بسم الله الرحمن الرحيم

الحلول المختارة لطلاب الهندسة والعمارة

السيطرة الآلية

اعداد

سلمان محمد العاكري

الاستاذ المشارك بكلية الهندسة والعمارة

الاسلامية بجامعة ام القرى

الاصدار الثالث

جمادى الآخرة ١٤١٧ هـ

تمهيد

الحمد لله رب العالمين والصلاة والسلام على سيد المرسلين نبينا محمد وعلى آله وصحبه أجمعين والتابعين بإحسان إلى يوم الدين

وبعد، فهذه مجموعة من المسائل المحلولة في مادة السيطرة الآلية لطلاب كلية الهندسة والعمارة الإسلامية، اخترتها خلال تدريسي لهذه المادة لجميع المنهج المقرر . وجميعها مأخوذة من كتاب:

Introduction to Control System: Analysis & Design

لمؤلفه : F. J. Hale

الطبعة الأولى لعام ١٩٧٣م، وغيره من واجبات وامتحانات .

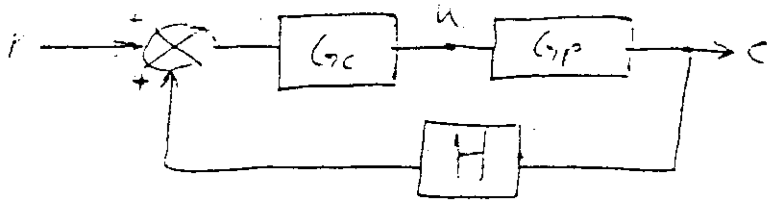
وقد قمت بتبويبها وفق تسلسل المراجع نفسها وتقسيمها، واكتفيت عند ذكر المسألة بذكر رقمها والصفحة التي وردت فيها في الكتاب المذكور أعلاه والمقرر لمادة السيطرة الآلية لطلاب قسم الهندسة الكهربائية والحاسبات بجامعة أم القرى ،

والله أسأل أن ينفع بهذا العمل وأن يجزيني به يوم تنقطع الأسباب والأنساب إنه جواد كريم .

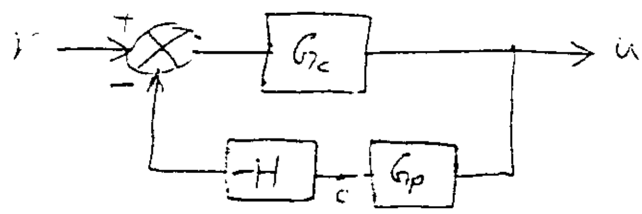
المدرس
١٠/٧/١٤١٣هـ

الفهرس

2b / 24

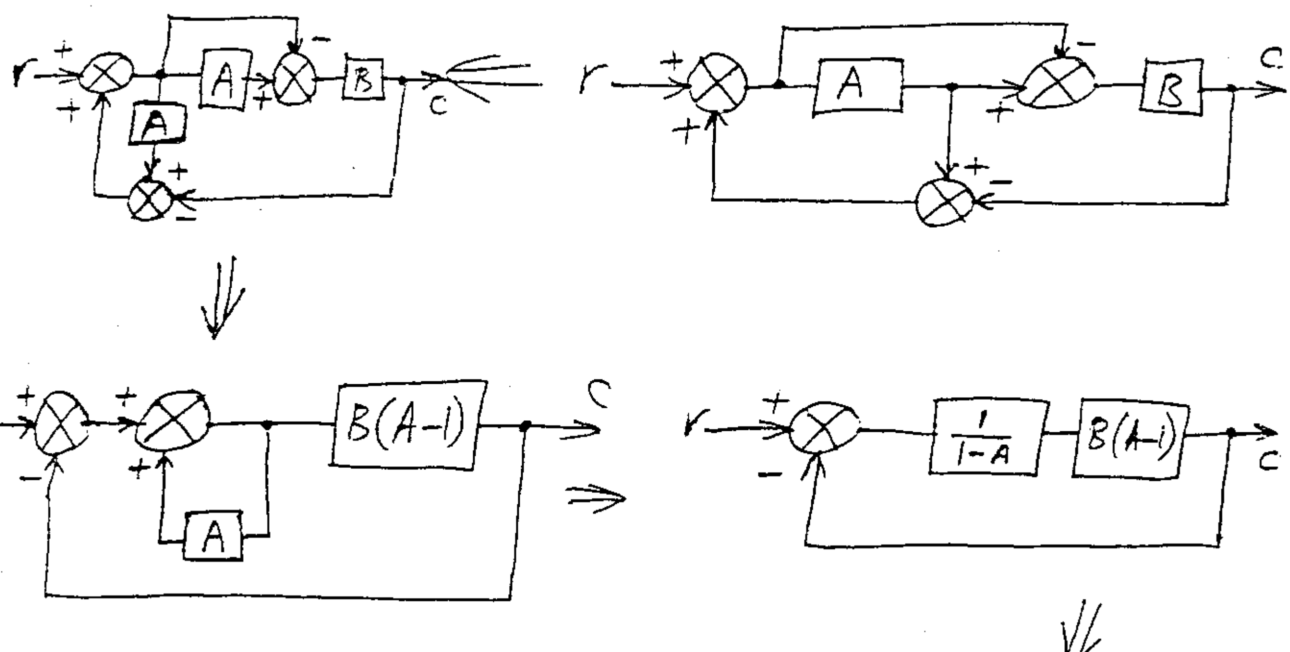


For u as output the block diagram becomes:

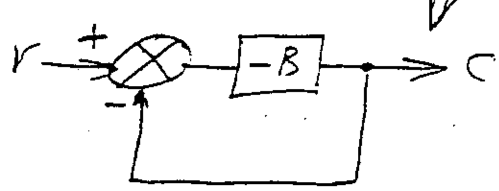


$$\therefore \frac{u}{r} = \frac{G_c}{1 - G_c G_p H}$$

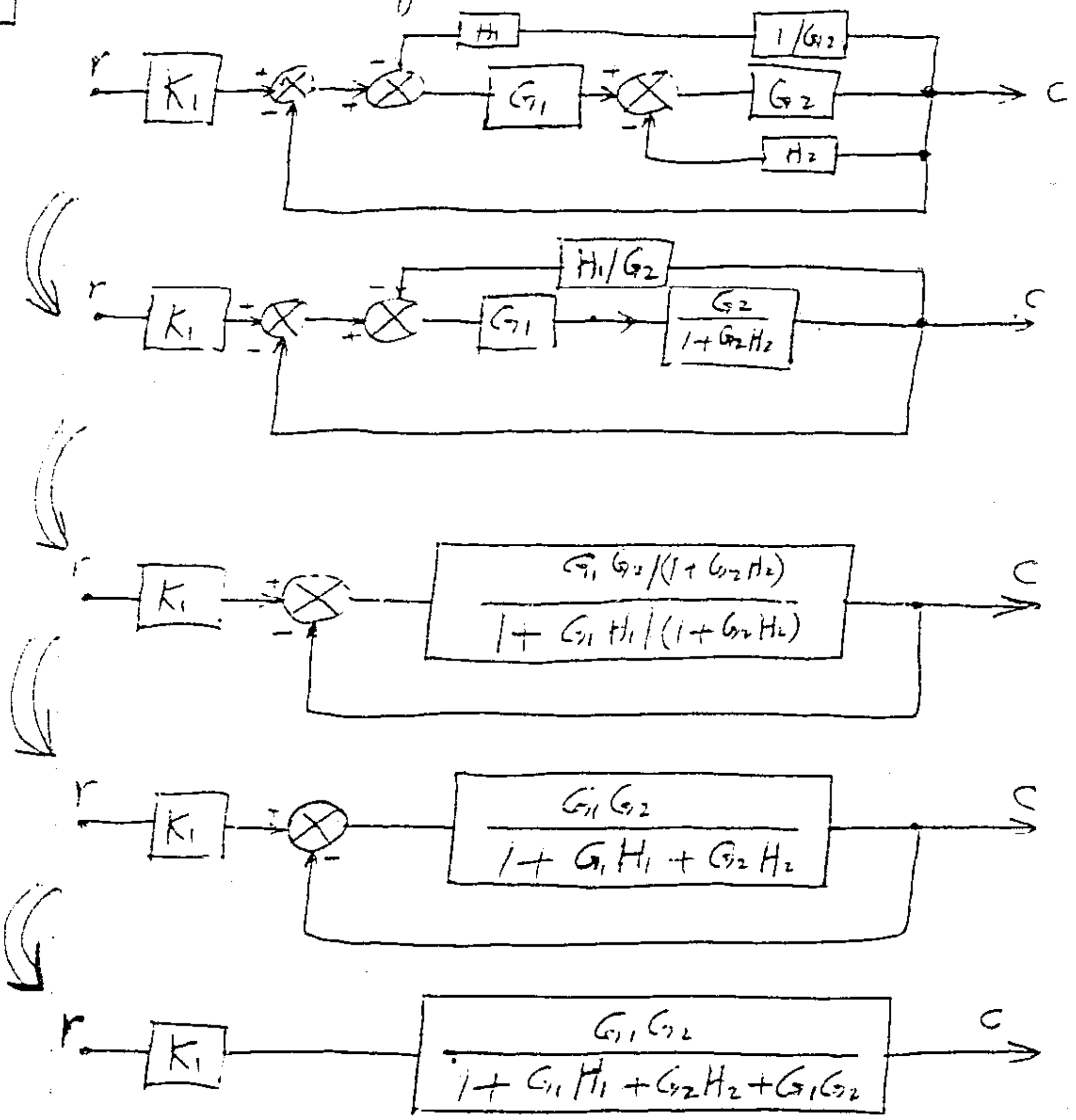
To find the transfer function $\frac{c}{r}$ of:



$$\therefore \frac{c}{r} = \frac{-B}{1-B} = \frac{B}{B-1}$$

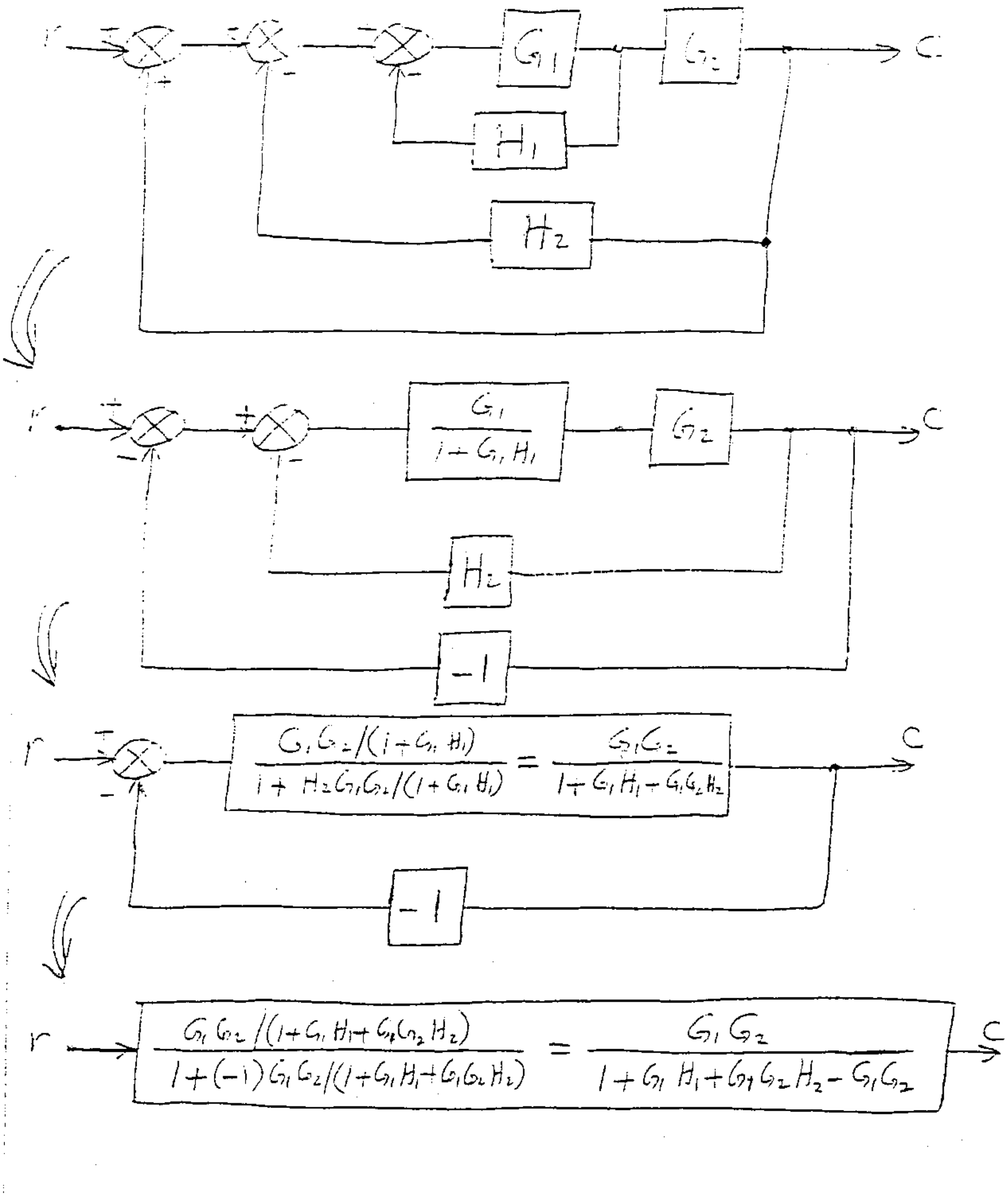


$\frac{3b}{24}$ The shown diagram is reduced to:



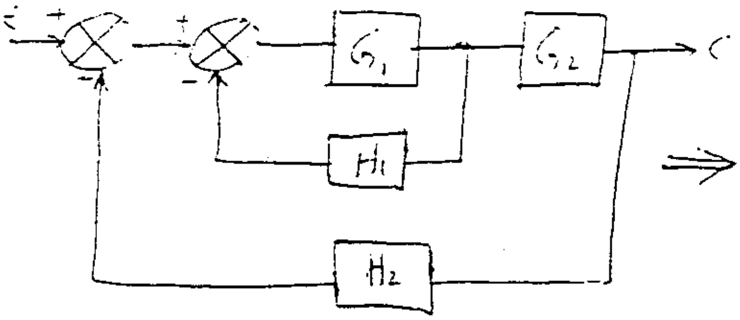
$$\therefore \quad \frac{c}{r} = \frac{K_1 G_1 G_2}{1 + G_1 G_2 + G_1 H_1 + G_2 H_2}$$

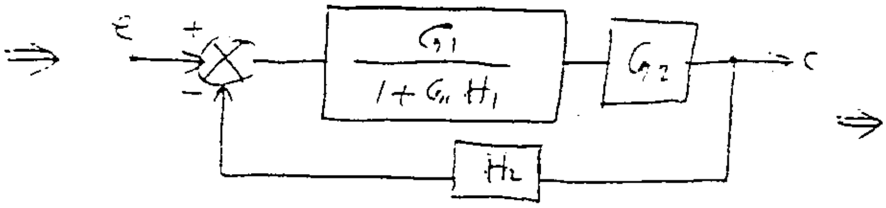
3C
24

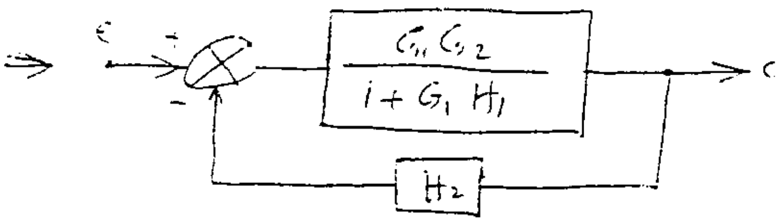


$$\therefore \frac{c}{r} = \frac{G_1 G_2}{1 + G_1 H_1 + G_1 G_2 H_2 - G_1 G_2}$$

$\boxed{\frac{3C}{24}}$ # To find $\frac{C}{E}$







$$\therefore \frac{C}{E} = \frac{G_1 G_2}{1 + G_1 H_1 + G_1 G_2 H_2}$$

$\frac{6}{26}$ Let the vertical axis represent $v(t)$ $\therefore v_{ss}(t) = 0.5$

1- $\therefore v(t_d) = .1 * v_{ss}(t) = .10 * .5 = .05$ $\therefore t_d = 2.2$ sec

2- $v(t_d + t_r) = .9 * v_{ss}(t) = .9 * .5 = .45$ $\therefore t_d + t_r = 5.2$ sec $\therefore t_r = 3$ sec

3- $v'(t_p) = 0$ $\therefore t_p = 7.7$ sec

4- $v(t \geq t_s) \in [.95, 1.05]\, v_{ss}(t)$ $\therefore t_s = 17.7$ sec

5- $v(t_p) = 0.83$ $\therefore r = \frac{.83 - .5}{.5} = \frac{.33}{.5} = .66 = 66\,\%$

6- $N = 2$ cycles

$\because$ System is second order:

$\therefore t_p = \frac{\pi}{\omega}$ (1), $\omega = \sqrt{1-\eta^2}\,\omega_n$ (2), $r = \exp(-\eta\pi/\sqrt{1-\eta^2})$ (3)

Hence;

From (3) $\therefore -\ln r = \eta\pi/\sqrt{1-\eta^2}$ $\therefore \eta\pi = \sqrt{1-\eta^2} \cdot \ln(\frac{1}{r})$

$\therefore \eta\pi/[\ln(\frac{1}{r})] = \sqrt{1-\eta^2}$ $\therefore \eta^2\pi^2/\ln^2\frac{1}{r} = 1 - \eta^2$

$\therefore \eta^2\left(\frac{\pi^2}{\ln^2\frac{1}{r}} + 1\right) = 1$ $\therefore \eta^2 = \frac{\ln^2(1/r)}{\pi^2 + \ln^2(1/r)}$, Hence:

7- $$\eta = \sqrt{\frac{1}{1+(\pi/\ln\frac{1}{r})^2}} = \sqrt{\frac{1}{1+[\pi/\ln(\frac{100}{66})]^2}} = 0.1311$$

8- From (2) $\therefore \omega_n = \frac{\omega}{\sqrt{1-\eta^2}} = 1.00871 * \omega$ but $\omega = \frac{2\pi}{T} = \frac{2\pi}{9.4}\frac{\text{rad}}{\text{s}}$

OR $\omega = \frac{\pi}{t_p} = \frac{\pi}{7.7}$ rad/sec $\therefore \omega \in [.4080, .6684]$ rad/sec

$\therefore \omega_n \in 1.00871 * \omega = [.41155, .67425]$ rad/sec.

9- Bandwidth of second order system is $\simeq \frac{\omega_n}{2\pi} \in \frac{1}{2\pi}[.41155, .67425]$ Hz

$\therefore B \in [.0655, .1073]$ Hz.

10- Cutoff rate is at -40 db/dec

11- J_∞ = area of square error curve between $0, \infty$ = (assuming parabolas)

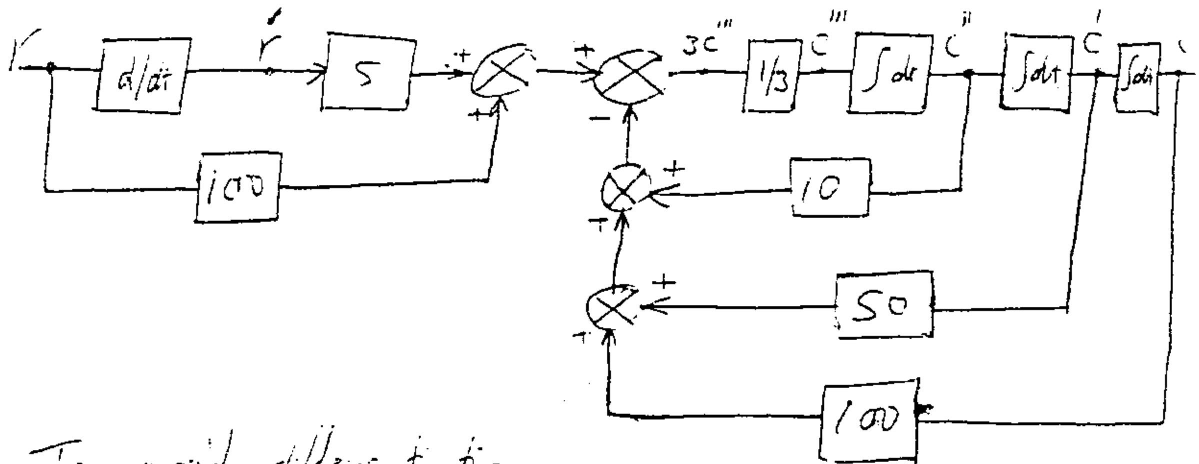
$= \frac{2}{3}\left(.5^2 * 5.4 + .33^2 * (11.3 - 5.4) + .07^2 * (15.6 - 11.3) + .03^2 * (18.7 - 15.6)\right) =$

$= \frac{2}{3}(1.35 + .64251 + .02107 + .00279) = 1.344$ sec.

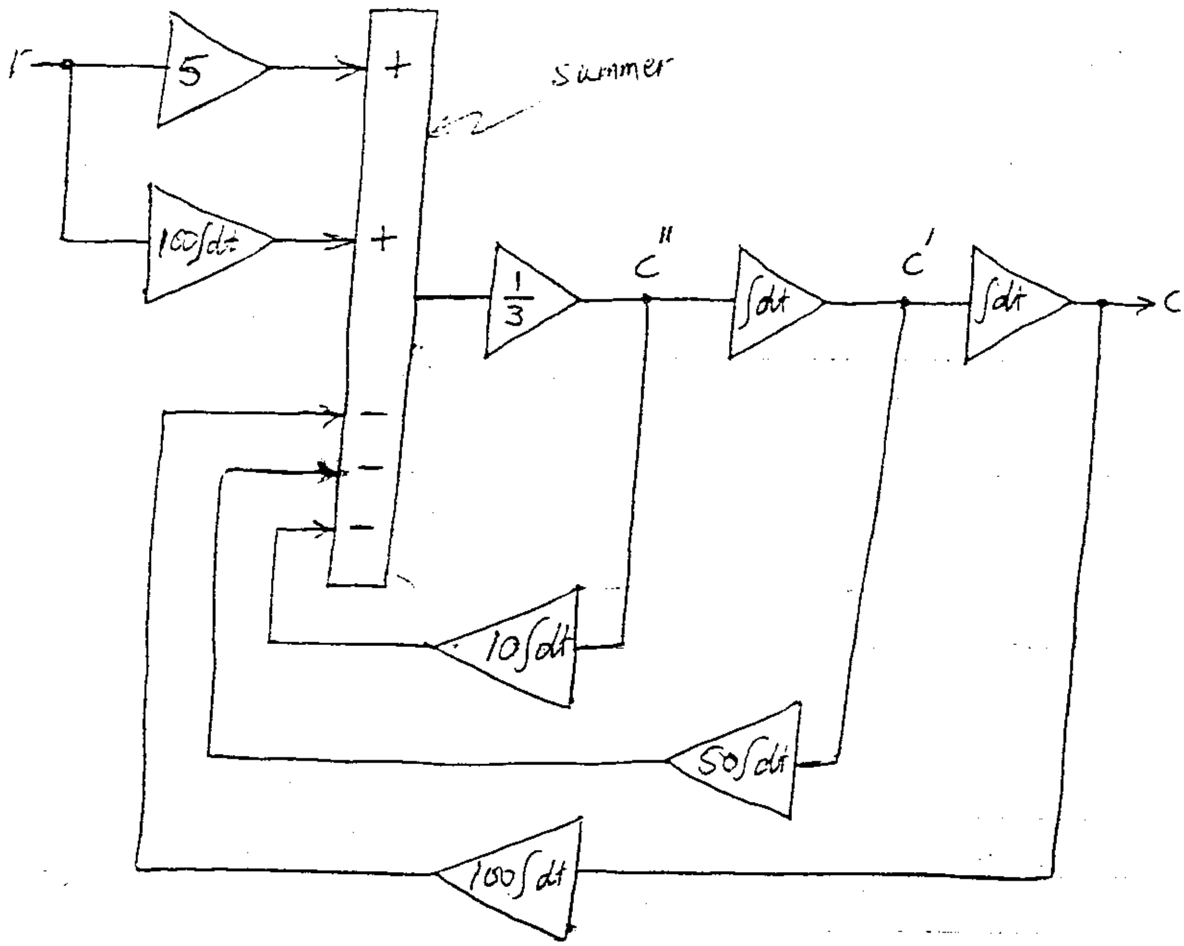
5/27

$$3c''' + 10c'' + 50c' + 100c = 5r' + 100r$$

$$\therefore 3c''' = 5r' + 100r - (10c'' + 50c' + 100c)$$



To avoid differentiation,
$\therefore$ passing the d/dt block into the first ∫dt block and accounting for changes gives:



$\boxed{\frac{4}{27}}$ $4t_c$ computer seconds = t real time seconds ∴ $4\,dt_c = dt$

∴ $\dot{c}_c = \frac{dc}{dt_c} = \frac{4\,dc}{4\,dt_c} = 4\frac{dc}{dt} = 4\dot{c}$

& $\ddot{c}_c = \frac{d\dot{c}_c}{dt_c} = \frac{4\,d\,4\dot{c}}{4\,dt_c} = 16\frac{d\dot{c}}{dt} = 16\ddot{c}$

& $\dddot{c}_c = 64\dddot{c}$

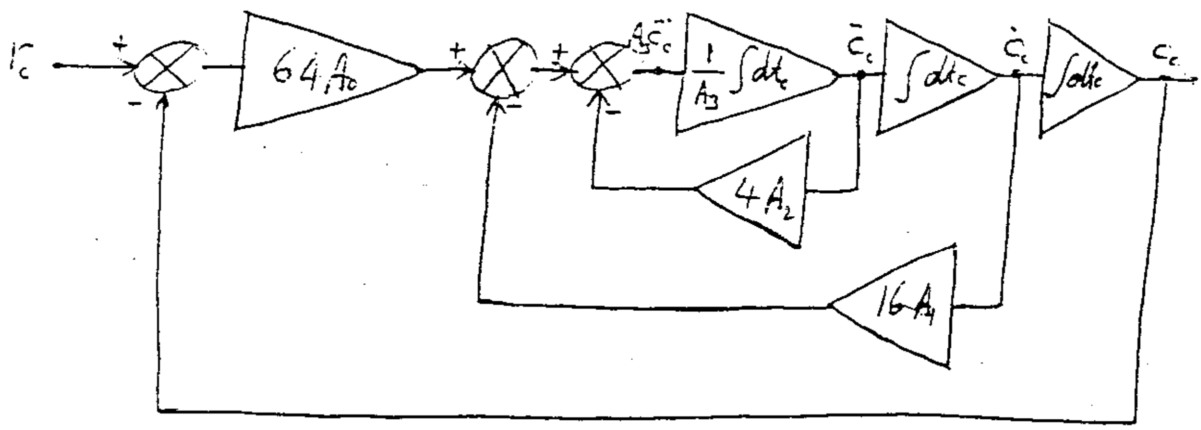
∴ The computer will see the differential equation as:

$$A_3\left(\frac{\dddot{c}_c}{64}\right) + A_2\left(\frac{\ddot{c}_c}{16}\right) + A_1\left(\frac{\dot{c}_c}{4}\right) + A_0 c_c = A_0 r_c$$

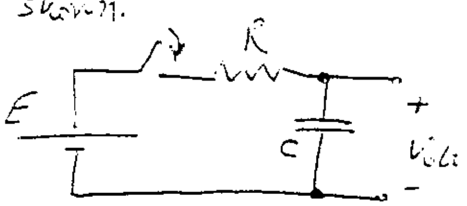
or $$A_3\dddot{c}_c + 4A_2\ddot{c}_c + 16A_1\dot{c}_c + 64A_0 c_c = 64A_0 r_c$$

∴ $$A_3\dddot{c}_c = 64A_0(r_c - c_c) - 16A_1\dot{c}_c - 4A_2\ddot{c}_c$$

The schematic diagram is shown below:



Finding t_d, t_r & t_s of $v_c(t)$ shown.



$$\therefore\ v_c(t) = E\left(1 - e^{-t/RC}\right)$$

$$\therefore\ v_c(\infty) = E$$

$$\therefore\ v_c(t_d) = 0.1\,E = E\left(1 - e^{-t_d/RC}\right)$$

$$\therefore\ e^{-t_d/RC} = 0.9 = \frac{9}{10} \Rightarrow e^{t_d/RC} = \frac{10}{9}$$

$$\therefore\ t_d = RC\ \ln\left(\frac{10}{9}\right) = 0.1054\ RC$$

$$\&\ v_c(t_d + t_r) = 0.9\,E = E\left(1 - e^{-(t_d+t_r)/RC}\right)$$

$$\therefore\ e^{-(t_d+t_r)/RC} = 0.1 = \frac{1}{10} \Rightarrow e^{(t_d+t_r)/RC} = 10$$

$$\therefore\ t_d + t_r = RC\ \ln 10 = 2.3026\ RC$$

$$\therefore\ t_r = RC\left(\ln 10 - \ln\frac{10}{9}\right) = RC\ \ln\left(\frac{10}{10/9}\right) = RC \ln 9$$

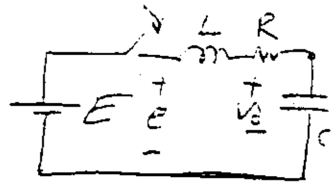
$$\therefore\ t_r = RC\ \ln 9 = 2.1972\ RC$$

$$\&\ v_c(t_s) = 0.95\,E = E\left(1 - e^{-t_s/RC}\right)$$

$$\therefore\ e^{-t_s/RC} = 0.05 = \frac{1}{20} \Rightarrow e^{t_s/RC} = 20$$

$$\therefore\ t_s = RC\ \ln 20 = 2.9957\ RC \approx 3\ RC$$

Finding t_d, t_r, t_p, t_s, r & J_∞ of $v_o(t)$ shown, given $RC = 2$ sec, $LC = 5$ sec^2 & $v_o(0)=0$



$$\frac{V_o(s)}{e(s)} = \frac{1/sC}{sL + R + 1/sC} = \frac{1}{s^2LC + sCR + 1} = \frac{1}{5s^2+2s+1}$$

$$\therefore 5s^2+2s+1=0 \Rightarrow s = \frac{-2\pm\sqrt{4-4(5)}}{10} = \frac{-2\pm\sqrt{-16}}{10} = -.2\mp j.$$

$$\therefore V_o(s) = e(s)\cdot \frac{1}{5(s+.2+j.4)(s+.2-j.4)} = \frac{E}{s}\cdot\frac{1}{5(s+.2+j.4)(s+.2-j.$$

$$= \frac{E}{5}\left[\frac{A}{s} + \frac{Bs+C}{(s+.2+j.4)(s+.2-j.4)}\right]$$

$$\therefore A(s^2+.4s+.2)+(Bs+C)s = 1$$

put $s=0$ $\quad\therefore .2A=1 \quad\therefore A=5$

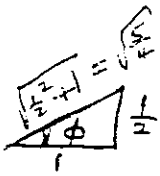
equate coefficients of s $\quad\therefore .4A+C=0 \quad\therefore C=-.4A=-2$

" " " s^2 $\quad\therefore A+B=0 \quad\therefore B=-A=-5$

$$\therefore V_o(s) = \frac{E}{5}\left[\frac{5}{s}+\frac{-5s-2}{s^2+.4s+.2}\right] = \frac{E}{5}\left(\frac{5}{s} - \frac{5(s+.2)+1}{(s+.2)^2+.4^2}\right)$$

$$\therefore V_o(t) = \frac{E}{5}\left(5-5e^{-.2t}\cos .4t - \frac{1}{.4}e^{-.2t}\sin .4t\right)$$

$$= E\left(1-e^{-.2t}\left(\cos .4t + \tfrac12 \sin .4t\right)\right)$$



$$= E\left(1-e^{-.2t}\cdot\tfrac{\sqrt5}{2}\left(\cos\phi\cos .4t+\sin\phi\sin .4t\right)\right)$$

$$= E\left(1-e^{-.2t}\cdot\tfrac{\sqrt5}{2}\cos(.4t-\phi)\right)$$

$$\therefore V_o(t) = E\left[1-\frac{\sqrt5}{2}\cdot e^{-.2t}\cdot\cos\left(.4t-\tan^{-1}\left(\tfrac12\right)\right)\right]$$

Hence, the steady-state value of $V_o(t)$ is $\lim_{t\to\infty} V_o(t) = E$.

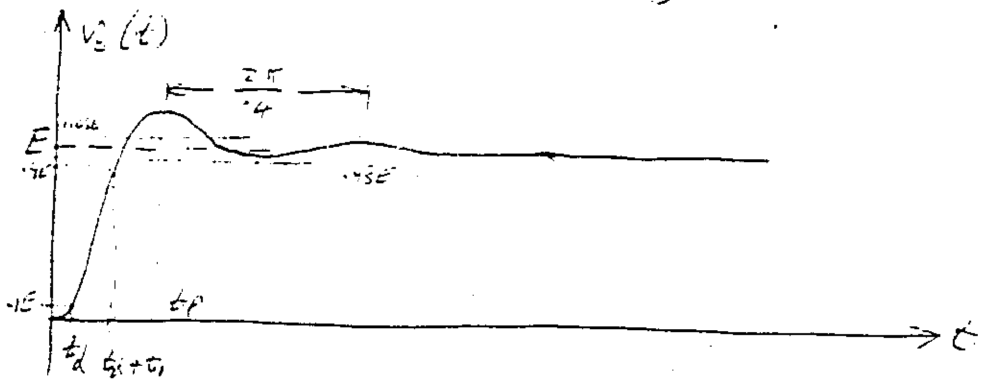
The time, required to reach αE is given by:—

$$\alpha E = E\left(1-\frac{\sqrt5}{2}e^{-.2\tau}\cos(.4\tau-\tan^{-1}.5)\right)$$

∴ $1-\alpha-\frac{\sqrt5}{2}e^{-.2\tau}\cos(.4\tau-\tan^{-1}.5) = f(\tau) = 0$, then by N.R.:

∴ $\tau' = \tau - \dfrac{f(\tau)}{f'(\tau)}$

solving for $\alpha = 0.1$ ∴ $\tau = t_d = 1.0823$ sec
when $\alpha = 0.4$ ∴ $\tau = t_d + t_r = 4.5284$ sec
when $\alpha = 0.95$ ∴ $\tau = 4.7938$ (only) sec
i.e one intersection with $.95E$ only.
when $\alpha = 1.05$ ∴ $\tau = 5.4175, 11.7262$ (only) sec
i.e two intersections with $1.05E$ only.



∴ $t_d = 1.0823$ sec
& $t_r = 4.5284 - 1.0823 = 3.4461$ sec
& t_s = last of $\{4.7938, 5.4175, 11.7262\} = 11.7262$ sec

now, $\dot{v}_o(t_p) = 0$ ∴ $0 = e^{-.2t_p}\left(-.4\sin(.4t_p-\tan^{-1}.5) - .2\cos(.4t_p-\tan^{-1}.5)\right)$

or $\tan(.4t_p - \tan^{-1}.5) = -.5$ ⇒ ∴ $\dfrac{\tan .4t_p - .5}{1+.5\tan .4t_p} = -.5$
∴ $\tan(.4t_p) - .5 = -.5 - .25\tan .4t_p$
∴ $\tan .4t_p = 0$ ∴ $0.4t_p = 0, \pi, 2\pi, \dots$ ∴ $t_p = 0, \frac{\pi}{.4}, \frac{2\pi}{.4}, \frac{k\pi}{.4}$ k int
∴ t_p (the maximum of maximas) = first occurrence after 0 = $\frac{\pi}{.4}$ sec
∴ $t_p = 7.8540$ sec
& $(1+r)E = v_i(t_p) = E\left(1-\frac{\sqrt5}{2}e^{-.2t_p}\cos(.4t_p-\tan^{-1}.5)\right) \Rightarrow 1+r = 1-\frac{\sqrt5}{2}e^{-\pi/2}\cos(\pi-\tan^{-1}.5) = 1.2079$

∴ $r = 0.2079$

$$\therefore\quad G(jw) = \left.\frac{1}{5s^2+2s+1}\right|_{s=jw} = \frac{1}{-5w^2+1+2jw} =$$

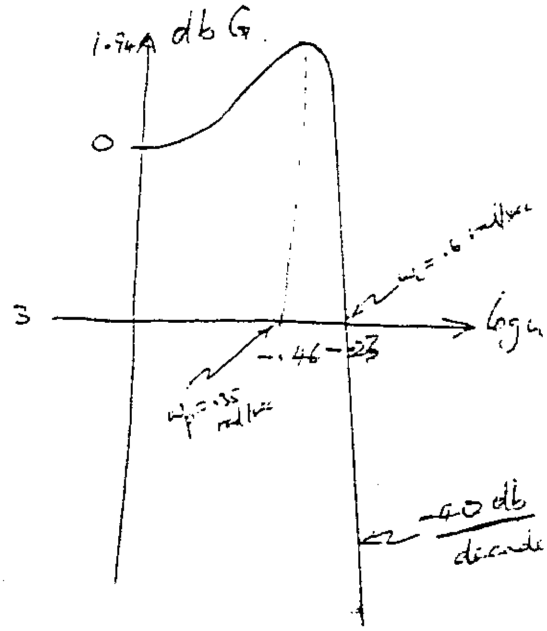
$$= \frac{1}{1-5w^2+j2w}$$

$$\therefore\quad |G(jw)| = \left[(1-5w^2)^2+4w^2\right]^{-1/2}$$

$$\therefore\quad db|G| = |20\log|G|| = 10\log\left[(1-5w^2)^2+4w^2\right]$$

$$db|G(0)| = 0 \text{ db}$$

$$\left(db|G(w_p)|\right)' = 0$$



$$\therefore\quad 2(1-5w_p^2)(-10w_p)+8w_p=0$$

$$\therefore\quad w_p(-12+100w_p^2)=0$$

$$\therefore\quad w_p = 0 \quad (\text{not accepted})$$

$$\text{or}\quad w_p = \sqrt{\frac{12}{100}} = .2\sqrt{3} \text{ rad/sec}$$

$$= .3464 \text{ rad/sec}$$

$$\therefore\quad W_p = 0.3464 \text{ rad/sec}$$

$$\text{or}\quad f_p = 0.05513 \text{ Hz}$$

$$\text{peak db} = db|G(w_p)| = 1.9382 \text{ db}$$

$$\text{if}\quad (1-5w_c^2)^2+(2w_c)^2 = 2 \quad\therefore\quad 25w_c^4-6w_c^2-1=0$$

$$\therefore\quad w_c^2 = \frac{6\pm\sqrt{36+100}}{50} = 2(6\pm\sqrt{136})/100 = (\text{-ve rejected})\ .3532 \quad\therefore\quad w_c = .5943 \text{ r/s}$$

(check : $db(w_c) = 3.0103$ ∴ OK)

$$\therefore\quad w_c = 0.5943 \text{ rad/sec} \qquad \text{i.e. } f_c = BW = 0.09459 \text{ Hz}$$

At very high frequencies:

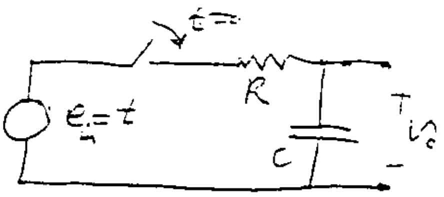
$$\therefore\quad db|G| = 10\log(-5w^2)^2 = 10\log(25w^4) = 10\log 25 + 40\log w.$$

∴ slope = 40 db / decade (...

∴ Assuming OIC find t_s.

$\therefore \frac{v_o}{e_{in}} = \frac{1/sC}{R+1/sC} = \frac{1}{1+sCR}$

$\therefore v_o = \frac{1}{1+sCR} \cdot e_{in}$



for $e_{in} = t$ $\therefore e_{in}(s) = \frac{1}{s^2}$

$\therefore V_o(s) = \frac{1}{1+sCR} \cdot \frac{1}{s^2} = \frac{A+Bs}{s^2} + \frac{d}{1+sCR}$

$\therefore (A+Bs)(1+sCR) + s^2 d = 1$

$s = 0$ $\quad\therefore\quad A = 1$

coeff s $\quad\therefore\quad B + ACR = 0 \quad \therefore B = -ACR = -RC$

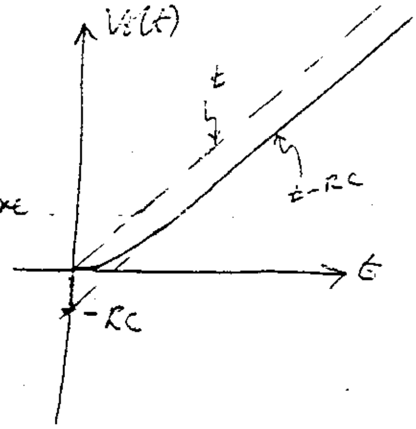
coeff s^2 $\quad\therefore\quad BCR + d = 0 \quad \therefore d = -BCR = R^2C^2$

$\therefore V_o(s) = \frac{1}{s^2} + \frac{-RC}{s} + \frac{R^2C^2}{1+sCR}$

$\therefore v_o(t) = t - RC + RC\, e^{-t/RC} = t - RC\left(1 - e^{-t/RC}\right)$

& $v_{oss}(t) = t - RC$

The graph is shown in the figure.



whereby the output is always less than t and approaches assymptotically $t-RC$ from above

$\therefore V_o(t_s) = 1.05(t_s - RC) = t_s - RC\left(1 - e^{-t_s/RC}\right)$

or $\quad 0.05\left(\frac{t_s}{RC} - 1\right) = e^{-t_s/RC}$

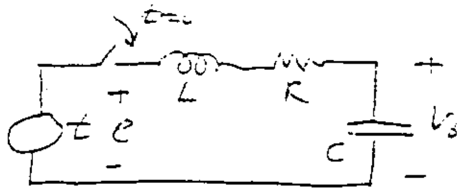
Let $t_s/RC = x$ $\quad\therefore$ Solving $1 + 20e^{-x} - x = 0$

$\therefore x = 2.555 \quad\therefore t_s = 2.555\,RC$

∴ The settling time t_s is $2.555\,RC$

Finding t_s:

from HW-2.



