frac{2\pi}{\cos^2\theta}$$

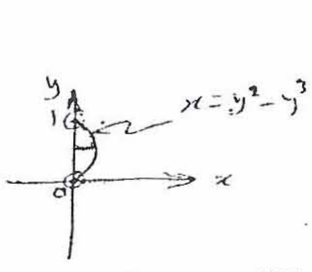


When the point is 1 mile off the light ∴ $\cos\theta = \frac{1/2}{1} = \frac{1}{2}$

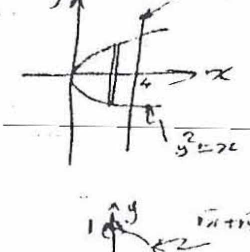
$$\therefore \frac{dx}{dt} = \frac{2\pi}{\cos^2\theta} = \frac{2\pi}{(\frac{1}{2})^2} = 8\pi \text{ mile/min} = 8\pi\times 60 \text{ mile/hr} = 480\pi \text{ mile/hr}$$

∴ The ray of light is moving along the shore at 480π mile/hr

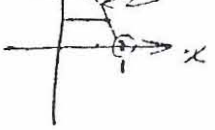
$\frac{3}{230}$ $$A = \int_0^1 x\,dy = \int_0^1 (y^2-y^3)\,dy = \frac{y^3}{3}-\frac{y^4}{4}\Big|_0^1 = \frac{1}{3}-\frac{1}{4} = \frac{1}{12}$$



$\frac{4}{230}$ $$A = \int_0^4 2y\,dx = \int_0^4 2\sqrt{x}\,dx = 2\frac{x^{3/2}}{3/2}\Big|_0^4 = \frac{4}{3}\cdot 4^{3/2} = \frac{4}{3}\cdot 8 = \frac{32}{3}.$$



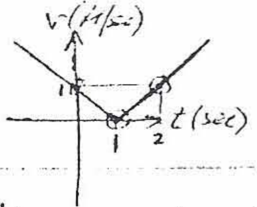
$\frac{11}{230}$ $$A = \int_0^1 x\,dy = \int_0^1 \left(1-\sqrt{y}\right)^2 dy = \int_0^1 \left(1-2\sqrt{y}+y\right)dy =$$



$$= y-2\cdot\frac{y^{3/2}}{3/2}+\frac{y^2}{2}\Big|_0^1 = 1-\frac{4}{3}+\frac{1}{2} = \frac{6-8+3}{6} = \frac{1}{6}$$

$\frac{4}{233}$

$v = |t-1| \quad 0 \leq t \leq 2$



a) v is positive always at any t

b) v is never negative at any t

$\therefore s = \int_a^b v\,dt = \int_0^2 |t-1|\,dt = 2 * \left|\int_0^1 (t-1)\,dt\right| = 2\left|\left[\frac{t^2}{2} - t\right]_0^1\right| = 2\left|\left[\frac{1}{2} - 1\right]\right| = 1\,ft.$

$\frac{\ }{233}$

$v = \int a\,dt = \int \frac{1}{\sqrt{4t+1}}\,dt = \int (4t+1)^{-1/2}\,dt = \frac{(4t+1)^{1/2}}{(1/2)(4)} + C = \frac{1}{2}\sqrt{4t+1} + C$

at $t = 0 \quad v = v_0 = 1 \quad \therefore 1 = \frac{1}{2}\cdot\sqrt{1} + C \quad \therefore C = \frac{1}{2} \quad \therefore v = \frac{1}{2}\left[\sqrt{4t+1} + 1\right]$

$\therefore s = \int_0^2 v\,dt = \int_0^2 \frac{1}{2}\left[\sqrt{4t+1} + 1\right] dt = \frac{1}{2}\cdot\left[\frac{(4t+1)^{3/2}}{(3/2)(4)} + t\right]_0^2 = \frac{1}{2}\left[\frac{9^{3/2} - 1^{3/2}}{6} + 2 - 0\right] = \frac{13+6}{6} = \frac{19}{6}\,ft.$

$\frac{14}{233}$

$f(t) = \frac{dQ}{dt} \qquad \therefore Q = \int f(t)\,dt$

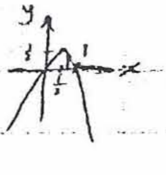
$\therefore Q_{0b}\left[\text{during } t = 0 \rightarrow t = b\right] = \int_0^b f(t)\,dt$

$\therefore$ The total amount of water = The amount already there at $t = 0$ + Q_{0b}

$\therefore Q = Q_0 + \int_0^b f(t)\,dt.$

$\frac{3}{239}$

$V = \int_0^1 \pi y^2\,dx = \pi\int_0^1 (x - x^2)^2\,dx = \pi\left[\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5}\right]_0^1$



$= \pi\cdot\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] = \frac{\pi}{30}(10 + 6 - 15) = \frac{\pi}{30}.$

$\boxed{\frac{4}{239}}$

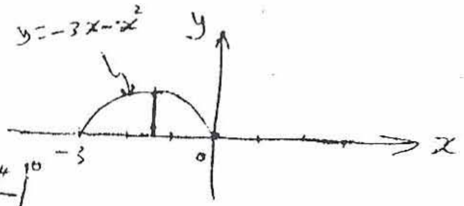
$$y = -3x - x^2 = -x(3+x)$$

$$V = \int_{-3}^{0} \pi y^2 \, dx = \pi \int_{-3}^{0} x^2 (3+x)^2 \, dx =$$

$$= \pi \int_{-3}^{0} \left(9x^2 + x^4 + 6x^3\right) dx = \pi \left[3x^3 + \frac{x^5}{5} + \frac{6x^4}{4}\right]_{-3}^{0}$$

$$= \pi \left[3(0+27) + \frac{0+3^5}{5} + \frac{3}{2}(0-81)\right] = \pi\left(81 + \frac{243}{5} - \frac{243}{2}\right) = \frac{\pi}{10}\left(810 + 486 - 1215\right)$$

$$= \pi \cdot \frac{81}{10} = \frac{81\pi}{10} .$$

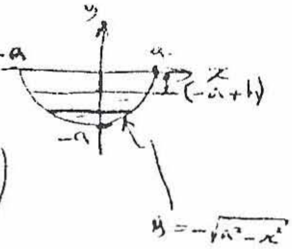


$\boxed{\frac{1}{239}}$

a) $$V = \int_{-a}^{-a+h} \pi x^2 \, dy = \pi \int_{-a}^{-a+h} \left(a^2 - y^2\right) dy = \pi \left[a^2 y - \frac{y^3}{3}\right]_{-a}^{-a+h} =$$

$$= \pi \left[a^2(-a+h+a) - \frac{(-a+h)^3 + a^3}{3}\right] = \pi \cdot \left(a^2 h - \frac{-a^3 + h^3 - 3ah^2 + 3ha^2 + a^3}{3}\right)$$

$$= \pi \cdot \left(a^2 h - \frac{h^3}{3} + ah^2 - ha^2\right) = \pi h^2 \left(a - \frac{h}{3}\right) = \frac{\pi h^2}{3} \cdot (3a - h)$$



b) $$\frac{dV}{dh} = \frac{\pi}{3} \cdot \left[2h(3a-h) + h^2(-1)\right] = \frac{\pi}{3} \cdot \left[6ah - 2h^2 - h^2\right] = \frac{\pi}{3}\left(6ah - 3h^2\right) = \pi h (2a-h)$$

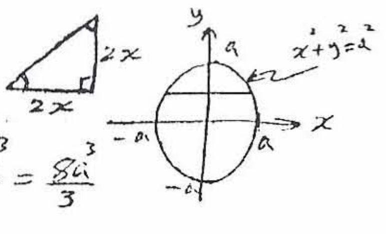
$\therefore \frac{dV}{dt} = \pi h \cdot (2a - h) \frac{dh}{dt}$ $\quad \therefore 0.2 = \pi * 4 * (2*5-4) \cdot \frac{dh}{dt}$ $\quad \therefore \frac{dh}{dt} = \frac{0.2}{24\pi}$ ft/sec

$\therefore dh/dt = \frac{0.2}{24\pi}$ ft/sec $= \frac{0.2 * 12}{24 * \pi * \frac{1}{60}}$ in/min $= \frac{6}{\pi}$ in/min.