$\therefore \frac{v_c}{e} = \frac{1}{5s^2+2s+1}$ & $s = -.2 \pm j.4$

$$\left.\begin{array}{l} RC = 2\,sec \\ LC = 5\,sec^2 \\ OTC \end{array}\right\}$$

$\therefore v_c(s) = \frac{e(s)}{5s^2+2s+1} = \frac{1}{s^2(5s^2+2s+1)} = \frac{A+Bs}{s^2} + \frac{d+es}{5s^2+2s+1}$

$\therefore (A+Bs)(5s^2+2s+1)+(d+es)(s^2)=1$

put $s=0$ $\therefore A=1$

coeff of s $\therefore 2A+B=0$ $\therefore B=-2A=-2$

coeff of s^2 $\therefore 5A+2B+d=0$ $\therefore d=-(5A+2B)=-(5-4)=-1$

coeff of s^3 $\therefore 5B+e=0$ $\therefore e=-5B=10$

$\therefore v_c(s) = \frac{1}{s^2} - \frac{2}{s} + \frac{-1+10s}{5s^2+2s+1} = \frac{1}{s^2} - \frac{2}{s} + \frac{1}{5}\cdot\frac{10(s+.2)-(\frac{3}{.4})(.4)}{(s+.2)^2+.4^2}$

$\therefore v_c(t) = t-2+\frac{10}{5}e^{-.2t}\cos .4t - \frac{3}{2}e^{-.2t}\sin .4t$

$= t-2+\sqrt{\left(\frac{10}{5}\right)^2+\left(\frac{3}{2}\right)^2}\cdot e^{-.2t}\cos\left(.4t+\tan^{-1}\left(\frac{3}{2}\cdot\frac{5}{10}\right)\right)$

$= t-2+\frac{5}{2}e^{-.2t}\cos\left(.4t+\tan^{-1}\frac{3}{4}\right)$

$\therefore V_{css}(t) = t-2$

$\therefore V(t_s) = .95\,V_{css} = .95(t_s-2) = t_s-2+\frac{5}{2}e^{-.2t_s}\cos\left(.4t_s+\tan^{-1}\frac{3}{4}\right)$

or $t_s-2+50e^{-.2t_s}\cos\left(.4t_s+\tan^{-1}\frac{3}{4}\right)=0$ $\therefore t_s \in \{2.346, 8.334\}$ sec only

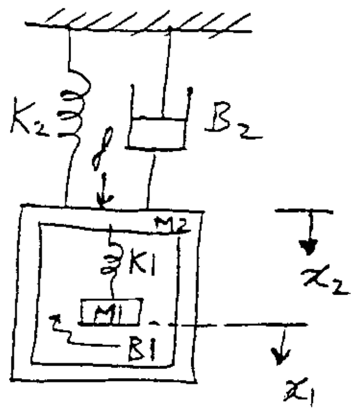
& $V(t_s) = 1.05\,V_{css} = 1.05(t_s-2) = t_s-2+\frac{5}{2}e^{-.2t_s}\cos\left(.4t_s+\tan^{-1}\frac{3}{4}\right)$

or $t_s-2-50e^{-.2t_s}\cos\left(.4t_s+\tan^{-1}\frac{3}{4}\right)=0$ $\therefore t_s \in \{2.295\}$ sec only

$\therefore t_s \in \{2.295, 2.346, 8.334\}$ sec only

$\therefore$ The settling time is $\underline{8.334}$ sec

The force f is opposed by 5 forces: those of K_2, B_2, M_2, K_1 & B_1. Noting the senses:



$$\therefore \; f = K_2 x_2 + B_2 \dot{x}_2 + M_2 \ddot{x}_2 + K_1 (x_2 - x_1) + B_1 (\dot{x}_2 - \dot{x}_1) \quad (1)$$

& The forces of K_1 & B_1 are opposed by the inertia of M_1

$$\therefore \; K_1 (x_2 - x_1) + B_1 (\dot{x}_2 - \dot{x}_1) = M_1 \ddot{x}_1 \quad (2)$$

Rearranging (1) & (2) after introducing the operator D,

$$\therefore \; \left[K_2 + K_1 + (B_2 + B_1)D + M_2 D^2\right] x_2 - (K_1 + B_1 D)\, x_1 = f$$

$$\& \; -(K_1 + B_1 D)\, x_2 + (K_1 + B_1 D + M_1 D^2)\, x_1 = 0$$

Arranged in a system as:

$$\begin{bmatrix} K_2 + K_1 + (B_2 + B_1)D + M_2 D^2 & -(K_1 + B_1 D) \\ -(K_1 + B_1 D) & K_1 + B_1 D + M_1 D^2 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

To find the roots of: $f(s) = s^3 + 15s^2 + 50s + 1 = 0$

Use Newton-Raphson: $s' = s - \frac{f(s)}{f'(s)}$

1: Smallest root; around $s = -\frac{1}{50}$ $\Rightarrow$ ∴ $s_1 = -.0201213$

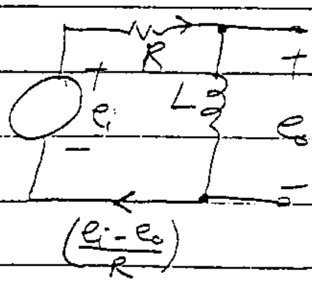
2: largest root; around $s = -15$ $\Rightarrow$ ∴ $s_2 = -10.0199$

3: medium root; around $s = -\frac{50}{15}$ $\Rightarrow$ ∴ $s_3 = -4.96000$

To find $Z = \frac{2+j4}{5\angle 30^\circ} \cdot \frac{j2}{1-j40}$ in rectangular & polar

∴ $Z = -0.037913 - j0.023693 = 0.0447074 \angle -148^\circ$

To find the diff eq of



$e_o = L(e_i - e_o)'/R$

∴ $R e_o = LD e_i - LD e_o$

∴ $(R + LD) e_o = LD e_i \Rightarrow e_o = \frac{LD}{R+LD} e_i$

For the differential equation:

$$\ddot{y}+4\dot{y}+4y = x(t) \quad ; \text{ with } x(t)=u(t),\ y_0=0 \ \& \ y'_0=.5$$

1.. the system may be represented by $x(t) \rightarrow \boxed{\text{System}} \rightarrow y(t)$

2.. $y(s)$ is obtained from:

$$s^2y(s) - s\overset{0}{\cancel{y_0}} - \overset{.5}{\cancel{y'_0}} + 4\left(sy(s) - \overset{0}{\cancel{y_0}}\right) + 4y(s) = x(s) = \frac{1}{s}$$

$$\therefore \ y(s) = \frac{\frac{1}{s}+.5}{s^2+4s+4} = \frac{2+s}{2s\,(s^2+4s+4)} = \frac{1}{2s\,(s+2)}$$

3.. $y(t) = \mathcal{L}^{-1}\left(\frac{1}{2s(s+2)}\right) = \mathcal{L}^{-1}\frac{1}{4}\left(\frac{1}{s} - \frac{1}{s+2}\right) = \frac{1}{4}\left(1-e^{-2t}\right).$

4 DC gain $= \frac{y_{ss}}{x_{ss}} = \frac{1/4}{1} = \frac{1}{4}$

5. $y(t_d) = .1\, y_{ss} \Rightarrow \frac{1}{4}\left(1-e^{-2t_d}\right) = .1 * \frac{1}{4} \Rightarrow t_d = 52.7$ msec

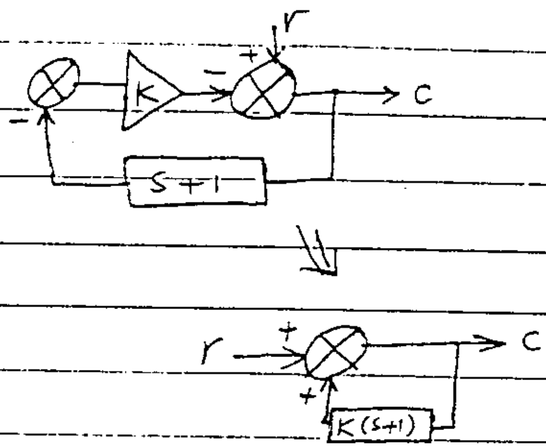
6. $y(t_d+t_r) = .9\, y_{ss} \Rightarrow \frac{1}{4}\left(1-e^{-2(t_d+t_r)}\right) = \frac{.9}{4} \Rightarrow t_r = 1.10$ sec

7. $y'(t_p) = 0 \Rightarrow \frac{2}{4}e^{-2t_p} = 0 \Rightarrow t_p = \infty$

8. $y(t_s) = .95\, y_{ss} \Rightarrow \frac{1}{4}\left(1-e^{-2t_s}\right) = \frac{.95}{4} \Rightarrow t_s = 1.50$ sec

9. $J_0 = \int_0^{\infty}\left[y_{ss} - y(t)\right]^2 dt = \int_0^{\infty} \frac{1}{16}\cdot e^{-4t}\, dt = \frac{e^{-4t}}{-64}\Big|_0^{\infty} = \frac{1}{64}$

$$\frac{c}{r} = \frac{1}{1-K(s+1)}$$



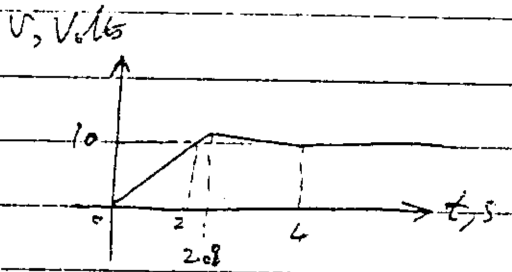
$$= \frac{1}{1-K-Ks}$$

Note that $G = 1$ &

$H = -K(s+1)$

in the standard form.

Eq. of segment through O is



$V = 5\,t$

a. ∴ $V(t_d) = .1\,V_{ss} = .1 * 10 = 1$

∴ $5\,t_d = 1 \Rightarrow t_d = 0.2$ sec

b. $V(t_d + t_r) = .9\,V_{ss} = 9 \Rightarrow 5\,(t_d + t_r) = 9$

∴ $t_d + t_r = 1.8 \Rightarrow t_r = 1.8 - .2 = 1.6$ sec

c. $t_p = 2.08$ sec with $V(t_p) = 5 * 2.08 = 10.4$ Volts

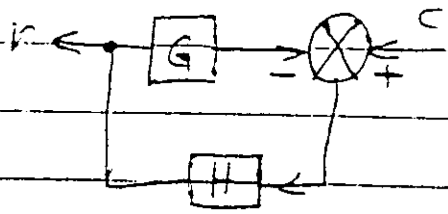
d. $V(t_s) = $
- $.95\,V_{ss} \Rightarrow 5\,t_s = 9.5 \Rightarrow t_s = \frac{9.5}{5} = 1.9$ sec
- $1.05\,V_{ss} = 10.5$, but max. $V(\text{at } t_p) = 10.4$ ∴ None

∴ $t_s = 1.9$ sec

e. $r = \frac{10.4 - 10}{10} = \frac{.4}{10} = 0.04$

For the block diagram shown

$$\frac{c}{r} = \frac{1}{r/c} = \frac{1}{\frac{H}{1+GH}}$$



$$\therefore \frac{c}{r}(s) = \frac{1+GH}{H}$$

For $y(t) = \begin{cases} 25 \sin^2 t & t \in (0, 2.2143) \\ 16 & t \geq 2.2143 \end{cases}$

$\therefore y(0) = 0$ & $y_{ss}(t) = 16$

$\therefore y(t_d) = 0 + .1 * 16 = 1.6 = 25 \sin^2 t_d \Rightarrow t_d = .2558$

& $y(t_d + t_r) = 0 + .9 * 16 = 14.4 = 25 \sin^2 (t_d + t_r) \Rightarrow t_r = .6059$

& $y(t_s) = 0 + \begin{cases} .95 * 16 = 15.2 = 25 \sin^2 t_s \Rightarrow t_s = .8943 \\ 1.05 * 16 = 16.8 = 25 \sin^2 t_s \Rightarrow t_s = .9610 \text{ or } 2.1806 \end{cases}$

$\therefore t_s = 2.1806$

& $y'(t_p) = 0 \Rightarrow \sin 2t_p = 0 \Rightarrow 2t_p = 0, \pi, \ldots \quad \therefore t_p = \frac{\pi}{2} = 1.5708$

& $r = \frac{y(t_p) - y_{ss}}{y_{ss} - y_0} = \frac{25-16}{16-0} = \frac{9}{16} = .5625 = 56.25\%$

$$+\ J_\infty = \int_0^\infty \left(y(t) - y_{ss}\right)^2 dt = \int_0^{2.2143} \left(25\sin^2 t - 16\right)^2 dt =$$

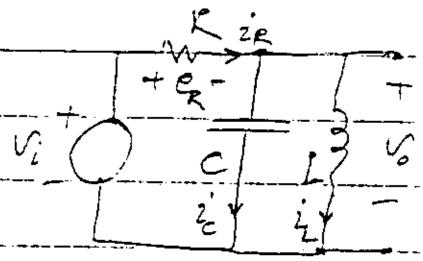
$$= \int_0^{2.2143} \left(-12.5\cos 2t - 3.5\right)^2 dt = \int_0^{2.2143} \left(12.5^2\cos^2 2t + 3.5^2 + 87.5\cos 2t\right) dt$$

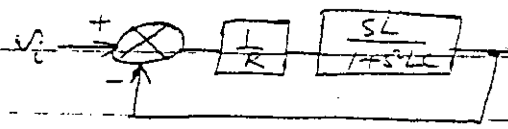
$$= \int_0^{2.2143} \left(\frac{12.5^2}{2}\cos 4t + 87.5\cos 2t + 3.5^2 + \frac{12.5^2}{2}\right) dt =$$

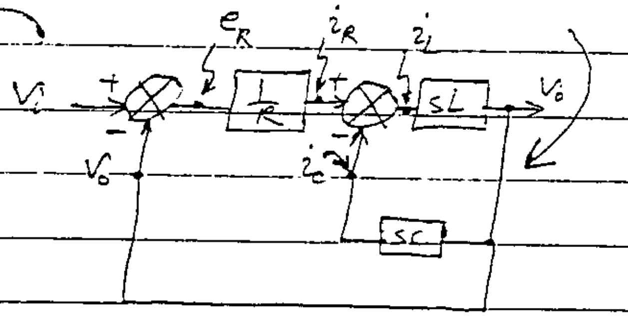
$$= \frac{12.5^2}{8}\sin 4t + \frac{87.5}{2}\sin 2t + \left(3.5^2 + \frac{12.5^2}{2}\right)t \Bigg|_0^{2.2143} = 168.62$$

$\therefore J_\infty = 168.62$

\# Using Block Diagram representation & reduction, the transfer function of the shown electrical network can be obtained as follows:





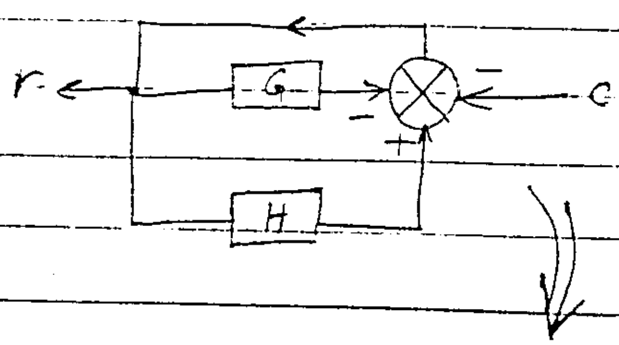


$$\therefore \frac{V_o}{V_i}(s) = \frac{sL}{R(1+s^2LC) + sL}$$

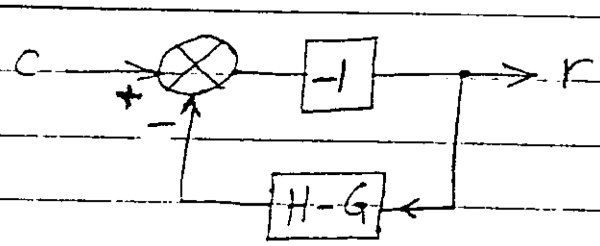
$\left(\text{OR as checking:}\right.$ $\dfrac{V_o}{V_i} = \dfrac{sL \parallel \frac{1}{sC}}{(sL \parallel \frac{1}{sC}) + R} = \dfrac{\frac{sL}{s^2LC+1}}{\frac{sL}{s^2LC+1} + R} = \dfrac{sL}{sL + R(1+s^2LC)}$

$\therefore$ OK $\left.\right)$

To find $\frac{c}{r}(s)$ of the system shown in the figure; we reduce it as shown



$$\therefore \frac{r}{c} = \frac{-1}{1+G-H}$$



$$\therefore \frac{c}{r} = H - G - 1$$

To find t_d, t_r, t_s, t_p, r & J_∞ for $y(t)$ given

$$y(t) = \begin{cases} 25 \sin^2 t &, t \in (0, .9542) \\ 16 &, t \geq .9542 \end{cases}$$

$\therefore y_{ss} = 16$ & $y_0 = 0 \Rightarrow \therefore$ Transition is 16

$\therefore 25 \sin^2 t_d = .1 * 16 + 0 \Rightarrow t_d = 0.25576$

& $25 \sin^2 (t_d + t_r) = .9 * 16 + 0 \Rightarrow t_r = 0.60593$

Max $y(t)$ is 25 at $t = \frac{\pi}{2} = 1.57 \notin (0, .9542)$

$\therefore$ Max. occurs at end of interval at $t = .9542 \Rightarrow t_p = .9542$

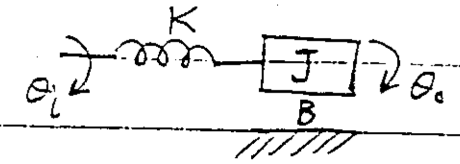
$\therefore y_{Max} = 25 \sin^2 .9542 = 16.640 \Rightarrow r = \frac{16.64 - 16}{16} = .04002$

$\therefore 25 \sin^2 t_s = .95 * 16 \Rightarrow t_s = 0.89426$ (No upper intersection)

& $J_\infty = \int_0^{.9542} (25 \sin^2 t - 16)^2 dt = \frac{12.5^2}{8} \sin 4t + \frac{87.5}{2} \sin 2t + \left(3.5^2 + \frac{12.5^2}{2}\right) t \Big|_0^{.9542}$

$\therefore J_\infty = 115.31$

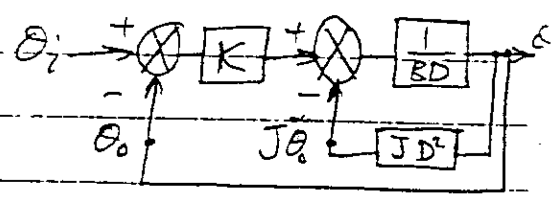
To find the DE of:



$$\therefore K(\theta_i - \theta_o) = J\ddot{\theta}_o + B\dot{\theta}_o$$

$$\therefore K\theta_i = (JD^2 + BD + K)\theta_o$$

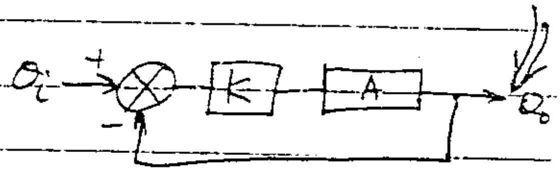
$$\therefore \frac{\theta_o}{\theta_i} = \frac{K}{K+BD+JD^2}$$



OR: by block diagram reduction:

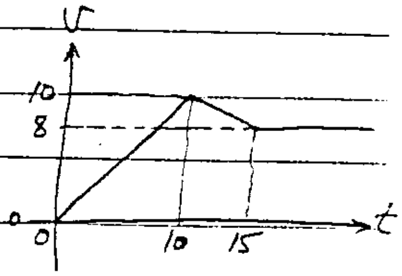
where

$$A = \frac{1/BD}{1+(1/BD)(JD^2)} = \frac{1}{BD+JD^2}$$



$$\therefore \frac{\theta_o}{\theta_i} = \frac{KA}{1+KA} = \frac{K}{K+BD+JD^2}$$

To find t_d, t_r, t_p, t_s, r & J_∞ of the response shown:



$$\therefore V = \begin{cases} t & , t \in (0,10) \\ 14 - .4t & , t \in [10,15) \\ 8 & t \geq 15 \end{cases}$$

$\therefore V_{ss} = 8$

$\therefore V(t_d) = .1 V_{ss} \Rightarrow t_d = .8$

& $V(t_d + t_r) = .9 V_{ss} \Rightarrow t_d + t_r = 7.2 \quad \therefore t_r = 6.4$

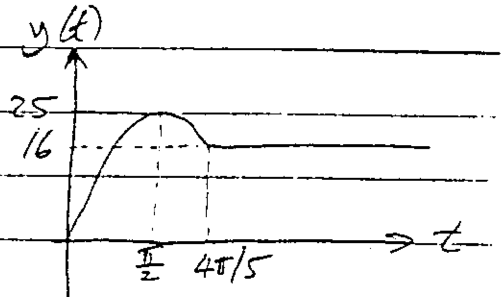
& $V(t_s) = 1.05 V_{ss} \Rightarrow 14 - .4 t_s = 8.4 \quad \therefore t_s = 14$

& $t_p = 10$, $r = \frac{2}{8} = 0.25$ & $J_\infty = \frac{1}{3}(64*8 + 4*7) = 180$

For $y(t) = \begin{cases} 100t(\pi-t)/\pi^2 & , t \in (0, 4\pi/5) \\ 16 & , t \geq 4\pi/5 \end{cases}$

t_d, t_r, t_p, t_s, r & J_∞ could be found as follows:

first, the function is plotted as shown.



$\therefore\ y_{ss} = 16$

$\therefore\ y(t_d) = .1 * 16 = 1.6$

$\because\ \pi^2 y = 100\pi t - 100t^2$

$\therefore\ 100t^2 - 100\pi t + \pi^2 y = 0 \Rightarrow t = \dfrac{100\pi \pm \sqrt{10^4\pi^2 - 400\pi^2 \cdot y}}{200}$

$\therefore\ t(y) = \frac{\pi}{2}\left(1 \pm \sqrt{1 - .04y}\right)$

$\therefore\ t_d(1.6) = \frac{\pi}{2}\left(1 - \sqrt{1 - .064}\right) = \frac{\pi}{2}\left(1 - \sqrt{.936}\right) = 0.05110$

& $y(t_d + t_r) = .9 * 16 = 14.4$

$\therefore\ t_d + t_r(14.4) = \frac{\pi}{2}\left(1 - \sqrt{1 - .576}\right) = \frac{\pi}{2}\left(1 - \sqrt{.424}\right) = 0.54797$

$\therefore\ t_r = 0.49687$

& $y'(t_p) = 0 \Rightarrow \pi - 2t_p = 0 \Rightarrow t_p = \frac{\pi}{2} = 1.57080$

$\therefore\ y_p = 100\left(\frac{\pi}{2}\right)^2/\pi^2 = 25 \quad \therefore\ r = \dfrac{25-16}{16} = \dfrac{9}{16} = 0.5625$

& $y(t_s) = 1.05 * 16 = 16.8$

$\therefore\ t_s(16.8) = \frac{\pi}{2}\left(1 + \sqrt{1 - .04 * 16.8}\right) = 2.47041$

& $J_\infty = \displaystyle\int_0^{4\pi/5} \left[\frac{100t(\pi - t)}{\pi^2} - 16\right]^2 dt = \left.\frac{100^2}{\pi^4} \cdot \frac{t^5}{5} + 16^2 t + \left(\frac{100^2}{\pi^2} + \frac{2 * 1600}{\pi^2}\right) \cdot \frac{t^3}{3} - \frac{20000}{\pi^3} \cdot \frac{t^4}{4} - \frac{3200 t^2}{\pi^2}\right|_0^{4\pi/5}$

$\therefore\ J_\infty = 128.680$

$\boxed{\frac{1c}{49}}$ $\ddot{X} + Y\cos X + \sin Y + \dot{Y} + Z = 0$, nonlinear due to sinusoidal terms of X & Y. To linearize, assume
$X = X_0 + x$, $Y = Y_0 + y$ & $Z = Z_0 + z$

$\therefore$ dc equation is $Y_0 \cos X_0 + \sin Y_0 + Z_0 = 0$

& ac equation is obtained from $Z(\ddot{X}, X, Y, \dot{Y}) = -\ddot{X} - Y\cos X - \sin Y - \dot{Y}$

$\therefore z = -1 \cdot \ddot{x} + Y_0 \sin X_0 \cdot x - \cos X_0 \cdot y - \cos Y_0 \cdot y - 1 \cdot \dot{y}$

$\therefore$ ac equation is $\ddot{x} - (Y_0 \sin X_0)x + (\cos X_0 + \cos Y_0)y + \dot{y} + z = 0$

and this is linear in x, y & z

$\boxed{\frac{1d}{49}}$ $3\dot{X} - \frac{Z}{XY} = \sin t$

The above equation is nonlinear due to the term $\frac{Z}{XY}$.

$\boxed{\frac{1e}{49}}$ $\dot{X} + 3X + \dot{Y} + Y\sin X - 2YZ = 0$

Let $X = X_0 + x$ + $Y = Y_0 + y$ & $Z = Z_0 + z$

$\therefore$ dc eqn. is $3X_0 + Y_0 \sin X_0 - 2Y_0 Z_0 = 0$

& ac eqn. is $\dot{x} + 3x + \dot{y} + (Y_0 \cos X_0)x + (\sin X_0)y - 2Y_0 z - 2Z_0 y = 0$

2d/49 Let $X = X_0 + x$ & $Y = Y_0 + y$ & $Z = Z_0 + z$

$\therefore \dot{X} = \dot{x}$

& $3\dot{X} - \dfrac{Z}{XY} = \sin t$ substituting

$\therefore 3\dot{x} - (Z_0 + z)/[(X_0 + x)(Y_0 + y)] = \sin t$ (Note: $\dfrac{1}{X_0 + x} \approx \dfrac{1}{X_0} - \dfrac{x}{X_0^2}$)

$\therefore 3\dot{x} - (Z_0 + z)\left(\frac{1}{X_0} - \frac{x}{X_0^2}\right)\left(\frac{1}{Y_0} - \frac{y}{Y_0^2}\right) = \sin t$

Equating constant terms in both sides:

$\therefore -Z_0\left(\frac{1}{X_0}\right)\left(\frac{1}{Y_0}\right) = 0$

$\therefore Z_0 = 0$ & $X_0 Y_0 \neq 0$

& Equating ac terms in both sides:

$\therefore 3\dot{x} + \frac{Z_0}{X_0}\cdot\frac{y}{Y_0^2} + \frac{Z_0 x}{X_0^2}\cdot\frac{1}{Y_0} - \frac{Z_0 x}{X_0^2}\cdot\frac{y}{Y_0^2} - \frac{z}{X_0}\cdot\frac{1}{Y_0} + \frac{z}{X_0}\cdot\frac{y}{Y_0^2} + \frac{zx}{X_0^2}\frac{1}{Y_0} - \frac{zx}{X_0^2} = \sin t$

Ignoring small terms relative to large terms:
(i.e terms of product of small changes)

$\therefore 3\dot{x} + \frac{Z_0}{X_0 Y_0^2}\cdot y + \frac{Z_0}{X_0^2 Y_0}\cdot x - \frac{1}{X_0 Y_0}\cdot z = \sin t$

$\therefore$ The steady state equations are: $Z_0 = 0$ & $X_0 Y_0 \neq 0$

& the dynamic equation is $3X_0^2 Y_0^2 \cdot \dot{x} + X_0 Z_0 \cdot y + Y_0 Z_0 \cdot x - X_0 Y_0 \cdot z = X_0^2 Y_0^2 \sin t$

3d/49 $3\dot{X} - \dfrac{Z}{XY} = W$ where $W = \sin t = W_0 + w$

$X = X_0 + x$, $Y = Y_0 + y$ & $Z = Z_0 + z$

$\therefore$ taking dc of the above eq: $0 - \frac{Z_0}{X_0 Y_0} = W_0 = 0$ $\therefore Z_0 = 0$ & $X_0 Y_0 \neq 0$

& taking ac: $\therefore w = 3\dot{x} + \frac{Z_0}{X_0^2 Y_0}\cdot x + \frac{Z_0}{X_0 Y_0^2}\cdot y - \frac{z}{X_0 Y_0}$

$\therefore 3X_0^2 Y_0^2 \cdot \dot{x} + Y_0 Z_0 \cdot x + X_0 Z_0 \cdot y - X_0 Y_0 \cdot z = X_0^2 Y_0^2 \cdot \sin t$

$\boxed{\frac{4}{49}}$ $V = \frac{4}{3}\pi R^3$. Let $V = V_o + v$ & $R = R_o + r$

a) $\therefore V_o = \frac{4}{3}\pi R_o^3$ dc equation

& $v = 4\pi R_o^2 r$ ac equation

b) $R_o = 10''$, $r = +1''$

$\therefore V = \frac{4}{3}\pi (11)^3 = 5575.28 \ ''^3$ (accurate volume)

& $V \approx V_o + v = \frac{4}{3}\pi (10)^3 + 4\pi (10)^2 (1) = 5445.43 \ ''^3$ (approximated vo

$\therefore$ % error $= \frac{5575.28 - 5445.43}{5575.28} \times 100 = 2.33$ %

c) $R_o = 20''$, $r = +1''$

$\therefore V = \frac{4}{3}\pi (21)^3 = 38792.39 \ ''^3$ (accurate volume)

& $V \approx V_o + v = \frac{4}{3}\pi (20)^3 + 4\pi (20)^2 (1) = 38536.87 \ ''^3$ (app. volume)

$\therefore$ % error $= 1 - \frac{38536.87}{38792.39} = 0.659$ %

Note:

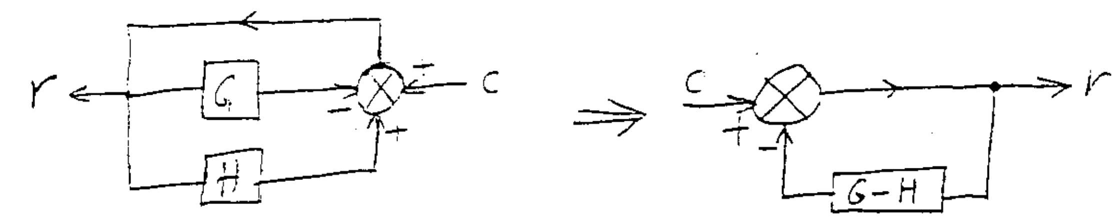
r changes 10 % in (b) & 5 % in (c)

therefore, we expect that the latter has smaller error

because, the smaller the change the more representative

the linearized equation is.

#



$\therefore \quad \frac{r}{c} = \frac{1}{1+G-H} \Rightarrow \frac{c}{r} = 1+G-H$

$y_0 = 0$, $y_{ss} = 16$ (see pp. 22)

$\therefore \; t_d = .25576$,

$t_r = .60593$,

$t_p = .9542$,

$t_s = .89426$,

$r = .04002$

$J_\infty = 115.31$

To linearize $F = I^2/Z^2$:

$\therefore \; f = \frac{2I}{Z^2} \cdot i - \frac{2I^2}{Z^3} \cdot z$ where:

f, i & z are the perturbations of F, I & Z respectively.

$\boxed{\frac{7a}{50}}$

$$14\dot{x} + 4x - 14\dot{y} - .07y = .25r \qquad (1)$$

$$.06\dot{x} + .7x + .5\ddot{y} + .2\dot{y} = .7r \qquad (2)$$

taking Laplace transform.

$\therefore$ (1) becomes: $14(sx - x_0) + 4x - 14(sy - y_0) - .07y = .25r$

or $(14s + 4)x - (14s + .07)y = .25r + 14x_0 - 14y_0 \qquad (3)$

& (2) becomes: $.06(sx - x_0) + .7x + .5(s^2y - sy_0 - y_0') + .2(sy - y_0) = .7r$

or $(.06s + .7)x + (.5s + .2)s \cdot y = .7r + .06x_0 + .5sy_0 + .5y_0' + .2y_0 \qquad (4)$

Solving (3) & (4) for x & y

$$\therefore \Delta = (14s + 4)(.5s + .2)s + (14s + .07)(.06s + .7)$$
$$= 7s^3 + (2.8 + 2.0 + .84)s^2 + (.8 + 9.8 + .0042)s + .049$$
$$= 7s^3 + 5.64s^2 + 10.6042s + .049$$

& $\Delta_x = (.5s + .2)s \cdot (.25r + 14x_0 - 14y_0) + (14s + .07)(.7r + .06x_0 + .5sy_0 + .5y_0' + .2y_0)$

$$= (.125s^2 + 9.85s + .049)r + s^2(7x_0 - 7y_0 + 7y_0)$$
$$+ s(2.8x_0 - 2.8y_0 + .84x_0 + 7y_0' + 2.8y_0 + .035y_0)$$
$$+ (.0042x_0 + .035y_0' + .014y_0)$$

For simplicity assume zero I.C.

$$\therefore \Delta_x = (.125s^2 + 9.85s + .049)r$$

& $\Delta_y = (14s + 4) * .7r - (.06s + .7) * .25r$

$$= (9.785s + 2.625)r$$

$$\therefore y = \frac{\Delta_y}{\Delta} = \frac{(9.785s + 2.625)r}{\Delta}, \quad x = \frac{\Delta_x}{\Delta} = \frac{(.125s^2 + 9.85s + .049)}{\Delta}$$

$$\therefore \frac{x}{r}(s) = \frac{.125s^2 + 9.85s + .049}{7s^3 + 5.64s^2 + 10.6042s + .049}$$

$$\& \quad \frac{y}{r}(s) = \frac{9.785s + 2.625}{7s^3 + 5.64s^2 + 10.6042s + .049}.$$

$\boxed{\frac{5C}{50}}$ $y(s) = \frac{2(s+2)}{s^3(s+4)} = \frac{A+Bs+Cs^2}{s^3} + \frac{D}{s+4}$

$\therefore (A+Bs+Cs^2)(s+4) + Ds^3 = 2(s+2)$

put $s=0 \Rightarrow \therefore 4A = 4 \quad \therefore A=1$

diff then put $s=0 \quad \therefore (B+2Cs)(s+4)+(A+Bs+Cs^2)+3Ds^2 = 2 \quad \therefore$

$\therefore 4B + A = 2 \quad \therefore B = \frac{2-A}{4} = \frac{2-1}{4} = \frac{1}{4}$

diff again then put $s=0 \quad \therefore 2C(4)+B+B=0 \quad \therefore C = \frac{-2B}{8} = -1/16$

put $s=-4 \quad \therefore D(-4)^3 = 2(-4+2) \quad \therefore D = -4/(-64) = \frac{1}{16}$

[note: you can obtain them by another way as follows:

$D = \lim_{s\to -4} (s+4)\,y(s) = \lim_{s\to -4} \frac{2(s+2)}{s^3} = \frac{2(-4+2)}{(-4)^3} = \frac{1}{16}$

$A = \lim_{s\to 0} s^3 y(s) = \lim_{s\to 0} \frac{2(s+2)}{s+4} = \frac{4}{4} = 1$

$B = \lim_{s\to 0} [s^3 y(s)]' = \lim_{s\to 0} \frac{2(s+4)-2(s+2)}{(s+4)^2} = \lim_{s\to 0} \frac{4}{(s+4)^2} = \frac{1}{4}$

& $C = \frac{1}{2}\lim_{s\to 0} [s^3 y(s)]'' = \frac{1}{2}\lim_{s\to 0} \frac{-8}{(s+4)^3} = \frac{1}{2}\cdot\frac{-8}{64} = \frac{-1}{16}$]

$\therefore y(s) = \frac{1}{s^3} + \frac{1}{4}\cdot\frac{1}{s^2} - \frac{1}{16}\cdot\frac{1}{s} + \frac{1}{16}\cdot\frac{1}{s+4}$

$\therefore y(t) = \frac{t^2}{2} + \frac{t}{4} - \frac{1}{16} + \frac{1}{16}e^{-4t} = .5t^2 + .25t - .0625(1-e^{-4t})$

8ii / S[illegible]

$$y(s) = \frac{2(s+4)}{s(s+2)^2(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{s+4}$$

$\therefore$ multiplying by s then putting $s=0 \Rightarrow \frac{8}{16} = A \Rightarrow A = \frac{1}{2}$.

" " $(s+2)^2$ " " $s=-2 \Rightarrow \frac{4}{-4} = C \Rightarrow C = -1$

" " " " differentiating then putting $s=-2 \Rightarrow 2\left(\frac{(-2)(2)-(2(-2)+4)(2)}{(-2)^2(2)^2}\right) = B$

$\Rightarrow B = \frac{-1}{2}$

" " $(s+4)$ then putting $s=-4 \Rightarrow 0 = D$

$$\therefore y(s) = \frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\cdot\frac{1}{s+2} - \frac{1}{(s+2)^2}$$

$$\therefore y(t) = \frac{1}{2} - \frac{1}{2}e^{-2t} - t e^{-2t}$$

$$\therefore y(t) = \frac{1}{2}\left[1 - e^{-2t}\cdot(1+2t)\right]$$

11e / S1

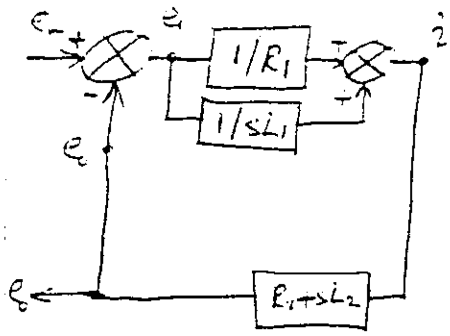
$$y_\infty = \lim_{s\to 0} s\,y(s) = \lim_{s\to 0} s\cdot\frac{2(s+4)}{s(s+2)^2(s+4)} = \lim_{s\to 0}\frac{2}{(s+2)^2} = \frac{1}{2}$$

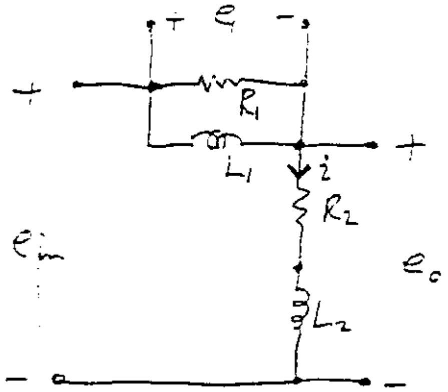
and $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \frac{1}{2}\left[1 - e^{-2t}(1+2t)\right] = \frac{1}{2}$

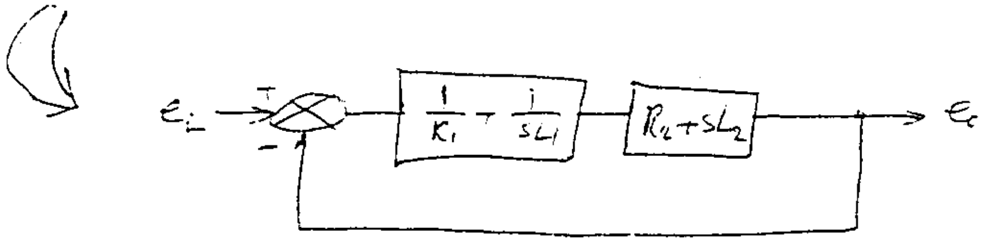
$\therefore$ both method agrees $\therefore$ OK for $y_{ss} = \frac{1}{2}$.

24/73 Block Diagram.

O.I.C







$$\therefore \quad \frac{e_c}{e_{in}} = \frac{\left(\frac{sL_1+R_1}{R_1\, sL_1}\right)\cdot(R_2+sL_2)}{1+\left(\frac{sL_1+R_1}{R_1\, sL_1}\right)(R_2+sL_2)} = \frac{(R_1+sL_1)(R_2+sL_2)}{(R_1+sL_1)(R_2+sL_2)+sR_1L_1}$$

$$\text{or} \quad \frac{e_c}{e_{in}}(s) = \frac{s^2L_1L_2+s(L_1R_2+L_2R_1)+R_1R_2}{s^2L_1L_2+s(L_1R_1+L_1R_2+L_2R_1)+R_1R_2}$$

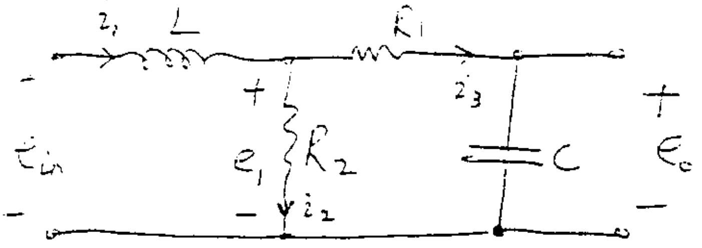
for $e_i = u(t)$ volts $\quad \therefore e_i(s) = \frac{1}{s}$

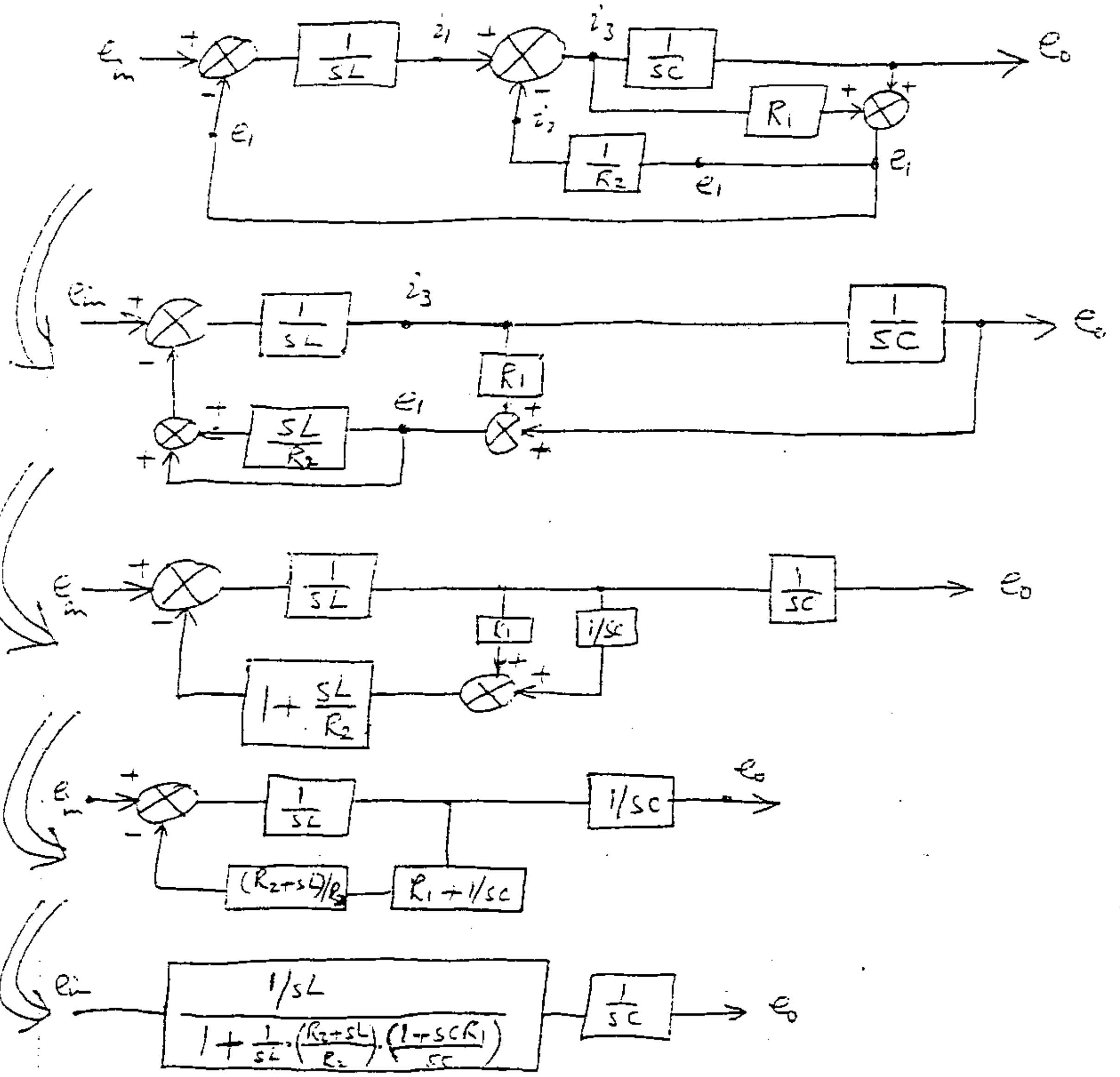
$\therefore e_o(s) = \left[\frac{e_o}{e_i}(s)\right]\cdot\frac{1}{s} \qquad$ or $\quad s\,e_o(s) = \frac{e_o}{e_i}(s)$

$$\therefore e_{oss} = \lim_{s\to 0} s\,e_o(s) = \lim_{s\to 0}\frac{e_o}{e_i}(s) = \frac{R_1R_2}{R_1R_2} = 1$$

$\therefore e_{oss} = 1$ volts.