$\boxed{\frac{16}{239}}$

$$V = \int_{-a}^{a} \frac{1}{2}(2x)(2x)\, dy = 2 \int_{-a}^{a} \left(a^2 - y^2\right) dy = 2\left(a^2 y - \frac{y^3}{3}\right)_{-a}^{a}$$

$$= 2\left(a^2(a+a) - \frac{a^3 + a^3}{3}\right) = 2\left(2a^3 - \frac{2a^3}{3}\right) = 2 \cdot \frac{4a^3}{3} = \frac{8a^3}{3}$$

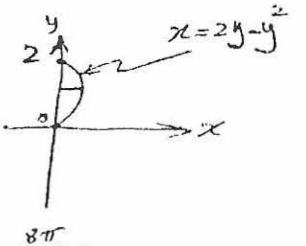


$\boxed{\frac{2}{245}}$

$x = 2y - y^2 = y(2-y)$ $\quad \therefore x = 0$ at $y = 0$ & $y = 2$

$$\therefore V = \int_{0}^{2} (2\pi y) x \cdot dy = 2\pi \int_{0}^{2} y \cdot \left(2y - y^2\right) dy = 2\pi \int_{0}^{2} \left(2y^2 - y^3\right) dy =$$

$$= 2\pi \left[\frac{2y^3}{3} - \frac{y^4}{4}\right]_{0}^{2} = 2\pi\left(\frac{16}{3} - \frac{16}{4}\right) = 32\pi \cdot \frac{4-3}{12} = \frac{32\pi}{12} = \frac{8\pi}{3} .$$



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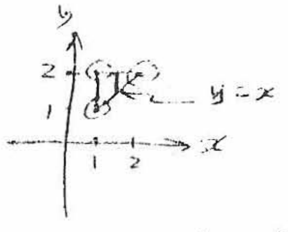
$\boxed{\frac{11}{245}}$ ⓐ $V = \int_1^2 \pi (2^2 - y^2) dx = \pi \int_1^2 (4 - x^2) dx =$

$= \pi \left[4x - \frac{x^3}{3} \right|_1^2 = \pi \left[4(2-1) - \frac{8-1}{3} \right] = \pi \left(4 - \frac{7}{3} \right) = \frac{5\pi}{3}.$

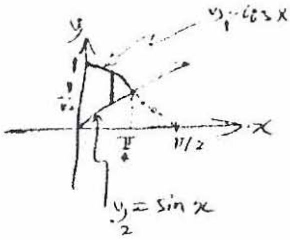


ⓑ $V = \int_1^2 2\pi x \cdot (2 - y) dx = 2\pi \int_1^2 x \cdot (2 - x) dx = 2\pi \cdot \int_1^2 (2x - x^2) dx = 2\pi \left[x^2 - \frac{x^3}{3} \right|_1^2 =$

$= 2\pi \left[(4-1) - \frac{8-1}{3} \right] = 2\pi \left(3 - \frac{7}{3} \right) = 2\pi \cdot \frac{2}{3} = \frac{4\pi}{3}.$

$\boxed{\frac{12}{246}}$ $V = \int_0^{\pi/4} \pi (y_1^2 - y_2^2) dx = \pi \int_0^{\pi/4} (\cos^2 x - \sin^2 x) dx = \pi \cdot \int_0^{\pi/4} \cos 2x \, dx$

$= \pi \cdot \frac{\sin 2x}{2} \Big|_0^{\pi/4} = \frac{\pi}{2} \cdot (1 - 0) = \pi/2 .$



$\boxed{\frac{4}{254}}$ $L = \int_1^3 \sqrt{1 + y'^2} \, dx = \int_1^3 \sqrt{1 + [x^2 - \frac{1}{4x^2}]^2} \, dx =$

$= \int_1^3 \sqrt{1 + \left(\frac{4x^4 - 1}{4x^2} \right)^2} \, dx = \int_1^3 \sqrt{\frac{16x^4 + 16x^8 - 8x^4 + 1}{(4x^2)^2}} \cdot dx = \int_1^3 \frac{1}{4x^2} \cdot \sqrt{16x^8 + 8x^4 + 1} \cdot dx$

$= \int_1^3 \frac{1}{4x^2} \cdot (4x^4 + 1) \, dx = \int_1^3 \left(x^2 + \frac{1}{4x^2} \right) dx = \frac{x^3}{3} - \frac{1}{4x} \Big|_1^3 = \frac{27-1}{3} - \frac{1}{4}\left(\frac{1}{3} - 1 \right) =$

$= \frac{26}{3} + \frac{1}{4} \cdot \frac{2}{3} = \frac{52}{6} + \frac{1}{6} = \frac{53}{6}.$

$\boxed{\frac{8}{255}}$ $L = \int_0^{2\pi} \sqrt{x'^2 + y'^2} \, dt = \int_0^{2\pi} \sqrt{(1 - \cos t)^2 + (\sin t)^2} \, dt = \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} \cdot dt$

$= \int \sqrt{2 - 2\cos t} \, dt = \sqrt{2} \int_0^{2\pi} \sqrt{1 - \cos t} \, dt = \sqrt{2} \cdot \int_0^{2\pi} \sqrt{2 \sin^2 \frac{t}{2}} \, dt = 2 \cdot \int_0^{2\pi} \sin \frac{t}{2} \, dt =$

$= 2 \cdot \left(\frac{-\cos \frac{t}{2}}{\frac{1}{2}} \right)_0^{2\pi} = -4 \left(\cos \pi - \cos 0 \right) = -4 (-1 - 1) = 8.$

$\frac{4}{260}$

$$S = \int_0^4 2\pi x \cdot \sqrt{x'^2 + y'^2}\, dt = 2\pi \int_0^4 (t+1)\sqrt{(1)^2 + (t+1)^2}\, dt =$$

$$= 2\pi \int_0^4 \left[1 + (t+1)^2\right]^{\frac{1}{2}} \cdot (t+1)\, dt = 2\pi \cdot \frac{\left[1+(t+1)^2\right]^{3/2}}{\left(\frac{3}{2}\right)(2)} \Big|_0^4 = \frac{2\pi}{3}\left[26\sqrt{26} - 2\sqrt{2}\right]$$

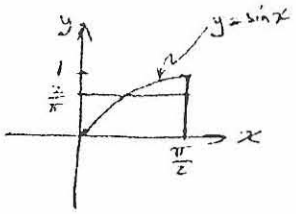
$$= \frac{4\sqrt{2}\,\pi}{3}\left(13\sqrt{13} - 1\right) = 271.74$$

$\frac{}{263}$

(a) $y = \sin x \quad , \; x \in \left[0, \frac{\pi}{2}\right]$

$\therefore A = \int_0^{\pi/2} y\, dx = \int_0^{\pi/2} \sin x\, dx = -\cos x \Big|_0^{\pi/2} = -(0-1) = 1$

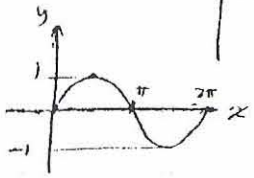
$\therefore y_{av} = \frac{1}{\pi/2} = \frac{2}{\pi}.$



(b) $y = \sin x \quad , \; x \in [0, 2\pi]$

$\therefore A = \int_0^{2\pi} \sin x\, dx = -\cos x \Big|_0^{2\pi} = -(1-1) = 0$

$\therefore y_{av} = \frac{0}{2\pi} = 0.$

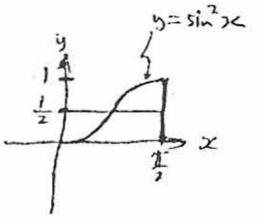


$\frac{2}{263}$

(a) $y = \sin^2 x \quad , \; x \in \left[0, \frac{\pi}{2}\right]$

$\therefore A = \int_0^{\pi/2} \sin^2 x\, dx = \int_0^{\pi/2} \frac{1 - \cos 2x}{2} \cdot dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{4}$

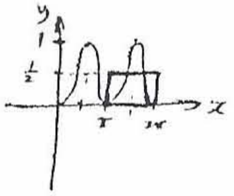
$\therefore y_{av} = \frac{\pi/4}{\pi/2} = \frac{1}{2}.$



(b) $y = \sin^2 x \quad , \; x \in [\pi, 2\pi]$

$\therefore A = \int_\pi^{2\pi} \sin^2 x\, dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_\pi^{2\pi} = \frac{1}{2}\left[\pi - \frac{0-0}{2}\right] = \pi/2$

$\therefore y_{av} = \frac{\pi/2}{\pi} = \frac{1}{2}.$



$\frac{2}{270}$

By symmetry $\therefore \bar{x} = 0$

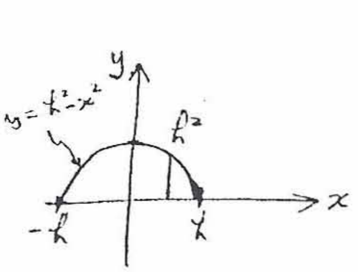
$\therefore A = \int_{-h}^{h} y\, dx = \int_{-h}^{h} (h^2 - x^2)\, dx = h^2 x - \frac{x^3}{3}\Big|_{-h}^{h} = h^2(h+h) - \frac{h^3 + h^3}{3} =$

$= 2h^3 - \frac{2h^3}{3} = 4\, h^3/3.$

$\therefore M_x = \frac{4h^3}{3} \cdot \bar{y} = \int_{-h}^{h} (y\, dx)\, \frac{y}{2} = \int_{-h}^{h} \frac{y^2}{2}\, dx = \frac{1}{2}\int (h^4 - 2h^2x^2 + x^4)\, dx = \frac{1}{2}\left[h^4 x - 2h^2 \frac{x^3}{3} + \frac{x^5}{5}\right]_{-h}^{h} =$

$= \frac{1}{2}\left[h^4(h+h) - \frac{2h^2}{3}(h^3 + h^3) + \frac{h^5 + h^5}{5}\right] = \frac{1}{2}\left[2h^5 - \frac{4h^5}{3} + \frac{2h^5}{5}\right] = \frac{h^5}{30}(30 - 20 + 6) = \frac{16h^5}{30} = \frac{8h^5}{15}$

$\therefore \bar{y} = \frac{8h^5}{15} \cdot \frac{3}{4h^3} = 2h^2/5 \quad \therefore$ Centre of mass is at $(0, 2h^2/5)$.

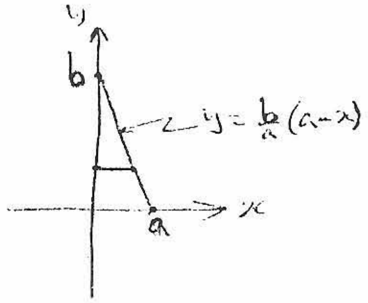


0.