2b / 73



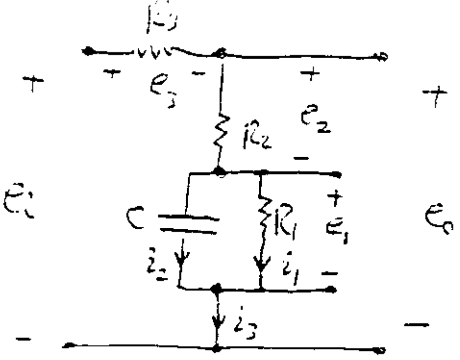


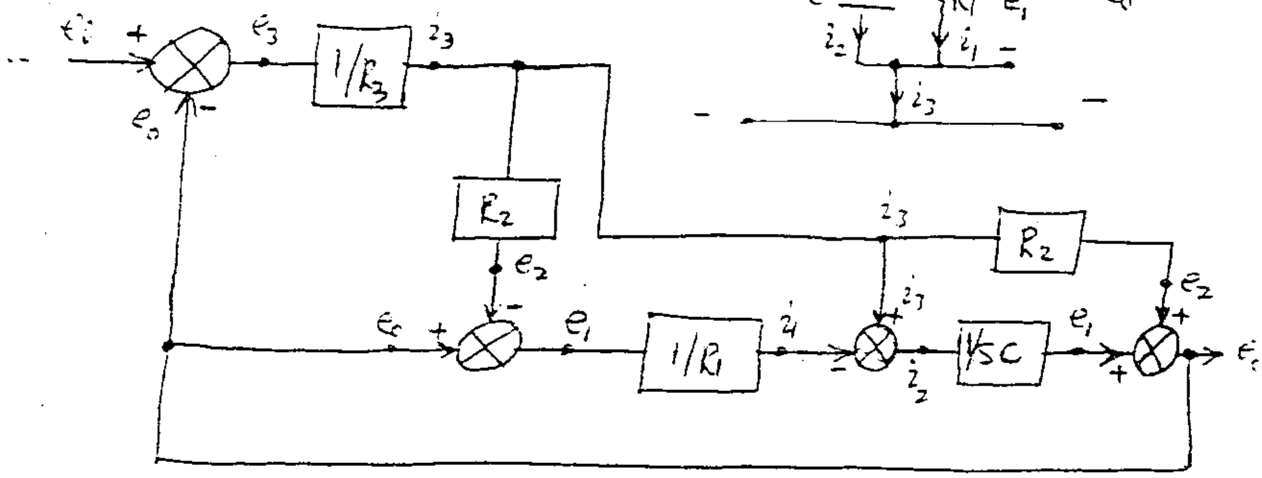
$$\therefore \frac{e_o}{e_i} = \frac{R_2}{R_2 s^2 LC + (1+sCR_1)(R_2+sL)} = \frac{R_2}{(R_2+R_1)LC\,s^2 + (L+CR_1R_2)s + R_2}$$

$$\left(\text{check } \frac{e_o}{e_i} = \frac{1/sC}{1/sC+R_1}\cdot\frac{(R_1+1/sC)R_2/(R_1+\frac{1}{sC}+R_2)}{sL+(R_1+1/sC)R_2/(R_1+\frac{1}{sC}+R_2)} = \frac{1}{1+sCR_1}\cdot\frac{(sCR_1+1)R_2/sC}{\frac{sL}{sC}(1+(R_1+R_2)sC)+(1+sCR_1)\frac{R_2}{sC}}\right.$$

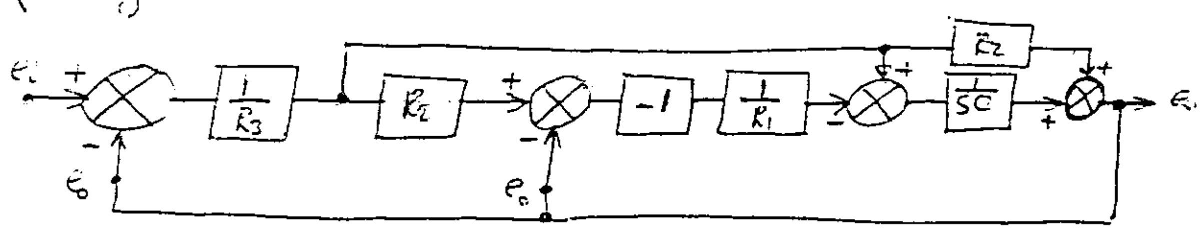
$$\left.= \frac{R_2}{(R_1+R_2)LCs^2+(L+CR_1R_2)s+R_2} \quad \therefore \text{OK}\right)$$

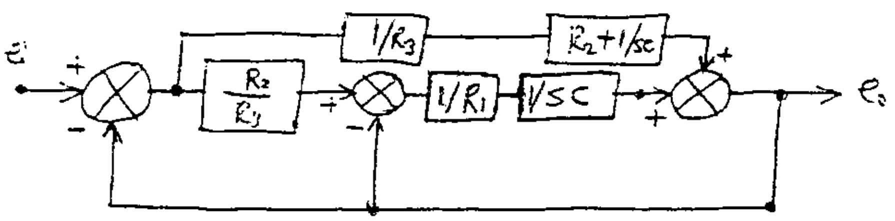
2C/74. Denoting the voltages and currents as shown in the figure; then the following block diagram is obtained:

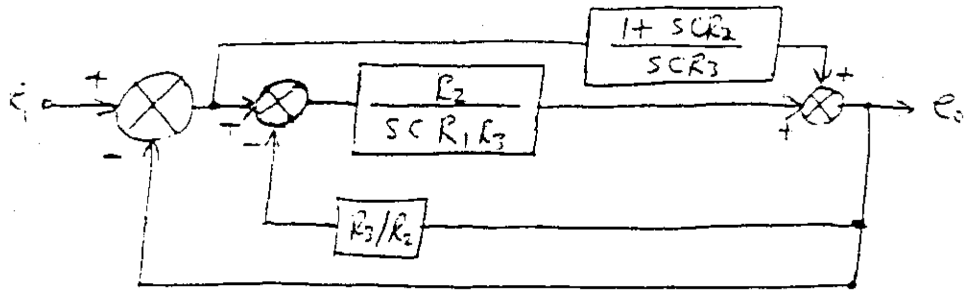


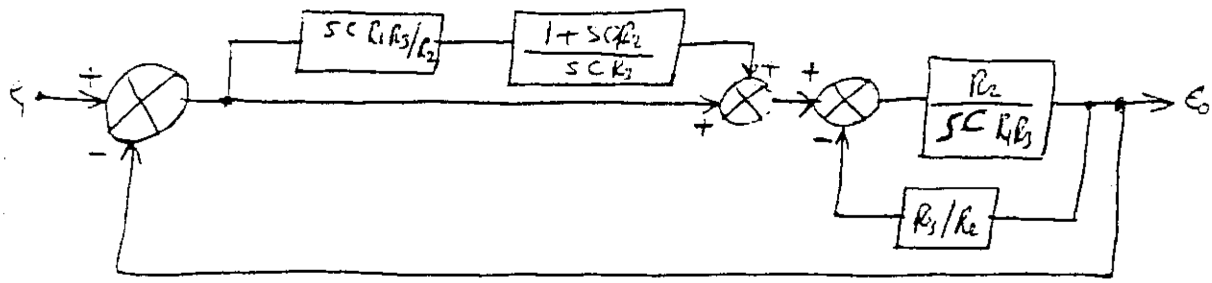


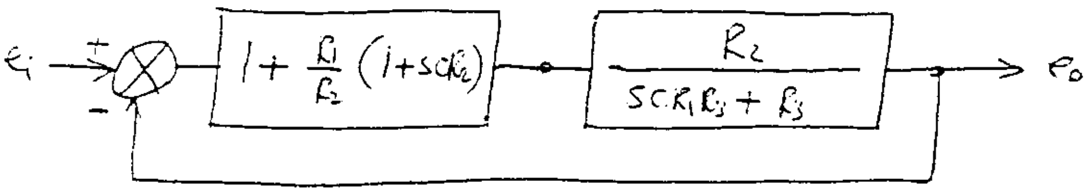
(Using reduction:

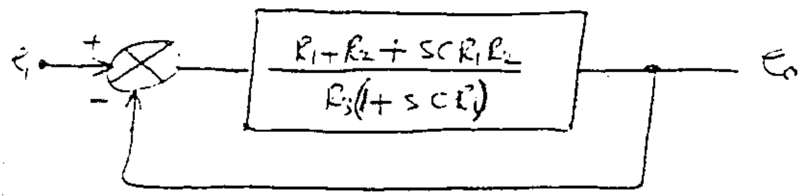




∴ 

∴ 

∴ 

∴ 

$$\therefore \frac{e_o}{e_i} = \frac{R_1+R_2+sCR_1R_2}{R_3(1+sCR_1)+R_1+R_2+sCR_1R_2} = \frac{R_1+R_2+sCR_1R_2}{R_1+R_2+R_3+sCR_1(R_2+R_3)}$$

(OK: by circuit rules: $\frac{e_o}{e_i} = \frac{R_2+\frac{R_1/sC}{R_1+1/sC}}{R_3+R_2+\frac{R_1/sC}{R_1+1/sC}} = \frac{R_2+\frac{R_1}{1+sCR_1}}{R_3+R_2+\frac{R_1}{1+sCR_1}} \Rightarrow$

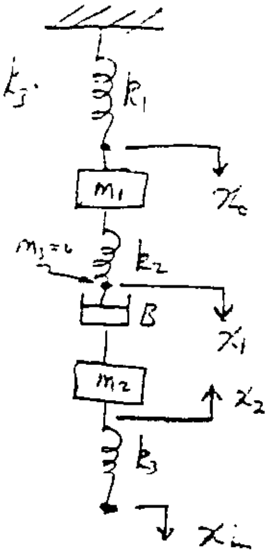
$$\therefore \frac{e_o}{e_i} = \frac{R_1+R_2+sCR_1R_2}{R_1+R_2+R_3+sCR_1(R_2+R_3)}$$

∴ OK)

\# Assume step input A ∴ $e_o = \frac{R_1+R_2}{R_1+R_2+R_3}\cdot A$ (c o/c)

$\boxed{\frac{3a}{74}}$ # Equations of reduction (assuming the shown symbols of sense

Don't be concerned with unconnected end of k_3.



$$k_3(x_{in}+x_2) = -m_2\ddot{x}_2 - B(\dot{x}_1+\dot{x}_2) \qquad (1)$$

$$B(\dot{x}_1+\dot{x}_2) + k_2(x_1-x_o) + \cancel{m_3}\ddot{x}_1 = 0 \qquad (2)$$

$$k_2(x_1-x_o) = m_1\ddot{x}_o + k_1x_o \qquad (3)$$

$V_{in} = \dot{x}_{in}$

From (3)

$$\therefore k_2x_1 = m_1\ddot{x}_o + (k_1+k_2)x_o$$

into (2)

$$\therefore B\dot{x}_2 = -\frac{B}{k_2}\left(m_1\dddot{x}_o + (k_1+k_2)\dot{x}_o\right) - m_1\ddot{x}_o - k_1x_o =$$

into (1)

$$\therefore k_3x_{in} = -\frac{m_2}{B}\left[-\frac{B}{k_2}\left(m_1x_o^{(4)} + (k_1+k_2)\ddot{x}_o\right) - m_1\dddot{x}_o - k_1\dot{x}_o\right] + m_1\ddot{x}_o + k_1x_o$$
$$-\frac{k_3}{B}\int\left[-\frac{B}{k_2}\left(m_1\dddot{x}_o + (k_1+k_2)\dot{x}_o\right) - m_1\ddot{x}_o - k_1x_o\right]dt$$

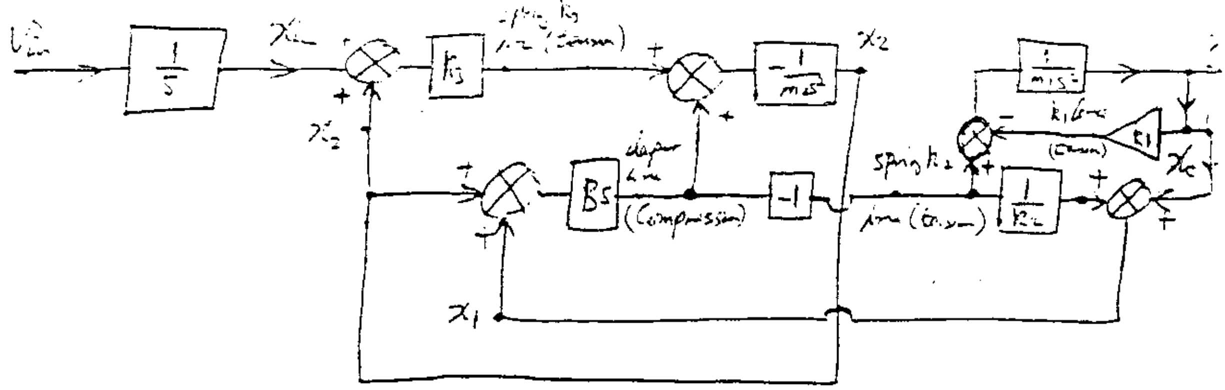
Laplace-transforming assuming 0 IC & reducing gives:

$$\therefore k_3x_{in}(s) = \left\{\left[\frac{m_2m_1}{k_2}s^4 + \frac{m_2(k_1+k_2)}{k_2}s^2 + \frac{m_1m_2}{B}s^3 + \frac{m_2k_1}{B}s\right]\left(1+\frac{k_3}{m_2s^2}\right) + m_1s^2 + k_1\right\}x_o(s)$$

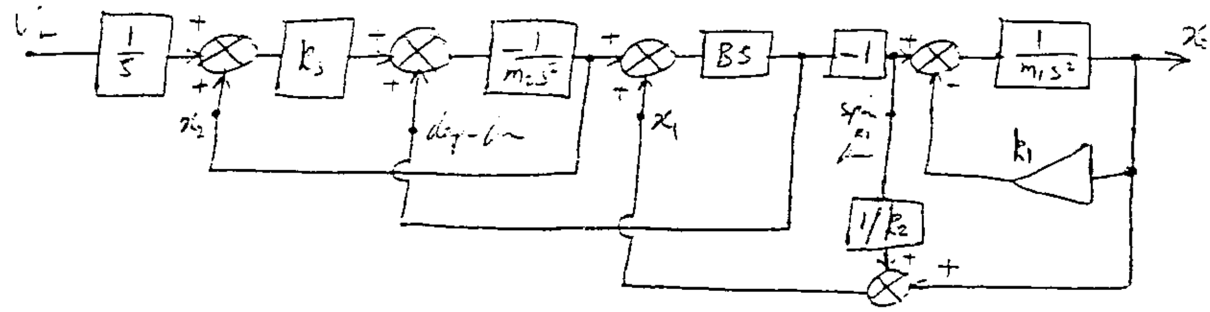
$\therefore$ Simplifying: $\therefore Bk_2k_3\,s\,x_{in}(s) = \Big\{\Big[Bm_1s^3 + B(k_1+k_2)s + m_1k_2s^2$
$$+ k_1k_2\Big](m_2s^2+k_3) + Bk_2s(m_1s^2+k_1)\Big\}x_o(s)$$

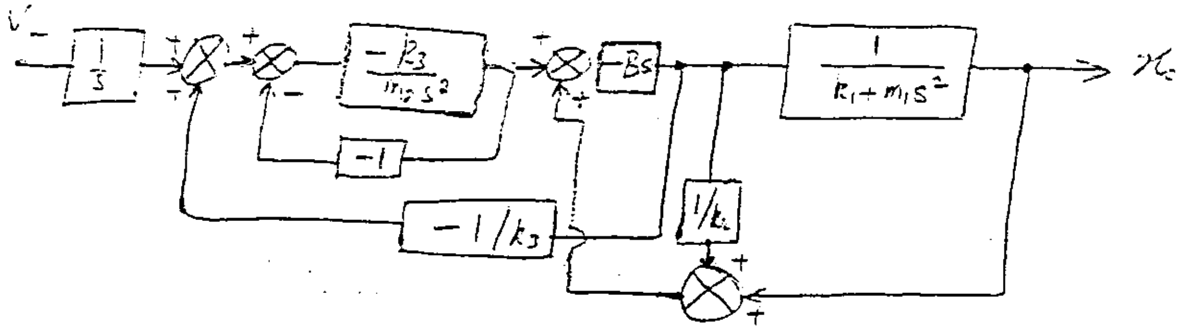
$$\therefore \frac{x_o(s)}{V_{in}(s)} = \frac{x_o(s)}{\dot{x}_{in}(s)} = \frac{x_o(s)}{s\,x_{in}(s)} = \frac{Bk_2k_3}{(Bm_1s^3+k_2m_1s^2+(k_1+k_2)Bs+k_1k_2)(m_2s^2+k_3)+(m_1s^2+k_1)Bk_2}$$

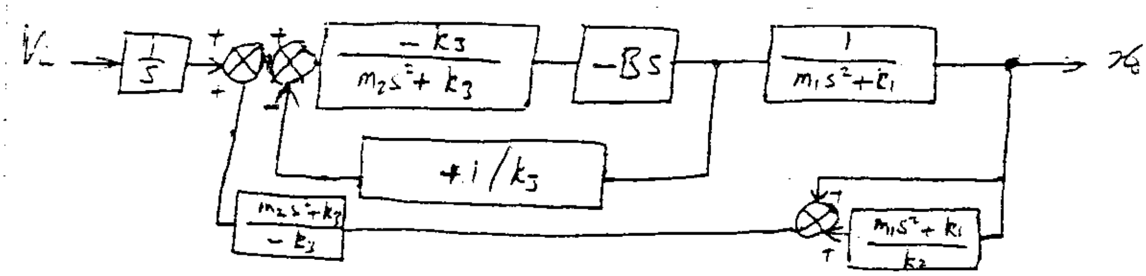
Block diagram & reduction (OIC)

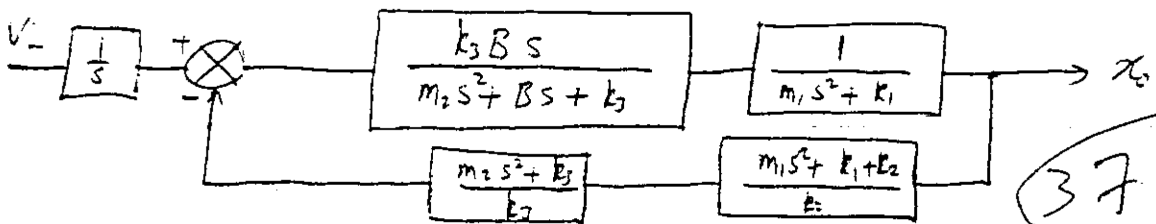


Redrawn to give









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$$\therefore \frac{x_c}{V_{in}}(s) = \frac{k_2 k_3 B}{k_2(m_2 s^2 + Bs + k_3)(m_1 s^2 + k_1) + (m_1 s^2 + k_1 + k_2)(m_2 s^2 + k_3)Bs} \quad (\therefore OK)$$

If $v_i(t) = \text{Constant} = V_o$

$\therefore \; V_i(s) = \dfrac{V_o}{s}$

$\therefore \; x_o(s) = \left(\dfrac{x_o}{V_i}(s)\right) \cdot V_i(s) = \left(\dfrac{x_o}{V_i}(s)\right) \cdot \dfrac{V_o}{s}$

$\therefore \; s\,x_o(s) = V_o \cdot \dfrac{x_o}{V_i}(s)$

$\therefore \; x_{o_{ss}} = \lim_{s \to 0} s\,x_o(s) = \lim_{s \to 0} V_o \dfrac{x_o}{V_i}(s) = V_o \dfrac{x_o}{V_i}(0) = V_o \cdot \dfrac{k_2 k_3 B}{k_1 k_2 k_3}$

$\therefore \; x_{o_{ss}} = B V_o / k_1$

$\frac{4}{75}$ a) The problem is a rotational dynamics one.

Consider first the section $K_1 J_1 B$:

$\therefore$ Assuming O.I.C with $\dot{\theta}_{in} = \omega_{in}$ $\therefore$ $s\,\theta_{in}(s) = \omega_{in}(s)$

$$\therefore\; K_1(\theta_{in} - \theta_1) = J_1\ddot{\theta}_1 + B(\dot{\theta}_1 - \dot{\theta}_2) \qquad (1)$$

Consider the section $B J_2 K_2$:

$$\therefore\; B(\dot{\theta}_1 - \dot{\theta}_2) = J_2\ddot{\theta}_2 + K_2(\theta_2 - \theta_{out}) \qquad (2)$$

Consider the section $K_2 J_3 K_3$:

$$\therefore\; K_2(\theta_2 - \theta_{out}) = J_3\ddot{\theta}_{out} + K_3\theta_{out} \qquad (3)$$

b) Taking the Laplace-transform of (1), (2) & (3) and rearranging:

$$\therefore\; K_1\theta_{in}(s) = (J_1 s^2 + Bs + K_1)\theta_1(s) - Bs\,\theta_2(s) \qquad (4)$$

$$\&\; Bs\,\theta_1(s) = (J_2 s^2 + Bs + K_2)\theta_2(s) - K_2\theta_{out}(s) \qquad (5)$$

$$\&\; K_2\theta_2(s) = (J_3 s^2 + K_2 + K_3)\theta_{out}(s) \qquad (6)$$

$\therefore$ Substituting for θ_1 & θ_2 from (5) & (6) into (4) gives

$$K_1\theta_{in} = \left(\frac{J_1 s^2 + Bs + K_1}{Bs}\right)\left[(J_2 s^2 + Bs + K_2)\theta_2 - K_2\theta_{out}\right] - Bs\,\theta_2 =$$

$$= -\frac{K_2}{Bs}\cdot(J_1 s^2 + Bs + K_1)\theta_{out} + \frac{1}{Bs}\left[\begin{matrix} J_1J_2 s^4 + B(J_1+J_2)s^3 + (J_1K_2 + J_2K_1)s^2 \\ + (K_1+K_2)Bs + K_1K_2 \end{matrix}\right]\theta_2$$

$$= -\frac{K_2^2}{K_2 Bs}(J_1 s^2 + Bs + K_1)\theta_{out} + \frac{1}{K_2 Bs}\left[J_1J_2 s^4 + B(J_1+J_2)s^3 + (J_1K_2 + K_1J_2)s^2\right.$$

$$\left. + (K_1+K_2)Bs + K_1K_2\right](J_3 s^2 + K_2 + K_3)\theta_{out}$$

$$= \frac{1}{K_2 Bs}\left[J_1J_2J_3 s^6 + BJ_3(J_1+J_2)s^5 + (J_1J_2K_3 + J_1K_2J_3 + K_1J_2J_3 + J_1J_2K_2)s^4\right.$$

$$+ B\left[(J_1+J_2)(K_2+K_3) + J_3(K_1+K_2)\right]s^3 + \left[(K_2+K_3)(J_1K_2 + K_1J_2) + K_1K_2J_3 - J_1K_2^2\right]s^2$$

$$\left. + \left((K_1+K_2)(K_2+K_3) - K_2^2\right)Bs + K_1K_2K_3\right]\theta_{out}$$

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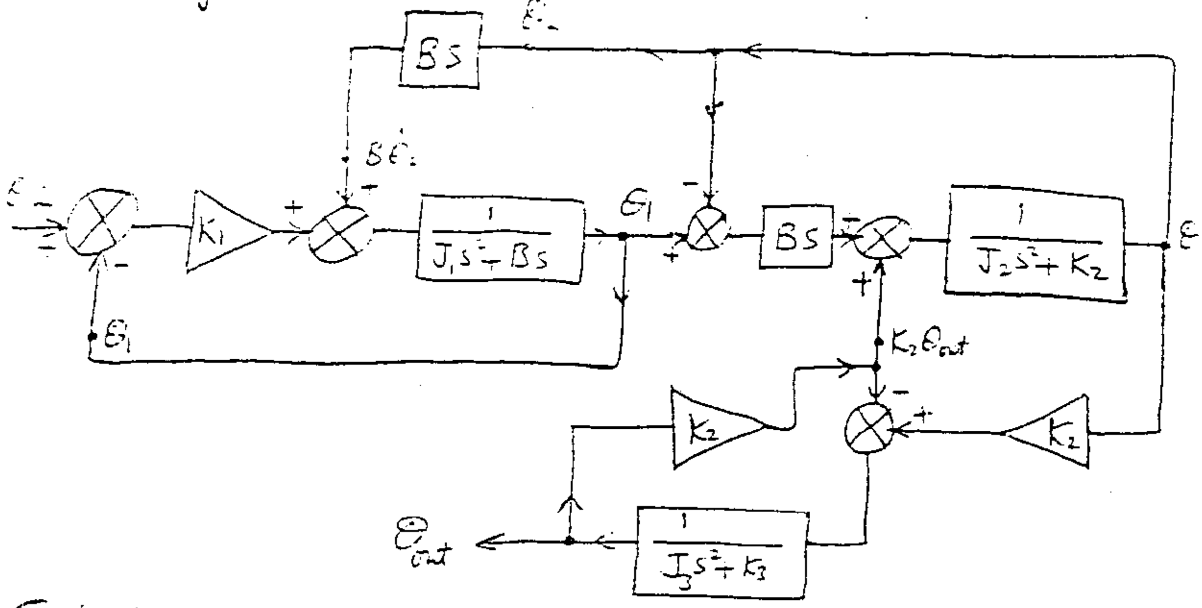
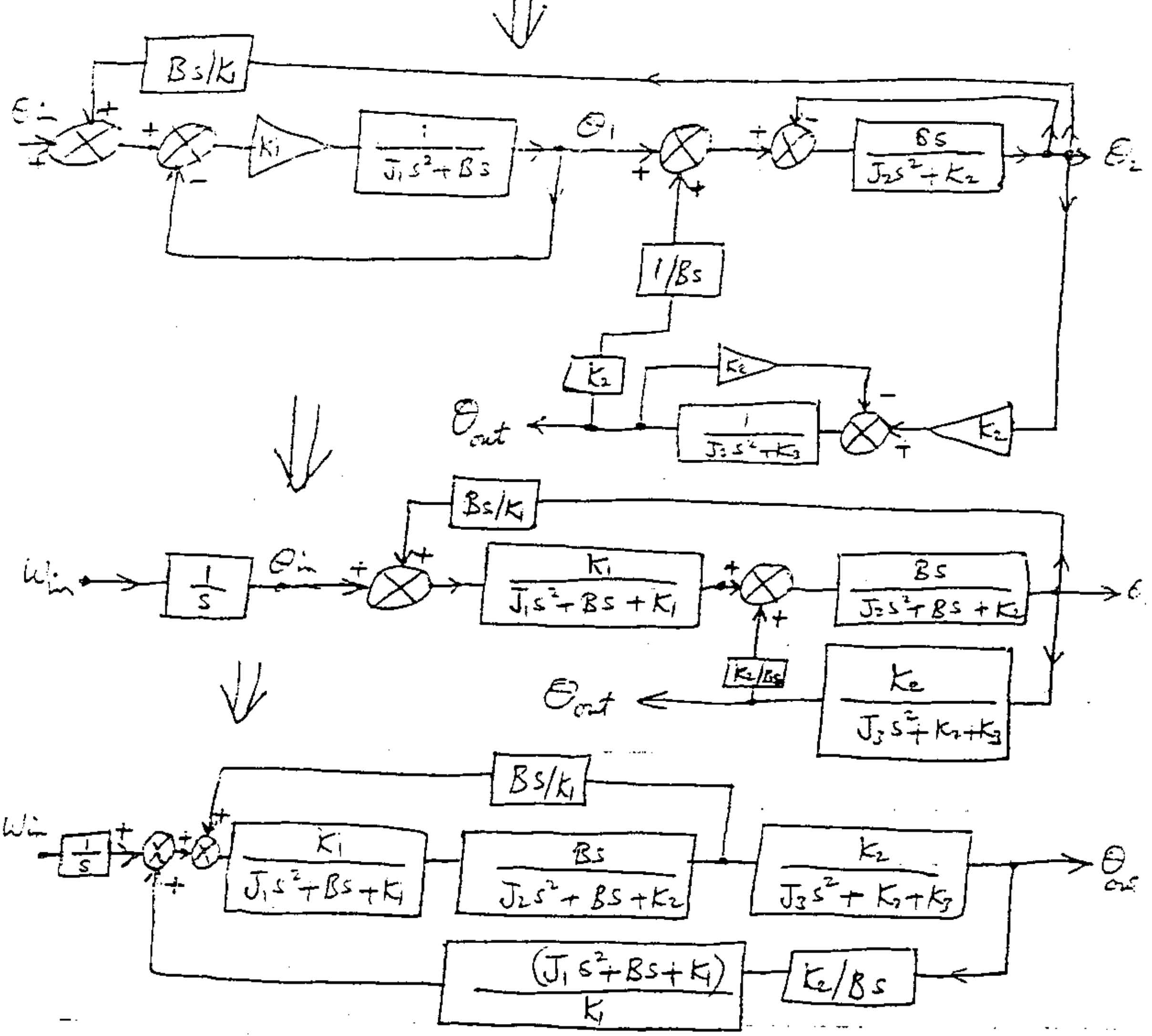
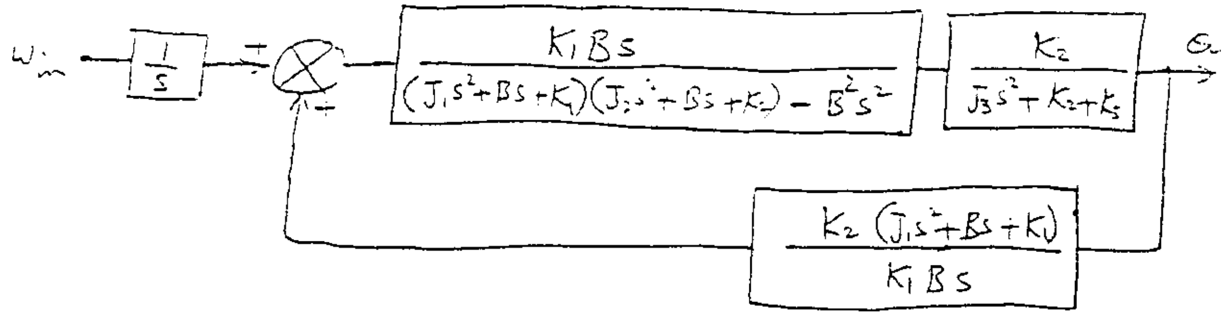
$$\therefore\; \frac{\theta_{out}(s)}{s\,\theta_{in}(s)} = \frac{\theta_{out}(s)}{\omega_{in}(s)} = \frac{K_1K_2B}{\left[\begin{matrix} J_1J_2J_3 s^6 + BJ_3(J_1+J_2)s^5 + (J_1J_2K_3 + J_1K_2J_3 + K_1J_2J_3 + J_1J_2K_2)s^4 \\ + B(J_1K_2 + J_1K_3 + J_2K_2 + J_2K_3 + K_1J_3 + K_2J_3)s^3 + \\ + (J_1K_2K_3 + K_1K_2J_2 + K_1J_2K_3 + K_1K_2J_3)s^2 + B(K_1K_2 + K_1K_3 + K_2K_3)s \end{matrix}\right]}$$

c) Block Diagram:



(Check:





$$\therefore \frac{\Theta_{out}}{\omega_{in}} = \frac{K_1 K_2 B}{\left[(J_1 s^2+Bs+K_1)(J_2 s^2+Bs+K_2)-B^2 s^2\right](J_3 s^2+K_2+K_3) - K_2^2 (J_1 s^2+Bs+K_1)}$$

or
$$\frac{\Theta_{out}(s)}{\omega_{in}(s)} = \frac{K_1 K_2 B}{\left[\begin{array}{l} J_1 J_2 J_3 s^6 + B J_3 (J_1+J_2) s^5 + (J_1 J_2 K_3 + J_1 K_2 J_3 + K_1 J_2 J_3 + J_1 J_2 K_2) s^4 + \\ + B(J_1 K_2 + J_1 K_3 + J_2 K_2 + J_2 K_3 + K_1 J_3 + K_2 J_3) s^3 + \\ + (J_1 K_1 K_3 + K_1 K_2 J_2 + K_1 J_2 K_3 + K_1 K_2 J_3) s^2 + B(K_1 K_2 + K_1 K_3 + K_2 K_3) s + K_1 K_2 K_3 \end{array}\right]}$$

$\therefore$ OK

d) For $K_3 << K_2$ the expression becomes

$$\frac{\Theta_{out}(s)}{\omega_{in}(s)} = \frac{K_1 K_2 B}{\left[\begin{array}{l} J_1 J_2 J_3 s^6 + B J_3 (J_1+J_2) s^5 + (J_1 K_2 J_3 + K_1 J_2 J_3 + J_1 J_2 K_2) s^4 + \\ + B(J_1 K_2 + J_2 K_2 + K_1 J_3 + K_2 J_3) s^3 + (J_1 K_2 K_3 + K_1 K_2 J_2 + K_1 K_2 J_3) s^2 + \\ + (K_1 + K_3) K_2 B s + K_1 K_2 K_3 \end{array}\right]}$$

Furthermore, if both K_1 & $K_2 >> K_3$ we'll have:

$$\frac{\Theta_{out}(s)}{\omega_{in}(s)} = \frac{K_1 K_2 B}{\left[\begin{array}{l} J_1 J_2 J_3 s^6 + B J_3 (J_1+J_2) s^5 + (J_1 K_2 J_3 + K_1 J_2 J_3 + J_1 J_2 K_2) s^4 + K_1 K_2 B s + \\ + B(J_1 K_2 + J_2 K_2 + K_1 J_3 + K_2 J_3) s^3 + (J_1 K_2 K_3 + K_1 K_2 J_2 + K_1 K_2 J_3) s^2 + K_1 K_2 K_3 \end{array}\right]}$$

For $\omega_{in}(t) = A u(t)$ $\therefore$ $\omega_{in}(s) = \frac{A}{s}$ $\therefore$ $\Theta_{out}(s) = \left(\frac{\Theta_{out}(s)}{\omega_{in}(s)}\right) \cdot \frac{A}{s}$

$\therefore$ $\Theta_{out}(t)$ at steady state $= \lim_{t\to\infty} \Theta_{out}(t) = \lim_{s\to 0} s\,\Theta_{out}(s) =$

$$= A\left(\frac{\Theta_{out}(s)}{\omega_{in}(s)}\right)\Bigg|_{s=0} = \frac{A K_1 K_2 B}{K_1 K_2 K_3} = AB/K_3.$$

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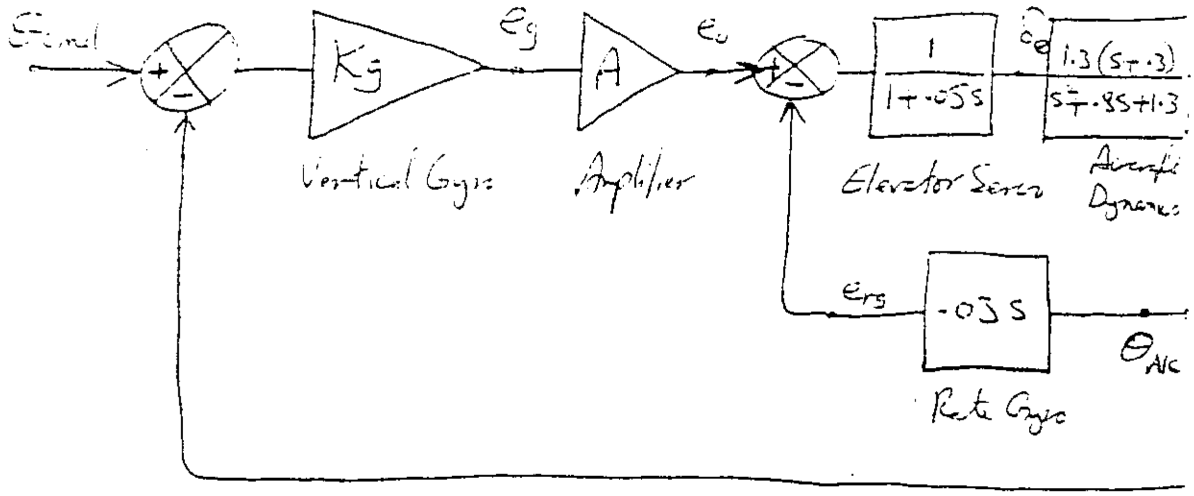
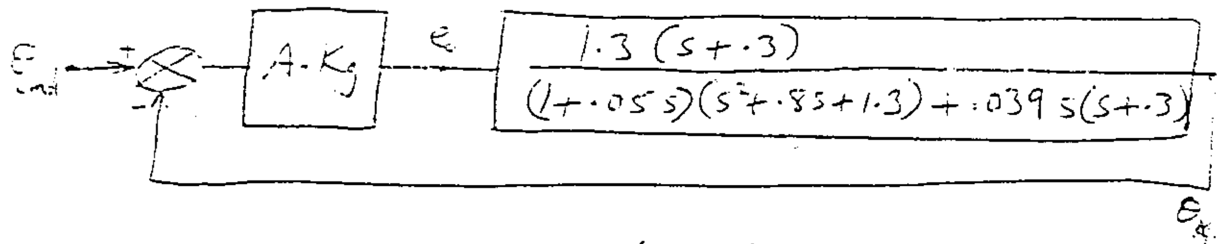
a) Block Diagram.



b) Reduction:



$$\therefore \frac{\theta_{A/c}}{\theta_{cmd}}(s) = \frac{1.3\,A \cdot K_g \cdot (s+.3)}{.05s^3+1.079s^2+.8767s+1.3+1.3AK_g(s+.3)}$$

$$\& \quad \frac{\delta_e}{\theta_{cmd}}(s) = \frac{A \cdot K_g \cdot (s^2+.8s+1.3)}{.05s^3+1.079s^2+(.8767+1.3AK_g)s+1.3(1+.3AK_g}$$

c) $\theta_{cmd}(t) = B\,u(t) \quad \therefore \theta_{cmd}(s) = \frac{B}{s} \quad \therefore \theta_{A/c}(s) = \frac{\theta_{A/c}}{\theta_{cmd}}(s) \cdot \frac{B}{s}$

$$\therefore \lim_{t \to \infty} \theta_{A/c}(t) = \lim_{s \to 0} s\,\theta_{A/c}(s) = \lim_{s \to 0} B\left(\frac{\theta_{A/c}}{\theta_{cmd}}(s)\right) = B\,\frac{.39AK_g}{1.3(1+.3AK_g)}$$

$\therefore$ Final value of $\theta_{A/c}(t)$ is $\frac{.3ABK_g}{1+.3AK_g}$

Likewise, Final value of $\delta_e(t)$ is $\frac{ABK_g}{1+.3AK_g}$

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$$\left(\frac{\theta_{A/c}}{\theta_{cmd}}\right)_{ss} = \lim_{s \to 0}\left(\frac{s\,\theta_{A/c}}{s\,\theta_{cmd}}(s)\right) = \frac{1.3AK_g * .3}{(1+.3AK_g)1.3} = \frac{1}{1+(1/(.3AK_g))} \neq 1 \therefore \text{No.}$$

a) $(D^2+9)x = 1$ with OIC $\qquad \therefore D = \pm j3$

$\therefore \quad x = A\sin 3t + B\cos 3t + \frac{1}{\cancel{D^2}+9}\cdot 1 \rightarrow \frac{1}{9} \Rightarrow$

$x(0)=0 \Rightarrow 0 = B + \frac{1}{9} \Rightarrow B = -\frac{1}{9}$

$\& \quad \dot{x}(0)=0 \Rightarrow 0 = 3A \Rightarrow A=0$

$\therefore$ Solution is $\quad x = \frac{1}{9}(1-\cos 3t)$

b) $(D^2+5D+4)x = 8$ with OIC $\qquad \therefore D = -4, -$

$\therefore \quad x = Ae^{-t} + Be^{-4t} + \frac{1}{\cancel{D^2+5D}+4}\cdot 8 \rightarrow 2 \qquad \Rightarrow$

$x(0)=0 \Rightarrow 0 = A+B+2$

$\& \quad \dot{x}(0)=0 \Rightarrow 0 = -A-4B$

$\therefore \quad A=-4B \quad \& \quad -4B+B+2=0 \Rightarrow B = \frac{2}{3} \quad \& \quad A = \frac{-8}{3}$

$\therefore$ Solution is $\quad x = \frac{2}{3}\left[e^{-4t} - 4e^{-t} + 3\right]$

c) $(D^2+D+4.25)x = t+1$ with OIC

$\therefore \quad D = \frac{-1\pm\sqrt{1-17}}{2} = \frac{-1\pm j4}{2}$

$\frac{1-4.25(t+1)}{-4.25^2} = \frac{4t}{17}+\frac{52}{17^2}$

$\therefore \quad x(t) = e^{-t/2}\cdot\left[A\sin 2t + B\cos 2t\right] + \frac{1}{D^2+\cancel{D+}4.25}\cdot(t+1)$

$x(0)=0 \Rightarrow 0 = B + \frac{52}{17^2} \Rightarrow B = \frac{-52}{17^2}$

$\dot{x}(0)=0 \Rightarrow 0 = \left[-\frac{1}{2}\cdot B + 2A\right] + \frac{4}{17} \Rightarrow A = \frac{B}{4} - \frac{2}{17} = \frac{-13-34}{17^2} = \frac{-47}{17^2}$

$\therefore$ Solution is : $\quad x(t) = \frac{4t}{17} + \frac{52}{17^2} - \frac{e^{-t/2}}{17^2}\cdot\left(47\sin 2t + 52\cos 2t\right).$

d) $(D^3+D^2+4D+4)x = 10\sin 10t$ with OIC

$\therefore \left[D^2(D+1)+4(D+1)\right]x = 10\sin 10t$

$\therefore (D^2+4)(D+1)x = 10\sin 10t \qquad \Rightarrow D=-1,\ \pm j2$

$$\therefore x(t) = Ae^{-t} + B\sin 2t + C\cos 2t + \frac{10}{(D^2+4)(D+1)}\cdot\sin 10t$$

$$= \underbrace{\qquad\qquad\qquad\qquad}_{''} + \frac{10(D-1)}{(D^2+4)(D^2-1)}\cdot\sin 10t$$

$$= \underbrace{\qquad\qquad\qquad\qquad}_{''} + \frac{10}{(-100+4)(-100-1)}\cdot(10\cos 10t-\sin 10t)$$

$$= Ae^{-t} + B\sin 2t + C\cos 2t + \frac{10}{9696}(10\cos 10t - \sin 10t)$$

but: $x(0)=0 \quad\Rightarrow\quad 0 = A + C + \frac{100}{9696} \qquad (1)$

& $\dot{x}(0)=0 \quad\Rightarrow\quad 0 = -A + 2B - \frac{100}{9696} \qquad (2)$

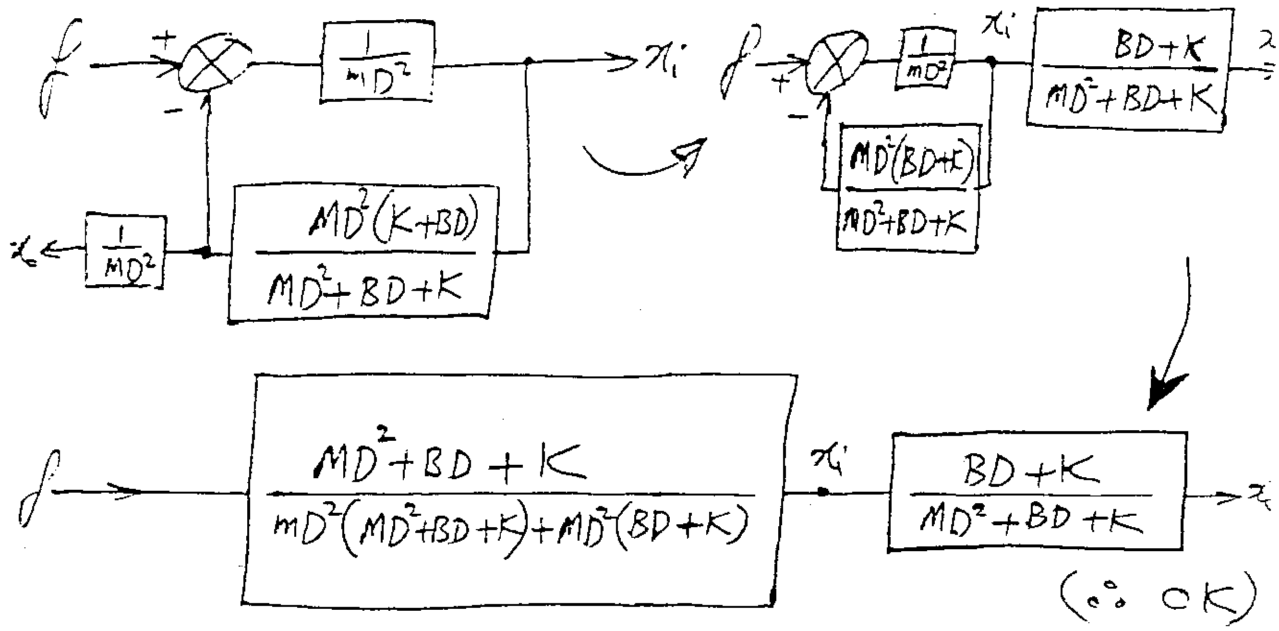
& $\ddot{x}(0)=0 \quad\Rightarrow\quad 0 = A - 4C - \frac{10000}{9696} \qquad (3)$

$\frac{(1)-(3)}{5} \quad\Rightarrow\quad C = \frac{-1}{5}\cdot\frac{10100}{9696} = \frac{-2020}{9696} = \frac{-5}{24} \qquad (4)$

$(1)\,\&\,(4) \quad\Rightarrow\quad A = \frac{2020-100}{9696} = \frac{1920}{9696} = \frac{20}{101} \qquad (5)$

$(2)\,\&\,(5) \quad\Rightarrow\quad B = \frac{960+50}{9696} = \frac{1010}{9696} = \frac{5}{48}$

$$\therefore x(t) = \frac{20}{101}e^{-t} + \frac{5}{48}(\sin 2t - 2\cos 2t) + \frac{5}{4848}(10\cos 10t - \sin 10t)$$



$(\therefore \text{ OK})$

To solve for impulse response when $f(t)=\delta(t)$ with OIC:

$\therefore\ f(s)=1$

$$\therefore\ X_o(s)=\frac{Bs+K}{s^2\left[mMs^2+(m+M)(Bs+K)\right]}\cdot f(s)=\frac{Bs+K}{s^2\left[mMs^2+(m+M)(Bs+K)\right]}=$$

$$=\frac{Bs+K}{s^2(m+M)\left[\frac{mM}{m+M}\cdot s^2+Bs+K\right]}$$

& since bumper mass m is usually much less than car mass M $\therefore\ m+M\approx M$

$$\therefore\ X_o(s)\approx\frac{Bs+K}{Ms^2(ms^2+Bs+K)}=\frac{Bs+K}{mMs^2(s-s_1)(s-s_2)}\text{, where:}$$

$s_1,s_2=\frac{-B\pm\sqrt{B^2-4mK}}{2m}$. Now usually bumper damper is of high damping factor whereas its spring is not so stiff.