3/270

Q. √

The right circular cone is produced by rotating the area shown about the y-axis.



By symmetry the centre of mass is on the y-axis.

$$V = \int_0^b \pi x^2 dy = \int_a^0 \pi x^2 \cdot \left(-\frac{b}{a} dx\right) = -\frac{\pi b}{a} \cdot \frac{x^3}{3}\Big|_a^0 =$$

$$= \frac{-\pi b}{3a}\left(0 - a^3\right) = \frac{\pi a^2 b}{3}$$

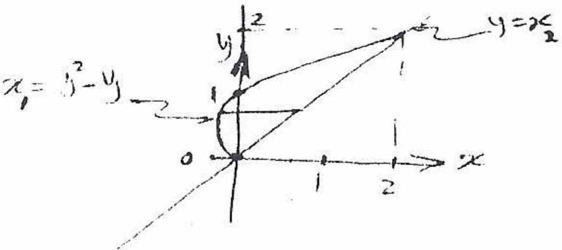
$$M_x = \frac{\pi a^2 b}{3} \cdot \bar{y} = \int_0^b (\pi x^2 dy) \cdot y = \pi \int_a^0 x^2 \cdot \frac{b}{a}(a-x)\left(-\frac{b}{a} dx\right) = -\frac{\pi b^2}{a^2} \int_a^0 (a x^2 - x^3) dx$$

$$= \frac{-\pi b^2}{a^2} \cdot \left(a\frac{x^3}{3} - \frac{x^4}{4}\right)_a^0 = -\frac{\pi b^2}{a^2}\left[0 - \frac{a^4}{3} - \frac{0 - a^4}{4}\right] = \pi b^2 a^2 \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{\pi b^2 a^2}{12}$$

$$\bar{y} = \frac{\pi b^2 a^2}{12} \cdot \frac{3}{\pi a^2 b} = \frac{b}{4}$$

∴ The centre of mass of a right circular cone lies on its axis and away from its base by quarter its height.

5/273



$$x = y^2 - y = y(y-1)$$

∴ Points of intersection are : $y = y(y-1)$

or $y = y^2 - y \Rightarrow y^2 - 2y = 0 \quad \therefore y(y-2) = 0$

$y = 0$ or $y = 2$ ∴ They are $(0,0)$ & $(2,2)$

$$\therefore A = \int_0^2 (x_2 - x_1) dy = \int_0^2 [y - (y^2 - y)] dy = \int_0^2 (-y^2 + 2y) dy = -\frac{y^3}{3} + y^2\Big|_0^2 = -\frac{8}{3} + 4 = \frac{4}{3}$$

$$\therefore \frac{4}{3} \cdot \bar{y} = M_x = \int_0^2 y(x_2 - x_1) dy = \int_0^2 y(y - y^2 + y) dy = \int_0^2 (-y^3 + 2y^2) dy =$$

$$= -\frac{y^4}{4} + \frac{2y^3}{3}\Big|_0^2 = -\frac{16}{4} + \frac{2 * 8}{3} = 16\left(-\frac{1}{4} + \frac{1}{3}\right) = 16 \cdot \frac{1}{12} = \frac{4}{3} \qquad \therefore \bar{y} = \frac{4}{3} \cdot \frac{3}{4} = 1$$

$$\& \quad \frac{4}{3} \cdot \bar{x} = M_y = \int_0^2 [(x_2 - x_1) dy] \cdot \left[\frac{x_2 - x_1}{2} + x_1\right] = \int_0^2 [(x_2 - x_1) dy] \cdot \frac{x_2 + x_1}{2} = \frac{1}{2}\int_0^2 (x_2 - x_1)(x_2 + x_1) dy$$

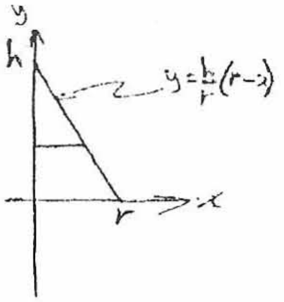
$$= \frac{1}{2}\int_0^2 (x_2^2 - x_1^2) dy = \frac{1}{2}\int_0^2 [y^2 - y^2(y-1)^2] dy = \frac{1}{2}\int_0^2 (y^2 - y^4 + 2y^3 - y^2) dy = \frac{1}{2}\left[-\frac{y^5}{5} + \frac{2y^4}{4}\right]_0^2 = \frac{1}{2}\left[-\frac{32}{5} + \frac{32}{4}\right]$$

$$= \frac{32}{2}\left(\frac{-4+5}{20}\right) = \frac{32}{40} = \frac{4}{5} \qquad \therefore \bar{x} = \frac{4}{5} \cdot \frac{3}{4} = \frac{3}{5}$$

∴ The centroid is at $\left(\frac{3}{5}, 1\right)$.

تابع

$\frac{9}{244}$ This right circular conical shell is produced by rotating the line shown about the y-axis.



$\therefore$ By symmetry, the centroid lies on the axis of rotation (y-axis).

$$\therefore\ S=\int_0^r 2\pi x\sqrt{1+y'^2}\,dx = 2\pi\int_0^r x\cdot\sqrt{1+\left(-\frac{h}{r}\right)^2}\cdot dx =$$

$$=2\pi\cdot\sqrt{1+\frac{h^2}{r^2}}\int_0^r x\,dx = \frac{2\pi}{r}\sqrt{r^2+h^2}\cdot\left.\frac{x^2}{2}\right|_0^r = \frac{\pi}{r}\cdot\sqrt{r^2+h^2}\cdot(r^2-0) = \pi r\cdot\sqrt{r^2+h^2}.$$

$$\therefore\ \pi r\cdot\sqrt{r^2+h^2}\cdot\bar{y} = M_x = \int_0^r y\left[2\pi x\sqrt{1+y'^2}\,dx\right] = 2\pi\int_0^r xy\cdot\sqrt{1+y'^2}\cdot dx =$$

$$=2\pi\int_0^r x\cdot\frac{h}{r}(r-x)\cdot\sqrt{1+\left(-\frac{h}{r}\right)^2}\,dx = \frac{2\pi h}{r}\sqrt{1+\frac{h^2}{r^2}}\cdot\int_0^r x(r-x)\,dx =$$

$$=\frac{2\pi h}{r^2}\cdot\sqrt{r^2+h^2}\cdot\int_0^r(rx-x^2)\,dx = \frac{2\pi h}{r^2}\cdot\sqrt{r^2+h^2}\cdot\left[r\cdot\frac{x^2}{2}-\frac{x^3}{3}\right]_0^r = \frac{2\pi h\sqrt{r^2+h^2}}{r^2}\left[\frac{r^3}{2}-\frac{r^3}{3}\right]=$$

$$=\frac{2\pi h\cdot\sqrt{r^2+h^2}\cdot r^3}{r^2}\cdot\frac{(3-2)}{6} = \frac{\pi h r\sqrt{r^2+h^2}}{3} \quad \therefore\ \bar{y}=\frac{\pi h r\sqrt{r^2+h^2}}{3}\cdot\frac{1}{\pi r\sqrt{r^2+h^2}}=\frac{h}{3}$$

$\therefore$ The centre of gravity is on the axis and away from the base by $\frac{h}{3}$.

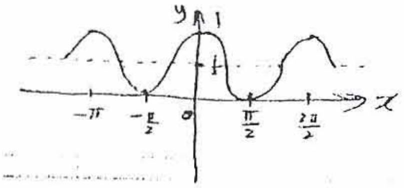
$\frac{4}{276}$ According to Pappus theorem, $\therefore$ The Volume, V
= ~~Area~~ A * distance travelled by centroid =

$$=\frac{\pi r^2}{2} * 2\pi\left(\frac{4r}{3\pi}+r\right) = \pi^2 r^2 r\cdot\left(\frac{4+3\pi}{3\pi}\right)=$$

$$=\frac{\pi^2 r^3(4+3\pi)}{3\pi} = \left(\frac{4+3\pi}{3}\right)\cdot\pi\cdot r^3 = 14.06\ r^3$$

OC

$\frac{10}{294}$ $y=\frac{1}{2}+\frac{1}{2}\cos 2x=\frac{1}{2}(1+\cos 2x)$



—

$\frac{13}{294}$ $y=\tan(3x^2) \quad \therefore y'=\sec^2(3x^2)\cdot 6x$

—

$\frac{22}{294}$ $y=\sec^4 x-\tan^4 x \quad \therefore y'=4\sec^3 x\cdot\sec x\tan x-4\tan^3 x\cdot\sec^2 x=$

$=4\sec^2 x\tan x\left(\sec^2 x-\tan^2 x\right)=4\sec^2 x\tan x$

—

$\frac{42}{294}$ $\int\tan^3 x\sec^2 x\,dx=\int u^3\,du=\frac{u^4}{4}+c=\frac{\tan^4 x}{4}+C.$ $\left\{\begin{array}{l}\text{Let } \tan x=u\\ \therefore \sec^2 x=\frac{du}{dx}\\ \therefore \sec^2 x\cdot dx=du\end{array}\right.$

—

$\frac{43}{294}$ $\int\sec^3 x\tan x\,dx=\int\sec^2 x\cdot\sec x\tan x\,dx=$ Let $\sec x=u$ $\therefore \sec x\tan x=\frac{du}{dx}$