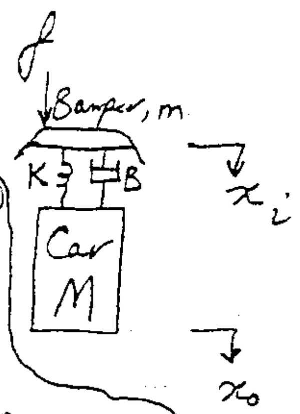
$\therefore$ B value is high whereas both m & K values are relatively low.

$$\therefore\ B^2\gg 4mK\ \Rightarrow\ \sqrt{B^2-4mK}=B\cdot\sqrt{1-\frac{4mK}{B^2}}\simeq\left(1-\frac{2mK}{B^2}\right)\cdot B$$

$$\therefore\ s_1,s_2\simeq\frac{-B\pm B\left(1-2mK/B^2\right)}{2m}=\frac{-B\pm(B-2mK/B)}{2m}=\begin{cases}-K/B\\ -\frac{B}{m}+\frac{K}{B}\approx-\frac{B}{m}\end{cases}$$

$\therefore$ Let s_1 = the smaller one $=-\frac{K}{B}$ & s_2 = the larger one $=-\frac{B}{m}$.

To derive the differential equations of the shown car bumper system:



$$\therefore \quad f - K(x_i - x_o) - B(\dot{x}_i - \dot{x}_o) = m\ddot{x}_i \quad (1)$$

$$K(x_i - x_o) + B(\dot{x}_i - \dot{x}_o) = M\ddot{x}_o . \quad (2)$$

$$(1) \Rightarrow f = (mD^2 + BD + K)x_i - (BD + K)x_o . \quad (3)$$

$$(2) \Rightarrow (MD^2 + BD + K)x_o = (BD + K)x_i . \quad (4)$$

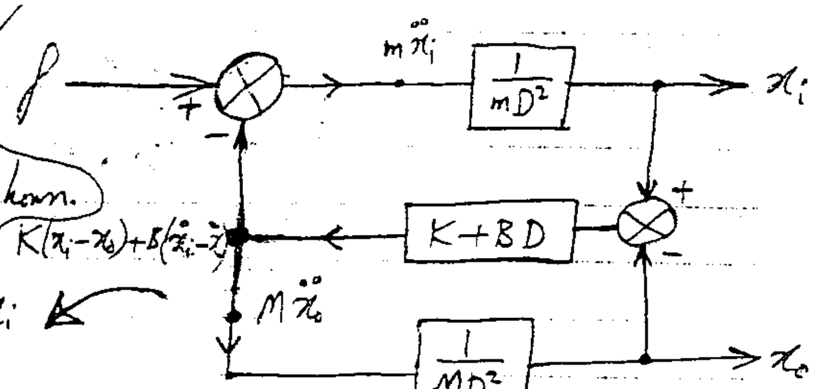
$$\therefore (4) \text{ into } (3) \Rightarrow f = x_i\left[mD^2 + BD + K - \frac{(BD+K)^2}{MD^2 + BD + K}\right]$$

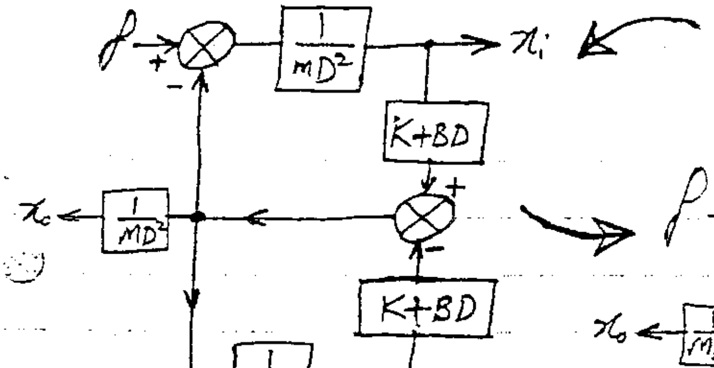
$$\therefore \quad x_i = \frac{MD^2 + BD + K}{mMD^4 + (BD+K)(m+M)D^2} \cdot f = \frac{MD^2 + BD + K}{D^2[mMD^2 + (m+M)(BD+K)]} f$$

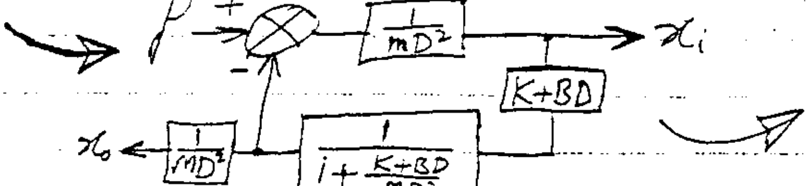
into (4) $\Rightarrow$

$$x_o = \frac{BD+K}{MD^2 + BD + K} \cdot \frac{MD^2 + BD + K}{D^2[mMD^2 + (m+M)(BD+K)]} \cdot f = \frac{BD+K}{D^2[mMD^2 + (m+M)(BD+K)]} f$$

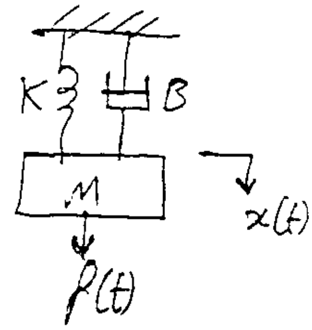
This is represented by the block Dm shown and can be reduced as is also shown.





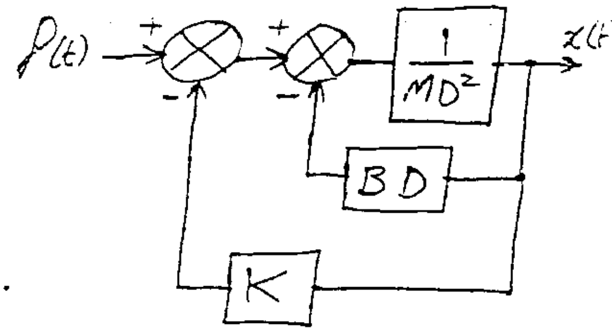


The mechanical system indicated has the following differential equation:



$$f(t) - B\dot{x}(t) - Kx(t) = M\ddot{x}(t)$$

This system can be represented by the shown block diagram.



It could be reduced to the following form:

$$(MD^2 + BD + K)\,x(t) = f(t).$$

It could be Laplace-transformed assuming 0 IC to

$$(Ms^2 + Bs + K)\,X(s) = F(s).$$

And: with $M = K = \frac{B}{2} = 1$, standard units
& $f(t) = 2 + .1\sin 3t$, ; then:

$$x(t) = x_c + \frac{1}{D^2+2D+1}\cdot f(t) = x_c + \frac{1}{(D+1)^2}(2 + .1\sin 3t) = (A+Bt)e^{-t} +$$

$$+ \frac{1}{(0+1)^2}\cdot(2) + .1 * \frac{(D-1)^2}{(D^2-1^2)^2}\cdot \sin 3t =$$

$$= (A+Bt)e^{-t} + 2 + .1 * \frac{-9-2D+1}{(-9-1)^2}\cdot \sin 3t =$$

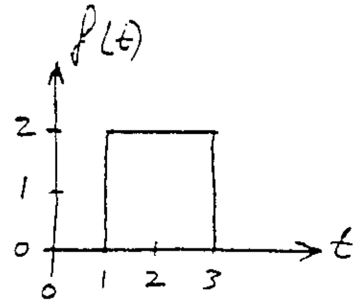
$$= (A+Bt)e^{-t} + 2 + .001 * (-8\sin 3t - 6\cos 3t)$$

$\because x_0 = 0 \Rightarrow 0 = A + 2 + .001 * (-6) \Rightarrow A = -1.994$

& $\dot{x}_0 = 0 \Rightarrow 0 = B - A + .001 * (-8 * 3) \Rightarrow B = A + .024 = -1.97$

$\therefore x(t) = 2 - .002(4\sin 3t + 3\cos 3t) - e^{-t}(1.994 + 1.97t).$

For the shown $f(t)$;

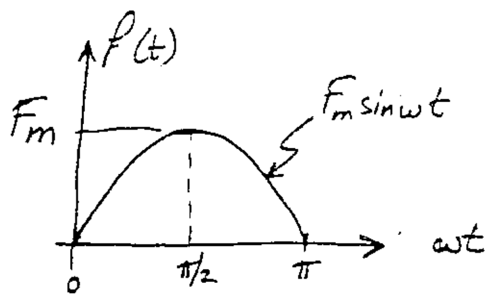


$$\therefore F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt = 2\int_1^3 e^{-st}\, dt$$

$$= 2 \cdot \frac{e^{-st}}{-s}\Big|_1^3 = \frac{2}{-s}\left[e^{-3s} - e^{-s}\right]$$

$$= \frac{2}{s}\left(e^{-s} - e^{-3s}\right).$$

For the shown $f(t)$:



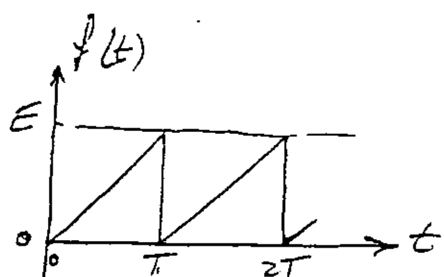
$$\therefore F(s) = F_m \int_{0/\omega}^{\pi/\omega} \sin\omega t\; e^{-st}\, dt =$$

$$= F_m \cdot e^{-st} \cdot \frac{1}{D+(-s)} \cdot \sin\omega t \Big|_0^{\pi/\omega} =$$

$$= F_m \cdot e^{-st} \cdot \frac{D+s}{-\omega^2 - s^2} \cdot \sin\omega t \Big|_0^{\pi/\omega} = -F_m \frac{e^{-st}}{\omega^2 + s^2} \cdot \left(\omega\cos\omega t + s\sin\omega t\right)\Big|_0^{\pi/\omega} =$$

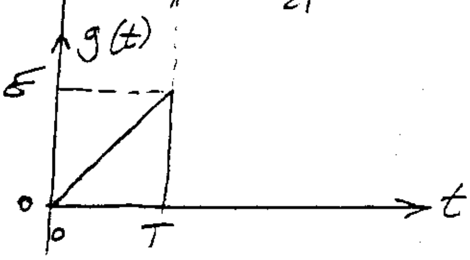
$$= -\frac{F_m}{\omega^2 + s^2} \cdot \left[e^{-s\pi/\omega} \cdot (-\omega + 0) - \omega\right] = \frac{\omega F_m}{\omega^2 + s^2}\left(1 + e^{-s\pi/\omega}\right).$$

For the shown $f(t)$:



$$\therefore f(t) = \sum_{n=0}^{\infty} g(t - nT),$$

where $g(t)$ is as shown.



$$\therefore F(s) = \sum_{n=0}^{\infty} G(s) \cdot e^{-nTs},$$

but: $G(s) = \int_0^T \frac{Et}{T} e^{-st}\, dt =$

$$= \frac{E}{T} \cdot e^{-st} \frac{1}{D-s} \cdot t \Big|_0^T = \frac{E}{T} \cdot e^{-st} \cdot \frac{D+s}{D^2 - s^2} \cdot t \Big|_0^T$$

$$= \frac{E}{T} \cdot e^{-st} \cdot \frac{1}{0 - s^2} \cdot (1 + st)\Big|_0^T = \frac{-E}{s^2 T}\left[e^{-sT} \cdot (1+sT) - 1\right] = \frac{E}{s^2 T} \cdot \left[1 - e^{-sT} \cdot (1+sT)\right]$$

Let $X = \sum_{n=0}^{\infty} e^{-nTs}$

$\therefore X = 1 + e^{-Ts} + e^{-2Ts} + e^{-3Ts} + \cdots\cdots + e^{-nTs} + \cdots\cdots$

$= \left(\text{by comparison to } \frac{1}{1-\alpha} = 1 + \alpha + \alpha^2 + \alpha^3 + \cdots + \alpha^n + \cdots \quad |\alpha|<1\right)$

$= \frac{1}{1-e^{-Ts}}$ provided $e^{-Ts} < 1$; or $Ts > 0$ i.e. $s > 0$

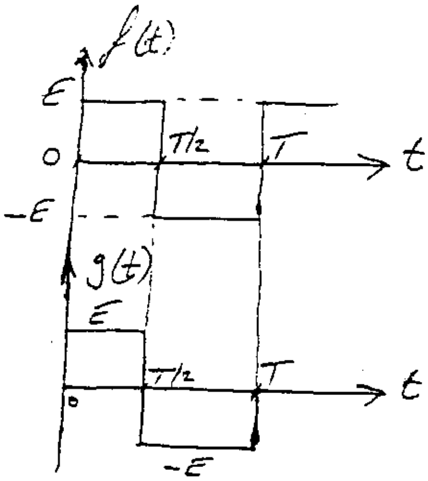
$$\therefore F(s) = \sum_{n=0}^{\infty} G(s)\cdot e^{-nTs} = G(s)\cdot X = \frac{G(s)}{1-e^{-Ts}} =$$

$$= \frac{E}{s^2 T}\cdot\frac{1-e^{-sT}-sTe^{-Ts}}{1-e^{-Ts}} = \frac{E}{s^2 T}\left[1 - \frac{sT\cdot e^{-sT}}{1-e^{-sT}}\right]$$

\# For the shown $f(t)$:

$\therefore f(t) = \sum_{n=0}^{\infty} g(t-nT)$, where:

$g(t)$ is a single period as shown.



$$\therefore F(s) = \sum_{n=0}^{\infty} G(s)\cdot e^{-nTs}.$$

But: $G(s) = \int_0^{T/2} E e^{-st} dt + \int_{T/2}^{T} (-E) e^{-st} dt =$

$$= E\left[\frac{e^{-st}}{-s}\Big|_0^{T/2} - \frac{e^{-st}}{-s}\Big|_{T/2}^{T}\right] =$$

$$= \frac{E}{s}\left[1 - e^{-sT/2} + e^{-sT} - e^{-sT/2}\right] = \frac{E}{s}\cdot\left(1-e^{-sT/2}\right)^2$$

$$\therefore F(s) = \sum_{n=0}^{\infty} G(s)\cdot e^{-nTs} = G(s)\cdot\sum_{n=0}^{\infty} e^{-nTs} = \frac{E}{s}\cdot\frac{\left(1-e^{-Ts/2}\right)^2}{1-e^{-Ts}}$$

$$= \frac{E}{s}\left[1 + 2\cdot\frac{e^{-Ts}-e^{-Ts/2}}{1-e^{-Ts}}\right] = \frac{E}{s}\cdot\left[1 - 2\cdot\frac{e^{-Ts/2}\cdot\left(1-e^{-Ts/2}\right)}{\left(1+e^{-Ts/2}\right)\left(1-e^{-Ts/2}\right)}\right]$$

$$\therefore F(s) = \frac{E}{s}\left[1 - \frac{2e^{-Ts/2}}{1+e^{-Ts/2}}\right]$$

$(D+7)x=0$ & $x_0=-1$ What is $X(s)$?

$\therefore \quad \dot{x}+7x=0 \Rightarrow sX(s)-x_0+7X(s)=0$

$\therefore \quad X(s)=\dfrac{x_0}{s+7}=\dfrac{-1}{s+7}$

$(D^2+2D+5)x=10$ & $x_0=2$ & $\dot{x}_0=0$ What is $X(s)$

$\therefore \quad \ddot{x}+2\dot{x}+5x=10 \Rightarrow s^2X(s)-sx_0-\dot{x}_0+2(sX(s)-x_0)+5X(s)=\dfrac{10}{s}$

$\therefore \quad X(s)=\dfrac{(s+2)x_0+\dot{x}_0+\frac{10}{s}}{s^2+2s+5}=\dfrac{2s^2+4s+10}{s(s^2+2s+5)}$

$(D^2+3D+1)x=t$ & $x_0=0$ & $\dot{x}_0=-2$ What is $X(s)$

$\therefore \quad s^2X(s)-sx_0-\dot{x}_0+3(sX(s)-x_0)+X(s)=\dfrac{1}{s^2} \Rightarrow$

$X(s)=\dfrac{(s+3)x_0+\dot{x}_0+\frac{1}{s^2}}{s^2+3s+1}=\dfrac{1-2s^2}{s^2(s^2+3s+1)}$

$(D^3+4D^2+8D+4)x=\sin 5t$ & $x_0=-4$ & $\dot{x}_0=1$ & $\ddot{x}_0=0$

$\therefore \quad s^3X(s)-s^2x_0-s\dot{x}_0-\ddot{x}_0+4(s^2X(s)-sx_0-\dot{x}_0)+8(sX(s)-x_0)+4X(s)=\dfrac{5}{s^2+5^2}$

$\therefore \quad X(s)=\dfrac{s^2\overset{-4}{x_0}+s\overset{1}{\dot{x}_0}+\overset{0}{\ddot{x}_0}+4s\overset{-4}{x_0}+4\overset{1}{\dot{x}_0}+8\overset{-4}{x_0}+\frac{5}{s^2+25}}{s^3+4s^2+8s+4}=$

$=\dfrac{(-4s^2-15s-28)(s^2+25)+5}{(s^3+4s^2+8s+4)(s^2+25)}=$

$=\dfrac{-4s^4-15s^3-128s^2-375s-695}{(s^2+25)(s^3+4s^2+8s+4)}$

a) $F(s) = \dfrac{10}{(s+1)^2(s^2+2s+2)}$, $f(t) = ?!$

$\therefore$ $F(s) = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{C+Ds}{s^2+2s+2}$, where:

$B = \lim\limits_{s\to -1} (s+1)^2 F(s) = \dfrac{10}{-1^2+2(-1)+2} = \dfrac{10}{1} = 10$

& $A = \lim\limits_{s\to -1} \dfrac{d}{ds}(s+1)^2 F(s) = \lim\limits_{s\to -1} \dfrac{-10\,(2s+2)}{(s^2+2s+2)^2} = 0$

& $\lim\limits_{s^2+2s+2\to 0} (s^2+2s+2)\,F(s) = \lim\limits_{s^2+2s+2\to 0} \dfrac{10}{(s+1)^2} = \lim\limits_{s^2+2s+2\to 0} C+Ds$

$\therefore$ $\lim\limits_{s^2+2s+2\to 0} \dfrac{10}{s^2+2s+1} = \lim\limits_{s^2+2s+2\to 0} C+Ds$ $\Rightarrow -10 = C+Ds$

$\Rightarrow C = -10$ & $D = 0$

$\therefore$ $F(s) = \dfrac{10}{(s+1)^2} + \dfrac{-10}{(s+1)^2+1}$ $\Rightarrow f(t) = 10te^{-t} - 10e^{-t}\sin t$

b). $F(s) = \dfrac{13}{s(s^2+4s+13)} = \dfrac{A}{s} + \dfrac{B+Cs}{s^2+4s+13}$ where

$A = \lim\limits_{s\to 0} sF(s) = 1$ &

$B + Cs = \lim\limits_{s^2+4s+13\to 0} (s^2+4s+13)\,F(s) = \lim\limits_{s\to \frac{-13}{s+4}} 13/s = -(s+4)$

$\therefore$ $B = -4$ & $C = -1$

$\therefore$ $F(s) = \dfrac{1}{s} - \dfrac{4+s}{(s+2)^2+3^2} = \dfrac{1}{s} - \dfrac{2+(s+2)}{(s+2)^2+3^2}$

$\therefore$ $f(t) = 1 - \frac{2}{3}e^{-2t}\sin 3t - e^{-2t}\cos 3t$

$\boxed{\frac{19}{105}}$ $(s+2)(s^2+6s+18)(s^2+4)^2=0$

$\therefore (s+2)((s+3)^2+3^2)(s^2+4)^2=0$

$\therefore \quad s=-2\ ,\ -3\pm j3\ ,\pm j2,\pm j2$

$\therefore$ The system is unstable due to the repetition of $\pm j2$ poles.

The response is $c_{tr}(t) = A_1 e^{-2t} + e^{-3t}(A_2 \sin 3t + A_3 \cos 3t) +$
$+ (A_4 \sin 2t + A_5 \cos 2t)(A_6 + A_7 t)$

$\boxed{\frac{2E}{105}}$

s^6	1	3	2	5	
s^5	2	1	6	0	
s^4	5	-2	10	0	$(*2)$
s^3	9	10	0		$(*5)$
s^2	-68	90	0		$(*9)$
s	1490	0			$(*68)$
1	90	0			$(*1490)$

$\therefore$ Two sign changes $\therefore$ Unstable with two poles in RHP.

$s^6+2s^5+3s^4+s^3+2s^2+6s+5 = f(s)=0$. Deflating :

$\therefore s_{1,2} = 0.814315 \pm j.913475$

& $s_{3,4} = -0.946463 \pm j.457357$

& $s_{5,6} = -0.867851 \pm j1.506139$

$\therefore$ OK

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$\boxed{\frac{25}{105}}$ $s^6 + 14 s^4 + 49 s^2 + 36 = 0$

I) The Routhian is:

s^6: 1 14 49 36
s^5: 0 0 0 0

The row of zeros is in the basic rows, so it can't be removed by auxilary equation of previous row.
However, if we did that, it would become:

s^6	1	14	49	36	
s^5	6	56	98	0	
s^4	7	49	54	0	(after multiplication by $\frac{6}{4}$)
s^3	49	181	0		(" " " $\frac{7}{2}$)
s^2	162	378	0		(" " " 7)
s	1	0			(" " " $\frac{3}{200}$)
1	1	0			(" " " $\frac{1}{378}$)

and this shows stable system since no sign change.

II) Assuming $s^2 = s'$ ∴ The equation becomes: $s'^3 + 14 s'^2 + 49 s' + 36$

s'^3	1	49	
s'^2	14	36	
s'^1	1	0	(after multiplication by 14/650)
1	1	0	(" " " 1/36)

∴ s' is stable.
∴ $s = \pm\sqrt{s'}$ is marginally stable or unstable and we can be sure it is not stable (due to ±)

SL

III) Deflating the equation: $s'^3 + 14s'^2 + 49s' + 36 = 0$

$\therefore$ $s' = -1$ & the equation becomes $(s'+1)(s'^2 + 13s' + 36) = 0$
$\therefore$ other roots are:

$$s' = \frac{-13 \pm \sqrt{169 - 144}}{2} = \frac{-13 \pm \sqrt{25}}{2} = \frac{-13 \pm 5}{2} = -9 \ \& \ -4$$

$\therefore$ The three roots are $-9, -4$ & -1 for s' (stable as in II)

$\therefore$ Roots for s are $\pm j3$, $\pm j2$ & $\pm j1$

The system is oscilatory

(Note: the first method is misleading and this expected because it is wrongly applied).

IV) Suppose we put $s' = -s''$
$\therefore$ The equation becomes. $-s''^3 + 14s''^2 - 49s'' + 36 = 0$

s''^3	:	-1	-49
s''^2	:	14	36
s''	:	$-652/14$	0
1	:	36	0

$\therefore$ Three sign changes $\therefore$ three positive-half s'' roots

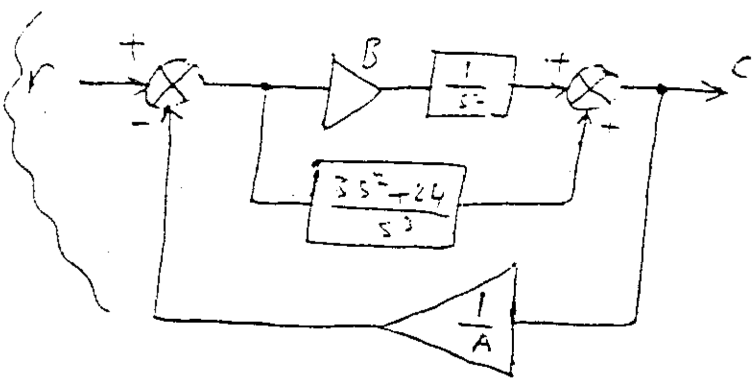
$\because$ $s' = -s''$ $\therefore$ three negative-half s' roots

$\therefore$ Stable for s', as before.

$\therefore$ OK

—

$\boxed{\frac{3b}{105}}$ ∴ $$\frac{c}{r}(s) = \frac{\frac{B}{s^2}+\frac{3s^2+24}{s^3}}{1+\frac{1}{A}\left[\frac{B}{s^2}+\frac{3s^2+24}{s^3}\right]} =$$



$$= \frac{3s^2+Bs+24}{3s^2+Bs+24+As^3}\cdot A$$

Stability region:

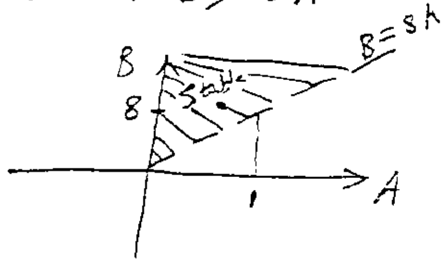
s^3	A	B
s^2	3	24
s	$3B-24A$	0
1	1	0

$A>0$

$3B-24A>0 \quad \therefore B>8A$

∴ B must be $>8A$ & $A>0$

∴ The region is given aside



dc gain $= \frac{c}{r}(0) = \frac{24A}{24} = A$

To see whether the system $W(s) = \dfrac{s^3+s^2+s+1}{s^6+s^5+s^4+s^3+s^2+s+1}$ is stable, then Routh is applied to its characteristic eqn.: $s^6+s^5+s^4+s^3+s^2+s+1=0$

∴

s^6	1	1	1	1
s^5	1	1	1	0
s^4	5	3	1	0
$s^3 (\times 5)$	2	4	0	0
s^2	−7	1	0	0
$s (\times 7)$	30	0	0	0
1	1			

(originally was zero and auxiliary eq. of previous row was differentiated)

∴ There are two changes in sign ⟹ System has 2 unstable poles.

[Solving ch. eq. gives the following poles: $-.222521 \pm j.974928$ (stable), $+.623490 \pm j.781831$ (those the unstable ones) & $-.900969 \pm j.433884$ (stable)]

$\frac{3e}{105}$

s^4	1	$K+95$	$100-K$
s^3	18	150	0
s^2	$18K+1560$	$18(100-K)$	0
s	$3024K+201600$	0	
1	$18(100-K)$	0	

$(\times 18)$

$(\times 18K+1560)$

∴ Conditions for stability are:

- s^2 leading coeff. $\quad 18K+1560>0 \quad (1)$
- s " " $\quad 3024K+201600>0 \quad (2)$
- 1 " " $\quad 100-K>0 \quad (3)$

From (1) ∴ $18K>-1560$ ∴ $K>\frac{-1560}{18}=-86.7$ ∴ $K\in(-86.7,$

From (2) ∴ $3024K>-201600$ ∴ $K>-66.7$ ∴ $K\in(-66.7,\infty)$

From (3) ∴ $K<100$ ∴ $K\in(-\infty,100)$

To satisfy all requirements ∴ $K\in(-86,\infty)\cap(-66.7,\infty)\cap(-\infty,$

∴ $K\in(-66.7,100)$ ∴ Stable if: $-66.7<K<100$

$\frac{4}{105}$ a) $\frac{c}{r}=\frac{N(s)}{s+4}$ ∴ stable with pole -4, $\tau=\frac{1}{4}$ sec

transient mode is e^{-4t} shape, response time $(=t_s)=\frac{3}{4}$ sec

system is stable (The above information applies for $N(-4)\neq 0$)

b) $\frac{c}{r}=\frac{N(s)}{s+4}$ ∴ $C=\frac{N\cdot r}{s+4}=\frac{4\cdot(\frac{1}{s})}{s+4}=\frac{4}{s(s+4)}=\frac{1}{s}-\frac{1}{s+4}$

∴ $c(t)=1-e^{-4t}$ & $\dot{c}(t)=4e^{-4t}$

$c(0)=0$, $c(\infty)=1$ & $\dot{c}(0)=4$ & $\dot{c}(\infty)=0$

c) Same as in (b) above but values are halfed.

∴ $c(t)=\frac{1}{2}(1-e^{-4t})$ & $\dot{c}(t)=2e^{-4t}$

$c(0)=0$, $c(\infty)=\frac{1}{2}$, $\dot{c}(0)=2$ & $\dot{c}(\infty)=0$

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d) $\frac{C}{R} = \frac{\cdot}{s+4}$ $\therefore C = \frac{Nr}{s+4} = \frac{.5(s+8)\left(\frac{1}{s}\right)}{s+4} = \frac{.5(s+8)}{s(s+4)} =$

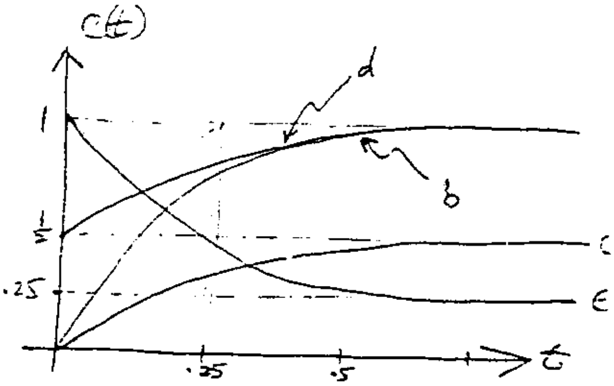
$= \frac{1}{s} - \frac{.5}{s+4}$ $\therefore c(t) = 1 - .5e^{-4t}$ & $\dot{c}(t) = 2e^{-4t}$

$\therefore c(0) = 0.5$, $c(\infty) = 1$, $\dot{c}(0) = 2$ & $\dot{c}(\infty) = 0$

e) $C = \frac{Nr}{s+4} = \frac{(s+1)}{(s+4)s} = \frac{.25}{s} + \frac{.75}{s+4}$

$\therefore c(t) = .25 + .75e^{-4t}$ & $\dot{c}(t) = -3e^{-4t}$
$c(0) = 1$, $c(\infty) = .25$ & $\dot{c}(0) = -3$ & $\dot{c}(\infty) = 0$

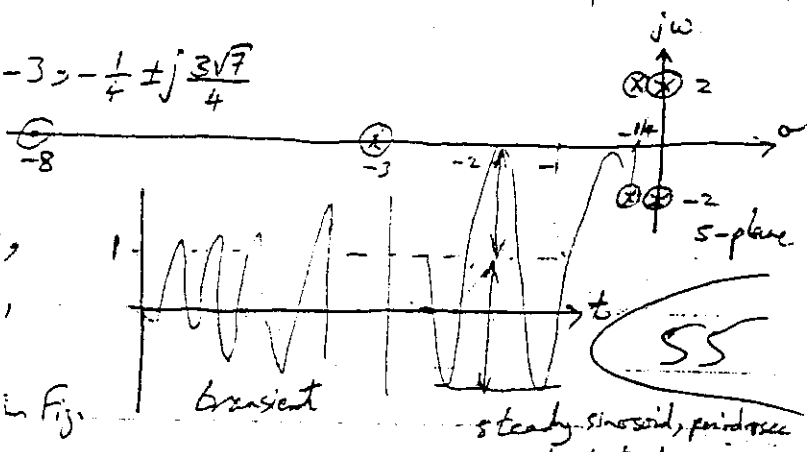
f) The responses are sketched aside. All of them have zero slope at ∞, time constant of 0.25 sec, stable as predicted in (a). The only difference between them is the initial & final values & initial slopes.



$\therefore$ $N(s)$ (when $N(-4) \neq 0$) can only influence the magnitudes of $c(0)$, $c(\infty)$ & $\dot{c}(0)$ but not the settle times (t_s) because, that is decided by the denominator which is not change

$\boxed{\frac{5.8}{107}}$ $r(t) = u(t)$ & $\frac{C}{R}\Big|_{s=0} = \frac{6*8}{4*3*4} = 1$ (dc gain of 1)

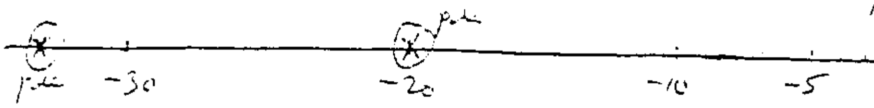
$\therefore$ $c(t) = u(t) + A_1 \sin 2t + A_2 \cos 2t + A_3 e^{-3t} + e^{-.25t}\left(A_4 \sin\frac{3\sqrt{7}}{4}t + A_5 \cos\frac{3\sqrt{7}}{4}t\right)$
$c(0) = \lim_{s\to\infty}\left(\frac{C}{R}\right) = 0$
roots are at $s = \pm j2, -3, -\frac{1}{4} \pm j\frac{3\sqrt{7}}{4}$
zeros at $s = -8$
Dominant poles are the nearest 4 conjugates, two of which are oscilatory, others are just stable.
Sketch is shown here in Fig.

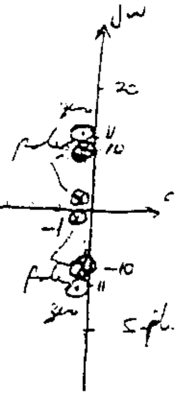


$\boxed{\dfrac{9e}{107}}$ zeros at $s = \dfrac{-.45 \pm \sqrt{.45^2 - 4(121)}}{2} = -.225 \pm j\ 11.0$

poles at $s = -33.3,\ -20,\ -.2 \pm j9.995,\ -.3 \pm j.954$

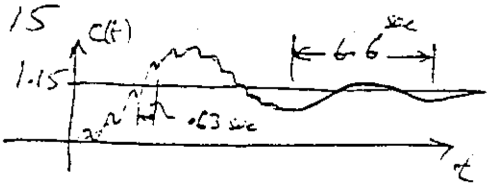
Located as shown:





The dominant poles are the last four, two of which are **diluted** by the nearby zeros leaving $-.3 \pm j.954$ as most dominant poles.

$\frac{c}{r}(0) = \text{dc gain} = \dfrac{.45 * 121}{100} = 1.15$



$\frac{c}{r}(\infty) = c(\text{at } t=0) = 0$

$\therefore\ c(t) = 1.15\, u(t) + e^{-.3t}(A_1 \sin .954t + A_2 \cos .954t) + e^{-.2t}(A_3 \sin 10t + A_4 \cos 10t)$

plotted as shown in the above Fig.

$\boxed{\dfrac{10}{107}}$ a) $\dfrac{c}{u} = G_p = \dfrac{2}{s(s-1)(s+3)}$, $\therefore$ plant is unstable due pole at 1

b) $G_c = A \quad \therefore \dfrac{c}{r} = \dfrac{2A}{2A + s(s-1)(s+3)} = \dfrac{2A}{s^3 + 2s^2 - 3s + 2A}$

$\therefore$ coeff. of s is -3 $\therefore$ It can't be made stable for any A.

c) $G_c = A(2s+1) \quad \therefore \dfrac{c}{r} = \dfrac{2A(2s+1)}{2A(2s+1) + s(s-1)(s+3)} = \dfrac{2A(2s+1)}{s^3 + 2s^2 + (4A-3)s + 2A}$

$\therefore$ The system can now be made stable. Range is obtained by

s^3	1	$4A-3$	
s^2	2	$2A$	
s	$6A-6$	0	$\therefore A-1>0 \quad \therefore A>1 \quad \therefore A \in (1,\infty)$
1	$2A$	0	$\therefore A>0 \quad \therefore A \in (0,\infty)$

$\therefore\ A \in (0,\infty) \cap (1,\infty) = (1,\infty)$

$\therefore$ Choosing A within $(1,\infty)$ ensures stability.

$\therefore$ $A>1$ gives stable systems

$\boxed{\frac{12}{108}}$ a) $\frac{C}{R} = \frac{10A}{s^2+3s+10A}$

$s^2 : 1 \quad 10A$

$s : 3 \quad 0$

$1 : 10A$

$\therefore$ Stable for $A>0$, Unstable if $A<$

b) $\frac{C}{R} = \frac{10A}{s^2+3s-10A}$

$s^2 : 1 \quad -10A$

$s : 3 \quad 0$

$1 : -10A$

$\therefore$ stable for $A<0$, Unstable if $A>0$

c) If $|A| = 1.825$, then the unit step response is

stable when $A = 1.825$ $\underline{-ve}$ FB OR $A = -1.825$ $\underline{+ve}$ FB

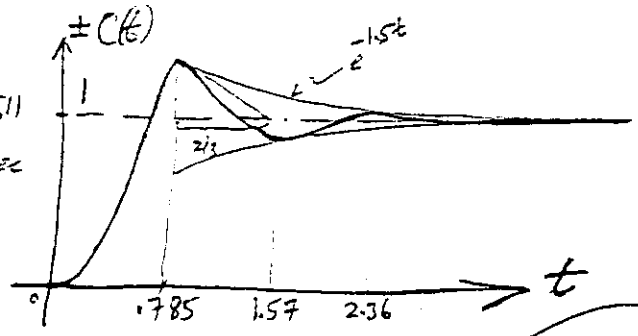
In either cases $\therefore C(s) = \frac{\pm 18.25}{s^2+3s+18.25} \cdot \frac{1}{s} = \pm\left[\frac{1}{s} - \frac{s+1.5+1.5}{(s+1.5)^2+4^2}\right]$

$$\therefore C(t) = \pm\left[1 - e^{-1.5t}\left(\cos 4t + \frac{1.5}{4}\sin 4t\right)\right] =$$

$$= \pm\left[1 - e^{-1.5t}\left(\cos 4t + .375 \sin 4t\right)\right] =$$

$$= \pm\left[1 - 1.068\, e^{-1.5t} \cdot \sin\left(4t + \underset{(=69.4^\circ)}{1.212}\right)\right]$$

Sketched as shown:
damped sinusoid $\eta = .3511$
$\omega = 4$ rad/sec, period $= 1.57$ sec
decaying at rate of $e^{-1.5t}$
to the steady state value
of unity, $r = 31\%$.



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$s^3+13s^2+33s+30=0$

$$\therefore \begin{array}{lll} s^3: & 1 & 33 \\ s^2: & 13 & 30 \\ s: & \frac{399}{13} & 0 \\ 1: & 30 & 0 \end{array}$$

$\therefore$ No RHP roots (No change of sign)

$s^3+4s^2+6s+4=0$

$$\therefore \begin{array}{lll} s^3: & 1 & 6 \\ s^2: & 4 & 4 \\ s: & 5 & 0 \\ 1: & 4 & 0 \end{array}$$

$\therefore$ No change of sign $\therefore$ Zero RHP roots

$s^4+2s^3+2s^2+3s+6=0$

$$\therefore \begin{array}{llll} s^4: & 1 & 2 & 6 \\ s^3: & 2 & 3 & 0 \\ s^2: & \frac{1}{2} & 6 & 0 \\ s: & -21 & 0 & 0 \\ 1: & 6 & 0 & 0 \end{array}$$

$\therefore$ 2 sign Changes $\Rightarrow$ 2 RHP zeros.

$s^4+s^3+2s^2+9s+5=0$

$$\therefore \begin{array}{llll} s^4: & 1 & 2 & 5 \\ s^3: & 1 & 9 & 0 \\ s^2: & -7 & 5 & 0 \\ s: & \frac{68}{7} & 0 & 0 \\ 1: & 5 & 0 & 0 \end{array}$$

$\therefore$ 2 Sign Changes $\Rightarrow$ 2 RHP roots.

\# $s^3+5s^2+8s+6=0$

$$\therefore \begin{array}{l|cc} s^3: & 1 & 8 \\ s^2: & 5 & 6 \\ s: & \frac{34}{5} & 0 \\ 1: & 6 & 0 \end{array}$$

$\therefore$ NO RHP Roots.

\# $s^3+7s^2+16s+10=0$

$$\therefore \begin{array}{l|cc} s^3: & 1 & 16 \\ s^2: & 7 & 10 \\ s: & \frac{102}{7} & 0 \\ 1: & 10 & 0 \end{array}$$

$\therefore$ No RHP roots.

\# $s^4+12s^3+54s^2+108s+80=0$

$$\therefore \begin{array}{l|ccc} s^4: & 1 & 54 & 80 \\ s^3: & 12 & 108 & 0 \\ s^2: & 45 & 80 & 0 \\ s: & \frac{260}{3} & 0 & 0 \\ 1: & 80 & 0 & 0 \end{array}$$

$\therefore$ No RHP roots

\# $s^4+7s^3+12.2s^2+11.05s=0$

$$\therefore \begin{array}{l|ccc} s^4: & 1 & 12.2 & 0 \\ s^3: & 7 & 11.05 & 0 \\ s^2: & \frac{74.35}{7} & 0 & 0 \\ s: & 11.05 & 0 & 0 \\ 1: & 0 & 0 & 0 \end{array}$$

$\therefore$ No RHP roots

(Note: The zero in the last element of the first column indicates marginal stability.)