$=\int u^2\,du=\frac{u^3}{3}+c=\frac{\sec^3 x}{3}+C$

—

$\frac{52}{294}$ $\lim_{y\to 0}2y\cot y=\lim_{y\to 0}2y\cdot\frac{\cos y}{\sin y}=\lim_{y\to 0}\frac{y}{\sin y}*2\cos y=1*2=2$

—

$\frac{53}{294}$ $\lim_{t\to\pi}\frac{t-\pi}{2\tan 3t}=L$, Let $t-\pi=u$ $\therefore t\to\pi\equiv u\to 0$

$\therefore L=\lim_{u\to 0}\frac{u}{2\tan 3(u+\pi)}=\lim_{u\to 0}\frac{u}{2\tan(3u+3\pi)}=\lim_{u\to 0}\frac{u}{2\tan 3u}=$

$=\lim_{u\to 0}\frac{u\cos 3u}{2\sin 3u}=\lim_{u\to 0}\frac{1}{3}\cdot\frac{3u}{\sin 3u}\cdot\frac{\cos 3u}{2}=\frac{1}{3}*1*\frac{1}{2}=\frac{1}{6}.$

—

$\frac{8}{249}$

(a) $\sin(\sin^{-1} 0.735) = 0.735$

(b) $\cos(\sin^{-1} 0.8) = \frac{\sqrt{1-.8^2}}{1} = .6$

(c) $\sin(2\sin^{-1} 0.8) = 2\sin(\sin^{-1} 0.8)\cos(\sin^{-1} 0.8) = 2 * 0.8 * 0.6 = 0.96$

(d) $\tan^{-1}(\tan\frac{\pi}{3}) = \frac{\pi}{3} \pm k\pi$ f k integer.

(e) $\cos^{-1}(-\sin\pi/6) = \cos^{-1}(\cos(\frac{\pi}{2}+\frac{\pi}{6})) = \cos^{-1}(\cos\frac{2\pi}{3}) = \pm(\frac{2\pi}{3} + 2k\pi)$ f k integer.

(f) $\sec^{-1}(\sec(-30^\circ)) = \pm(-\frac{\pi}{6} + 2k\pi)$ f k integer

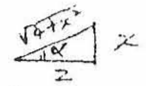
---

$\frac{5}{302}$

$y = \sin^{-1}(\frac{x}{2}) \quad \therefore y' = \frac{(1/2)}{\sqrt{1-(\frac{x}{2})^2}} = \frac{1}{\sqrt{4-x^2}}$

$\frac{9}{302}$

$y = \cot^{-1}\frac{2}{x} + \tan^{-1}\frac{x}{2} = \alpha + \alpha = 2\alpha$



$\frac{y}{2} = \alpha \quad \therefore \sec\frac{y}{2} = \sec\alpha = \frac{\sqrt{4+x^2}}{2} \quad \therefore \quad \sec\frac{y}{2}\cdot\tan\frac{y}{2}\cdot\frac{y'}{2} = \frac{1}{2}\cdot\frac{2x}{2\sqrt{4+x^2}}$

$\therefore \sec\alpha\tan\alpha\cdot\frac{y'}{2} = \frac{x}{2\sqrt{4+x^2}} \quad \therefore y' = \frac{x}{\sqrt{4+x^2}}\cdot\cos\alpha\cdot\cot\alpha = \frac{x}{\sqrt{4+x^2}}\cdot\frac{2}{\sqrt{4+x^2}}\cdot\frac{2}{x}$

$\therefore y' = \frac{4}{4+x^2}$

$\frac{10}{302}$

$y = \sin^{-1}\frac{x-1}{x+1} \quad \therefore \sin y = \frac{x-1}{x+1} \quad \therefore \cos y\cdot y' = \frac{x+1-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$

$\therefore y' = \frac{2}{(x+1)^2}\cdot\frac{1}{\cos y} = \frac{2}{(x+1)^2}\cdot\frac{1}{\sqrt{1-\sin^2 y}} = \frac{2}{(x+1)^2}\cdot\frac{1}{\sqrt{1-(\frac{x-1}{x+1})^2}} = \frac{2/(x+1)}{\sqrt{(x+1)^2-(x-1)^2}} =$

$= \frac{2}{x+1}\cdot\frac{1}{\sqrt{(x+1+x-1)(x+1-x+1)}} = \frac{2}{x+1}\cdot\frac{1}{\sqrt{2x\cdot 2}} = \frac{1}{\sqrt{x}\cdot(x+1)}.$

$\frac{21}{303}$

$\int_{\frac{1}{\sqrt{3}}}^{1}\frac{dx}{x\sqrt{4x^2-1}} = I$, Let $2x = \sec u \quad \therefore \sqrt{4x^2-1} = \sqrt{\sec^2 u - 1} = \tan u$

f $2dx = \sec u\tan u\,du$ f at $x = 1 \therefore u = \frac{\pi}{3}$
at $x = \frac{1}{\sqrt{3}} \therefore u = \pi/6$

$\therefore I = \int_{\pi/6}^{\pi/3}(\frac{1}{2}\sec u\tan u\,du)\cdot\frac{2}{\sec u}\cdot\frac{1}{\tan u} = \int_{\pi/6}^{\pi/3} du$

$= u\Big|_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}.$

ج٢

$\boxed{\frac{23}{303}}$ ⓐ $\lim_{x\to0}\frac{\sin^{-1}2x}{x}=\lim_{x\to0}\frac{2x}{x}=2$

ⓑ $\lim_{x\to0}\frac{2\tan^{-1}3x}{5x}=L$ Let $3x=\tan u$ $\therefore x\to0\equiv u\to0$

$\therefore L=\lim_{u\to0}\frac{2\tan^{-1}(\tan u)}{\frac{5}{3}\tan u}=\frac{6}{5}\cdot\lim_{u\to0}\frac{u\cos u}{\sin u}=\frac{6}{5}\cdot1\cdot1=\frac{6}{5}.$

ⓒ $\lim_{x\to0}x^{-3}(\sin^{-1}x-x)=\lim_{x\to0}\frac{\sin^{-1}x-x}{x^3}=L$

Let $\sin^{-1}x-x=u$ $\therefore\sin(\sin^{-1}x-x)=\sin u$

$\therefore x\cos x-\cos(\sin^{-1}x)\cdot\sin x=\sin u\Rightarrow x\cos x-\sqrt{1-\sin^2(\sin^{-1}x)}\cdot\sin x=\sin u$

$\therefore x\cos x-\sqrt{1-x^2}\cdot\sin x=\sin u\;\therefore u=\sin^{-1}\left[x\cos x-\sqrt{1-x^2}\sin x\right]=\sin^{-1}x-x$

$\therefore L=\lim_{x\to0}\frac{\sin^{-1}\left[x\cos x-\sqrt{1-x^2}\cdot\sin x\right]}{x^3}=$

$=\lim_{x\to0}\frac{x\cos x-\sqrt{1-x^2}\sin x}{x^3}=\lim_{x\to0}\frac{x\cos x-\sqrt{1-x^2}\sin x}{x^3}\cdot\frac{x\cos x+\sqrt{1-x^2}\cdot\sin x}{x\cos x+\sqrt{1-x^2}\cdot\sin x}$

$=\lim_{x\to0}\frac{x^2\cos^2x-(1-x^2)\sin^2x}{x^3\cdot(x\cdot1+\sqrt{1}\cdot x)}=\lim_{x\to0}\frac{x^2\cos^2x-\sin^2x+x^2\sin^2x}{2x^4}=$

$=\lim_{x\to0}\frac{x^2(\sin^2x+\cos^2x)-\sin^2x}{2x^4}=\lim_{x\to0}\frac{x^2-\sin^2x}{2x^4}=\lim_{x\to0}\frac{(x+\sin x)(x-\sin x)}{2x^4}$

$=\lim_{x\to0}\frac{(x+x)(x-\sin x)}{2x^4}=\lim_{x\to0}\frac{x-\sin x}{x^3}=\lim_{x\to0}\frac{1-\cos x}{3x^2}=\lim_{x\to0}\frac{\sin x}{6x}=\frac{1}{6}$

Or, straight $\lim_{x\to0}\frac{\sin^{-1}x-x}{x^3}=\lim_{x\to0}\frac{\frac{1}{\sqrt{1-x^2}}-1}{3x^2}=\lim_{x\to0}\frac{-\frac{1}{2}(1-x^2)^{-\frac{3}{2}}(-2\cancel{x})}{6\cancel{x}}=\frac{1}{6}$

ⓓ $\lim_{x\to0}x^{-3}(\tan^{-1}x-x)=\lim_{x\to0}\frac{\tan^{-1}x-x}{x^3}=\lim_{x\to0}\frac{\frac{1}{1+x^2}-1}{3x^2}=\lim_{x\to0}\frac{-(1+x^2)^{-2}(2x)}{6x\;3}=-\frac{1}{3}$

ج ٥

$\boxed{\frac{1}{307}}$ $y = \ln(x^2+2x) \quad \therefore y' = \frac{2x+2}{x^2+2x} = \frac{2(x+1)}{x(x+2)}.$

$\boxed{\frac{11}{307}}$ $y = \ln\left(\frac{x}{2+3x}\right) = \ln x - \ln(2+3x) \therefore y' = \frac{1}{x} - \frac{(3)}{2+3x} = \frac{2+\cancel{3x}-\cancel{3x}}{x(2+3x)} = \frac{2}{x(2+3x)}.$

$\boxed{\frac{14}{307}}$ $y = x\ln^3 x \quad \therefore y' = \ln^3 x + 3\ln^2 x . \frac{1}{x} . x = (\ln x)^2((\ln x)+3).$

$\boxed{\frac{22}{307}}$ $\int \frac{\sin x}{2-\cos x}\,dx = I$, Let $2-\cos x = u \quad \therefore \sin x\,dx = du$

$\therefore I = \int \frac{du}{u} = \ln u + C = \ln(2-\cos x) + C$

$\boxed{\frac{28}{307}}$ $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = I$, Let $1+\sqrt{x} = u \quad \therefore \frac{du}{dx} = \frac{1}{2\sqrt{x}} \therefore du = \frac{dx}{2\sqrt{x}}$

$\therefore I = \int \frac{2du}{u} = 2\ln u + C = 2\ln(1+\sqrt{x}) + C$

$\boxed{\frac{32}{307}}$ $\int \frac{dx}{x\ln x} = I$, Let $\ln x = u \quad \therefore \frac{du}{dx} = \frac{1}{x} \quad \therefore du = \frac{dx}{x}$

$\therefore I = \int \frac{du}{u} = \ln u + C = \ln(\ln x) + C$

$\boxed{\frac{2}{315}}$ $y = x^2 e^x \quad \therefore \quad y' = 2xe^x + x^2 e^x = xe^x(2+x)$

$\boxed{\frac{17}{315}}$ $e^{2x} = \sin(x+3y) \quad \therefore \quad 2e^{2x} = \cos(x+3y)\cdot(1+3y')$

$\therefore \quad y' = \dfrac{2e^{2x} - \cos(x+3y)}{3\cos(x+3y)} = \frac{2}{3}\cdot e^{2x}\cdot\sec(x+3y) - \frac{1}{3}.$

$\boxed{\frac{18}{315}}$ $y = e^{\frac{1}{x}} \quad \therefore y' = e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right) = -\dfrac{e^{(1/x)}}{x^2}.$

$\boxed{\frac{24}{315}}$ $\int e^{\sin x}\cos x\,dx = I \quad , \quad$ Let $\sin x = u \quad \therefore \quad du = \cos x\,dx$

$\therefore \quad I = \int e^u du = e^u + C = e^{\sin x} + C$

$\boxed{\frac{25}{315}}$ $\int e^{x/3}\,dx = \dfrac{e^{x/3}}{(1/3)} + C = 3e^{(x/3)} + C$

$\boxed{\frac{27}{315}}$ $\int \dfrac{e^x}{1+2e^x}\cdot dx = I \quad , \quad$ Let $1+2e^x = u \quad \therefore \quad \dfrac{du}{dx} = 2e^x$

$\therefore \quad I = \int \dfrac{du/2}{u} = \frac{1}{2}\ln u + C = \frac{1}{2}\ln(1+2e^x) + C$

$\boxed{\frac{8}{320}}$ $y = x^{\ln x} \quad \therefore \quad \ln y = (\ln x)\cdot(\ln x) = (\ln x)^2 \quad \therefore \quad y = e^{(\ln x)^2}$

$\therefore \quad y' = e^{(\ln x)^2}\cdot(2\ln x)\cdot\frac{1}{x} = y\cdot\dfrac{2\ln x}{x} = \dfrac{2y}{x}\cdot\ln x$

$\boxed{\frac{12}{320}}$ $\int_0^{1.2} 3^x dx = I \quad , \quad$ let $u = 3^x \quad \therefore \quad \ln u = x\ln 3 \quad \therefore \quad u = e^{x\ln 3} = 3^x$

$\therefore \quad I = \int_0^{1.2} e^{x\ln 3} dx = \dfrac{e^{x\ln 3}}{\ln 3}\Big|_0^{1.2} = \dfrac{e^{1.2\ln 3} - e^0}{\ln 3} = \dfrac{e^{1.2\ln 3} - 1}{\ln 3} = 2.492$

$\boxed{\frac{14}{321}}$ $\int_0^1 5^{2t-2}dt = \int_0^1 e^{(2t-2)\ln 5}dt = \dfrac{e^{(2t-2)\ln 5}}{2\ln 5}\Big|_0^1 = \dfrac{e^0 - e^{-2\ln 5}}{2\ln 5} = \dfrac{1-e^{-2\ln 5}}{2\ln 5} = 0.2482$

OV

$\boxed{\frac{2}{404}}$ $C(-2,0)$, $r=3$ $\therefore (x+2)^2+y^2=9$ or $x^2+y^2+4x=5$

$\boxed{\frac{10}{404}}$ $2x^2+2y^2+x+y=0$ $\therefore x^2+y^2+\frac{x}{2}+\frac{y}{2}=0$ $\therefore (x+\frac{1}{4})^2+(y+\frac{1}{4})^2-\frac{2}{16}=0$

$\therefore (x+\frac{1}{4})^2+(y+\frac{1}{4})^2=(\frac{\sqrt{2}}{4})^2$ $\therefore C(-\frac{1}{4},-\frac{1}{4})$ & $r=\frac{\sqrt{2}}{4}$.

$\boxed{\frac{13}{404}}$ $C(2,2)$ & $r=\sqrt{(2-4)^2+(2-5)^2}=\sqrt{4+9}=\sqrt{13}$

$\therefore$ The equation is $(x-2)^2+(y-2)^2=13$ or $x^2+y^2-4x-4y=5$

$\boxed{\frac{15}{404}}$ $(x-h)^2+(y-k)^2=r^2$ is the equation of circle centre (h,k) & radius r

$\because A(2,-2)$ is on the circle $\therefore (2-h)^2+(-2-k)^2=r^2$ (1)

$\because B(3,4)$ is on the circle $\therefore (3-h)^2+(4-k)^2=r^2$ (2)

$\because C(h,k)$ is on $x+y=2$ $\therefore h+k=2$ (3)

$(1)-(2) \Rightarrow (2-h)^2+(2+k)^2-(3-h)^2-(4-k)^2=0$ $\therefore 4-4h+\cancel{h^2}+4+4k+\cancel{k^2}-9+6h-\cancel{h^2}-16+8k-\cancel{k^2}=0$

$\therefore 2h+12k=17$ but from (3) $k=2-h$

$\therefore 2h+12(2-h)=17 \Rightarrow 2h+24-12h=17$ $\therefore -10h=17-24=-7$ $\therefore h=\frac{7}{10}$

$\therefore k=2-\frac{7}{10}=\frac{13}{10}$ $\therefore r^2$ (from (1)) $=(2-\frac{7}{10})^2+(2+\frac{13}{10})^2=\frac{13^2+33^2}{100}=\frac{1258}{100}$

$\therefore$ The equation is $(x-\frac{7}{10})^2+(y-\frac{13}{10})^2=\frac{1258}{100}$ or $(10x-7)^2+(10y-13)^2=1258$

$\therefore 100x^2-140x+49+100y^2-260y+169=1258$ $\therefore 100x^2+100y^2-140x-260y=1040$

OR $5x^2+5y^2-7x-13y=52$.

$\boxed{\frac{17}{404}}$ Let the equation be $x^2+y^2+ax+by+c=0$

$\therefore A(2,3)$ is on the circle $\therefore 4+9+2a+3b+c=0$ $\therefore 2a+3b+c=-13$ (1)

$\therefore B(3,2)$ " " " " $\therefore 9+4+3a+2b+c=0$ $\therefore 3a+2b+c=-13$ (2)

$\therefore D(-4,3)$ " " " " $\therefore 16+9-4a+3b+c=0$ $\therefore -4a+3b+c=-25$ (3)

$(1)-(2)$ $\therefore -a+b=0$ $\therefore a=b$ (5) & $(2)-(3)$ $\therefore 7a-b=12$ using (5)

$\therefore 7a-a=12$ $\therefore 6a=12$ $\therefore a=2$ $\therefore b=2$ $\therefore c$ from (1) $=-13-4-6=-23$

CA

2.V

$\therefore$ The equation is $x^2+y^2+2x+2y-23=0$ or $(x+1)^2+(y+1)^2=25$.

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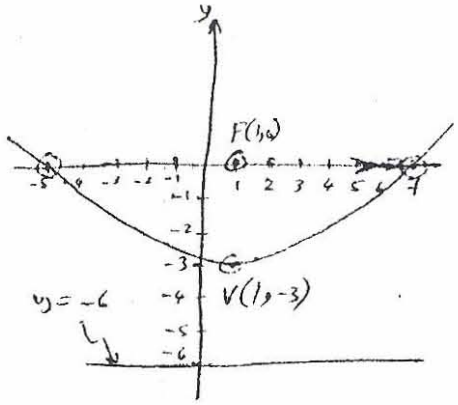
$p=3$ & axis // y-axis & concave up

$\therefore$ $(x-1)^2 = 12(y+3)$ is its equation

& $y=-6$ is its directrix



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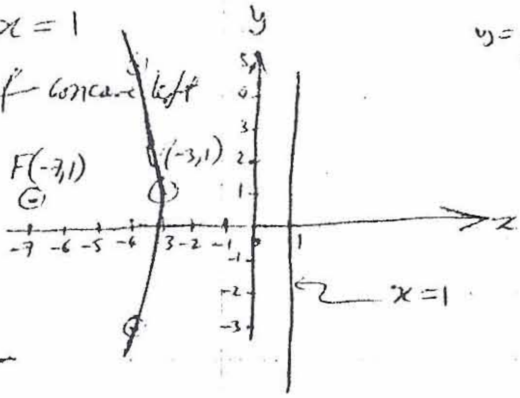
$V(-3,1)$ & directrix $x=1$

$\therefore$ $p=4$ & axis // x-axis & concave left

$\therefore$ $F(-7,1)$

$\therefore$ $(y-1)^2=-16(x+3)$

is its equation



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$3x^2-8y-12x=4$

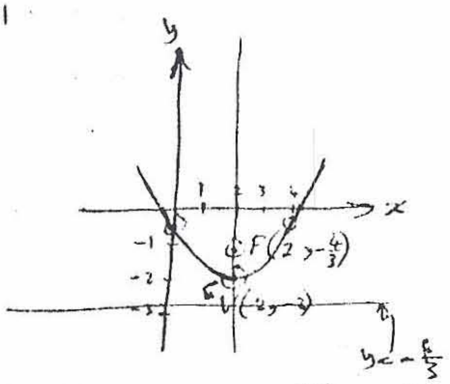
$3(x^2-4x) = 4+8y$

$3[(x-2)^2-4] = 4+8y$

$(x-2)^2 = 4+\frac{4+8y}{3} = \frac{16+8y}{3} = \frac{8}{3}(y+2)$

$\therefore$ Vertex is $(2,-2)$ & $p=\frac{2}{3}$ & axis // to y-axis (equation $x=2$) & concave up, $F(2,-\frac{4}{3})$

& directrix is $y=-\frac{8}{3}$.