\# $s^5 + 3s^4 + 28s^3 + 226s^2 + 600s + 400 = 0$

$$
\therefore
\begin{array}{lccccccc}
s^5: & 1 & 28 & 600 & & & & \\
s^4: & 3 & 226 & 400 & & & & \\
s^3: & -\frac{142}{3} & \frac{1400}{3} & 0 & \Rightarrow & -142 & 1400 & 0 \\
s^2: & \frac{36292}{142} & 400 & 0 & \Rightarrow & 36292 & 56800 & 0 \\
s: & \frac{5887\,4400}{36292} & 0 & 0 & \Rightarrow & 588744 & 0 & 0 \\
1: & 56800 & 0 & 0 & & & &
\end{array}
$$

$\therefore$ 2 sign changes $\Rightarrow$ 2 RHP roots.

\# Characteristic Eq: $s^4 + 2s^3 + 2s^2 + (K+3)s + K = 0$

$K \in$?

$$
\therefore
\begin{array}{lccccl}
s^4: & 1 & 2 & K & & \\
s^3: & 2 & K+3 & 0 & & \\
s^2: & \frac{1-K}{2} & K & 0 & \Rightarrow & \therefore\ 1-K \text{ must be} > 0 \Rightarrow K < 1 \quad (1) \\
s: & \frac{(1-K)(K+3)-4K}{1-K} & 0 & 0 & \Rightarrow & \therefore\ -K^2 - 6K + 3 \ \text{''} > 0 \quad (2) \\
1: & K & 0 & 0 & \Rightarrow & \therefore\ K > 0 \quad (3)
\end{array}
$$

a) $\therefore$ When $K = 6$ condition (1) is violated and system is unstable

b) $\therefore$ Conditions (1) & (3) $\Rightarrow$ $K \in (0, 1)$;

whereas condition (2) implies $K^2 + 6K - 3 < 0$

$(K + 3 + 2\sqrt{3})(K + 3 - 2\sqrt{3}) < 0$ $\Rightarrow$ by studying the sign

	$K < -3-2\sqrt{3}$	$-3-2\sqrt{3}$	$-3-2\sqrt{3} < K < -3+2\sqrt{3}$	$-3+2\sqrt{3}$	$K > -3+2\sqrt{3}$
Factor $K+3+2\sqrt{3}$	$- - - -$	0	$+ + + + +$		$+ + +$
Facto $K+3-2\sqrt{3}$	$- -$		$- - -$	0	$+ + + +$
Product	$+ + +$		$- - - -$		$+ + + +$

$\therefore$ Condition (2) implies that K must be $\in (-3-2\sqrt{3}, -3+2\sqrt{3})$

$\therefore$ To satisfy all conditions $\therefore K \in (0,1) \cap (-3-2\sqrt{3}, \underbrace{-3+2\sqrt{3}}_{.464})$

$\therefore K \in (0, 2\sqrt{3} - 3) \simeq (0, 0.464)$

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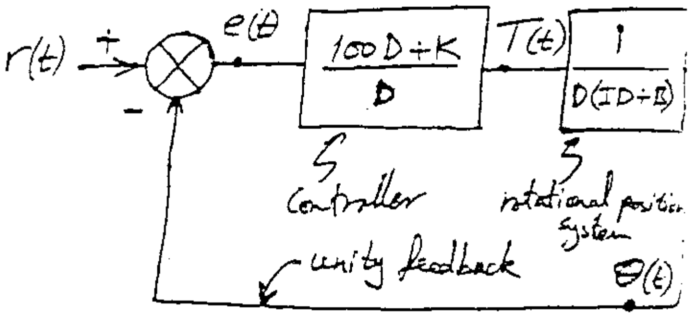
$I = 4$ Slug ft^2
$B = 2$ lb-ft/(rad/sec)



$$T(t) = \left(100 + \frac{K}{D}\right) e(t)$$

$$\therefore\ T(t) = I\ddot{\theta} + B\dot{\theta}$$

$$\therefore\ T(t) = (ID^2 + BD)\,\theta(t)$$

$$\therefore\ \theta(t) = \frac{1}{D(ID+B)} \cdot T(t)$$

$\therefore$ For stability study $1+GH=0 \Rightarrow 1 + \left(\frac{100D+K}{D}\right)\left(\frac{1}{D(ID+B)}\right) = 0$

Assuming OIC $\Rightarrow$ ch. eq. is $1 + \frac{100s+K}{s} \cdot \frac{1}{s(Is+B)} = 0$

OR: $s^2(Is+B) + 100s + K = 0$ with values:

$$\therefore\ 4s^3 + 2s^2 + 100s + K = 0$$

$\therefore$

s^3 :	4	100
s^2 :	2	K
s :	$100-2K$	0
1 :	K	0

$\therefore$ For stability $100-2K$ must be >0 & $K>0$

OR $K < 50$ & $K > 0$

$\therefore$ $K \in (0, 50)$ gives <u>stable</u> system

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1. Ch. eq. is:

$$s^5 + s^4 + 2s^3 + s^2 + (K+1)s + K + 2 = 0$$

2. For stability, the Routhian matrix is:

$$\begin{array}{llll}
s^5: & 1 & 2 & K+1 \\
s^4: & 1 & 1 & K+2 \\
s^3: & 1 & -1 & 0 \\
s^2: & 2 & K+2 & 0 \\
s: & \frac{-(4+K)}{2} & 0 & 0 \\
1: & K+2 & 0 & 0
\end{array}$$

This gives the following conditions:

$$4 + K < 0 \quad \& \quad K + 2 > 0$$

3. Solving the inequalities:

$\therefore$ $K < -4$ & $K > -2$

$\therefore$ $K \in (-\infty, -4) \cap (-2, \infty) = \{\phi\}$

$\therefore$ The system is unstable and no matter of K value it can't be brought to stability.

$G = \dfrac{K(s^2+2s+10)}{s^2(s-4)}$ & $H=1$

$r(t) = 3 + t - t^2$

a. The system is type 2.

b. $K_p = K_v = \infty$ & $K_a = \lim_{s\to 0} s^2 G = \dfrac{-10K}{4} = -2.5K$

d. $e_{ss} = (r-c)_{ss} = \dfrac{3}{1+K_p} + \dfrac{1}{K_v} - \dfrac{2}{K_a} = \dfrac{-2}{-10K/4} = 0.8/K$

c. Limits of K for stability by Routh:

Ch. eq. is: $s^2(s-4) + K(s^2+2s+10) = 0$

or: $s^3 + (K-4)s^2 + (2K)s + 10K = 0$

s^3:	1	$2K$		
s^2:	$K-4$	$10K$	$K-4>0$	(1)
s:	$2K(K-4)-10K$	0	$2K(K-9)>0$	(2)
1:	$10K$	0	$10K>0$	(3)

(1) $\Rightarrow K \in (4,\infty)$

(2) $\Rightarrow K \in (-\infty,0) \cup (9,\infty)$

(3) $\Rightarrow K \in (0,\infty)$

$\therefore$ For stability $K \in (4,\infty) \cap \{(-\infty,0)\cup(9,\infty)\} \cap (0,\infty)$

$\therefore K \in (9,\infty)$

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e. dc gain of the system $= \dfrac{G(0)}{1+G(0)H(0)} = \dfrac{1}{\frac{1}{G(0)}+1} = 1$, unity.

\# $\frac{c}{r}(s) = \frac{1}{(s+1)(s+5)}$, $r(t) = u(t)$ & OIC

$\therefore$ 1. $c(t) = 0.2 + Ae^{-t} + Be^{-5t}$ where: $A = \left.\frac{1}{s(s+5)}\right|_{-1} = -0.25$

& $B = \left.\frac{1}{s(s+1)}\right|_{s=-5} = 0.05$

$\therefore$ $c(t) = 0.2 - .25e^{-t} + .05e^{-5t}$

C_{max} at t_p: $.25e^{-t_p} - .25e^{-5t_p} = 0 \Rightarrow e^{t_p} = e^{5t_p} \Rightarrow t_p = 0$ i.e no peak

$\therefore$ 2. t_s: $c(t_s) \leq C_{ss} * .95 = .19$

$\therefore$ $.19 = .2 - .25e^{-t_s} + .05e^{-5t_s} \Rightarrow t_s = 3.2189$

3. Approximating $\frac{c}{r}(s) \Rightarrow \frac{c}{r}(s) \approx \frac{1}{s+1}\left[\left.\frac{1}{s+5}\right|_{s=0}\right] = \frac{0.2}{s+1}$

4. $\therefore$ Approximate response is: $c(t) \approx 0.2 + Ae^{-t}$ $c(0) = 0 \Rightarrow A = -.2$

$\therefore$ $c(t) \approx 0.2\,(1 - e^{-t})$

5. Hence, approximate settling time is t^* given from:

$c(t^*) \lessapprox 0.95 * C_{ss} = .19$

$\therefore$ $.19 \simeq .2\,(1 - e^{-t^*}) \Rightarrow t^* = \ln 20 = 2.9957$

6. Hence, error due to approximation by dominant poles is, e:

$e = \frac{3.2189 - 2.9957}{3.2189} * 100 = 6.93\ \%$

$\therefore$ Accepting 10% error validates the approximation.

\# $c(t) = .2 - .25e^{-t} + .05e^{-5t}$, $C_{ss} = 0.2$ & $C_o = 0$

∴ $c(t_d) = .1C_{ss} \Rightarrow .02 = .2 - .25e^{-t_d} + .05e^{-5t_d} \Rightarrow t_d = .2530$

& $c(t_d + t_r) = .9C_{ss} \Rightarrow .18 = .2 - .25e^{-t_d - t_r} + .05e^{-5t_d - 5t_r} \Rightarrow t_r = 2.2727$

∴ $c(t) \approx .2(1 - e^{-t})$, $C_{ss} = .2$ & $C_o = 0$

∴ $c(t_d) \approx .1C_{ss} \Rightarrow .02 \simeq .2(1 - e^{-t_d}) \Rightarrow t_d \simeq .1054$

& $c(t_d + t_r) \approx .9C_{ss} \Rightarrow .18 \simeq .2(1 - e^{-t_d - t_r}) \Rightarrow t_r = 2.197$

∴ Error in approximating t_d is: 58 %

& " " " t_r is: 3.32 %

∴ App. is unvalid for t_d.

To find order, stability & dominant roots of

$$f(s) = (s+1)(s+2)(s+3)(s+4)(s^2+2s+10)(s^2+s-12) = 0$$

$\therefore$ order of system is 8,

roots are : $-1, -2, -3, -4, -4, 3, -1 \pm j3$

The system is unstable due to root at 3.

This is the only dominant root

$-K \pm j$

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$$T - D - W\sin\gamma = m\dot{V} \qquad (1)$$
$$L - W\cos\gamma = mV\dot{\gamma} \qquad (2)$$

a) The initial steady-state equations occur when the plane has just taken off the ground at a constant shoot-off angle.

$\therefore\ \gamma = \text{constant} \quad \therefore\ \dot{\gamma} = 0$

$\therefore$ From (2) $\ \therefore\ L - W\cos\gamma = 0 \quad \therefore\ L = W\cos\gamma \qquad (3)$

into (1)

$\therefore\ T - D - W \cdot \frac{\sin\gamma}{\cos\gamma} \cdot \cos\gamma = m\dot{V}$

$\therefore\ T - D - L\tan\gamma = m\dot{V} \qquad (4)$

Equations (3) & (4) are linear in V and represent the plane at steady shoot-off.

b) $W = 200000$ lb & $L = 15D$, $\gamma = 20°$

minimum thrust occurs when it is climbing at constant speed

$\therefore\ \dot{V} = 0$ & $T = D + L\tan\gamma = \frac{L}{15} + L\tan\gamma = \frac{L}{15}(1 + 15\tan\gamma)$

$= \frac{W\cos\gamma}{15}(1 + \tan\gamma \cdot 15) = \frac{\cos 20 + 15\sin 20}{15} * 200{,}000 = 80{,}933$ lb

$\therefore$ The minimum thrust to enable climbing is 80,933 lb

when $\gamma = 0$, the minimum thrust to keep the plane is also at $\dot{V} = 0$

$\therefore\ T = D = \frac{L}{15} = \frac{W}{15} = \frac{200{,}000}{15} = 13{,}333$ lb

$\therefore$ min. thrust to enable flying at constant altitude is 13,300 lb

c) $T\ (\text{at } \underset{V=\text{const}}{\gamma = 0}) = D$ & $L = W \quad \therefore\ T/W = D/L$

$\therefore\ \frac{T}{W} = 1/(L/D)$

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$\therefore$ system type is '3'. (i.e. type 3 system)

$K_c = \frac{40}{50}$ (the dc gain of $s^3 G(s)$)

$\because$ type $= 3$ $\therefore$ $\underset{(unit)}{K_\alpha} = \frac{40}{50} = 0.8$ $\therefore K_a = K_v = K_p = \infty$

& $\underset{(unit)}{e_{\alpha_{ss}}} = \frac{50}{40} = 1.25$ whereas $e_{a_{ss}} = \frac{1}{K_a} \approx e_{v_{ss}} = \frac{1}{K_v} = 0$ & $e_{p_{ss}} = \frac{1}{1+K_p} = 0$

$\boxed{\frac{6e}{134}}$ $G^* = \frac{G}{1+G(b_0-1)} = \frac{20(s^3+4s^2+5s+2)}{s^5+15s^4+(50+20(b_0-1))s^3+80(b_0-1)s^2+100(b_0-1)s+40(b_0-1)}$

For $b_0 = 1$ it is as in $\boxed{\frac{5e}{134}}$ above

For $b_0 \neq 1$: check stability: $\therefore 1+GH = s^5+15s^4+(50+20b_0)s^3+80b_0s^2+100b_0s+40b_0$

$\therefore s^5$	1	$50+20b_0$	$100b_0$		
s^4	15	$80b_0$	$40b_0$		
s^3	$75+22b_0$	$146b_0$	0	$(\times 15/10)$	$\therefore b_0 > -\frac{75}{22} = -3.4$
s^2	$176b_0^2+381b_0$	$4b_0(75+22b_0)$	0	$(\times(75+22b_0)/10)$	$\therefore b_0 \in$ [set 1]
s	$b_0\begin{pmatrix}23760b_0^2\\+42426b_0\\-22500\end{pmatrix}$	0		$(\times(176b_0^2+381b_0))$	$\therefore b_0 \in$ [set 2]
1	$4b_0(75+22b_0)$				$\therefore b_0 \in$ [set 3]

$\therefore$ set 1 is $176b_0^2+381b_0 > 0 \Rightarrow b_0(176b_0+381) > 0$

$\therefore b_0 \in (0,\infty) \cap (-\frac{381}{176},\infty)$ OR $(-\infty,0) \cap (-\infty, \frac{-381}{176})$

i.e. $b_0 \in (0,\infty)$ OR $(-\infty, \frac{-381}{176}) =$ set 1 $= (0,\infty)$ OR $(-\infty,-2.16)$

& set 2 is $b_0(23760b_0^2+42426b_0-22500) > 0$ $\therefore b_0(b_0+2.21)(b_0-.428) > 0$

$\therefore b_0 \in (0,\infty) \cap (-2.21,\infty) \cap (.428,\infty)$ OR $(-2.21,0) = (.428,\infty)$ OR $\underset{= \text{set } 2}{(-2.21,0)}$

& set 3 is $b_0(75+22b_0) > 0$ $\therefore b_0 \in (0,\infty) \cap (-3.41,\infty)$ OR $(-\infty,0) \cap (-\infty,-3.41)$

$\therefore b_0 \in (0,\infty)$ OR $(-\infty,-3.41) =$ set 3

$\therefore$ Combining the four conditions

$\therefore b_0 \in (-3.41,\infty) \cap [(0,\infty) \text{ OR } (-\infty,-2.16)] \cap [(.428,\infty) \text{ OR } (-2.21,0)] \cap [(0,\infty) \text{ OR } (-\infty,-3.41)]$

$\therefore b_0 \in (.428,\infty)$

$\therefore$ Stable iff $b_0 > .428$

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$\therefore$ System type in this case is zero

$\therefore K_c = \lim_{s \to 0} G^*(s) = \dfrac{1}{b_0 - 1} \qquad (b_0 \neq 1)$

$\therefore K_p = K_c = \dfrac{1}{b_0 - 1}$ & $K_v = K_a = 0$

& $e_{p_{ss\,unit}} = \dfrac{1}{1+K_p} = \dfrac{b_0 - 1}{b_0} = 1 - \dfrac{1}{b_0}$ & $e_{v_{ss}} = e_{a_{ss}} = \infty$

$\therefore$ Error is finite only for step inputs and is minimum at:

$|e_{p_{ss}}|' = 0$ or $\left(\sqrt{\left(\dfrac{b_0 - 1}{b_0}\right)^2}\right)' = 0$ or $\dfrac{1}{2} \cdot \dfrac{2\left(\frac{b_0-1}{b_0}\right)\left(\frac{1}{b_0^2}\right)}{\sqrt{\left(\frac{b_0-1}{b_0}\right)^2}} = 0$

i.e $\dfrac{(b_0-1)/b_0}{\sqrt{(b_0-1)^2/b_0^2}} = 0$ but $\dfrac{(b_0-1)/b_0}{\sqrt{(b_0-1)^2/b_0^2}}$ is >0 when $b_0 > 1$ & <0 when $b_0 < 1$

$\therefore$ no extrema

$\therefore$ min. occurs at $.428$ or 1 or ∞

$|e_{p_{ss\,unit}}|\Big|_{b_0 = .423} = |-1.336| = 1.34$, $|e_{p_{ss\,unit}}|\Big|_{b_0 \to 1 = 1-\epsilon} = 1 - \dfrac{1}{1-\epsilon} = 1 - (1 + \epsilon + \epsilon^2 + \ldots) = -\epsilon - \epsilon^2 \ldots$

$|e_{p_{ss\,unit}}|\Big|_{b_0 = \infty} = 1 - \dfrac{1}{\infty} = 1$

$\therefore$ Minimum error in this case is when b_0 approaches unity whereby $|e_{p_{ss\,unit}}| = |1 - b_0| \approx 0$

$\boxed{\dfrac{10}{135}}$ $G = \dfrac{50}{s^2 + 3s + 10}$, $H = 1$ $\therefore W(s) = \dfrac{\epsilon(s)}{r(s)} = 1 - \dfrac{c}{r}(s) = \dfrac{1}{1+G} = \dfrac{s^2+3s+10}{s^2+3s+60}$

$\therefore W(0) = \dfrac{10}{60}$ & $W'(0) = \dfrac{(2s+3)(s^2+3s+60 - s^2 - 3s - 10)}{(s^2+3s+60)^2}\Big|_0 = \dfrac{150}{60^2}$, $W''(0) = \dfrac{100 \times 60^2 - 2 \times 60 \times 3 \times 150}{60^4}$

$\therefore e_{ss}(t) = \dfrac{10}{60} r(t) + \dfrac{150}{60^2} r'(t) + \dfrac{85}{60^2} \dfrac{r''(t)}{2} = \dfrac{r(t)}{6} + \dfrac{r'(t)}{24} + \dfrac{17 r''(t)}{1440} + \ldots$

for $r(t) = 2 + 3t + .5t^2$ $\therefore r'(t) = 3 + t$ & $r''(t) = 1$ & r''' or higher $= 0$

$\therefore e_{ss}(10) = \dfrac{2 + 30 + 50}{6} + \dfrac{3 + 10}{24} + \dfrac{17}{1440} = \dfrac{82 \times 240 + 60 \times 13 + 17}{1440} = \dfrac{20477}{1440} \simeq 14.22$

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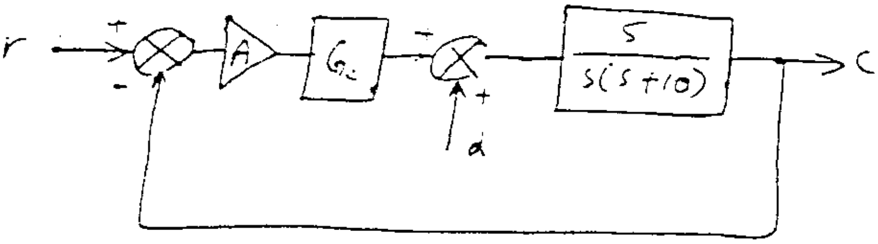


a) $G_c = 1$, $r = R$ & $d = D$ $\quad e_{ss_{tot}} = ?$ at $A = 10$

$$e_{ss_{tot}} = e_{ss}\Big|_{d=0} + e_{ss}\Big|_{r=0}$$

$$e(s)\Big|_{d=0} = \frac{r-c}{r}\cdot r = \left(1-\frac{c}{r}\right)r = \left(1 - \frac{5A}{5A + s(s+10)}\right)r = \frac{s(s+10)}{s^2+10s+5A}\cdot R/s$$

$$\therefore\ e_{ss}\Big|_{d=0} = \lim_{s\to 0} s\,e(s)\Big|_{d=0} = 0$$

$$\&\ e(s)\Big|_{r=0} = \frac{r-c}{d}\cdot d = \left(0 - \frac{c}{d}\right)d = \left(0 - \frac{5}{5A+(s+10)s}\right)d = \frac{-5}{s^2+10s+5A}\,\frac{D}{s}$$

$$\therefore\ e_{ss}\Big|_{r=0} = \lim_{s\to 0} s\,e(s)\Big|_{r=0} = \left(-\frac{1}{A}\right)D$$

$$\therefore\ e_{ss_{tot}} = 0 + \left(-\frac{1}{A}\right)D = \left(-1/A\right)D = (\text{for } A=10)\ -0.1\,D$$

b) $G_c = \frac{2s+1}{s}$, $r = R$, $d = D$ $\quad e_{ss_{tot}} = ?$ at $A = 10$

$$e(s)\Big|_{d=0} = \left(1 - \frac{c}{r}\right)r = \frac{R}{s}\cdot\frac{s^2(s+10)}{s^3+10s^2+5A(2s+1)}$$

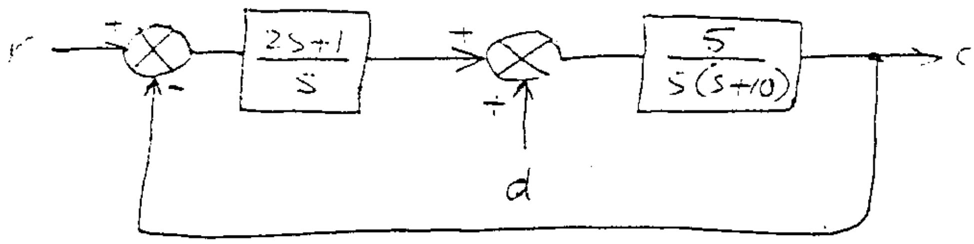
$$\therefore\ e_{ss}\Big|_{d=0} = \lim_{s\to 0} s\,e(s)\Big|_{d=0} = 0$$

$$\&\ e(s)\Big|_{r=0} = \left(\frac{\cancel{r}-c}{d}\right)d = \frac{-D}{s}\cdot\frac{5s}{s^3+10s^2+5A(2s+1)} = \frac{-5D}{s^3+10s^2+5A(2s+1)}$$

$$\therefore\ e_{ss}\Big|_{r=0} = \lim_{s\to 0} s\,e(s)\Big|_{r=0} = D\cdot 0 = 0$$

$$\therefore\ e_{ss_{tot}} = 0 + 0 = 0$$ (Note how PI controller decouple disturbance)

$\frac{16.b}{136}$



$r(t) = A_1 + B_1 t + C_1 t^2$ & $d(t) = A_2 + B_2 t + C_2 t^2$

$$\therefore\ C(s) = \frac{c}{r}(s)\Big|_{d=0} r(s) + \frac{c}{d}(s)\Big|_{r=0} d(s) =$$

$$= \frac{5(2s+1)}{s^2(s+10)+5(2s+1)} \cdot r(s) + \frac{5s}{s^2(s+10)+5(2s+1)} \cdot d(s)$$

$$= \frac{5}{s^3+10s^2+10s+5} \cdot \left[(2s+1)r(s) + s\,d(s)\right] \quad \text{Assuming OIC:}$$

$$\because\ r(s) = \frac{A_1}{s} + \frac{B_1}{s^2} + \frac{2C_1}{s^3} \quad \& \quad d(s) = \frac{A_2}{s} + \frac{B_2}{s^2} + \frac{2C_2}{s^3}$$

$$\therefore\ C(s) = \frac{5}{s^3+10s^2+10s+5}\left[(2s+1)\left(\frac{A_1}{s} + \frac{B_1}{s^2} + \frac{2C_1}{s^3}\right) + s\left(\frac{A_2}{s} + \frac{B_2}{s^2} + \frac{2C_2}{s^3}\right)\right] =$$

$$\therefore\ C_{ss}(s) = (\text{Note that } C(s) \text{ is stable}) \frac{\lim_{s\to 0} s^3 C(s)}{s^3} + \frac{\lim_{s\to 0}(s^3C(s))'}{s^2} + \frac{\lim_{s\to 0}(s^3C(s))''}{2s}$$

$$\therefore\ \lim_{s\to 0} s^3 C(s) = \frac{5\left[(2s+1)(A_1s^2+B_1s+2C_1) + s(A_2s^2+B_2s+2C_2)\right]}{s^3+10s^2+10s+5}\Bigg|_{s=0} =$$

$$= \frac{5\left[s^3(2A_1+A_2)+s^2(A_1+2B_1+B_2)+s(B_1+4C_1+2C_2)+2C_1\right]}{s^3+10s^2+10s+5}\Bigg|_0 = \frac{10C_1}{5} = 2C_1$$

$$\& \lim_{s\to 0}(s^3C(s))' = \frac{5\left[\left(3s^2(2A_1+A_2)+2s(A_1+2B_1+B_2)+B_1+4C_1+2C_2\right)(s^3+10s^2+10s+5)-(3s^2+20s+10)\begin{pmatrix} s^3(2A_1+A_2) \\ +s^2(A_1+2B_1+B_2) \\ +s(B_1+4C_1+2C_2) \\ +2C_1\end{pmatrix}\right]}{(s^3+10s^2+10s+5)^2}\Bigg|_0$$

$$= \frac{5(B_1+4C_1+2C_2)-10(2C_1)}{5}$$

$$= B_1 + 2C_2$$

$$\& \lim_{s\to 0}(s^3C(s))'' = \frac{5\left[5^2 * \left(10(B_1+4C_1+2C_2)+10(A_1+2B_1+B_2)-10(B_1+4C_1+2C_2)-20(2C_1)\right)-\begin{pmatrix}B_1+4C_1+2C_2 \\ -10(2C_1)\end{pmatrix}5\right]}{5^4}$$

$$= \frac{25(20B_1-40C_1+10A_1+10B_2)-(5B_1+10C_2)(100)}{5^3} = \frac{-40C_1+10A_1+10B_2-40C_2}{5}$$

$$= 2(A_1+B_2-4C_1-4C_2)$$

$$\therefore \quad C_{ss}(s) = \frac{2C_1}{s^3} + \frac{B_1 + 2C_2}{s^2} + \frac{A_1 + B_2 - 4C_1 - 4C_2}{s}$$

$$\therefore \quad C_{ss}(t) = A_1 + B_2 - 4C_1 - 4C_2 + (B_1 + 2C_2)t + C_1 t^2$$

$$\therefore \quad e_{ss} = r - C_{ss}(t) = A_1 + B_1 t + C_1 t^2 - A_1 - B_2 + 4C_1 + 4C_2 - (B_1 + 2C_2)t - C_1 t^2$$
$$= -B_2 + 4C_1 + 4C_2 - 2C_2 t$$
$$= 4(C_1 + C_2) - B_2 - 2C_2 t$$

OR

\# Apply superposition to all six inputs

$$\therefore \quad C_{ss}(t)\Big|_{A_1} = \frac{5(2s+1)}{s^3+10s^2+10s+5}\Big|_{s=0} * A_1 = A_1$$

$$\& \quad C_{ss}(t)\Big|_{A_2} = \frac{5s}{s^3+10s^2+10s+5}\Big|_{s=0} * A_2 = 0$$

$$\& \quad C_{ss}(t)\Big|_{B_1 t} = \frac{5(2s+1)}{s^3+10s^2+10s+5}\Big|_{s^2=0} \cdot B_1 t = \frac{5(2s+1)}{5(1+2s)} B_1 t = B_1 t$$

$$\& \quad C_{ss}(t)\Big|_{C_1 t^2} = \frac{5(2s+1)}{s^3+10s^2+10s+5}\Big|_{s^3=0} C_1 t^2 = \frac{5(2s+1)}{5(1+2s(s+1))} C_1 t^2 = \left[1 - 2s(s+1) + 4s^2(s+1)^2\right](4C_1 t + C_1 t^2)$$
$$= C_1\left[1 - 2s + 2s^2\right](4t + t^2)$$
$$= C_1(4t + t^2 - 8 - 4t + 4) = C_1(t^2 - 4)$$

$$\& \quad C_{ss}(t)\Big|_{B_2 t} = \frac{5s}{s^3+10s^2+10s+5}\Big|_{s^2=0} \cdot B_2 t = \frac{5}{1+2s} B_2 t = B_2(1-2s)s\,t = B_2(1-2s)1 = B_2$$

$$\& \quad C_{ss}(t)\Big|_{C_2 t^2} = \frac{5s}{s^3+10s^2+10s+5}\Big|_{s^3=0} \cdot C_2 t^2 = \frac{5s}{5[1+2s(s+1)]} C_2 t^2 = \left[1 - 2s(s+1) + 4s^2(s+1)^2\right] 2C_2 t$$
$$= (1 - 2s + 2s^2)\, 2C_2 t = 2C_2(t - 2)$$

$$\therefore \quad C_{ss}(t) = A_1 + B_1 t + B_2 + C_1(t^2 - 4) + 2C_2(t - 2) \qquad (\therefore \text{OK})$$

then find $e_{ss}(t)$ by $e_{ss}(t) = r - C_{ss}(t)$

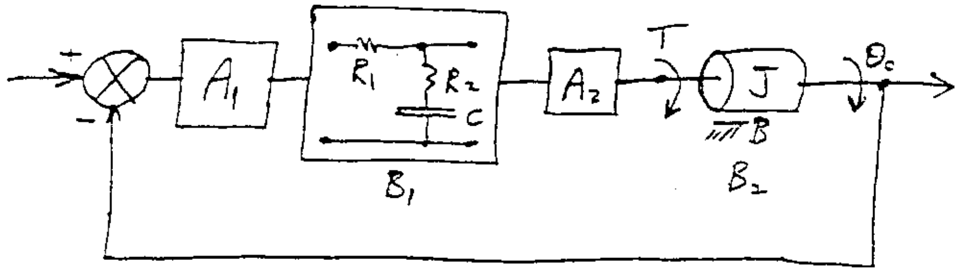
OR

\# $$G^*\Big|_{d=0} = \frac{5(2s+1)}{s^2(s+10)} \quad \therefore K_E = K_a = \frac{1}{2} \quad \therefore e_{ss}\Big|_{d=0} = 0 * A_1 + 0 * B_1 + 2(2C_1) = 4C_1$$

$$\& \quad e_{ss}(t)\Big|_{r=0} = \frac{\overset{0}{r} - C}{d} \cdot d = -\frac{C}{d} \cdot d = \frac{-5s\, d(t)}{s^3+10s^2+10s+5} = \frac{-5}{s^3+10s^2+10s+5}\, d'(t) =$$
$$= \frac{-5}{s^3+10s^2+10s+5}(B_2 + 2C_2 t) = \frac{-5}{5(1+2s)}(B_2 + 2C_2 t) = -(1-2s)(B_2 + 2C_2 t)$$
$$= -(B_2 + 2C_2 t - 4C_2)$$

$$\therefore \quad e_{ss}(t) = 4C_1 - B_2 - 2C_2 t + 4C_2 \qquad (\therefore \text{OK}) \quad \text{then } C_{ss}(t) = r - e_{ss}(t)$$

#



$$\therefore \quad B_1 = \frac{R_2 + \frac{1}{sC}}{R_2 + \frac{1}{sC} + R_1} = \frac{1 + sCR_2}{1 + sC(R_1 + R_2)}$$

& $B_2 = \frac{\theta_o}{T}$ where: $T = J\ddot{\theta}_o + B\dot{\theta}_o$ & assuming OIC

$$\therefore \quad B_2 = \frac{\theta_o}{T}(s) = \frac{1}{Js^2 + Bs}$$

$\therefore$ a). Open Loop Transfer function (i.e when feedback link is cut) is

$$A_1 \cdot B_1 \cdot A_2 \cdot B_2 = \frac{A_1 A_2 (1 + sCR_2)}{s(Js + B)(1 + sC(R_1 + R_2))}$$

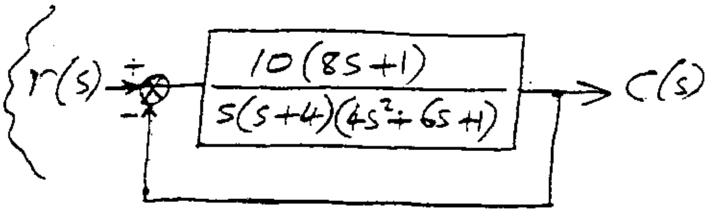
& b). Closed Loop Transfer function (i.e when feedback link is there).

$$\frac{\theta_o}{\theta_i} = \frac{A_1 B_1 A_2 B_2}{1 + A_1 B_1 A_2 B_2} = \frac{A_1 A_2 (1 + sCR_2)}{s(Js + B)[1 + sC(R_1 + R_2)] + A_1 A_2 (1 + sCR_2)}$$

\# a) $K_0 = K_p = G(0) = \infty$

$K_1 = K_v = \lim_{s \to 0} sG(s) = 2.5$

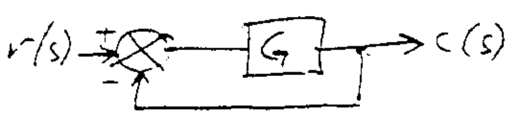
$K_2 = K_a = \lim_{s \to 0} s^2 G(s) = 0$



b). The system is type '1'

c). For parabolic $r(t)$, the system will <u>not</u> perform satisfactorily. This is because $e_{ss} = \frac{1}{K_a} = \infty$

$G(s) = \frac{20(s+2)}{s(s+3)(s+4)}$;



a) The dc gain $= G(0) = \infty$ & System gain, $K_n = \frac{10}{3}$

b) $K_p = G(0) = \infty$

$$K_v = \lim_{s \to 0} sG(s) = \frac{10}{3}$$

$$K_a = \lim_{s \to 0} s^2 G(s) = 0$$

c) For $r(t) = (3+5t)u(t)$; then, using superposition:

$$e_{ss}(t) = \frac{1}{1+K_p} * 3 + \underline{\frac{1}{K_v} * 5} = \frac{1}{\infty} * 3 + \frac{3}{10} * 5 = 1.5$$

For $G(s) = \frac{8}{s(s+1)}$ & $H(s) = 1$:

a. system type is '1'.

∴ b. $K_p = G(0) = \infty$, $K_v = \lim_{s \to 0} sG(s) = 8$ & $K_a = \lim_{s \to 0} s^2 G(s) = 0$

c. Stability is decided by c.q. $1 + GH(s) = 0$ or $1 + \frac{8}{s(s+1)} = 0$

OR $s(s+1) + 8 = 0 \Rightarrow s^2 + s + 8 = 0 \Rightarrow s = -\frac{1}{2} \pm j\sqrt{7.75}$

∴ System is stable.

d. $e_{ss}(t)$ for $r(t) = t$ is found from:

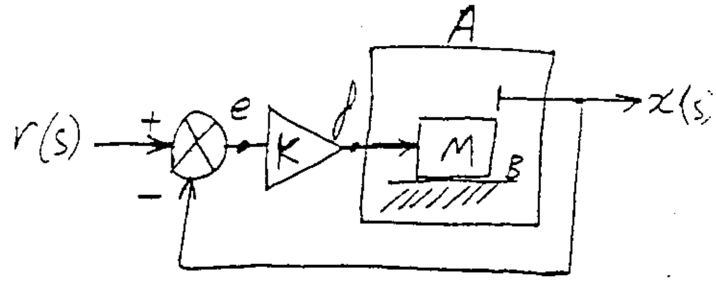
$$e_{ss}(t) = W_0 r + W_0' r' \quad \text{with } W(s) = \frac{1}{1+G_1(s)} = \frac{s^2+s}{s^2+s+8}\bigg|_0 = 0$$

$$\therefore W'(s)\Big|_0 = \frac{1*8 - 1*0}{8^2} = \frac{1}{8} = .125$$

$$\therefore e_{ss}(t) = 0 * t + .125 * 1 = .125$$

OR equally: $e_{ss}(t) = \underline{\frac{1}{K_v} * 1} = \frac{1}{8} * 1 = .125$

\# To find $A(s) = \frac{x}{f}(s)$



$\therefore f = M\ddot{x} + B\dot{x}$

$= D(MD + B)x$

$\therefore$ Assuming 0 IC

$\therefore A = \frac{x}{f}(s) = \frac{1}{s(B+Ms)}$

$\therefore$ Ch. eq. for stability: $1 + KA = 0 \Rightarrow K + s(B+Ms) = 0$

Using Routh:

s^2 :	M	K
s :	B	0
1 :	K	

$\therefore K$ must be > 0

& $G(s) = \frac{K}{s(B+Ms)}$ which indicates type '1' system.

Hence giving: $K_p = G(0) = \infty$,

$K_v = \lim_{s \to 0} sG(s) = \frac{K}{B}$ &

$K_a = \lim_{s \to 0} s^2 G(s) = 0$

To find $e_{ss}(t)$ for $r(t) = 7t^2$, using Taylor formula:

$\therefore W(s) = \frac{1}{1+G} = \frac{s(B+Ms)}{K+s(B+Ms)}\Big|_{s=0} = 0$

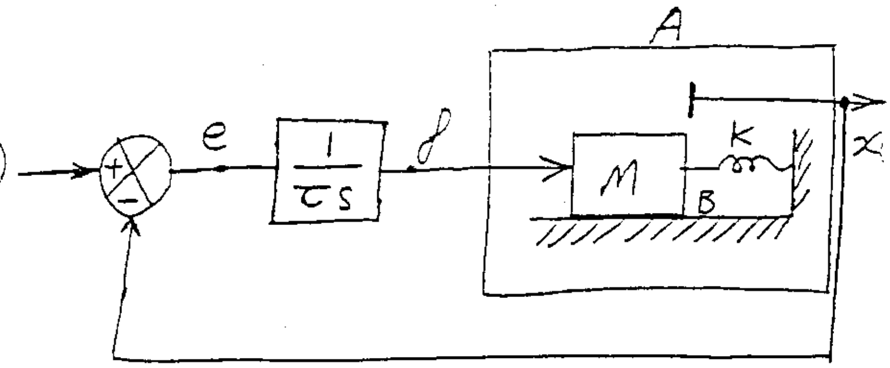
& $W'(s) = \frac{(B+2Ms)(K+Bs+Ms^2) - (B+2Ms)s(B+Ms)}{(K+Bs+Ms^2)^2} = \frac{(B+2Ms)K}{(K+Bs+Ms^2)^2}\Big|_{s=0} = \frac{B}{K}$

& $W''(s)\Big|_{s=0} = \frac{2MK^3 - 2K^2B^2}{K^4} = \frac{2}{K^2}(MK - B^2)$

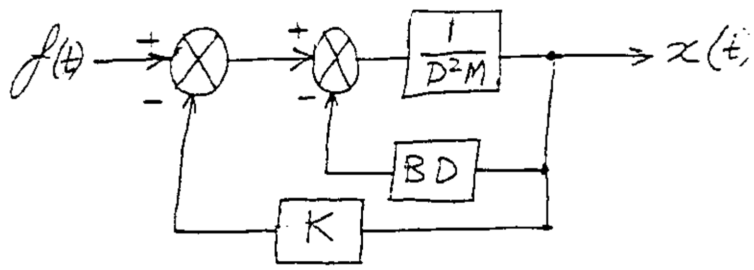
$\therefore e_{ss}(t) = W_0 r(t) + W_0' r'(t) + \frac{W_0''}{2} r''(t) = 0 + \frac{B}{K} \times 14t + \frac{1}{K^2}(MK - B^2) \times 14$

$\lim_{t \to \infty} e_{ss} = \infty$ $\therefore$ system can not cope with input for all K.

\# 1. For stage A, $r(s)$ the block diagram can be drawn using its differential equation: $f = M\ddot{x} + B\dot{x} + Kx$.



2. This gives assuming OIC:



$f(s) = (Ms^2 + Bs + K)\,x(s)$

$\therefore\ A = \frac{x}{f}(s) = \frac{1}{Ms^2 + Bs + K}$.

$\therefore\ \frac{x}{r}(s) = \frac{A/\tau s}{1 + A/\tau s} = \frac{1}{1 + \tau s(Ms^2 + Bs + K)}$.

3. Ch. eq. is: $1 + \tau s(Ms^2 + Bs + K) = 0$ OR in better form:

$Ms^3 + Bs^2 + Ks + \frac{1}{\tau} = 0$. Using Routh:

$\therefore$

s^3:	M	K
s^2:	B	$\frac{1}{\tau}$
s:	$\frac{BK - M/\tau}{B}$	0
1:	$\frac{1}{\tau}$	0

$\therefore$ For stability: BK must be $> \frac{M}{\tau}$ & $\frac{1}{\tau} > 0$

$\therefore\ \frac{1}{\tau} \in \left(-\infty, \frac{BK}{M}\right) \cap (0, \infty) = \left(0, \frac{BK}{M}\right)$

$\therefore\ \tau \in \left(\frac{M}{BK}, \infty\right)$ gives stable system.

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4. $G(s) = \dfrac{1}{\tau s(Ms^2+Bs+K)}$, system type is '1'

$\therefore K_p = \infty$, $K_v = \dfrac{1}{\tau K}$ & $K_a = 0$

5. $\therefore$ (a) The system can cope with step input, $R_0 u(t)$ giving zero error at steady-state.

(b) It can also cope with ramp input, $R_1 t u(t)$ giving a steady-state error of $\tau K R_1$

(c) Whereas it could not cope with acceleration input as it results in infinite steady-state error.