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$3x-2y^2-4y+7=0$

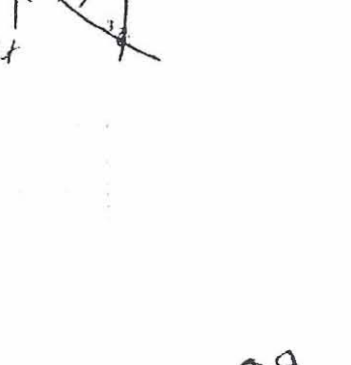
$\therefore$ $-2(y^2+2y) = -3x-7$

$\therefore$ $y^2+2y = \frac{3x+7}{2}$ $\therefore$ $(y+1)^2-1 = \frac{3x+7}{2}$

$(y+1)^2 = \frac{3x+9}{2} = \frac{3}{2}(x+3)$

$\therefore$ Vertex $(-3,-1)$ & $p=\frac{3}{8}$ & axis // to x-axis $(y=-1)$ & concave right

$F(-\frac{21}{8},-1)$ directrix $x=-\frac{27}{8}$.



Q6

$\frac{28}{410}$ (a) $//$ to x-axis $\therefore (y-k)^2 = 4p(x-h)$

$(-1,2) \Rightarrow (2-k)^2 = 4p(-1-h)$ (1)

$(1,-1) \Rightarrow (-1-k)^2 = 4p(1-h)$ (2)

$(2,1) \Rightarrow (1-k)^2 = 4p(2-h)$ (3)

$\frac{(1)}{(2)} \therefore \left(\frac{2-k}{1+k}\right)^2 = \frac{h+1}{h-1} \Longrightarrow \left(\frac{3}{1+k} - 1\right)^2 = 1 + \frac{2}{h-1} \Rightarrow \frac{9}{(1+k)^2} - \frac{6}{1+k} = \frac{2}{h-1}$

$\therefore \frac{9-6(1+k)}{(1+k)^2} = \frac{2}{h-1} \Longrightarrow \frac{9-6-6k}{(1+k)^2} = \frac{2}{h-1}$ (4) $\therefore h = 1 + \frac{2(1+k)^2}{3(1-2k)}$ (5)

& $\frac{(3)}{(2)} \therefore \left(\frac{1-k}{1+k}\right)^2 = \frac{h-2}{h-1} = 1 - \frac{1}{h-1} = (\text{from (4)})\ 1 - \frac{3-6k}{2(1+k)^2} = \frac{2(1+k)^2 - 3 + 6k}{2(1+k)^2}$

$\therefore 2(1-k)^2 = 2(1+k)^2 - 3 + 6k \Longrightarrow \cancel{2} - 4k + \cancel{2k^2} = \cancel{2} + 4k + \cancel{2k^2} - 3 + 6k$

$\therefore 14k = 3 \quad \therefore k = \frac{3}{14} \quad \therefore h = (\text{from 5})\ 1 + \frac{2}{3} \cdot \frac{\left(\frac{17}{14}\right)^2}{\left(\frac{8}{14}\right)} = 1 + \frac{2}{3} \cdot \frac{17^2 * 14}{14^2 * 8} = 1 + \frac{289}{168} = \frac{457}{168}$

from (1) $\therefore \left(2 - \frac{3}{14}\right)^2 = 4p\left(-1 - \frac{457}{168}\right) \therefore p = \left(\frac{25}{14}\right)^2 \cdot \frac{168}{-4(168+157)} = \frac{-42}{14^2} = \frac{-3}{14}$

$\therefore$ Equation is $\left(y - \frac{3}{14}\right)^2 = -\frac{3*4}{14}\left(x - \frac{457}{168}\right)$

$\therefore y^2 - \frac{6}{14}y + \frac{9}{196} = -\frac{12}{14*168}(168x - 457) = -\frac{168x - 457}{196}$

$\therefore 196y^2 - 84y + 9 = -168x + 457 \quad \therefore 196y^2 - 84y + 168x - 448 = 0$

OR the equation is $7y^2 - 3y + 6x - 16 = 0$

This is one way to solve the problem, another easier way is to put the equation of the parabola as $y^2 + ay + bx + c = 0$

$\therefore (-1,2) \Rightarrow 4 + 2a - b + c = 0$ (6)

& $(1,-1) \Rightarrow 1 - a + b + c = 0$ (7)

& $(2,1) \Rightarrow 1 + a + 2b + c = 0$ (8)

$(8) - (7) \Rightarrow 2a + b = 0$ (9) & $(7) - (6) \Rightarrow -3 - 3a + 2b = 0$ (10)

$\therefore$ (9) into (10) $\therefore -3 - 3a + 2(-2a) = 0 \quad \therefore a = \frac{-3}{7} \quad \therefore (\text{from (9)})\ b = \frac{6}{7}$

$\therefore$ (from (7)) $1 + \frac{3}{7} + \frac{6}{7} + c = 0 \quad \therefore c = \frac{-16}{7}$

$\therefore$ The equation is $y^2 - \frac{3}{7}y + \frac{6}{7}x - \frac{16}{7} = 0$ OR $7y^2 - 3y + 6x - 16 = 0$

which is the same as above.

Therefore, this way of solving is going to be used later.

(b) // to y-axis $\therefore x^2 + ax + by + c = 0$

$(-1,2) \Rightarrow 1 - a + 2b + c = 0$ (11)

$(1,-1) \Rightarrow 1 + a - b + c = 0$ (12)

$(2,1) \Rightarrow 4 + 2a + b + c = 0$ (13)

$\therefore$ (12) − (11) $\Rightarrow 2a - 3b = 0$ (14) & (13) − (12) $\Rightarrow 3 + a + 2b = 0$ (15)

(14) into (15) $\Rightarrow 3 + a + 2\left(\frac{2a}{3}\right) = 0 \quad \therefore \frac{7a}{3} + 3 = 0 \quad \therefore a = \frac{-9}{7}$

$\therefore$ (from (14)) $b = \frac{2a}{3} = \frac{2}{3} \times \frac{-9}{7} = \frac{-6}{7}$ & from (11) $\therefore 1 + \frac{9}{7} - \frac{12}{7} + c = 0$

$\therefore c = \frac{-4}{7}$ $\therefore$ The equation is $x^2 - \frac{9}{7}x - \frac{6}{7}y - \frac{4}{7} = 0$

or $7x^2 - 9x - 6y - 4 = 0$

$\boxed{\frac{4}{417}}$

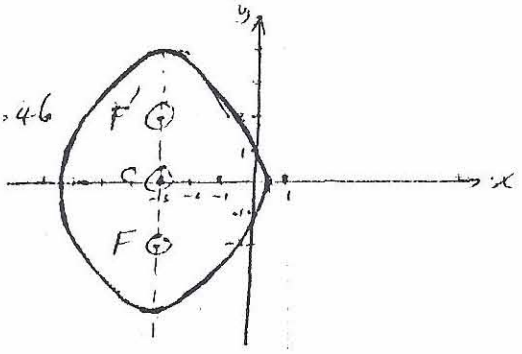
$C(-3,0)$, $F(-3,-2)$, $a = 4$

$b = \sqrt{a^2 - c^2} = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3} = 3.46$

$\therefore$ The equation is: $\frac{(x+3)^2}{(2\sqrt{3})^2} + \frac{y^2}{(4)^2} = 1$

or $\frac{(x+3)^2}{12} + \frac{y^2}{16} = 1$

$ea = c \quad \therefore e = \frac{c}{a} = \frac{2}{4} = \frac{1}{2}$



$\boxed{\frac{6}{417}}$

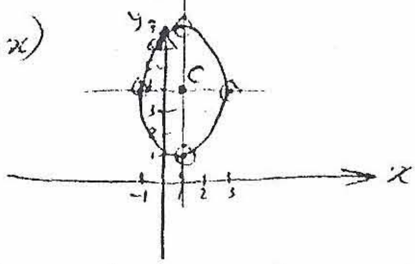
$\therefore$ C is $(1,4)$ and $a = 3$ (on y) & $b = 2$ (on x)