6. For τ at the verge of stability $\tau = \dfrac{M}{BK}$ giving with OIC:

$$\frac{x}{r}(s) = \frac{BK}{BK + MKs + MBs^2 + M^2s^3} = \frac{BK}{K(B+Ms)+Ms^2(B+Ms)}$$

$$= \frac{BK}{(B+Ms)(K+Ms^2)}$$

a) $\therefore$ dc gain $= 1$

b) oscilation frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{K}{M}}$, where $\omega = \sqrt{\dfrac{K}{M}}$

c) $\therefore x(s)\left[\text{for } r(t) = u(t)\right] = \dfrac{BK}{s(B+Ms)(K+Ms^2)} = \dfrac{C_1}{s} + \dfrac{C_2}{B+Ms} + \dfrac{C_3+C_4 s}{K+Ms^2}$, where

$C_1 = \lim_{s\to 0} s\,x(s) = 1$,

$$C_2 = \lim_{s\to -\frac{B}{M}} (B+Ms)x(s) = \frac{BK}{-\frac{B}{M}\cdot(K+B^2/M)} = \frac{-KM^2}{B^2+KM}$$

$$C_3 + C_4 s = \lim_{K+Ms^2\to 0} (K+Ms^2)\,x(s) = \lim_{Ms^2\to -K} \frac{BK}{Ms^2+Bs} = \lim_{Ms^2\to -K} \frac{BK(Bs+K)}{(Bs-K)(Bs+K)}$$

$$= \lim_{s^2\to -\omega^2} \frac{BK(Bs+K)}{B^2s^2-K^2} = \frac{-BK(Bs+K)}{B^2\omega^2+K^2} = -\frac{BM\cdot(Bs+K)}{B^2+KM}$$

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$$\therefore\ x(s) = \frac{1}{s} - \frac{KM^2}{B^2+KM}\cdot\frac{1}{B+Ms} - \frac{BM}{B^2+KM}\cdot\frac{Bs+K}{K+Ms^2} =$$

$$= \frac{1}{s} - \frac{KM}{B^2+KM}\cdot\frac{1}{s+B/M} - \frac{B}{B^2+KM}\cdot\frac{B\cdot s+K}{s^2+\omega^2}$$

$$\therefore\ x(t) = 1 - \frac{KM}{B^2+KM}\cdot e^{-Bt/M} - \frac{B}{B^2+KM}\cdot\left(B\cos\omega t + \frac{K}{\omega}\sin\omega t\right)$$

$\therefore$ For $B=K=M=1$ standard unit:

$$\therefore\ x(t) = 1 - \frac{1}{2}\left[e^{-t} + \cos t + \sin t\right]$$

$$\therefore\ x(t_d) = 0.1 \Rightarrow \quad 1.8 - e^{-t_d} - \cos t_d - \sin t_d = 0 \quad \Rightarrow t_d = 0.9199 \text{ standard unit}$$

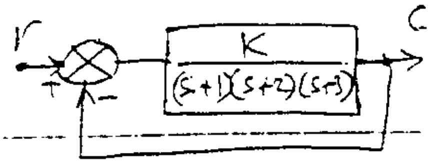
$$x(\underbrace{t_d+t_r}_{t^*}) = 0.9 \Rightarrow 0.2 - e^{-t^*} - \cos t^* - \sin t^* = 0 \Rightarrow \text{solving for } t^*$$

$$\therefore\ t_d + t_r = t^* = 2.2866 \quad \Rightarrow \quad t_r = 1.3667 \text{ standard units}$$

As the output oscilates with amplitude at steady-state reaching $\frac{\sqrt{2}}{2} \simeq 0.7071$; then there are ∞ crossing points with the .95 & 1.05 levels $\therefore\ t_s = \infty$

i.e. the system will never settle.

To find $e_{ss}(t)$ & limits of K for stability for the given network when $r(t) = 1 + t + t^2$:



$$\therefore \frac{c}{r}(s) = \frac{K}{(s+1)(s+2)(s+3) + K}$$

ch. eq. $(s+1)(s+2)(s+3) + K = 0 \Rightarrow s^3 + 6s^2 + 11s + 6 + K = 0$

$\therefore$ The Routhian is :

$$\begin{array}{l|ll} s^3 & 1 & 11 \\ s^2 & 6 & 6+K \\ s & 66-6-K & \\ 1 & 6+K & \end{array} \qquad \begin{array}{l} \\ \\ \Rightarrow 60 - K > 0 \quad (1) \\ \Rightarrow 6 + K > 0 \quad (2) \end{array}$$

$\therefore$ For stability, using (1) & (2)

$$\therefore K \in \{60 - K > 0\} \cap \{6 + K > 0\} \equiv \{K < 60\} \cap \{K > -6\}$$
$$\equiv (-\infty, 60) \cap (-6, \infty) \equiv (-6, 60)$$

$\therefore K \in (-6, 60)$ gives stable system

$$\frac{\epsilon}{r}(s) = \frac{1}{1 + \frac{K}{(s+1)(s+2)(s+3)}} = \frac{(s+1)(s+2)(s+3)}{(s+1)(s+2)(s+3) + K} = \frac{s^3 + 6s^2 + 11s + 6}{s^3 + 6s^2 + 11s + 6 + K} = W$$

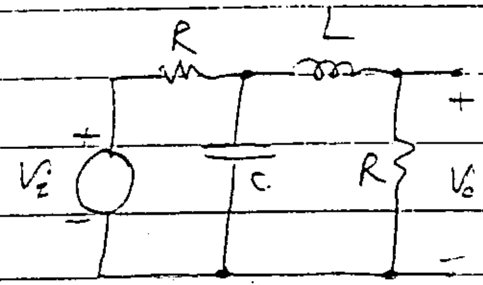
$$\therefore e_{ss}(t) = W_0 \cdot r + W_0' r' + W_0'' \frac{r''}{2} + \cdots$$

$$= \frac{6}{6+K} \cdot (1 + t + t^2) + \left(\frac{(3s^2 + 12s + 11) K}{(s^3 + 6s^2 + 11s + 6 + K)^2} \right) \Bigg|_{s=0} \cdot (1 + 2t) +$$

$$+ \frac{12(6+K)^2 - 11^2(6+K) * 2}{(6+K)^4} \cdot K \cdot 1 + 0 = \frac{6}{6+K}(1 + t + t^2) + \frac{11K(1+2t)}{(6+K)^2} + \frac{2K(6K-85)}{(6+K)^3}$$

$$\therefore e_{ss}(t) = \frac{6t^2}{6+K} + \frac{4(9+7K)t}{(6+K)^2} + \frac{216 - 32K + 29K^2}{(6+K)^3}, \; K \in (-6, 60)$$

1. To obtain the differential equation of the network shown:



$$\therefore \frac{V_o}{V_i} = \frac{R}{R+DL} \cdot \frac{(R+DL)\,//\,\frac{1}{DC}}{\left[(R+DL)\,//\,\frac{1}{DC}\right]+R}$$

$$= \frac{R}{R+DL} \cdot \frac{1}{1+\frac{R}{\frac{(R+DL)/DC}{R+DL+\frac{1}{DC}}}} = \frac{R}{R+DL} \cdot \frac{1}{1+\frac{(R+DL)DC+1}{R+DL}\cdot R}$$

$$= \frac{R}{R+DL+(R+DL)DCR+R} = \frac{R}{(R+DL)(1+DCR)+R}$$

$$\therefore \left[D^2LCR + D(L+CR^2) + 2R\right]V_o = R\,V_i$$

$\therefore$ The differential eq. is $LCR\,V_o'' + (L+CR^2)V_o' + 2R\,V_o = R\,V_i$

Or, you can obtain it using circuit laws as follows:

current through $L = \frac{V_o}{R}$

voltage across $L = L\frac{V_o'}{R}$

voltage across $C = V_o + \frac{L}{R}V_o'$

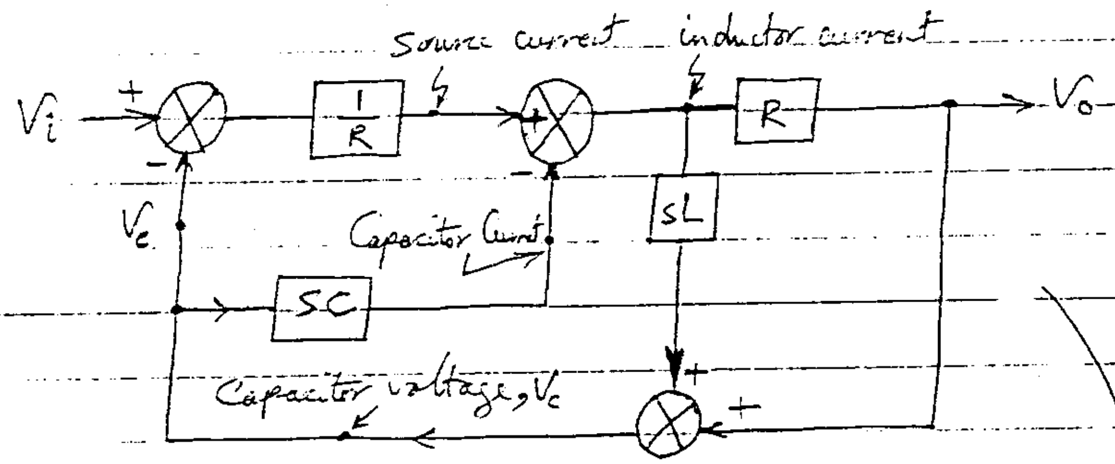
current through $C = CV_o' + \frac{CL}{R}V_o''$

$$\text{Source current} = \frac{V_o}{R} + CV_o' + \frac{CL}{R}V_o''$$

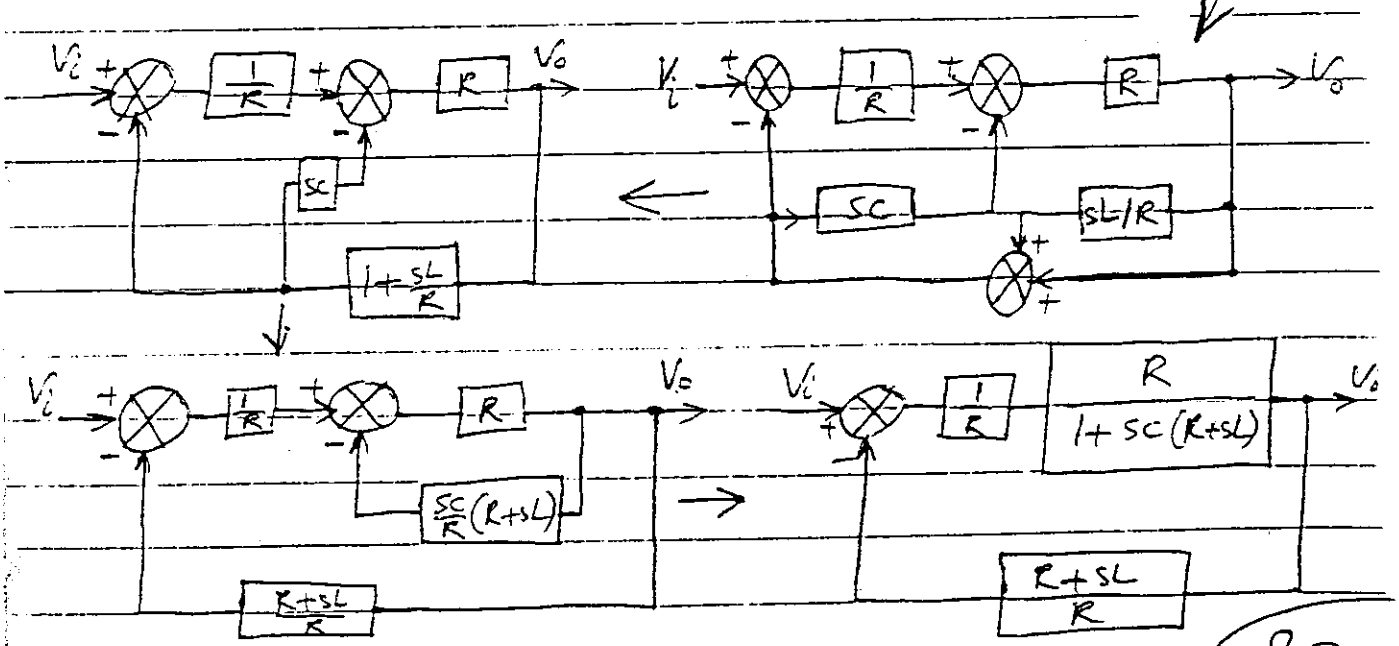
$$\therefore V_i = R\left(\frac{V_o}{R} + \frac{RCV_o'}{R} + \frac{LC}{R}V_o''\right) + V_o + \frac{L}{R}V_o'$$

$$\therefore RV_i = LCR\,V_o'' + (L + CR^2)V_o' + 2RV_o \qquad (\therefore \text{OK})$$

2. To represent the network using block diagram:



This could be reduced to obtain $\frac{V_o}{V_i}$ as follows:



$$\therefore \frac{V_o}{V_i}(s) = \frac{\dfrac{1}{1+sC(R+sL)}}{1+\dfrac{1}{1+sC(R+sL)}\cdot\dfrac{R+sL}{R}} = \frac{R}{R[1+sC(R+sL)]+R+sL}$$

$$= \frac{R}{R+(R+sL)(1+RsC)} \qquad (\therefore \text{OK})$$

3. For $R = L = C = V_i(t) = 1$ unit & OIC on C & L

$$\therefore V_o(t) = \frac{1}{D^2+2D+2}\cdot V_i(t)$$

$$= (A\sin t + B\cos t)\cdot e^{-t} + \frac{1}{2}$$

$$\left\{ \begin{aligned} & D^2+2D+2 = 0 \Rightarrow \\ & D = \frac{-2\pm\sqrt{4-8}}{2} \\ & \quad = -1 \pm j1 \end{aligned} \right.$$

$$\therefore \frac{V_o}{R}(0) = 0 \Rightarrow 0 = B + \tfrac{1}{2} \Rightarrow B = -\tfrac{1}{2}$$

$$\& \; V_c(0) = 0 \Rightarrow L\left(\frac{V_o}{R}\right)' + V_o \Big|_{t=0} = 0 \Rightarrow V_o'(0) = 0$$

$$\therefore 0 = A\cdot 1 - B\cdot 1 = A + \tfrac{1}{2} \Rightarrow A = -\tfrac{1}{2}$$

$$\therefore V_o(t) = \frac{1}{2}\left[1 - e^{-t}(\sin t + \cos t)\right] = 0.5\left[1 - \sqrt{2}\, e^{-t} \sin(t + \pi/4)\right]$$

4. To find $t_d, t_r, t_p, t_s, r, J_\infty$ & $e_{ss}(t)$:

$$\therefore V_{o_{ss}}(t) = \lim_{t\to\infty} V_o(t) = 0.5 \quad \& \; V_o(0) = 0$$

$$\therefore V_o(t_d) = 0.1 * (.5 - 0) + 0 = .05 \Rightarrow t_d = 0.3574 \text{ units}$$

$$\& \; V_o(t_d + t_r) = .9 * .5 = .45 \Rightarrow t_d + t_r = 1.8763 \Rightarrow t_r = 1.5189 \text{ ''}$$

$$\& \; V_o'(t_p) = 0 \Rightarrow \cos(t_p + \pi/4) = \sin(t_p + \pi/4) \Rightarrow \tan(t_p + \pi/4) = 1 \;\therefore t_p + \tfrac{\pi}{4} = \tfrac{\pi}{4} + k\pi$$

$$\therefore t_p = \underset{\text{min}}{0}, \underset{\text{max}}{\pi}, 2\pi, \ldots \Rightarrow \therefore t_p = \pi = 3.1416 \text{ units}$$

$$\therefore V_o(t_p) = 0.5\left[1+\sqrt{2}\, e^{-\pi} \cdot \frac{1}{\sqrt{2}}\right] = 0.5\left(1+e^{-\pi}\right)$$

$$\therefore r = e^{-\pi} = 0.043214 = 4.3214\ \%$$

$$\& \quad V_o(t_s) = \begin{cases} .95 * .5 = .475 \Rightarrow t_s = 2.07171 \\ \text{OR} \\ 1.05 * .5 = .525 \Rightarrow t_s = \{\phi\} \ (\text{at overshoot } 1.04 * .5) \end{cases}$$

$$\therefore t_s = 2.07171 \text{ units}$$

$$\& \quad J_\infty = \int_0^\infty \left[.5 * \sqrt{2}\, e^{-t} \sin(t+\pi/4)\right]^2 dt = \frac{1}{D} \cdot \frac{1}{2} \cdot e^{-2t} \sin^2(t+\pi/4) =$$

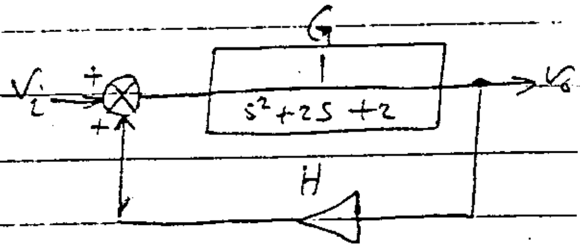
$$= \frac{1}{2} \cdot e^{-2t} \cdot \frac{1}{D-2} \cdot \left[1-\cos(2t+\pi/2)\right]/2 = \frac{1}{4} \cdot e^{-2t} \frac{D+2}{D^2-4} \left[1+\sin 2t\right]$$

$$= \frac{1}{4} \cdot e^{-2t} \left[-\frac{1}{2} + \frac{2\cos 2t + 2\sin 2t}{-4-4}\right]_0^\infty = 0 - \frac{1}{4} \cdot 1 \left[-\frac{1}{2} - \frac{1}{4}\right] = \frac{3}{16}$$

$$\& \quad e_{ss}(t) = r(t) - v_{o_{ss}}(t) = \frac{1}{2}$$

5. $$G(s) = \frac{V_o}{V_i}(s) = \frac{R}{s^2 LCR + s(L+CR^2) + 2R} = \frac{1}{s^2+2s+2}$$

$\therefore$ For the feedback loop shown



$$\frac{V_o}{V_i} = \frac{G}{1-GH} = \frac{1}{s^2+2s+2-H}$$

$\therefore$ Ch. eq. is $s^2+2s+2-H=0$ $\quad\therefore$ $s^2 : 1 \quad 2-H$

$\therefore$ Stable for $2-H>0 \Rightarrow H \in (-\infty, 2)$ $\quad s : 2$ $\quad 1 : 2-H$

$\therefore$ The system will preserve stability provided $H \in (-\infty, 2)$.

6. The value of H required to make t_p 50% less, i.e. $\frac{\pi}{2}$ is obtained as follows:

$$\frac{V_o}{V_i}(s) = \frac{1}{s^2+2s+2-H} \Rightarrow V_o(t) = \frac{1}{D^2+2D+2-H} \cdot V_i(t)$$

$$\therefore V_o(t) = \frac{1}{2-H} + e^{-t}(A\sin\omega t + B\cos\omega t)$$

$$\left\{ \begin{aligned} D &= \frac{-2\pm\sqrt{4-4(2-H)}}{2} \\ &= -1 \pm j\sqrt{1-H} \\ &= -1 \pm j\omega \\ \text{where } \omega &= \sqrt{1-H} \end{aligned} \right.$$

$$V_o(0) = V_o'(0) = 0 \Rightarrow B = \omega A = -\frac{1}{2-H}$$

$$\therefore V_o(t) = \frac{1}{2-H}\left[1 - e^{-t}\left(\frac{\sin\omega t}{\omega} + \cos\omega t\right)\right]$$

To find t_p: $\therefore V_o'(t_p) = 0 \Rightarrow -\left(\frac{\sin\omega t_p}{\omega} + \cancel{\cos\omega t_p}\right) + \cancel{\cos\omega t_p} - \omega\sin\omega t_p = 0$

$$\therefore -\left(\frac{1}{\omega}+\omega\right)\sin\omega t_p = 0 \Rightarrow \sin\omega t_p = 0 \Rightarrow \omega t_p = k\pi$$

$$\therefore t_p = \underset{min}{0}, \underset{max}{\frac{\pi}{\omega}}, \frac{2\pi}{\omega}, \ldots \quad \therefore t_p = \frac{\pi}{\omega} \text{ and should be } = \frac{\pi}{2}$$

$$\therefore \omega = 2 \Rightarrow \sqrt{1-H} = 2 \Rightarrow H = -3$$

This will give $\epsilon_{ss}(t) = 1 - \frac{1}{2-H} = \frac{1-H}{2-H} = \frac{4}{5}$

Ch. eq. of unity feedback system having

$$G(s) = \frac{K(s+1)(s+3)}{s(s+2)(s+4)} \quad \text{is:} \quad K(s+1)(s+3) + s(s+2)(s+4)$$

$$\therefore \quad s^3 + 6s^2 + 8s + K(s^2+4s+3) = 0$$

$$\therefore \quad s^3 + (6+K)s^2 + (8+4K)s + 3K = 0$$

$\therefore$ The Routhian:

$$\begin{array}{l|lll}
s^3 & 1 & 8+4K & \\
s^2 & 6+K & 3K & 6+K>0 \quad (1) \\
s & \frac{(6+K)(8+4K)-3K}{6+K} & 0 & \times(1) \Rightarrow 4K^2+29K+48 > 0 \quad (2) \\
1 & 3K & 0 & \Rightarrow K>0 \quad (3)
\end{array}$$

$\therefore$ (1) & (3) gives $K \in (0,\infty) \cap (-6,\infty) \equiv (0,\infty)$ (4)

& (2) gives: $K = \frac{-29 \pm \sqrt{29^2 - 768}}{8} = \frac{-29 \pm \sqrt{73}}{8} = -2.557, -4.693$

$\therefore$ K satisfying (2) $\in (-\infty, -4.693) \cup (-2.557, \infty)$ (5)

$\therefore$ (1) & (2) & (3) gives (4) & (5) gives $K \in (0,\infty)$ i.e. $K>0$

a) $\therefore$ System is stable for $K>0$

b) & System is type '1' with $K_p = \infty$, $K_v = \frac{3K}{8}$ & $K_a = 0$; hence it can cope with position & ramp inputs and can't cope with acc.

c) For acc input $r(t) = t^2$, error e_{ss} is ∞ &

$$e_{ss}(t) = W_0 r + W_1' r' + W_1'' \frac{r''}{2} \Rightarrow W = \frac{1}{1+G} = \frac{s^3+6s^2+8s}{s^3+(6+K)s^2+4(2+K)s+3K}$$

$$\therefore e_{ss}(t) = 0 + \frac{(12s+8)(4(2+K)s+3K) - [2(6+K)s+4(2+K)]8s}{[s^3+(6+K)s^2+4(2+K)s+3K]^2}\Bigg|_{s=0} 2t + \frac{[32(2+K)+36K-32(2+K)]9K^2 - 2\cdot 3K \cdot 4(2+K)\cdot 24K}{(3K)^4}$$

$$\therefore e_{ss}(t) = \frac{16t}{3K} + \frac{324K^3 - 576K^2(K+2)}{81K^4} = \frac{16t}{3K} - \frac{252K^3 + 1152K^2}{81K^4}$$

$$\therefore e_{ss}(t) = \frac{16t}{3K} - \frac{28K+128}{9K^2}.$$

1,β / 164.

$H = 1$, $G = \dfrac{K(s+4)^2}{s^2(s+20)}$. Locus is symmetric about Re-axis

Consider +ve half:

$\therefore\ GH = \dfrac{K(s+4)^2}{s^3(s+20)}$, poles at $0,0,0,-20$ & zeros at $-4,-4$

Branches: 4

Difference $= 4 - 2 = 2$

$\therefore$ Two zeros are at ∞

Assymptotes:

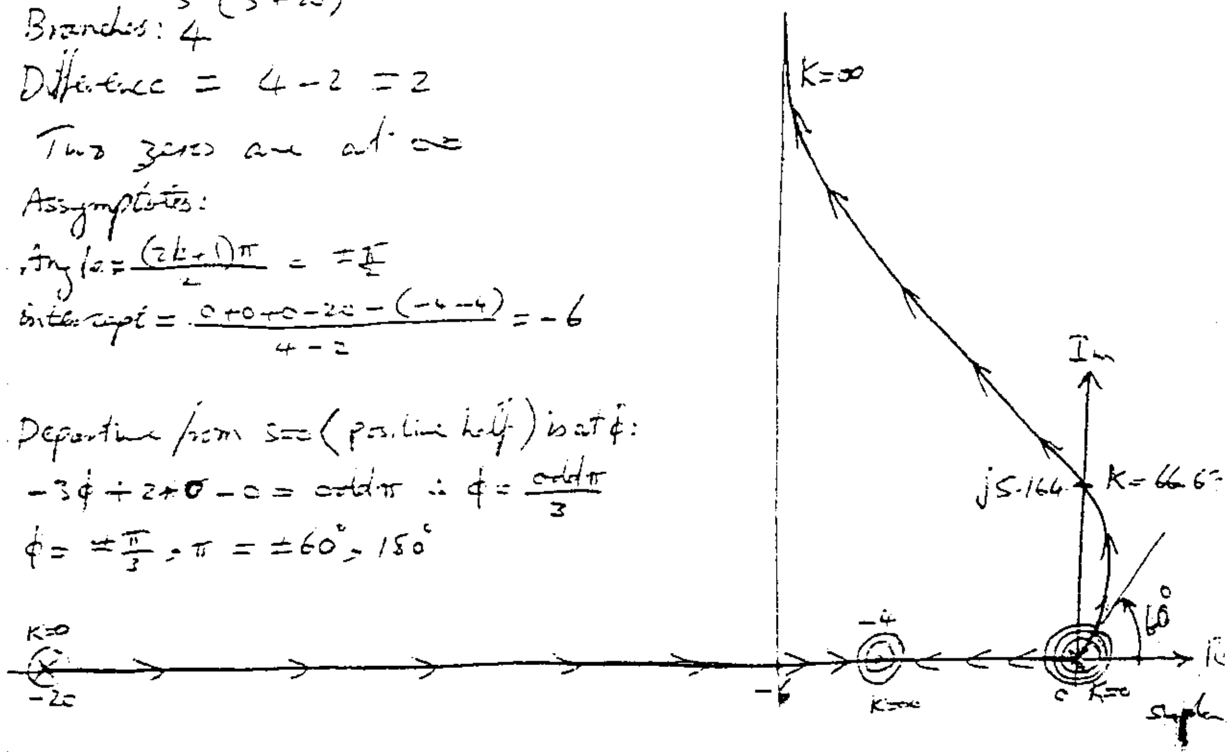
Angle $= \dfrac{(2k+1)\pi}{2} = \pm\dfrac{\pi}{2}$

intercept $= \dfrac{0+0+0-20-(-4-4)}{4-2} = -6$

Departure from $s=0$ (positive half) is at ϕ:

$\therefore\ -3\phi + 2 \times 0 - 0 = \text{odd}\,\pi \quad \therefore\ \phi = \dfrac{\text{odd}\,\pi}{3}$

$\therefore\ \phi = \pm\dfrac{\pi}{3}, \pi = \pm 60^\circ, 180^\circ$



$\therefore\ 1 + GH = 0$ at $f(s) = s^3(s+20) + K(s+4)^2 = 0$

when $s > 0$ $f(s) > 0$ always on the real line

$\therefore$ branch off $s=0$ is heading up

$\therefore$ intercept with Im-axis is at $j\omega$

$\therefore\ s^4 + Ks^2 + 16K + 20s^3 + 8Ks = f(s) = 0$ at $s = j\omega$

$\therefore\ \omega^4 - K\omega^2 + 16K + j\omega(8K - 20\omega^2) = 0 + j0$

$\because\ \omega \neq 0 \quad \therefore\ 8K - 20\omega^2 = 0 \quad \therefore\ K = \dfrac{5\omega^2}{2}$

& $\omega^4 - \left(\dfrac{5\omega^2}{2}\right)\omega^2 + 16\left(\dfrac{5\omega^2}{2}\right) = 0$

$\therefore\ 3\omega^2 - 80 = 0 \quad \therefore\ \omega = \sqrt{80/3} = 5.164$

$\therefore\ K = 66.67$

$\therefore$ Locus is shown for positive half and negative is a mirror image

$\therefore$ System is stable for $K > 66.67$ only.

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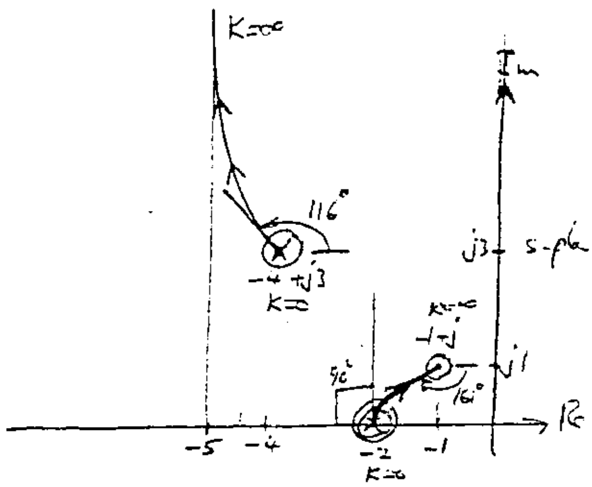
$\boxed{\dfrac{3C}{165}}$

$H = 1 \quad . \quad G = \dfrac{K(s^2+2s+2)}{(s^2+8s+25)(s^2+4s+4)}$

Locus is symmetric about Re-axis and upper half is considered only

$\therefore \; GH = \dfrac{K(s^2+2s+2)}{(s^2+8s+25)(s+2)^2}$.

poles at $-2, -2$ & $-4 \pm j3$ & zeros at $-1 \pm j$



Branches: 4

Difference $= 4 - 2 = 2$

$\therefore$ Two zeros at ∞

Asymptotes:

Angle $= \frac{2k-1}{2}\pi = \pm \pi/2$

Intercept $= \dfrac{-2-2-8-(-2)}{4-2} = -5$

No locus on real line.

Departure from $s=-2$ is at ϕ:

$\therefore -2\phi - 0 + 0 = \text{odd}\,\pi \quad \therefore \phi = \frac{\text{odd}\,\pi}{2} = \pm \pi/2$

Departure from $s = -4 + j3$ is at α:

$\therefore -\alpha - \frac{\pi}{2} - 2(\pi - \tan^{-1}\frac{3}{2}) + (\pi - \tan^{-1}\frac{2}{3}) + (\pi - \tan^{-1}\frac{4}{3}) = \text{odd}\,\pi \quad \therefore \alpha = \text{odd}\,\pi - \frac{\pi}{2} + 2\tan^{-1}\frac{3}{2} - \tan^{-1}\frac{2}{3} - \tan^{-1}(4/3)$

$\therefore \alpha = 115.8°$

Arrival into $s = -1 + j$ is at β:

$\therefore \beta + \frac{\pi}{2} - 2\tan^{-1}(1/1) + \tan^{-1}\frac{2}{3} - \tan^{-1}\frac{4}{3} = \text{odd}\,\pi \quad \therefore \beta = -160.6°$

$\therefore$ System is stable for any $K > 0$

$\boxed{\frac{3d}{165}}$ $H = 1$ & $G = \dfrac{K(s+4)(s+8)}{s(s+1)(s+12)(s^2+6s+25)}$

Utilize symmetry to get lower half.

$\therefore GH = \dfrac{K(s+4)(s+8)}{s(s+1)(s+12)(s^2+6s+25)}$

$f(s) = s(s+1)(s+12)(s^2+6s+25) + K(s+4)(s+8)$

$= s^5 + 19s^4 + 115s^3 + 347s^2 + 300s + K(s+4)(s+8)$

Poles: at $0, -1, -12, -3 \pm j4$

Zeros: at $-4, -8$

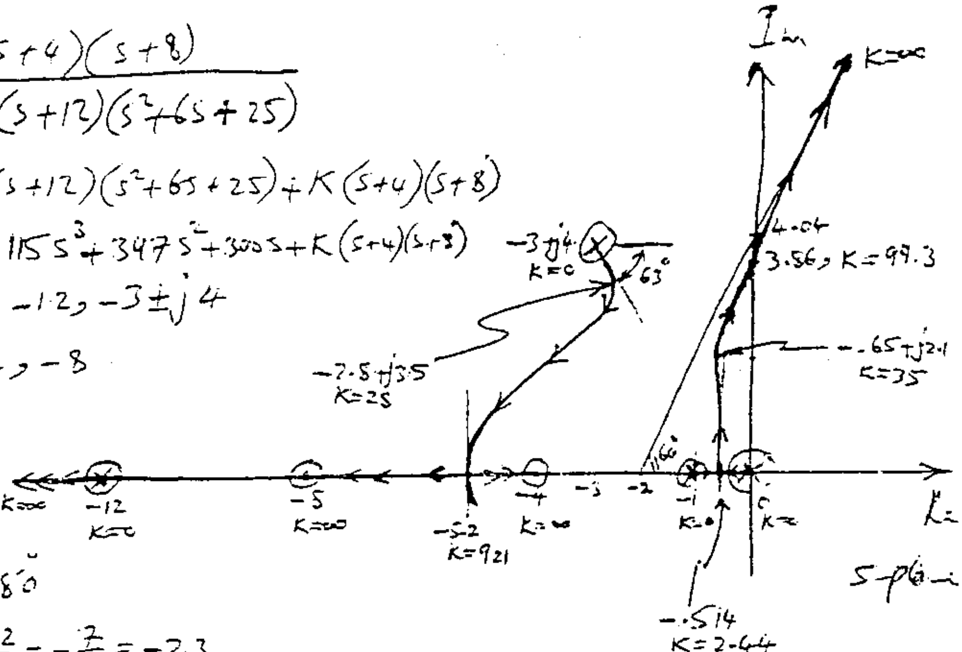
Branches: 5

3 zeros at ∞

Asymptote:

Angle: $\pm 60°, 180°$

intercept: $\frac{-19+12}{3} = -\frac{7}{3} = -2.3$



Departure from $-3+j4$ at α:

$\therefore -\alpha - 90° - (180° - \tan^{-1}\frac{4}{3}) - (180 - \tan^{-1}\frac{4}{2}) + \tan^{-1}\frac{4}{1} + \tan^{-1}\frac{4}{5} - \tan^{-1}\frac{4}{9} = \text{odd}\,(180°)$

$\therefore \alpha = -62.78°$

Breakaway point at x on $(-1, 0)$: $\therefore f(x) = f'(x) = 0 \therefore x = -.5136,\ K = 2.439$

Break-in point at y on $(-8, -4)$ $\therefore f(y) = f'(y) = 0 \therefore y = -5.176,\ K = 920.9$

Im-axis intercept at ω $\therefore f(j\omega) = 0 + j0 \therefore \omega = 3.859,\ K = 99.25$

Side of left loop at $f(s) = 0$, $Re\left(\frac{ds}{dK}\right) = 0$ $\therefore s = -2.831 + j3.545,\ K = 27.84$

Side of right loop at $f(s) = 0$, $Re(ds/dK) = 0$ $\therefore s = -.646 + j2.09,\ K = 35.0$

$\therefore$ System is Stable for $K \in [0, 99.3)$.

(Note: to know the direction of locus after departure assume $s = -3 + j4 + \epsilon e^{j\alpha}$

where $\alpha = -63° + \delta°$ $\therefore s = -3 + j4 + \epsilon\cos(63-\delta) - j\epsilon\sin(63-\delta) =$

$= -3 + \underbrace{\epsilon\cos 63° + \epsilon\delta\sin 63°}_{b} + j(4 \underbrace{- \epsilon\sin 63 + \epsilon\delta\cos 63°}_{a})$. Expand the angle relation by Taylor Series

$\therefore -\delta + \frac{3a-4b}{25} + \frac{2a-4b}{20} + \frac{a-4b}{17} + \frac{5a-4b}{41} - \frac{9a-4b}{52} = 0$

$\therefore -\delta + a(.228) + b(-.770) = 0 \implies -\delta + .228(.454\epsilon\delta - .891\epsilon) - .770(.454\epsilon + .891\epsilon\delta) = 0$

$\therefore \delta(-1 - .583\epsilon) - .553\epsilon = 0$ $\therefore -\delta - .553\epsilon = 0$ $\therefore \delta = -.553\epsilon < 0$ $\therefore$ heads left.

As a rule of thumb poles & zeros are like charges & in here, nearby -4 $\underbrace{\text{rat}}_{89}$

$\boxed{\frac{4d}{165}}$ $f(s) = s^4 + 4s^3 + (20+K)s^2 + 4Ks + 18K = 0$

$\therefore 1 + GH = 1 + \dfrac{K(s^2+4s+18)}{s^4+4s^3+20s^2} = 0$

$\therefore GH = \dfrac{K(s+3)(s+6)}{s^2(s^2+4s+20)}$

Locus is symmetric about Re-axis, and upper half is only considered

$\therefore$ poles at $0, 0, -2 \pm j4$ & zeros at -3 & -6

Branches : 4

Difference : 2

$\therefore$ Two zeros at ∞

Asymptotes

Angle $= \frac{2k+1}{2}\pi = \pm\pi/2$

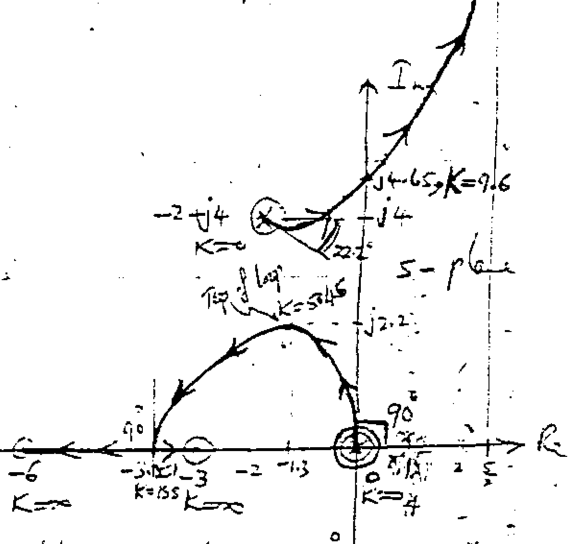
Intercept $= \frac{0-4-(-3-6)}{4-2} = \frac{5}{2}$

Departure from $s=0$ is at ϕ:

$\therefore -2\phi + 0 = \text{odd}\ \pi \ \therefore \phi = \pm\pi/2$

Departure from $s=-2+j4$ is at α:

$\therefore -\alpha - \frac{\pi}{2} - 2\left(\frac{\pi}{2} + \tan^{-1}\frac{2}{4}\right) + \tan^{-1}\frac{4}{1} + \tan^{-1}\frac{4}{4} = \text{odd}\ \pi \quad \therefore \alpha = -22.2^\circ$



$\because f(s) > 0$ for $s > 0$ & real $\therefore$ The locus will dive down then climb up without hitting the real axis. But it may hit $\text{Re}\,s = 5/2$ line at $\left(\frac{5}{2} \pm j\omega\right) = s$

If it hits $\therefore f\left(\frac{5}{2}+j\omega\right) = 0 \quad \therefore \left(\frac{5}{2}+j\omega\right)^4 + 4\left(\frac{5}{2}+j\omega\right)^3 + (20+K)\left(\frac{5}{2}+j\omega\right)^2 + 9K\left(\frac{5}{2}+j\omega\right) + 18K = 0$

$\therefore (5+2j\omega)^4 + 8(5+2j\omega)^3 + 4(20+K)(5+2j\omega)^2 + 72K(5+2j\omega) + 288K = 0$

$\therefore 625 + 16\omega^4 - 600\omega^2 + 1000 - 480\omega^2 + 4(20+K)(25-4\omega^2) + 360K + 288K = 0 \quad (1)$

& $1000\omega - 160\omega^3 - 64\omega^3 + 1200\omega + 80\omega(20+K) + 144K\omega = 0 \quad (2)$

$\because \omega \neq 0 \quad \therefore (2) \Rightarrow -224\omega^2 + 3800 + 224K = 0 \qquad \therefore K = \omega^2 - \frac{475}{28} \quad (3)$

(3) into (1) $\Rightarrow 1625 + 16\omega^4 - 1080\omega^2 + 4\left(20+\omega^2-\frac{475}{28}\right)(25-4\omega^2) + 648\left(\omega^2 - \frac{475}{28}\right) = 0$

$\therefore \omega^4(\cancel{16}-\cancel{16}) + \omega^2\left(-1080 + 100 - 16 \times \frac{55}{28} + 648\right) + \left(1625 + \frac{8500}{28} - \frac{648 \times 475}{28}\right) = 0$

$\therefore \omega^2 = \dfrac{-253800}{10656} = -ve \quad \therefore$ no intersection with $\text{Re}\,s = 5/2$

Im-axis intercept:

$\therefore f(j\omega) = 0 \quad \therefore +\omega^4 - (20+K)\omega^2 + 18K + j\omega(9K - 4\omega^2) = 0 + j0 \quad \because \omega \neq 0$

$\therefore K = 4\omega^2/9$ & $\omega^4 - \left(20 + \frac{4\omega^2}{9}\right)\omega^2 + 18\left(\frac{4\omega^2}{9}\right) = 0 \quad \therefore \frac{5\omega^4}{9} = 12\omega^2 \quad \therefore \omega = \sqrt{\frac{108}{5}} = 4.65$

$\therefore$ intersect at $s = j4.65$ where $K = 9.6$ — 92

Break-in point on real axis, at $s = x$: $\therefore f(x) = f'(x) = 0$

$\therefore f(x) = x^4 + 4x^3 + (20+K)x^2 + 9Kx + 18K = 0$ (1)

$\& \ f'(x) = 4x^3 + 12x^2 + 2(20+K)x + 9K = 0$ (2)

$\therefore K = -\dfrac{4x^3 + 12x^2 + 40x}{2x+9}$ (3)

$\therefore$ Using Newton–Raphson for root of $f(x)$ with the aid of (1) & (
and initial start of $x_i \in (-6, -3)$

$\therefore x = -3.851 \quad \& \quad K = 157.5$

Top of loop:

$\because \dfrac{ds}{dK}$ is real $\quad \therefore Im\,(ds/dK) = 0$

$\because s^4 + 4s^3 + (20+K)s^2 + 9Ks + 18K = 0$

$\therefore 4s^3\dfrac{ds}{dK} + 12s^2\dfrac{ds}{dK} + s^2 + 2s\dfrac{ds}{dK}(20+K) + 9s + 9K\dfrac{ds}{dK} + 18 = 0$

$\therefore \dfrac{ds}{dK} = -\dfrac{s^2 + 9s + 18}{4s^3 + 12s^2 + 2(20+K)s + 9K} = \text{real}$

putting $s = \sigma + j\omega$

$\therefore Im\left(-\dfrac{\sigma^2 + 2\omega\sigma j - \omega^2 + 9\sigma + 9j\omega + 18}{4\sigma^3 4j\omega^3 + 12\sigma^2 j\omega - 12\sigma\omega^2 + 12\sigma^2 12\omega^2 + 24\sigma\omega j + 2(20+K)(\sigma+j\omega) + \right.$

$\therefore Im\left(\dfrac{\sigma^2 - \omega^2 + 9\sigma + 18 + j\omega(9+2\sigma)}{4\sigma^3 12\sigma\omega^2 + 12\sigma^2 12\omega^2 + 2(20+K)\sigma + 9K + j\omega(-4\omega^2 + 12\sigma^2 + 24\sigma + 40 + \right.$

$\therefore (\sigma^2 - \omega^2 + 9\sigma + 18)\,\omega\,(-4\omega^2 + 12\sigma^2 + 24\sigma + 40 + 2K) = \omega(9+2\sigma)(4\sigma^3 - 12\sigma\omega^2 + 12\sigma^2 - 12\omega^2 + 2(20+K)\sigma + 9K)$

$\because \omega \neq 0 \quad \therefore$ The above equation can be used together with $f(\sigma + j\omega) = 0$ (3 equations) to solve for σ, ω & K (3 unknowns):

$\therefore \sigma = -1.299 \quad , \quad \omega = 2.159 \quad \& \ K = 5.451$

$\therefore$ System is Stable for $0 < K < 9.6$ i.e. $K \in [0, 9.6)$

$\frac{6d}{166}$ $H=1$, $G = \dfrac{K(s+24)}{s(s+5)(s+6)(s+10)}$

$\therefore GH = \dfrac{K(s+24)}{s(s+5)(s+6)(s+10)}$, Locus is symmetric & upper half is considered

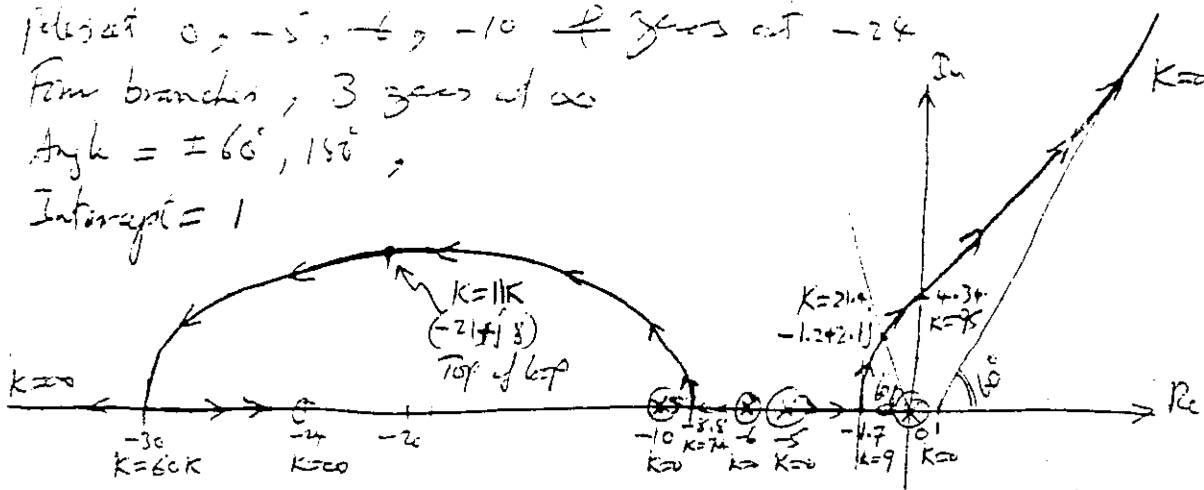
& $f(s) = s(s+5)(s+6)(s+10) + K(s+24) = 0$

poles at $0, -5, -6, -10$ & zeros at -24

Four branches, 3 zeros at ∞

Angle $= \pm 60^\circ, 180^\circ$,

Intercept $= 1$



Breakaway on real axis at $(-5,0)$ is x_1

$\therefore f(x_1) = f'(x_1) = 0 \implies$

$f'(s) = [s(s^2+11s+30)(s+10) + K(s+24)]' =$

$= [s^4 + 21s^3 + 140s^2 + 300s + Ks + 24K]' = 4s^3 + 63s^2 + 280s + 300 + K = 0$

$\therefore x_1 = -1.6551$ & $K = 8.983$

Breakaway on real axis at $(-10, -6)$ is x_2

$\therefore x_2 = -8.7619$ & $K = 7.397$ (top of loop at $f(s)=0$ & $\mathrm{Im}\left(\frac{ds}{dK}\right) = 0$ $\therefore s = -21+j8, K = 11K$)

Break-in on real axis at $(-\infty, -24)$ is x_3

$\therefore x_3 = -30.081$ & $K = 59996 \simeq 60K$

Locus doesn't intersect $\therefore$ nearby poles come to nearby zeros.