$\therefore$ The equation is $\frac{(x-1)^2}{(2)^2} + \frac{(y-4)^2}{(3)^2} = 1$

or, $\frac{(x-1)^2}{4} + \frac{(y-4)^2}{9} = 1$

$c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5} = 2.24$

$\therefore F_1$ is $(1, 4-\sqrt{5})$ & F_2 is $(1, 4+\sqrt{5})$ or $F_1(1, 1.76)$ & $F_2(1, 6.24)$



$\boxed{\frac{7}{417}}$

$25x^2 + 9y^2 - 100x + 54y - 44 = 0$

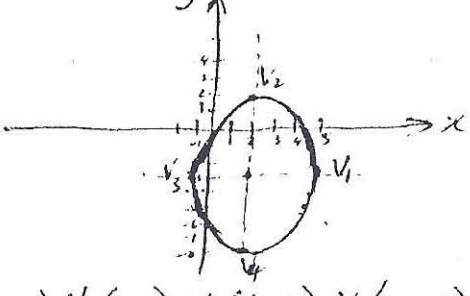
$\therefore 25(x^2 - 4x) + 9(y^2 + 6y) = 44$

$\therefore 25\left[(x-2)^2 - 4\right] + 9\left[(y+3)^2 - 9\right] = 44$

$\therefore 5^2(x-2)^2 + 3^2(y+3)^2 = 44 + 100 + 81 = 225 = 15^2$

$\therefore \frac{(x-2)^2}{3^2} + \frac{(y+3)^2}{5^2} = 1 \quad \therefore$ Centre is $(2,-3)$, $V_1(5,-3)$, $V_2(2,2)$, $V_3(-1,-3)$, $V_4(2,-8)$

$c = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4 \quad \therefore F(2, -3 \pm 4)$ or $F_1(2,-7)$, $F_2(2,1)$.



ﻋﻟ

$\frac{16}{418}$ $4x^2+y^2+Ax+By+C=0$

$\therefore$ $8x+2yy'+A+By'=0$ $\because$ tangent to x $\therefore$ $y'=0$ and $x=0$ & $y=0$

$\therefore$ $8(0)+0+A+B(0)=0$ $\therefore$ $A=0$

$(0,0) \Rightarrow$ $C=0$

$(-1,2) \Rightarrow$ $4+4-\cancel{A}^{0}+2B+\cancel{C}^{0}=0$ $\therefore$ $B=-4$

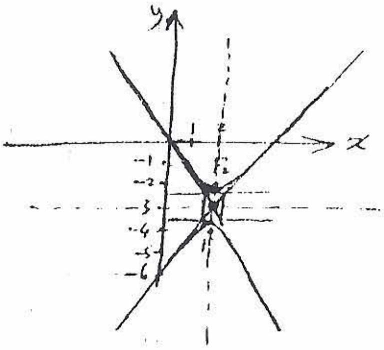
$\therefore$ The equation is $4x^2+y^2-4y=0$ or $4x^2+(y-2)^2=4$ or $x^2+\frac{(y-2)^2}{4}=1$

$\frac{4}{425}$ $4(y+3)^2-9(x-2)^2=1$

$\therefore$ $\frac{(y+3)^2}{(\frac{1}{2})^2}-\frac{(x-2)^2}{(\frac{1}{3})^2}=1$ $\therefore$ $a=\frac{1}{2}$, $b=\frac{1}{3}$ $\therefore$ $c=\frac{\sqrt{13}}{6}\approx .6$

assymptotis are $4(y+3)^2-9(x-2)^2=0$

or $2(y+3)=\pm 3(x-2)$ i.e $y=-3\pm\frac{3}{2}(x-2)$



$\frac{5}{425}$ $5x^2-4y^2+20x+8y=4$

$\therefore$ $5(x^2+4x)-4(y^2-2y)=4$

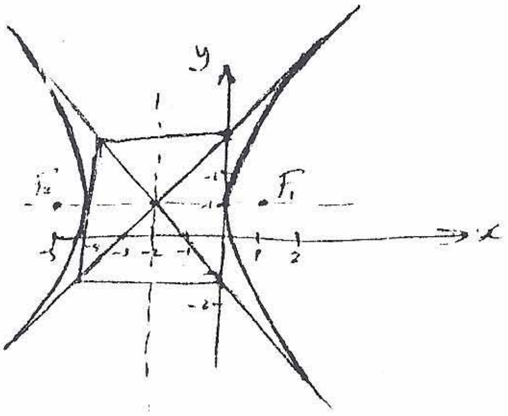
$\therefore$ $5\left[(x+2)^2-4\right]-4\left[(y-1)^2-1\right]=4$

$5(x+2)^2-4(y-1)^2=4+20-4=20$

$\frac{(x+2)^2}{4}-\frac{(y-1)^2}{5}=1 \Rightarrow \frac{(x+2)^2}{(2)^2}-\frac{(y-1)^2}{(\sqrt{5})^2}=1$

$\therefore$ assymptots at $(y-1)^2=\frac{5}{4}(x+2)^2$

or $y=1\pm\frac{\sqrt{5}}{2}(x+2)$, $a=\sqrt{5}$, $b=2$, $c=3$



$\frac{12}{426}$ $F_1(0,$ j , $(0,4)$ $\therefore$ C is $(0,2)$ & real $\because$ the y axis

$\therefore$ Its equation is $-\frac{(x-0)^2}{a^2}+\frac{}{b^2}=1$ or $\frac{(y-2)^2}{b^2}-\frac{x^2}{a^2}=1$ (1)

$\because$ $(12,9)$ is on it $\therefore$ $\frac{(9-2)^2}{b^2}-\frac{}{a^2}=1$ or $\frac{49}{b^2}-\frac{144}{a^2}=1$ (2)

... $c^2=a^2+b^2$ $\therefore$ $(\frac{4}{2})^2=a^2+b^2$ .. $a^2=4-b^2$ (3)

(3) into (2) $\therefore$ $\frac{49}{}-\frac{144}{}=1$ $\therefore$ $49(4-b^2)-144b^2=b^2(4-b^2)$

$\therefore$ $b^4-b^2(4+144+49)+4\times 49=0$. $b^4-197b^2+196=0$ $\therefore$ $(b^2-196)(b^2-1)=0$

$\therefore$ $b^2=196$ $\therefore$ a^2 (from (3)) $=4-196=-192$ X or $b^2=1$ $\therefore$ $a^2=3$

$\therefore$ The . , is (from (1)) $\frac{(y-2)^2}{1}-\frac{}{3}=1$, or $3(y^2-4y+4)-x^2=3$ $\therefore$ $3y^2-x^2-12y+9=0$

7C

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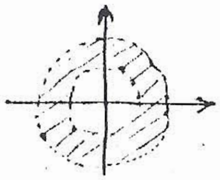
$1<r<2$. This describes the region between the two circles $r=1$ and $r=2$.



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(a) $(3,\frac{\pi}{4})$ $\therefore r=3, \theta=\frac{\pi}{4}$ $\therefore x=3\cos\frac{\pi}{4}=\frac{3}{\sqrt{2}}, y=3\sin\frac{\pi}{4}=\frac{3}{\sqrt{2}}$ $\therefore (x,y)=(\frac{3}{\sqrt{2}},\frac{3}{\sqrt{2}})$

(b) $(-3,\frac{\pi}{4})$ $\therefore r=-3, \theta=\frac{\pi}{4}$ $\therefore x=-3\cos\frac{\pi}{4}=\frac{-3}{\sqrt{2}}, y=-3\sin\frac{\pi}{4}=\frac{-3}{\sqrt{2}}$ $\therefore (x,y)=(\frac{-3}{\sqrt{2}},\frac{-3}{\sqrt{2}})$

(c) $(3,\frac{-\pi}{4})$ $\therefore r=3, \theta=\frac{-\pi}{4}$ $\therefore x=3\cos(\frac{-\pi}{4})=\frac{3}{\sqrt{2}}, y=3\sin(\frac{-\pi}{4})=\frac{-3}{\sqrt{2}}$ $\therefore (x,y)=(\frac{3}{\sqrt{2}},\frac{-3}{\sqrt{2}})$

(d) $(-3,\frac{-\pi}{4})$ $\therefore r=-3, \theta=\frac{-\pi}{4}$ $\therefore x=-3\cos(\frac{-\pi}{4})=\frac{-3}{\sqrt{2}}, y=-3\sin(\frac{-\pi}{4})=\frac{-3}{-\sqrt{2}}=\frac{3}{\sqrt{2}}$ $\therefore (x,y)=(\frac{-3}{\sqrt{2}},\frac{3}{\sqrt{2}})$

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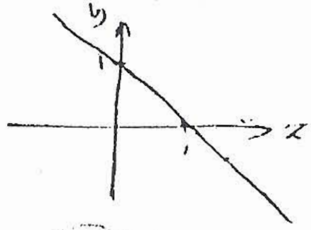
$r\cos\theta+r\sin\theta=1$

$\because r\cos\theta=x, r\sin\theta=y$

$\therefore x+y=1$, straight line.



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$r\sin\theta=r\cos\theta$, $\therefore y=x$

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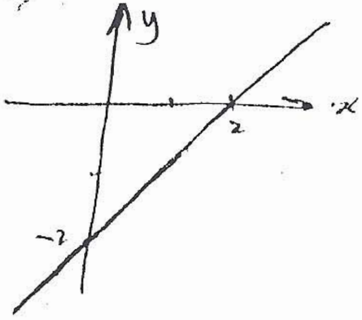
$r\sin(45^\circ-\theta)=\sqrt{2}$

$r(\sin 45\cos\theta-\cos 45\sin\theta)=\sqrt{2}$

$r(\frac{1}{\sqrt{2}}\cdot\cos\theta-\frac{1}{\sqrt{2}}\sin\theta)=\sqrt{2}$

$\therefore r\cos\theta-r\sin\theta=\sqrt{2}\cdot\sqrt{2}=2$

$\therefore x-y=2$, straight line.



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$r=a\cos 2\theta$, $r=a(1+\cos\theta)$ $\therefore a=0$, or then $a\cos 2\theta=a(1+\cos\theta)$

$\therefore 2\cos^2\theta-1=1+\cos\theta$ $\therefore 2\cos^2\theta-\cos\theta-2=0$

$\therefore \cos\theta=\frac{1\pm\sqrt{1-4(2)(-2)}}{4}=\frac{1\pm\sqrt{17}}{4}=$ (+ is not a solution) $\frac{1-\sqrt{17}}{4}=-.781$

$\therefore \theta=\pm 141.3^\circ\pm 2k\pi$, or $a=0$

$\therefore$ The points are:

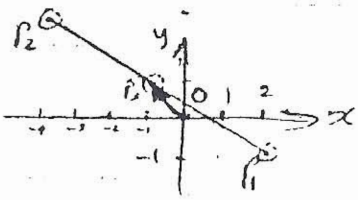
1- $a=0$ $\therefore r=0$, the origin,

2- $\theta=\pm 141.3^\circ\pm 2k\pi$ $\therefore r=a(1+(-.781))=a(1-.781)=.22a$

$\boxed{\frac{2}{487}}$

$O(0,0)$, $P_3\left(\frac{2-4}{2}, \frac{-1+3}{2}\right) = (-1, 1)$

$\therefore \overline{OP_3} = \langle -1-0, 1-0 \rangle = \langle -1, 1 \rangle$

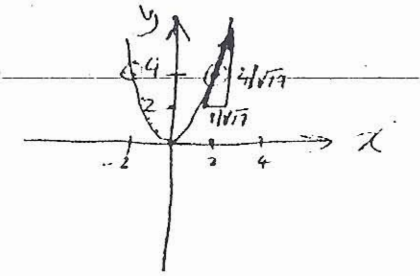


$\boxed{\frac{8}{487}}$

$y = x^2 \quad \therefore y' = 2x = \Big|_{(2,4)} 4$

$\therefore$ A tangent vector is $\langle 1, 4 \rangle$

$\therefore$ a unit vector is $\dfrac{\langle 1,4 \rangle}{\sqrt{1^2+4^2}} = \left\langle \dfrac{1}{\sqrt{17}}, \dfrac{4}{\sqrt{17}} \right\rangle$

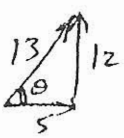


$\boxed{\frac{15}{487}}$

$|\langle 5, 12 \rangle| = \sqrt{5^2+12^2} = \sqrt{25+144} = \sqrt{169} = 13$

Length $= 13$

$\tan\theta = \frac{12}{5} = 2.4 \quad \therefore \theta = \tan^{-1} 2.4 = 67.4°$



$\boxed{\frac{1}{A3}}$

$$\begin{aligned} 2x - 3y + 4z &= -19 \\ 6x + 4y - 2z &= 8 \\ x + 5y + 4z &= 23 \end{aligned} \Longrightarrow \begin{bmatrix} 2 & -3 & 4 \\ 6 & 4 & -2 \\ 1 & 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -19 \\ 8 \\ 23 \end{bmatrix}$$

$\boxed{\frac{1}{A11}}$

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 2 & 1 \end{vmatrix} =$$

ⓐ $= 2\begin{vmatrix} 0 & 3 \\ 2 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2(-6) + 1(1) + 2(2) = -12 + 1 + 4 = -7$

ⓑ $= -(-1)\begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 1(1) + 0 - 2(4) = 1 - 8 = -7$

$\boxed{\frac{4}{A12}}$

$$\begin{vmatrix} 1 & -1 & 2 & 3 \\ 2 & 1 & 2 & 6 \\ 1 & 0 & 2 & 3 \\ -2 & 2 & 0 & -5 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 & 3 \\ 0 & 3 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 1 \end{vmatrix} = \frac{1}{3}\begin{vmatrix} 1 & -1 & 2 & 3 \\ 0 & 3 & -2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \frac{1}{3} \times 1 \times 3 \times 2 \times 1 = 2$$

$\boxed{\frac{1}{A3}}$

ⓐ By reduction: $\left[\begin{array}{ccc|c} 2 & -3 & 4 & -19 \\ 6 & 4 & -2 & 8 \\ 1 & 5 & 4 & 23 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & -3 & 4 & -19 \\ 0 & 13 & -14 & 65 \\ 0 & -13 & -4 & -65 \end{array}\right] \Rightarrow \left[\begin{array}{cccc} 2 & -3 & 4 & -19 \\ 0 & 13 & -14 & 65 \\ 0 & 0 & -18 & 0 \end{array}\right]$

$\therefore -18z = 0 \ \therefore z = 0$, $13y - 14\overset{0}{z} = 65 \ \therefore y = 5$, $2x - 3y + 4\overset{0}{z} = -19 \ \therefore x = -2$

(b) By inverse:

1- by reduction:

$$\left[\overbrace{\begin{matrix}2&-3&4\\6&4&-2\\1&5&4\end{matrix}}^{A}\middle|\overbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}^{I}\right] \Rightarrow \left[\begin{array}{ccc|ccc}2&-3&4&1&0&0\\0&13&-14&-3&1&0\\0&-13&-4&1&0&-2\end{array}\right] \Rightarrow \begin{bmatrix}2&-3&4&1&0&0\\0&13&-14&-3&1&0\\0&0&-18&-2&1&-2\end{bmatrix}$$

$$\Downarrow$$

$$\left[\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\ \underbrace{\begin{matrix}\frac{52}{468}&\frac{64}{468}&\frac{-20}{468}\\ \frac{-13}{117}&\frac{2}{117}&\frac{14}{117}\\ \frac{2}{18}&\frac{-1}{18}&\frac{2}{18}\end{matrix}}_{A^{-1}}\right] \Leftarrow \begin{bmatrix}1&0&\frac{10}{26}&\frac{4}{26}&\frac{3}{26}&0\\0&1&0&\frac{-26}{234}&\frac{4}{234}&\frac{28}{234}\\0&0&1&\frac{2}{18}&\frac{-1}{18}&\frac{2}{18}\end{bmatrix} \Leftarrow \begin{bmatrix}1&-\frac{3}{2}&2&\frac{1}{2}&0&0\\0&1&-\frac{14}{13}&\frac{-3}{13}&\frac{1}{13}&0\\0&0&1&\frac{2}{18}&\frac{-1}{18}&\frac{2}{18}\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{468}\begin{bmatrix}52&64&-20\\-52&8&56\\52&-26&52\end{bmatrix}=\frac{1}{234}\begin{bmatrix}26&32&-10\\-26&4&28\\26&-13&26\end{bmatrix}$$

2- by determinant. $\therefore A^{-1}=\frac{C^{t}}{|A|}=$

$$=\frac{\begin{bmatrix}26&-26&26\\+32&4&-13\\-10&+28&26\end{bmatrix}}{2\begin{vmatrix}4&-2\\5&4\end{vmatrix}+3\begin{vmatrix}6&-2\\1&4\end{vmatrix}+4\begin{vmatrix}6&4\\1&5\end{vmatrix}}=\frac{\begin{bmatrix}26&32&-10\\-26&4&28\\26&-13&26\end{bmatrix}}{2(26)+3(26)+4(26)}=\frac{1}{234}\cdot\begin{bmatrix}26&32&-10\\-26&4&28\\26&-13&26\end{bmatrix}$$

$$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{234}\begin{bmatrix}26&32&-10\\-26&4&28\\26&-13&26\end{bmatrix}\begin{bmatrix}-19\\8\\23\end{bmatrix}=\frac{1}{234}\begin{bmatrix}-468\\1170\\0\end{bmatrix}=\begin{bmatrix}-2\\5\\0\end{bmatrix}$$

$\therefore\ x=-2\ ,\ y=5$ & $z=0$

(c) By Cramer Rule

$$\begin{bmatrix}2&-3&4\\6&4&-2\\1&5&4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-19\\8\\23\end{bmatrix}\ \therefore\ \underset{\text{(as before)}}{\Delta=234}\ ,\ \Delta_x=-19\begin{vmatrix}4&-2\\5&4\end{vmatrix}-8\begin{vmatrix}-3&4\\5&4\end{vmatrix}+23\begin{vmatrix}-3&4\\4&-2\end{vmatrix}$$

$$=-468\ ,\ \Delta_y=+19\begin{vmatrix}6&-2\\1&4\end{vmatrix}+8\begin{vmatrix}2&4\\1&4\end{vmatrix}-23\begin{vmatrix}2&4\\6&2\end{vmatrix}=1170$$

$$,\ \Delta_z=-19\begin{vmatrix}6&4\\1&5\end{vmatrix}-8\begin{vmatrix}2&-3\\1&5\end{vmatrix}+23\begin{vmatrix}2&-3\\6&4\end{vmatrix}=0$$

$$\therefore\ x=\frac{\Delta x}{\Delta}=\frac{-468}{234}=-2\ ,\ y=\frac{\Delta y}{\Delta}=\frac{1170}{234}=5\ ,\ z=\frac{\Delta z}{\Delta}=0$$

$\therefore\ x=-2,\ y=5$ & $z=0$

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