Intercept of branch heading to $\pm 60^\circ$ asymptotes is at $(0, j\omega)$

$\therefore \omega^4 - 140\omega^2 + 24K = 0$ & $-21\omega^2 + 300 + K = 0$ $\therefore K = 21\omega^2 - 300$

$\therefore \omega^4 - 140\omega^2 + 24(21\omega^2 - 300) = 0$ $\therefore \omega^2 = \dfrac{-364 \pm \sqrt{364^2 + 4(24)(300)}}{2} = 18.8084$

$\therefore \omega = 4.3369$, $K = 94.976$

$\therefore$ Stable for $K \in [0, 95)$

$\because \eta = 0.5 = \cos\phi$ $\therefore \frac{\omega}{\sigma} = \tan\phi = \tan\cos^{-1}\eta = \sqrt{3}$ where $s = -\sigma + j\omega$

$\therefore \omega = \sqrt{3}\,\sigma$ & $f(s) = f(-\sigma + j\omega) = 0$ 3 equations and 3 unknowns

OR $-120^\circ - \tan^{-1}\dfrac{\omega}{5-\sigma} - \tan^{-1}\dfrac{\omega}{6-\sigma} - \tan^{-1}\dfrac{\omega}{10-\sigma} + \tan^{-1}\dfrac{\omega}{24-\sigma} = 180^\circ$

$\therefore \sigma = 1.1969$ & $\omega = 2.0732$ & $K = 21.425$

$\therefore K = 21.425$ for which $s = -1.1969 + j2.0732$

$\frac{1C}{167}$ (a) $H = 0$ ∴ Closed Loop Gain $= \dfrac{AK(s+6)}{(s+36)(s^2+2s+5)}$

$f(s) = (s+36)(s^2+2s+5) + AK(s+6) = 0$

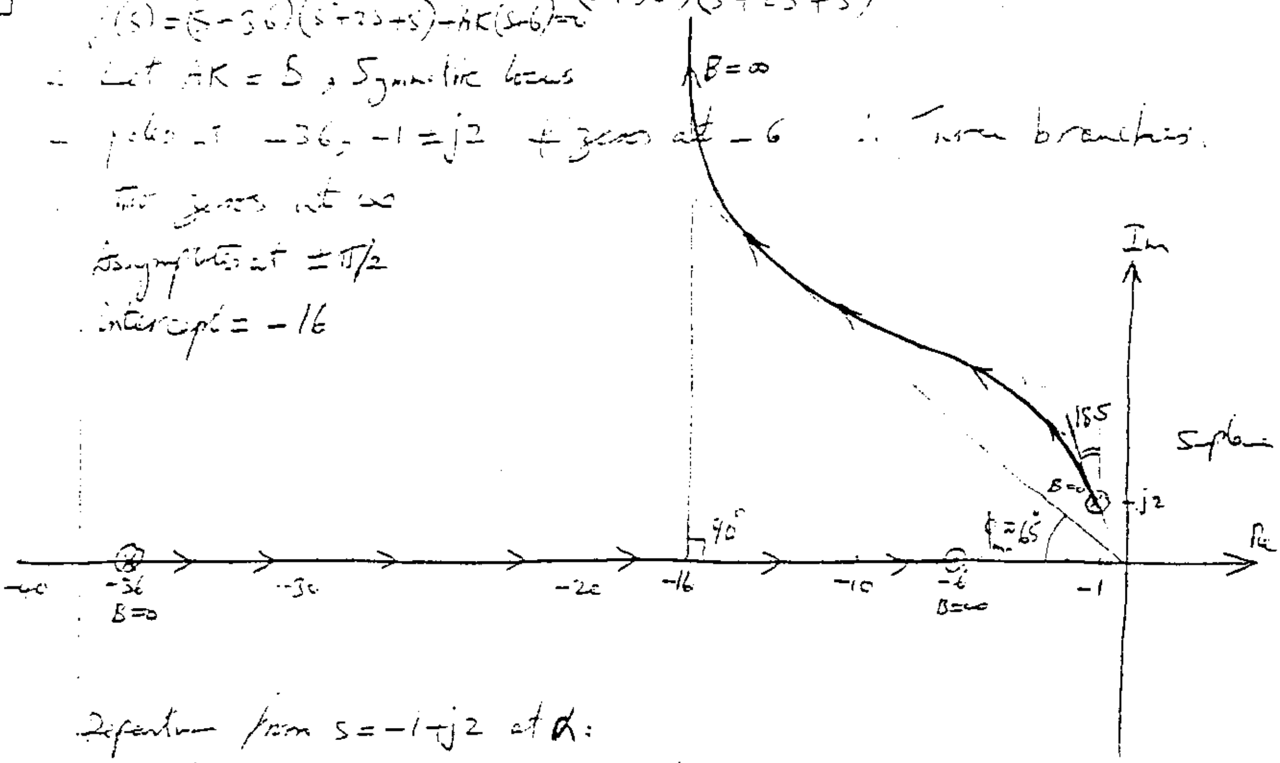
∴ Let $AK = B$, Symmetric locus

– poles at $-36, -1 \pm j2$ + zeros at -6 ∴ Three branches.

Two zeros at ∞

Asymptotes at $\pm\pi/2$

Intercept $= -16$



Departure from $s = -1 + j2$ at α:

$-\alpha - 90° + \tan^{-1}\frac{2}{5} - \tan^{-1}\frac{2}{35} = 180$ ∴ $\alpha = 108.53$

The specification of $\eta = .866 = \cos\phi \Rightarrow \phi = 30°$ seems to be impossible to meet by any value of $B > 0$.

To be sure, however, let's find maximum η possible to achieve

∴ $\eta_{max} = \cos\phi_{min}$ and this is just tangent to locus from below.

∴ $\underline{\angle ds/dB} = \pi - \phi_{min}$ where $s = -\sigma + j\omega$ & $\frac{\omega}{\sigma} = \tan\phi_{min}$

There are four unknowns: σ, ω, B & ϕ_{min} & four equations

$f(-\sigma + j\omega) = 0 + j0$ & $\angle ds/dB = \pi - \phi_{min}$ & $\frac{\omega}{\sigma} = \tan\phi_{min}$

Solved to give:

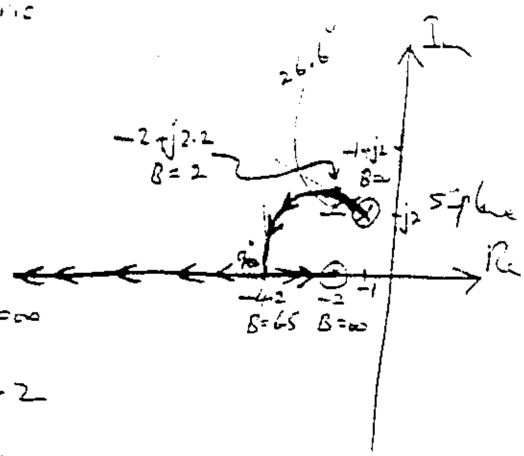
$\phi_{min} = 64.61°$, $s = -1.2878 + j2.7134$ & $B = 23.2595$

∴ It is impossible to have $\eta = 0.866$.

b) $H = .5s + 1 = \frac{s+2}{2}$ ∴ $GH\big|_{\text{inner loop}} = \frac{K(s+2)}{2(s^2+2s+5)}$

Let $K/2 = B$. Locus is symmetric

∴ $GH\big|_{\text{inner loop}} = \frac{B(s+2)}{(s^2+2s+5)}$



∴ $f(s) = s^2+2s+5+B(s+2) = 0$ $B=\infty$
Poles at $-1 \pm j2$ & zeros are at -2
Two branches, difference = one
∴ one asymptote at $\theta = \pi$
Break in on real line at $s \in (-\infty, -2)$
∴ $f(s) = f'(s) = 0$ ∴ $f'(s) = 2s+2+B = 0$ ∴ $B = -2(s+1)$
∴ $f(s) = s^2+2s+5+(-2)(s+1)(s+2) = s^2+2s+5-2s^2-6s-4 =$
$= -s^2-4s+1 = 0$ ∴ $s = \frac{4 \pm \sqrt{16+4}}{-2} = -2 \pm \sqrt{5} = -2-\sqrt{5}$
∴ $s = -4.2361$ & $B = 6.472$
Departure from $s = -1+j2$ at α:
∴ $-\alpha - 90^\circ + \tan^{-1} 2 = -180^\circ$ ∴ $\alpha = 153.4^\circ$
Top of loop:
∴ $f(s) = 0$ & $\text{Im}\,\frac{ds}{dB} = 0$ ∴ $\frac{dB}{ds} = -(2s+2+B)/(s+2)$
∴ $\frac{ds}{dB} = -(s+2)/(2s+2+B)$

∴ $B = 2$ & $s = -2+j\sqrt{5} = -2+j2.2361$

∴ When $K = 16$ ∴ $B = K/2 = 8$

∴ $f(s) = 0 \Rightarrow s = -3, -7$ ∴ roots are at $-3, -7$.

∴ The inner loop will now be $\frac{K}{s^2+2s+5+K(.5s+1)} = \frac{16}{(s+3)(s+7)}$

c) Use $H = 1$ & $G = \dfrac{A(s+6)}{s+36} \cdot \dfrac{16}{(s+3)(s+7)}$

$\therefore GH = \dfrac{16A(s+6)}{(s+3)(s+7)(s+36)}$ Let $16A = B$

Locus is symmetric & $f(s) = (s+3)(s+7)(s+36) + B(s+6) = 0$

poles at $-3, -7, -36$

zeros at -6

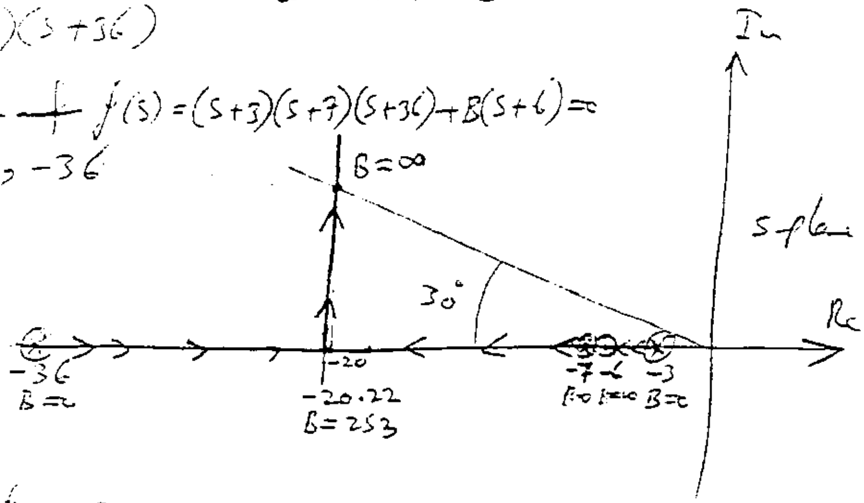
Three branches

difference 2

$\therefore$ Asymptotes:

angle $= \pm \pi/2$

intercept $= \dfrac{-3-7-36+6}{2} = -20$



Breakaway on $(-36, -7)$ is x:

$\therefore f(x) = f'(x) = 0$ $\therefore B = 252.6$ & $x = -20.222$

The branching point is very near to the assymptote-intercept.
(distance $= .222$)

$\therefore$ It is possible to achieve $\eta = 0.866$ now. ($\phi = 30°$)

$f(s) = 0$ & $\angle s = 180 - 30 = 150°$ (3 eq. & 3 unknowns).

$\therefore$ $s = -20.134 + j\,11.625$ & $B = 390.315$

$\therefore$ with $K = 16$ & $A = \dfrac{B}{16} = 24.395$ we can

get $\eta = .866$ from the indicated system.

$$\# \quad GH = \frac{K(s^2+4s+5)}{s^2(s+1)(s+3)} \quad , \text{ch. fn. } 1+GH=0 \Rightarrow$$

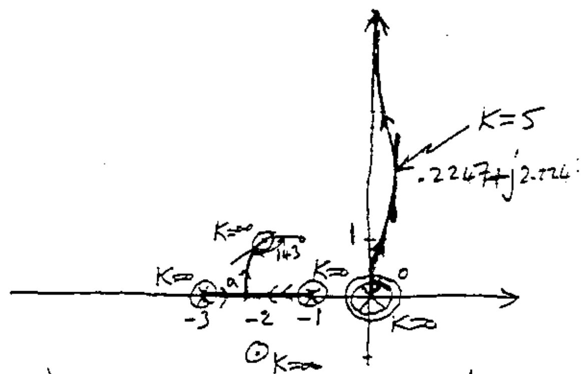
$\therefore$ Ch. fn. $f(s) = s^2(s+1)(s+3) + K(s^2+4s+5) = 0$

$\therefore$ 4 poles at $0, 0, -1, -3$

& 2 zeros at $-2 \pm j1$

$\therefore$ 2 Zeros are at ∞ radiating from $\frac{-4-(-4)}{2} = 0$ at $\pm\frac{\pi}{2}$

Utilizing symmetry, locus is as shown, where:



- breakaway point a is at:
  $f(a) = f'(a) = 0$ giving:
  $a = -2.224$ & $K = 4.474$
- arrival angle at zeros, α is:
  $2(180° - \tan^{-1}\frac{1}{2}) + (180° - \tan^{-1} 1) + \tan^{-1} 1 - (\alpha + 90°) = \text{odd} * 180°$

$\therefore$ $\alpha = -2\tan^{-1}\frac{1}{2} - 45° + 45° - 90 = -143.1°$

Always Unstable

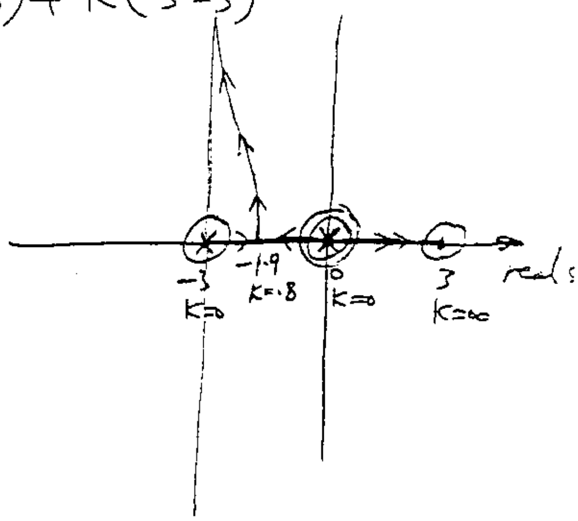
- departure angle at $s=0$, β is:
  $2\beta = \text{odd } \pi \Rightarrow \beta = \pm\pi/2$
- peak of branch at $Re\left(\frac{ds}{dK}\right) = 0$

$$\therefore \left(\frac{dK}{ds}\right) = -\frac{4s^3+12s^2+2(3+K)s+4K}{s^2+4s+5} = 0$$

$$\therefore Re\left(\frac{2s^3+6s^2+(3+K)s+2K}{s^2+4s+5}\right) = 0 \Rightarrow K=5 \text{ \& } s = .2247 \pm j2.2247$$

To plot Locus for $KGH(s) = \dfrac{K(s-3)}{s^2(s+3)}$

$\therefore\ f(s) = 1 + KGH(s) = 0 = s^2(s+3) + K(s-3)$



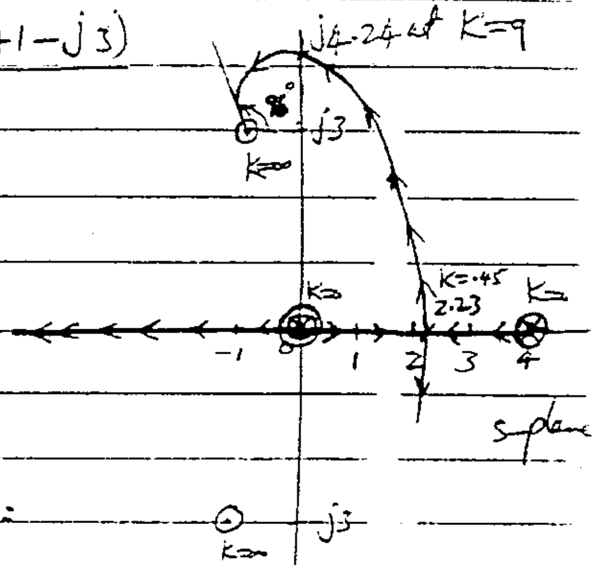
3 poles at $0, 0, -3$
1 zero at 3
no. of branches $= 3$
Symmetrical about real line
2 Assymptotes originating
from $\sigma = \dfrac{-3-3}{3-1} = -3$
at $\pm \pi/2$.
Break away at x:
$f(x) = f'(x) = 0$

$\therefore\ x = -1.8541$ & $K = 0.81153$

$\therefore$ System is always unstable due to RHS branch.

Root Locus for $GH = \dfrac{K(s^2+2s+10)}{s^2(s-4)}$

$\therefore GH = \dfrac{K(s+1+j3)(s+1-j3)}{s^2(s-4)}$



2 zeros at $-1 \pm j3$
3 poles at $0, 0, 4$
$\therefore$ 1 assymptotic zero at ∞
Utilizing symmetry;
$\therefore$ Locus is as shown, where:
– breakaway point a satisfies:
$f(a) = f'(a) = 0$ with:

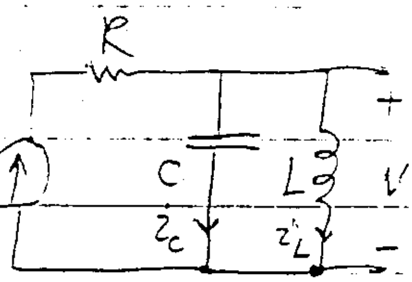
$f(s) = s^2(s-4) + K(s^2+2s+10) = 0$
& $f'(s) = s(3s-8) + 2K(s+1)$

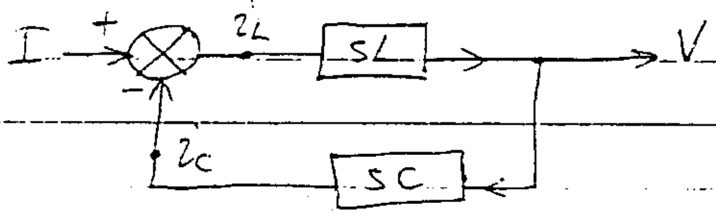
$\therefore a = 2.229$ at $K = .4529$
– vertical intercept at $j\omega$ with: $f(j\omega) = 0 \Rightarrow (j\omega)^3 + (K-4)(j\omega)^2 + 2Kj\omega + 10K = 0$
$\therefore -(K-4)\omega^2 + 10K = 0$ (1) & $-j\omega(-\omega^2 + 2K) = 0$ (2)
(2) $\Rightarrow K = \omega^2/2$ into (1) $\Rightarrow -\left(\frac{\omega^2}{2} - 4\right)\omega^2 + 5\omega^2 = 0$
$\therefore 5 = \frac{\omega^2}{2} - 4 \Rightarrow \omega^2 = 18 \Rightarrow \omega = 3\sqrt{2} = 4.243$ at $K=9$
– arrival angle, α, at complex zero is given by:

$180° - \tan^{-1}\frac{3}{5} + 2(180° - \tan^{-1}3) - [90° + \alpha] = 180°$ (odd)
$\therefore \alpha = 270° - \tan^{-1}.6 - 2\tan^{-1}3 = 95.91°$

$\therefore$ Plot is as shown with system stable for $K > 9$

To obtain the transfer function $\frac{V}{I}(s)$ of the given network by block diagram representation and reduction:





$$\therefore \quad \frac{V}{I} = \frac{sL}{1+s^2LC}$$

Ch. eq. of unity feedback system having:

$$G(s) = \frac{K(s^2+4s+13)}{(s-2)(s-4)(s+10)} \text{ is:}$$

$$f(s) = K(s^2+4s+13)+(s-2)(s-4)(s+10) = 0$$

$$\therefore f(s) = s^3+(4+K)s^2+4(K-13)s+13K+80 = 0$$

(a) ∴ The Routhian:

s^3: 1 $\quad 4(K-13)$

s^2: $4+K$ $\quad 13K+80$ $\Rightarrow 4+K>0$ (1)

s: $4K^2-49K-288$ $\quad 0$ $\Rightarrow 4K^2-49K-288>0$ (2)

1: $13K+80$ $\quad 0$ $\Rightarrow 13K+80>0$ (3)

$(2) \Rightarrow K = \frac{49 \pm \sqrt{49^2+4*4*288}}{2*4} = 16.590$ & -4.340 $\therefore K \in (-\infty, -4.34) \cup (16.59, \infty)$

$(1) \Rightarrow K \in (-4, \infty) \Rightarrow (1)$ & (2) gives $K \in (16.59, \infty)$

$(3) \Rightarrow K \in (-\frac{80}{13}, \infty) \Rightarrow (3)$ & (1) & (2) gives $K \in (16.59, \infty)$

∴ System is stable for $K > 16.59$

(b) System type is '0' & $K_p = \frac{13K}{80}$, $K_v = K_a = 0$

Hence, the system can cope with step only &

for $r(t) = 3t$ it will not cope.

(c) $e_{ss}(t) = W_0 \cdot r(t) + W_0' \cdot r'(t) + 0$, where $W = \frac{1}{1+G} = \frac{(s-2)(s-4)(s+10)}{f(s)}$

$\therefore e_{ss}(t) = \left.\frac{(s-2)(s-4)(s+10)}{f(s)}\right|_{s=0} * 3t + \left.\frac{-52f(0) - f'(0)*80}{f^2(s)}\right|_{s=0} * 3 =$

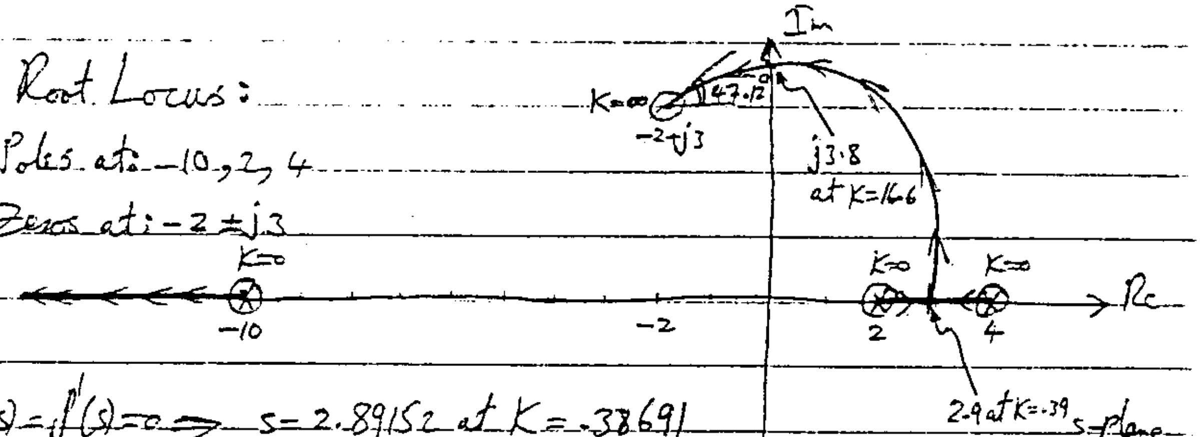
$= \frac{80}{80+13K} * 3t - \frac{52*(80+13K) + 4(K-13)*80}{(80+13K)^2} * 3$

$\therefore e_{ss}(t) = \frac{240}{80+13K} t - \frac{2988K}{(80+13K)^2}$ & $K > 16.59$

(d) Root Locus:

Poles at: $-10, 2, 4$

Zeros at: $-2 \pm j3$



$f(s) = f'(s) = 0 \Rightarrow s = 2.89152$ at $K = .38691$ | Symmetry Utilized

$f(j\omega) = 0 \Rightarrow -(4+K)\omega^2 + 13K + 80 = 0$ (1) &

$-\omega^2 + 4(K-13) = 0$ (2)

$\therefore K = 13 + \frac{\omega^2}{4}$ into (1) $\Rightarrow -\omega^2\left(4 + 13 + \frac{\omega^2}{4}\right) + 13\left(13 + \frac{\omega^2}{4}\right) + 80 = 0$

$\therefore -\frac{\omega^4}{4} - \frac{55}{4}\omega^2 + 249 = 0 \Rightarrow \omega^4 + 55\omega^2 - 996 = 0 \Rightarrow \omega^2 = 14.360$

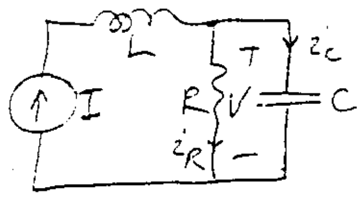
$\therefore K = 16.590$ & $\omega = 3.789$ $\therefore$ Stable for $K > 16.59$

Arrival angle α: $\alpha + 90° - \left(180° - \tan^{-1}\frac{3}{6} + 180° - \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{8}\right) = $ odd $180°$

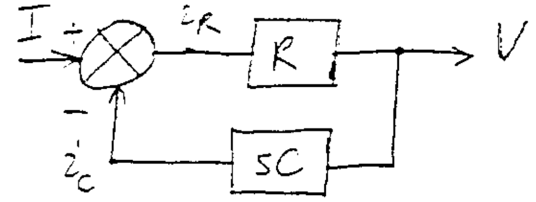
$\therefore \alpha = 47.12°$, Locus is as plotted.

100

To obtain the transfer function of $\frac{V}{I}$ by block diagram reduction of :



$\therefore$



$\therefore \quad \frac{V}{I} = \frac{R}{1 + sCR}.$

(pp 94) $H = 1$ & $G = \frac{K(s+2)}{s^2+2s+5} = \frac{K(s+2)}{(s+1)^2 + 2^2}$

$\therefore \quad \frac{c}{r} = \frac{G}{1+GH} = \frac{K(s+2)}{(s+1)^2 + K(s+2) + 4} = \frac{K(s+2)}{s^2 + (K+2)s + 2K+5}$

a) $\therefore$ dc gain $= \frac{c}{r}(0) = \frac{2K}{2K+5}$ & cut-off at -20 db/dec.

b) Routh $\Rightarrow f(s) = s^2 + (K+2)s + 2K+5 = 0$

$\therefore$

$s^2 : \quad 1 \qquad 2K+5$

$s : \quad K+2 \qquad \Rightarrow K+2 > 0 \quad \Rightarrow K \in (-2, \infty)$

$1 : \quad 2K+5 \qquad \Rightarrow 2K+5 > 0 \quad \Rightarrow K \in (-2.5, \infty)$

$\therefore$ For stability $K \in (-2, \infty) \cap (-2.5, \infty) \equiv (-2, \infty)$

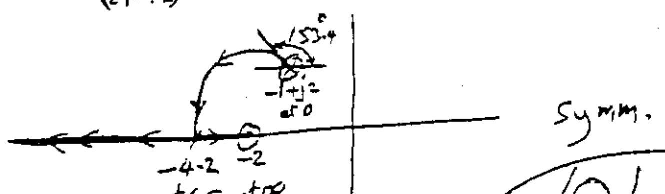
c) system type '0' $\Rightarrow K_p = \frac{2K}{5}$ & $K_v = K_a = 0$: $K > -2$

d) The system can not cope for $r(t) = t$,

and $e_{ss}(t) = \left. \frac{s^2+2s+5}{s^2+(K+2)s+2K+5} \right|_0 \cdot t + \frac{2(2K+5) - (K+2)5}{(2K+5)^2} = \frac{5t}{2K+5} - \frac{K}{(2K+5)^2} \xrightarrow{} \infty$ as $t \to \infty$.

e) RL as shown:

always stable for $K > 0$



1b / 193 Since ZIC

$$\therefore (s+1)c = 3r \qquad \therefore \frac{c}{r}(s) = \frac{3}{1+s}$$

$$\therefore \frac{c}{r}(j\omega) = \frac{3}{1+j\omega} = \frac{3}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$$

2c / 193

$$\frac{c}{r}(s) = W(s) = \frac{50}{(s^2+s+2)(s^2+6s+25)}$$

$$\therefore \frac{c}{r}(j\omega) = \frac{50}{(2-\omega^2+j\omega)(25-\omega^2+j6\omega)} = \frac{50 \angle -\tan^{-1}\left(\frac{\omega}{2-\omega^2}\right) - \tan^{-1}\left(\frac{6\omega}{25-\omega^2}\right)}{\sqrt{[(2-\omega^2)^2+\omega^2]\cdot[(25-\omega^2)^2+36\omega^2]}}$$

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$$W(j\omega) = \frac{40}{(2+j\omega)^2(10-\omega^2+j3\omega)} \qquad \left[\text{since } W(s) = \frac{40}{(s+2)^2(s^2+3s+10)}\right]$$

$$= \frac{40 \angle -2\tan^{-1}(\omega/2) - \tan^{-1}\left(\frac{3\omega}{10-\omega^2}\right)}{(4+\omega^2)\sqrt{(10-\omega^2)^2+9\omega^2}}$$

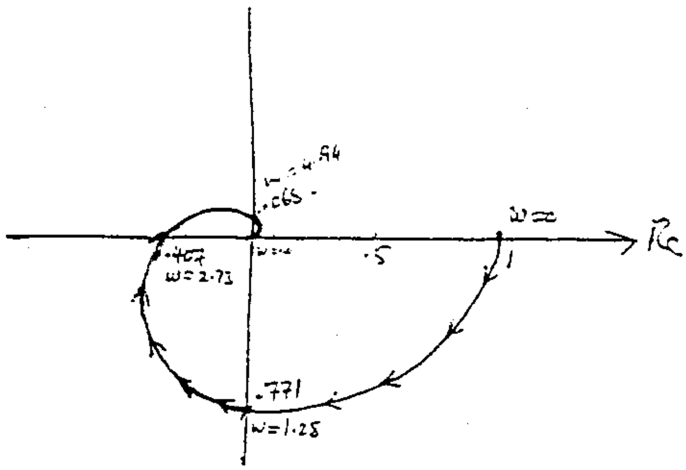
$$\therefore \text{Magnitude } M(j\omega) = \frac{40}{(4+\omega^2)\sqrt{(10-\omega^2)^2+9\omega^2}}$$

$$\text{\& Angle } \phi(j\omega) = -2\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{3\omega}{10-\omega^2}\right)$$

Substituting for $\omega \in [0, \infty)$ in the above two expressions, we'll get the polar plot that starts when $\omega = 0$ at $1\angle 0$ and ends at $\omega = \infty$ at the origin. This plot is the image of the positive imaginary axis of the s-plane under the transformation $W(s)$. The traditional way to do the plot is to draw point-by-point.

... more slowly for $\omega \in [0,10]$ & faster afterwards as follows:

ω	M	$\phi(^\circ)$
0	1.00	0.00
.1	.995	−7.44
.3	.983	−22.3
.5	.954	−36.8
.7	.915	−51.0
1	.843	−71.6
1.2	.792	−84.7
1.25	.771	−90
1.4	.740	−97.6
1.6	.689	−110
1.8	.639	−123
2.0	.589	−135
2.3	.515	−154
2.7	.415	−178
2.73	.407	−180
3.0	.340	−196
3.5	.229	−223
4.0	.149	−243
4.5	.097	−259
4.94	.063	−270
5.0	.065	−271
5.5	.045	−281
6	.032	−288
7	.017	−300
8	.010	−308
9	.006	−314
10	.004	−319
20	3E-4	−340
30	5E-5	−347
50	6E-6	−352
70	2E-6	−354
100	4E-7	−356
1000	4E-11	−360
10000	4E-15	−360



Polar Plot of $W(j\omega)$

$\boxed{\frac{9b}{196}}$

$$s^3+(K-4)s^2+2(K+2)s+K=0$$

$$\therefore\ s^3-4s^2+4s+K(s^2+2s+1)=f(s)=0$$

$$\therefore\ 1+\frac{K(s+1)^2}{s(s-2)^2}=0$$

$$\therefore \text{ for } K=1,\ GH=W(s)=\frac{(s+1)^2}{s(s-2)^2}$$

$$\therefore\ W(j\omega)=\frac{(j\omega+1)^2}{j\omega(j\omega-2)^2}=\frac{1+\omega^2}{\omega(4+\omega^2)}\angle 2\tan^{-1}\omega-90^\circ-2\left(180-\tan^{-1}\left(\tfrac{\omega}{2}\right)\right)=$$

$$=\frac{\omega^2+1}{\omega(\omega^2+4)}\angle 2\tan^{-1}\omega+2\tan^{-1}(\omega/2)-90^\circ=M\angle\phi^\circ$$

$\therefore\ \omega$	M	$\phi\,(^\circ)$
0	∞	-90.0
.001	250	-89.8
.01	25.0	-88.3
.1	2.52	-72.9
.2	1.29	-56.0
.3	.888	-39.5
.4	.697	-23.8
.5	.558	-8.80
.56	.544	Zero
.6	.520	$+5.33$
.7	.474	$+18.6$
.8	.442	$+30.9$
.9	.418	$+42.4$
1.0	.400	$+53.1$
1.3	.364	$+80.9$
1.4	.355	90
1.7	.332	$+110$
2.0	.313	$+127$
2.5	.283	$+149$
3.0	.256	$+165$
3.5616	.2303	$+180$
4.0	.213	$+189$
6.0	.154	$+214$
10.0	.097	$+236$
100.0	1E-2	$+267$

(short steps in $[0,10]$ then long)

$\therefore\ \phi_{\omega\to 0}=-90\quad\therefore$ Assymptote // Imax

$\therefore$ Intercept at $\lim_{\omega\to 0}\operatorname{Re} W=$

$$=\lim_{\omega\to 0}\frac{\omega^2+1}{\omega(\omega^2+4)}\cos\left[2\tan^{-1}\omega+2\tan^{-1}\omega/2)-90^\circ\right]=$$

$$=\lim_{\omega\to 0}\frac{\omega^2+1}{\omega(\omega^2+4)}\cdot\sin(2\tan^{-1}\omega+2\tan^{-1}\omega/2)$$

$$=\lim_{\omega\to 0}\frac{1}{4\omega}\cdot(2)(\tan^{-1}\omega+\tan^{-1}(\omega/2))$$

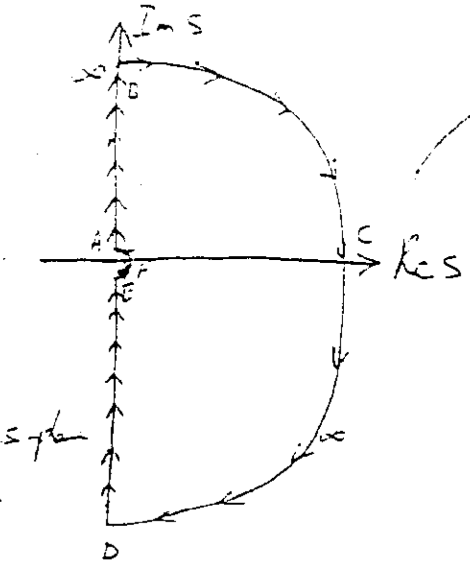
$$=\lim_{\omega\to 0}\frac{1}{2\omega}(\omega+\omega/2)=3/4$$

The polar plot for $\omega<0$ is same as for $\omega>0$ but mirror immage about the real axis, since M is just same but $\phi_{\omega<0}$ is $-\phi_{\omega>0}$

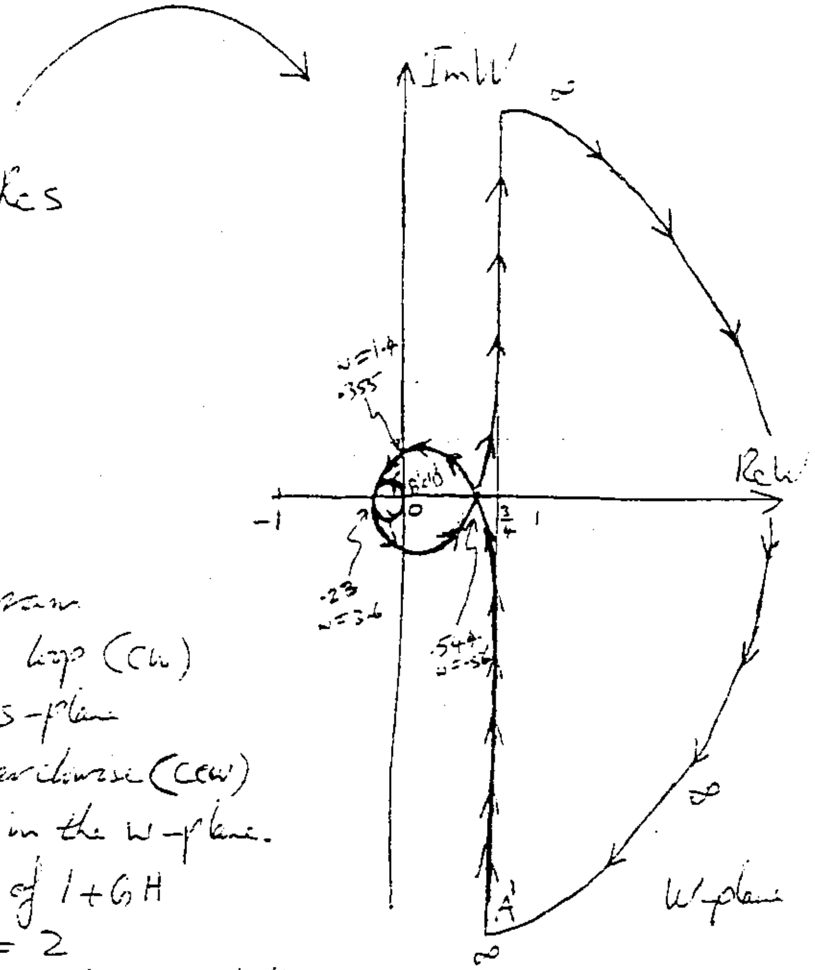
The plot near $\omega=0$ is considered to be the immage of right half ϵ circle $\therefore |s|=\epsilon$ $\angle s\in[-90^\circ,90^\circ]$. Using symmetry consider only $|s|=\epsilon$, $\angle s\in[0,90^\circ]$ $\therefore W(s)=\frac{1}{4s}=\frac{1}{4\epsilon}\angle{-\angle s}$

$\therefore |W|=\infty$ & $\angle W\in[0,-90^\circ]$ ... [illegible]

The maps are shown below



$$W' = W'(s) = \frac{(s+1)^2}{s(s-2)^2}$$



It is seen from diagram that one clockwise loop (CW) about the right half s-plane results at zero counterclockwise (CCW) loops about the -1 in the W-plane.

Hence, $0 = n_z - n_p$ of $1+GH$

∴ $n_p = n_z \Rightarrow n_z = n_p = 2$

here n_z no of zeros of $(1+GH)$ in right half s-plane, n_p no of poles there

∴ $1+GH$ has two right half plane roots, hence system is unstable

To make it stable we must have as many loops about -1 in the CCW direction as n_p i.e. 2

Increasing K from unity will magnify the whole W plot by K, hence, when K becomes $\frac{1}{.2303} = 4.34233$ the side of the loop will hit the -1 point of the W-plane. As K becomes slightly larger the -1 will be encircled CCW twice, hence becoming stable

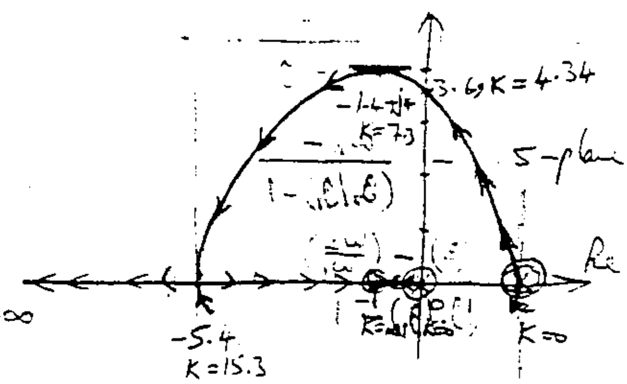
∴ $K > 4.34233$ gives stable system.

∴ The system is stable for $K \in (4.34233, \infty)$

Verifying with a root locus sketch:

$GH = \frac{K(s+1)^2}{s(s-2)^2}$. $F(s) = s(s-2)^2 + K(s+1)^2 = 0$

Symmetry gives negative half.
Positive half is considered.
poles at 0, +2, +2
zeros at −1, −1
Three branches.
One zero at ∞
Break-in at (−∞, −1) is at x



$\therefore f(x) = f'(x) = 0$

$\therefore x = -5.3723$ & $K = 15.274$

Angle of departure from s=2, θ

$\therefore -2\theta - 0 - 0 = \pi$ $\therefore \theta = \pm\pi/2$

Top of loop

$\therefore f(s) = 0$ & $\mathrm{Im}\left(\frac{ds}{dK}\right) = 0$ where $\frac{ds}{dK} = \frac{-(s+1)^2}{3s^2 - 8s + 4 + 2K(s+1)}$

$\therefore s = -1.434 + j3.913$

& $K = 7.2867$

Axis-intercept:

$f(j\omega) = 0$ $\therefore$ $j\omega(j\omega - 2)^2 + K(j\omega + 1)^2 = 0$

$\therefore j\omega(-\omega^2 + 4 - 4j\omega) + K(-\omega^2 + 1 + 2j\omega) = 0$

$\therefore j\omega(4 - \omega^2 + 2K) + 4\omega^2 + K(1 - \omega^2) = 0$ $\omega \neq 0$

$\therefore 4 - \omega^2 + 2K = 0 \Rightarrow \omega^2 = 4 + 2K$

$4\omega^2 + K(1-\omega^2) = 0 \Rightarrow 4(4+2K) + K(1-4-2K) = 0$

$\therefore -2K^2 + 5K + 16 = 0$ $\therefore K = \frac{-5 \pm \sqrt{25+128}}{-4} = \frac{+5 + \sqrt{153}}{4} = 4.34233$

$\therefore \omega^2 = 4 + 2K = 12.68$ $\therefore \omega = 3.562$

∴ System is stable for $K > 4.34233$.

∴ OK, both ways confirm that for stability $K > 4.34233$

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$\therefore GH = \dfrac{K}{s(s+3)(s+10)} = W(s)$. Consider $K=1$ $\left(f(s) = s^3 + 13s^2 + 30s + K\right)$

Routh:

s^3	1	30	
s^2	13	K	
s	$390-K$	0	$\therefore K < 390$
1	K		$\therefore K > 0$

$\therefore K \in (0, 390)$ for stability.

$\therefore GH(j\omega) = \dfrac{1}{j\omega(j\omega+3)(j\omega+10)} = M \angle \phi$

$\therefore M = \dfrac{1}{\omega\sqrt{(\omega^2+9)(\omega^2+100)}}$

$\& \ \phi = -90° - \tan^{-1}\frac{\omega}{3} - \tan^{-1}\frac{\omega}{10}$

ω	M	ϕ (°)
.001	$33.\bar{3}$	-90.02
.01	$3.\bar{3}$	-90.25
.0333	1.0	-90.83
.1	.33	-92.5
1	3.15E-2	-114.1
3	7.53E-3	-151.7
5	3.07E-3	-175.6
5.477	2.56E-3	-180
7	1.54E-3	-191.8
8	1.14E-3	-198.1
9	8.71E-4	-203.6
10	6.77E-4	-208.3
12	4.31E-4	-216.2
14	2.9E-4	-222.4
20	1.11E-4	-234.9
40	1.5E-5	-251.7
100	4.95E-7	-262.6
1000	1E-9	-269.3
∞	0	-270.0

Nyquist plot is symmetric w.r.t Re ax.

$\therefore \phi_{\omega=0} = -90°$ $\therefore$ Asymp. // Im axis

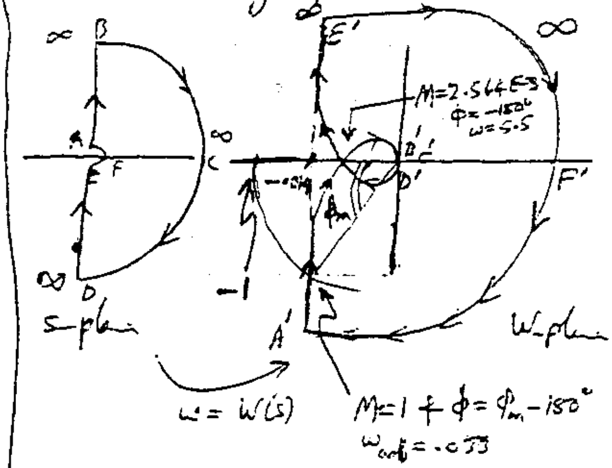
$\therefore$ intercept at $\lim_{\omega\to 0} \mathrm{Re}\, GH(j\omega) =$

$\lim_{\omega\to 0} \dfrac{1}{\omega\sqrt{(\omega^2+9)(\omega^2+100)}} \cos\left(90 + \tan^{-1}\frac{\omega}{3} + \tan^{-1}\frac{\omega}{10}\right) =$

$= \dfrac{1}{\omega\sqrt{900}}\left[-\sin\left(\frac{\omega}{3} + \frac{\omega}{10}\right)\right] = \dfrac{-\frac{\omega}{30}(10+3)}{\omega * 30}$

$= -13/900 = -.014$

and as you go around 0 with $s = \epsilon e^{j\theta}$, $\theta \in [0, 90]$ $\therefore$ plot goes at ∞ from angle $\phi = 0 \to -90°$.



No. of poles on RHS = 0 $\therefore$ no. of loops (around -1) $= N_z^{\circ} - N_p^{\circ} = 0$ for stability

$\therefore$ System is stable till K is $= \dfrac{1}{2.564E-3} = 390$ after which it is unstable

$\therefore$ Stable at $K \in [0, 390)$

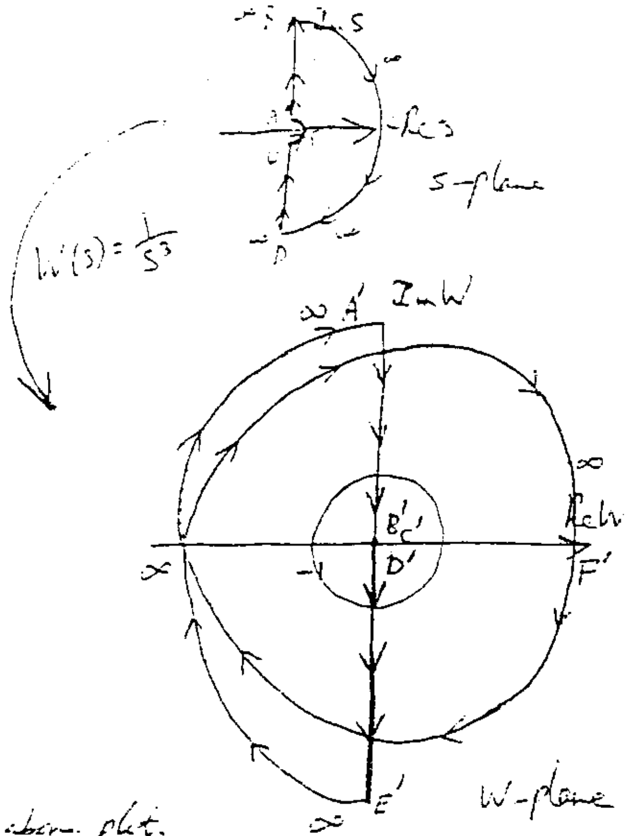
At $K=1$ gain margin $= 390$ & $\omega_{-180} = 5.5$ & phase margin $= 89.17$ & $\omega_{crit} = .033$

$\frac{1Ce}{196}$ Take $K = 1$

$\therefore GH = \frac{(s+1)^2}{s^3(s+1)^2} = \frac{1}{s^3} = W(s)$

$\therefore W(s) = \frac{1}{s^3}$

$\therefore W(j\omega) = \frac{1}{\omega^3} \angle -270^\circ$



when $\omega \in (0, \infty)$

$\therefore |W(j\omega)| \in (\infty, 0)$

& $\angle W(j\omega) = -270^\circ$

for region near zero

$\therefore s = \epsilon \angle \theta, \ \theta \in [0, 90^\circ]$

$\therefore |W| = \infty, \ \angle W \in [0, -270^\circ]$

The symmetry is used to give the above plot.

Asymptote: // Im axis

$\therefore \lim_{\omega \to 0} \operatorname{Re} W = \lim_{\omega \to 0} \frac{1}{\omega^3}\cos(-270^\circ) = \lim_{\omega \to 0} \frac{0}{\omega^3} = 0$ $\therefore$ The Im axis is the assymptote

$\therefore$ System is unstable since $2 = n_Z - 0 \Rightarrow$ Two right-half-plane roots for any K. No phase margine & no gain margine.

(Note: Checking by root locus

$\therefore GH = \frac{K}{s^3}$, $f(s) = s^3 + K$

poles: 0, 0, 0

roots: none

assymptotes three:

Angle: $\pm 60^\circ, 180^\circ$

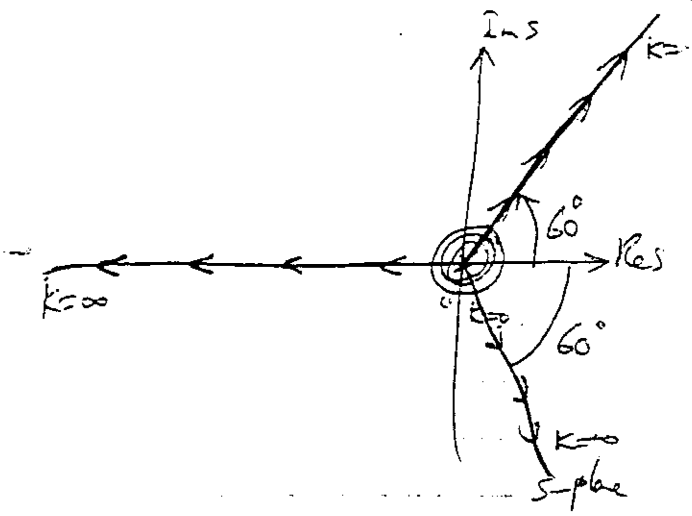
Intercept $\frac{0-0}{3} = 0$

departure from pole of 0 at θ:

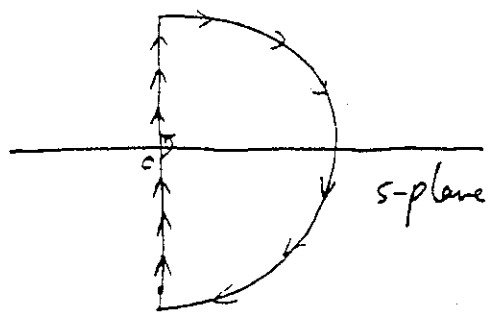
$-3\theta = \pm \pi$ $\therefore \theta = \pm 60^\circ, 180^\circ$

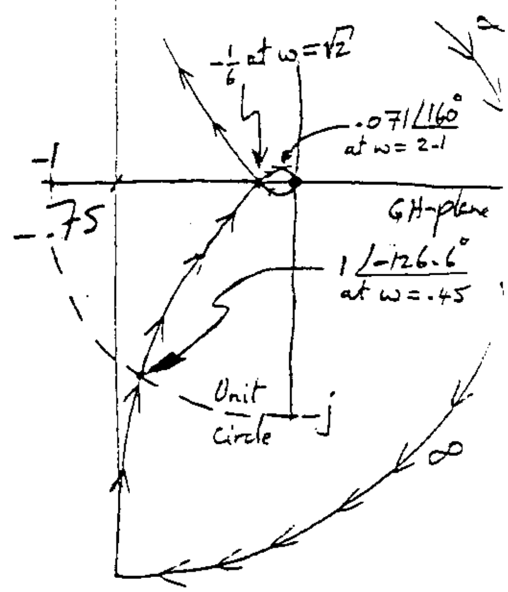
$\therefore$ Root coincide with assymptotes.

$\therefore$ Unstable for any $K > 0$ $\therefore$ OK)



$GH(s) = \dfrac{K}{s(s+1)(s+2)}$. Taking $K=1$ and solving by Nyquist:





The right semi-circle at ∞ goes to **zero** on GH-plane, whereas that at $s \to 0$ maps into an ∞ $\frac{1}{2}$circle at 4th & 1st quadrants of GH-plane. The immaginary axis maps into rest of picture which utilizes symmetry. Direction of contour is as indicated.

Special points are:

- Assymptote parallel to Im-axis is at $Re\, GH = \lim_{s \to j\epsilon \to 0} Re \dfrac{1}{s(s+1)(s+2)}$
$= \underset{\epsilon \to 0}{Re} \dfrac{1}{j\epsilon(j\epsilon+1)(j\epsilon+2)} = \underset{\epsilon \to 0}{Re} \dfrac{(j\epsilon-1)(j\epsilon-2)}{j\epsilon * 2^2} =$
$= \underset{\epsilon \to 0}{Re} \dfrac{2-3j\epsilon}{4j\epsilon} = -.75$

- Vertical intercepts at $\underset{s=j\omega}{Re}\, GH = 0 \Rightarrow Re. \dfrac{1}{j\omega(1+j\omega)(2+j\omega)} = 0$
$\Rightarrow Re\left[j\omega(1+j\omega)(2+j\omega)\right] = 0 \Rightarrow -3\omega^2 = 0$, $\because \omega \neq 0 \therefore$ None.

- Horizontal intercept at $\underset{s=j\omega}{Im}\, GH = 0 \Rightarrow Im\left[(j\omega)(1+j\omega)(2+j\omega)\right] = 0$
$\Rightarrow 2-\omega^2 = 0 \Rightarrow \omega = \sqrt{2} \xrightarrow{s=j\omega} GH(s=j\sqrt{2}) = -1/6$

- Assymptote intercept at $\underset{s=j\omega}{Re}\, GH = -.75 \Rightarrow Re\left[j\omega(2-\omega^2+3j\omega)\right]^{-1} = -\dfrac{3}{4}$
$\therefore \dfrac{-3}{4} = \dfrac{-3\omega^2}{9\omega^4+(2-\omega^2)^2\omega^2} \Rightarrow \omega \in \{\phi\} \therefore$ None.

- Intercept with unit circle at $|GH|_{s=j\omega} = 1 \Rightarrow \omega = 0.4457$ & $GH = 1\angle -126.6°$

- Top of loop at $Im \dfrac{dGH}{d\omega} = 0 \Rightarrow \omega = 2.095 \Rightarrow GH = 7.101 \times 10^{-2} \angle 159.2°$

- Side of loop at $Re\, dGH/d\omega = 0 \Rightarrow Re\, j\frac{d}{ds}\frac{1}{s^3+3s^2+2s} = 0$

$$\Rightarrow 0 = Im\,\frac{-3s^2-6s-2}{(s^3+3s^2+2s)^2} \Rightarrow Im\left[\frac{3\omega^2-6j\omega-2}{-\omega^6+9\omega^4-4\omega^2+4\omega^4+6j\omega^5-12j\omega^3}\right]=0$$

$$\therefore\ Im\left\{(3\omega^2-2-6j\omega)(-\omega^6+13\omega^4-4\omega^2-6j\omega^3(\omega^2-2))\right\}=0$$

$$\therefore\ (3\omega^2-2)6\omega^3(\omega^2-2)+6\omega(-\omega^6+13\omega^4-4\omega^2)=0 \Rightarrow \because \omega \neq 0 \quad \div\omega$$

$$\therefore -\omega^6+13\omega^4-4\omega^2+3\omega^6-8\omega^4+4\omega^2=0 \qquad \div\omega^2 \Rightarrow$$

$$2\omega^4+5\omega^2=0 \Rightarrow 2\omega^2=-5 \quad \therefore \text{None.}$$

- gain margin $= \frac{1}{1/6} = 6$
- phase margin $= 180^\circ + (-126.6^\circ) = 53.4^\circ$.
- Hence, for system stability:

$\therefore$ No of CW rotations of GH due to 1 CW rotation about RH s-pl.

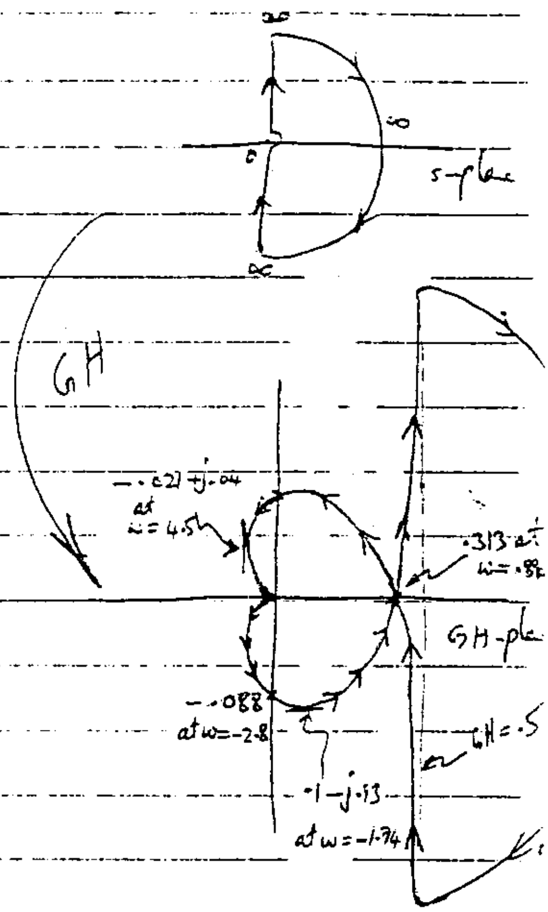
$=$ No. of ~~unstable~~ zero^0 - poles ~~at RH~~^{0} s-plane $= 0$

$\therefore$ K must be < 6 $\quad\therefore K \in (0, 6)$

At $K > 6$ two unstable poles of system (zeros of ch. eq) emerge.

At $K < 0$ one unstable pole of system emerges.

$GH = \dfrac{K(s+1)}{s(s-2)^2}$. To check stability take $K=1$ and plot the immage of a clockwise contour around RH s-plane



∴ s	$\lvert GH\rvert$	$\angle GH$ °
∞	0	—
$-j10^4$	10^{-8}	-179.97
$-j10^3$	10^{-6}	-179.7
$-j10^2$	10^{-4}	-177.1
$-j10$	10^{-2}	-151.7
$-j5$	.035	-125.1
$-j3$	.081	-94.2
$-j2.82843$	.088	-90.0
$-j2$	.140	-63.4
$-j1.5$	.192	-40.0
$-j1$	.283	-8.13
$-j.9$	.311	$-.44$
$-j.89442$	.313	0.000
$-j.8$	.345	7.74
$-j.7$	.388	16.43
$-j.6$	.446	25.64
$-j.5$	.526	35.4
$-j.4$	.647	45.6
$-j.3$	.851	56.2
$-j.2$	1.26	67.3
$-j.1$	2.51	78.6
$-j.05$	5.00	84.3
$-j.01$	25.0	88.9
$-j.001$	250	89.9

∴ Immage is plotted as shown.

— Vertical assymptotes at $GH =$

$$= \operatorname{Re}_{s\to j\epsilon\to 0} \frac{s+1}{s(s-2)^2} = \operatorname{Re}_{\epsilon\to 0} \frac{1+j\epsilon}{j\epsilon(2-j\epsilon)^2}$$

$$= \operatorname{Re}_{\epsilon\to 0} \frac{(1+j\epsilon)(4+4j\epsilon)}{j\epsilon(4-4j\epsilon)(4+4j\epsilon)} = \operatorname{Re}_{\epsilon\to 0} \frac{4+8j\epsilon}{j\epsilon\, 16} = 0.5$$

— Side of loop at $\omega = \pm 4.473$ & $GH = -.0208 \pm j.0373$

///

- Top of loop at $\omega = \pm 1.738$ & $G.H = .1010 \pm j\,.1296$

For stability, the image must rotate about:

a) (-1) for positive K 2 CCW rotations; or

b) $(+1)$ for negative K 2 CCW rotations.

As no magnification will result in this, then the system is always unstable for any K

It would have 2 unstable roots for $K > 0$, and for $K < 0$ it would have:

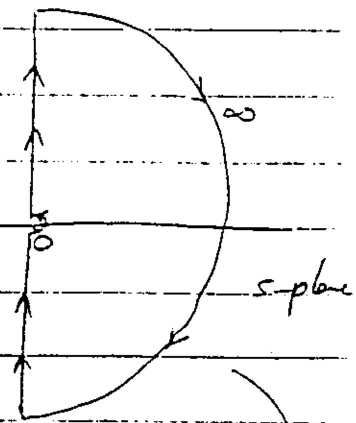
a) one unstable root, when $K < -\frac{1}{-.313} = -3.2$

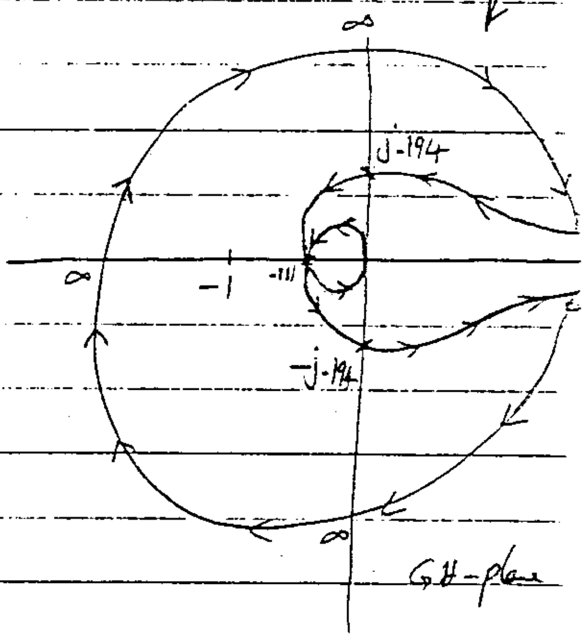
b) three unstable roots, when $K \in (-3.2, 0)$

Gain & Phase margins are meaningless, here.

Nyquist Polar Plot for $GH = \dfrac{s^2+2s+10}{s^2(s-4)}$

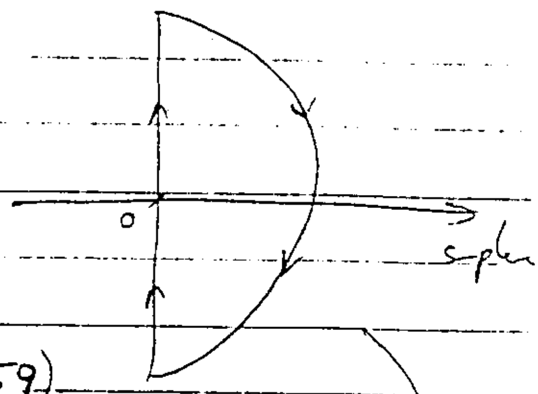
$j\omega$	$GH(j\omega)$
$-j\infty$	0
$-j10^3$	$.001 \angle 90.3^\circ$
$-j10$	$.086 \angle 124.3^\circ$
$-j8$	$.098 \angle 133.1^\circ$
$-j6$	$.1105 \angle 148.5^\circ$
$-j5$	$.1126 \angle 162.3^\circ$
$-j4.5$	$.1119 \angle 172.9^\circ$
$-j4.24264$	$.1111 \angle 180^\circ$
$-j4$	$.1105 \angle -171.9^\circ$
$-j3$	$.135 \angle -117.4^\circ$
$-j2.58199$	$.194 \angle -90^\circ$
$-j2$	$.403 \angle -60^\circ$
$-j1.41421$	$1.000 \angle -38.9^\circ$
$-j1$	$2.236 \angle -26.6^\circ$
$-j.8$	$3.637 \angle -21.0^\circ$
$-j.6$	$6.67 \angle -15.6^\circ$
$-j.4$	$15.35 \angle -10.4^\circ$
$-j.2$	$62.2 \angle -5.2^\circ$
$-j.1$	$250 \angle -2.6^\circ$
$-j.01$	$25000 \angle -.26^\circ$





The quarter circle at 0 results with a CW semi-circle at GH-plane due to s^2 term. Utilizing symmetry, the shown plot is obtained. System is stable for no of CW rotations about -1 = 0 − 1 i.e CCW rotation about the −1 point should be 1 ∴ K must be higher than $\frac{1}{.1111} = 9$

Hence, plot is as shown:



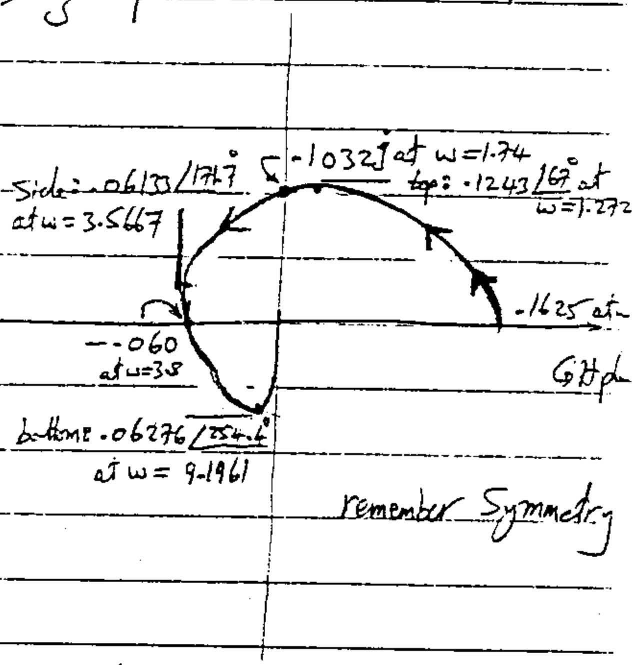
This gives $n_z - n_p = 0$

$\therefore n_z = n_p = 2$ $\therefore$ System has two unstable roots for $K=1$.

In fact it has 2 unstable roots for $K \in \left(0, \frac{1}{.060277}\right) \equiv (0, 16.59)$

For $K > 16.59$: $n_z - n_p = -2 \Rightarrow n_z = n_p - 2 = 0$

Hence, system becomes stable for $K > 16.59$ (i.e. OK)



remember Symmetry

As for $K < 0$; system has again 2 RH roots at $K=-1$. In fact, this is the case for $K \in \left(-\frac{1}{.1625}, 0\right)$

$\equiv (-6.154, 0)$

For $K < -6.154$: $n_z - n_p = -1$

$\therefore n_z = n_p - 1 = 2 - 1 = 1$

$\therefore$

K	RH roots
$(-\infty, -6.154)$	1
$(-6.154, 16.59)$	2
$(16.59, \infty)$	non

$\frac{1C}{2 \cdot 3}$ $H=1$, $G = \frac{2000}{s(s+10)(s+20)}$ $\therefore GH = \frac{K}{s(s+10)(s+20)}\Big|_{K=2000}$

$\therefore GH(j\omega) = \frac{2000}{j\omega(10+j\omega)(20+j\omega)} = \frac{10}{j\omega(1+j\omega/10)(1+j\omega/20)}$

$\therefore$ 2 db-corners at 10 & 20

& 4 ϕ-corners at 1, 2, 100 & 200

Take 3 decades 0.5 $\Rightarrow$ 500

db line.

$\omega = 0.5$ $\therefore GH = 26.02$db, slope is -20 db/dec till

$\omega = 10$ $\therefore GH = 0$ after which slope is -40 db/dec till

$\omega = 20$ $\therefore GH = -12.04$db after which slope is -60 db/dec "

$\omega = 500$ $\therefore GH = -95.92$ and keeps at -60db/dec till ∞

0 db occurs at $\omega = 10$.

ϕ line:

$\omega = 0.5$ $\therefore \phi = -90°$, slope 0°/dec till

$\omega = 1$ $\therefore \phi = -90°$ after which slope is $-45°$/dec till

$\omega = 2$ $\therefore \phi = -103.5°$ after which slope is $-90°$/dec till

$\omega = 100$ $\therefore \phi = -256.5°$ " " " $-45°$/dec "

$\omega = 200$ $\therefore \phi = -270.0°$ " " " 0°/dec till

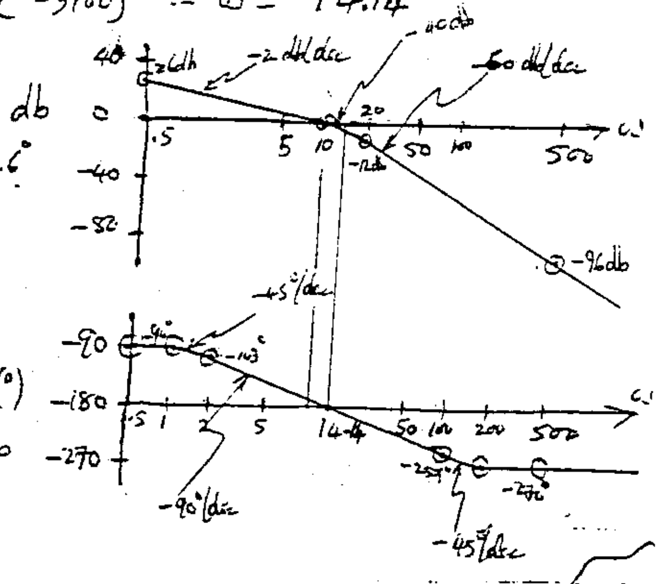
$\omega = 500$ $\therefore \phi = -270.0°$ and keeps so till ∞

$\therefore$ $-180°$ phase occurs at $\omega \in [2, 100]$ $\therefore \omega = 14.14$

$GM = -GH\big|_{\omega_{-180}} = -GH\big|_{14.2} = +6.02$db

$= 2.00$

$\phi_m = 180 + \phi\big|_{\omega_{0db}} = 180 + \phi\big|_{10} = 180 + (-166.4) = 13.6°$



$\therefore$ Gain margine $= 6.02$db $= 2.00$

& phase margin $= 13.6°$

System is stable and can become unstable if K is $> K_c = 2000 * 2.00 = 4000$

$\therefore$ Stable for $K \in (0, 4000)$

Routh: $P(s) = s^3 + 30s^2 + 200s + K$

s^3 1 200
s^2 30 K
s^1 $\frac{6000-K}{30}$ 0
s^0 K

$\therefore K < 6000$ & $K > 0$
$\therefore K \in (0, 6000)$ $\therefore$ OK within Bode approximation to lines.

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$\boxed{\dfrac{1d}{223}}$ $H = 1$. $G = \dfrac{20,000\,(s+5)}{s^2(s+10)(s+20)(s+50)}$

$\therefore GH(j\omega) = \dfrac{20,000\,(5+j\omega)}{(-\omega^2)(10+j\omega)(20+j\omega)(50+j\omega)} = \dfrac{10\,[1+j(\omega/5)]}{(-\omega^2)[1+j(\omega/10)][1+j(\omega/20)][1+j(\omega/50)]}$

$\therefore$ We have 4 corners for db lines at 5, 10, 20 & 50

& " " 8 corners for phase lines at .5, 1, 2, 5, 50, 100, 200 & 500

$\therefore$ We need at least three decades at $\log\omega$ axis. Take 4 decades [.1, 1000]

\# db line: At $\omega = 0.1$ all square brackets of GH are dead (=1) $\therefore$ db$|GH| = 20\log\frac{10}{.1^2} = 60$

At $\omega \in [.1, 5)$ slope of line is -40 db/decade

$\therefore$ At $\omega = 5$ $\Rightarrow$ db$G = 60$ db $- 40\frac{db}{decade} * \log\left(\frac{5}{.1}\right)$ decade $= -7.96$ db

At $\omega \in (5, 10)$ $\therefore$ corner 5 is alive & hence slope of line $= -40 + 20 = -20\frac{db}{dec}$

$\therefore$ At $\omega = 10$ $\Rightarrow$ db$G = -7.95$ db $- 20 * \log\left(\frac{10}{5}\right) = -13.98$ db

At $\omega \in (10, 20)$ .. corner 10 is alive & hence slope of line $= -40 + 20 - 20 = -40\frac{db}{dec}$

$\therefore$ At $\omega = 20$ $\Rightarrow$ db$G = -13.98 - 40\log\left(\frac{20}{10}\right) = -26.02$ db

At $\omega \in (20, 50)$ $\therefore$ corner 20 is alive & slope $= -40 + 20 - 20 - 20 = -60$ db/dec

$\therefore$ At $\omega = 50$, db$G = -26.02 - 60\log\left(\frac{50}{20}\right) = -49.90$ db

At $\omega \in (50, \infty)$ $\therefore$ corner 50 is alive & slope $= -60 - 20 = -80$ db/decade

$\therefore$ At $\omega = 10^3$, db$G = -49.90 - 80\log\left(\frac{1000}{50}\right) = -153.98$ db

The special point of 0 db occurs at $\omega \in (.1, 5)$ $\therefore \dfrac{60.0 - 0}{\log .1 - \log\omega} = -40$

$\therefore \dfrac{60.0}{-40} = \log\dfrac{0.10}{\omega} \Rightarrow \dfrac{0.10}{\omega} = 10^{-60.0/40} \Rightarrow \omega = .10 * 10^{+60/40} = 3.16 \Rightarrow \omega_{0db} = 3.16$

$\therefore$ We need the db range to be at least $= (-154, 60)$, take it $[-160, 60]$

\# ϕ line: At $\omega = .1$ all square brackets are dead & $\angle GH = \phi = -180°$

At $\omega \in [.1, .5]$ " " " still " & $\phi = -180°$

At $\omega \in (.5, 1)$ corner 5 is alive & hence slope of line $= +45°$/decade

$\therefore$ At $\omega = 1 \Rightarrow \phi = -180 + 45 * \log\left(\frac{1}{.5}\right) = -166.45°$

At $\omega \in (1, 2]$ corners 5 & 10 are alive $\therefore$ slope $= 0$ & $\phi = -166.45°$

At $\omega \in (2, 5)$ corners 5, 10 & 20 " " $\therefore$ slope $= -45°$/decade

$\therefore$ At $\omega = 5 \Rightarrow \phi = -166.45 - 45\log\left(\frac{5}{2}\right) = -184.36°$

At $\omega \in (5, 50)$ corners 5, 10, 20 & 50 are all alive $\therefore$ slope $= -90°$/decade

$\therefore$ At $\omega = 50 \Rightarrow \phi = -184.36 - 90 * \log\left(\frac{50}{5}\right) = -274.36°$

At $\omega \in (50, 100)$ corners 10, 20 & 50 are alive $\therefore$ slope $= -135°$/decade

$\therefore$ At $\omega = 100$, $\phi = -274.36 - 135\log\left(\frac{100}{50}\right) = -315.00°$

At $\omega \in (100, 200)$ corners at 50 are dead $\therefore$ slope $= -90°$/decade

$\therefore$ At $\omega = 200 \Rightarrow \phi = -315.00 - 90 \times \log\left(\frac{200}{100}\right) = -342.09°$

At $\omega \in (200, 500)$ corner 50 is only alive $\therefore$ slope $= -45°$/decade

At $\omega = 500$ $\therefore \phi = -342.09 - 45\log\left(\frac{500}{200}\right) = -360.00°$

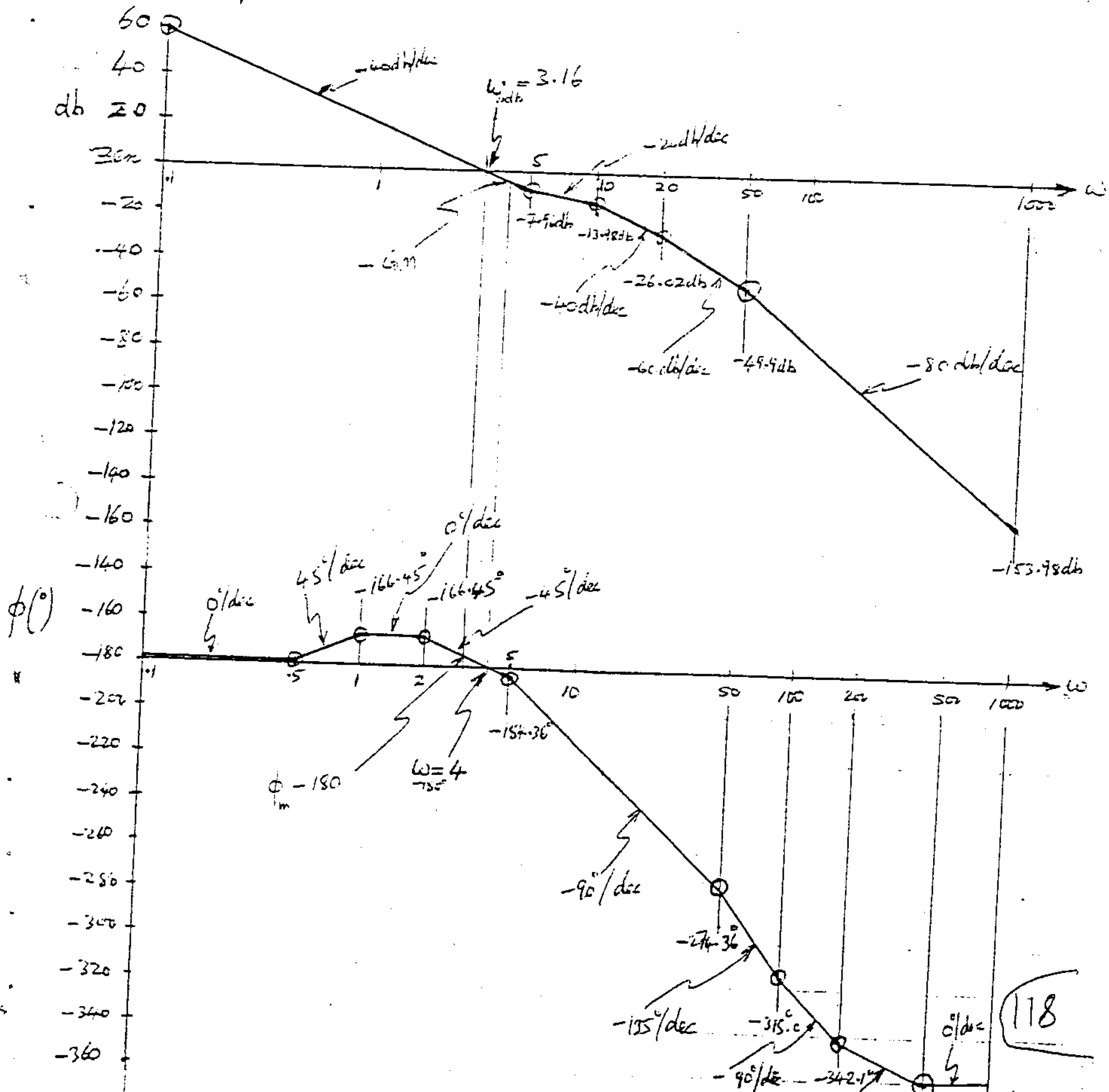
At $\omega \in (500, \infty)$ All corners are dead again $\therefore$ slope $= 0°$ & $\phi = -360°$

The second point of $-180°$ crossing is at $\omega \in (2, 5)$: $\frac{-166.45-(-180)}{\log 2 - \log \omega} = \ldots$

$\therefore \frac{13.55°}{\log\left(\frac{2}{\omega}\right)} = -45 \Rightarrow \log\left(\frac{2}{\omega}\right) = -\frac{13.55}{45} \therefore \omega = 2 \times 10^{13.55/45} = 4.00 \therefore \omega_{-180°} = 4.0$

$\therefore$ We need the angle range to be at least $= (-166°, -360°)$, take it $[-140°, 360°]$

$\therefore$ The plot is shown below:



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The Gain Margin, $G_m = 0 - db\, GH\Big|_{\omega_{-180}} \cong 0 - db\, GH\Big|_{\omega = 4 \in [1, 5)} = -\left(60 - 40 \log \frac{4}{.1}\right) = 4.08$ db

∴ The gain margin = $+4.08$ db $= 1.60$

The phase margin, $\varphi_m = \varphi\Big|_{\omega_{0db}} + \pi = \varphi\Big|_{3.16 \in (2, 5)} + 180° = 180 + \left(-166.45 - 45 \log\left(\frac{3.16}{2}\right)\right)$

$= 4.61°$

∴ Since φ_m is positive, system is stable. However, it can be made unstable if ω_{0db} is brought to right, to be after $\omega_{-180°} (= 4)$, whereby phase margin is then negative.
To do this, the db graph must be brought up by GM i.e by 1.60 db, i.e GH is multiplied by 1.60 where system becomes marginally stable.

∴ $GH = 1.6 * (\text{old } GH)$ is marginally stable

∴ $$1.6 * \frac{20000(s+5)}{s^2(s+10)(s+20)(s+50)} = \frac{32000(s+5)}{s^2(s+10)(s+20)(s+50)} = \frac{K(s+5)}{s^2(s+10)(s+20)(s+50)}\Bigg|_{K=32000}$$

is marginally stable ∴ $K \cong 32{,}000$ is critical.

∴ System is stable for $K < 32{,}000$, where the crossing 0db point will move to the left and as K is infinitely reduced it will correspond to very small phase margins (+ve) since the phase plot approaches the $-180°$ from above assymptotically.

∴ System is stable but can be made unstable when $K > 32{,}000$ (K is the number in the numerator corresponding to unity s coeffecients)

Checking by Routh: ∴ $f(s) = s^2(s+10)(s+20)(s+50) + K(s+5)$

s^5	1	1700	K		
s^4	80	10,000	5K		
s^3	1680	K	0	$(*80/75)$	
s^2	$210{,}000 - K$	105K	0	$(*1680/80)$	∴ $K \in (-\infty, 210{,}000)$
s	$-K^2 + 33600K$	0		$(*(210000 - K))$	∴ $K \in (0, 33600)$
1	105K				∴ $K \in (0, \infty)$

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∴ Stable if $K \in (0, 33{,}600)$ i.e $K < 33600$ cf 32000

∴ OK within Bode approximation assumptions

2d
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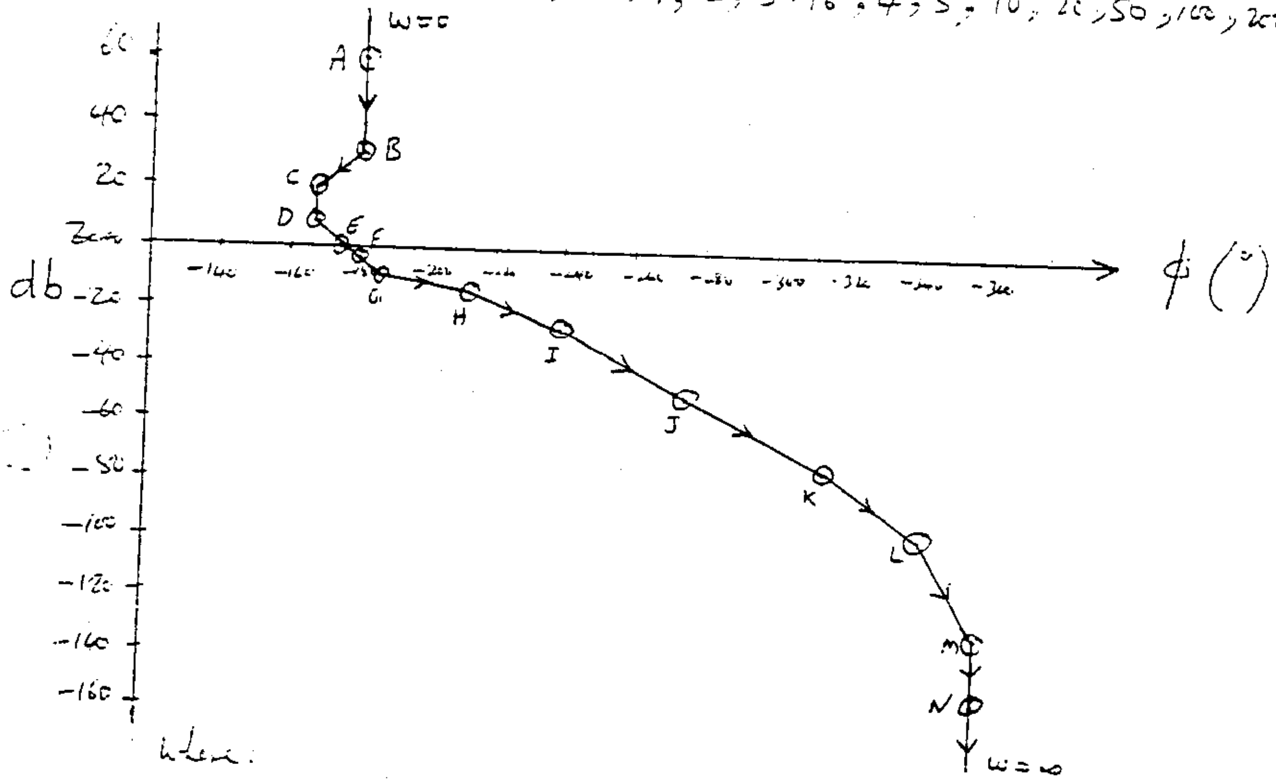
Gain-Phase plot is obtained by putting gain on vertical axis + phase on horizontal.

Since the relation is linear, then corner points are important; these are at $\omega = .1, .5, 1, 2, 3.16, 4, 5, 10, 20, 50, 100, 200, 500$ Hz



where:

$A = (db, \phi)|_{.1} = (60, -150)$

$B = (db, \phi)|_{.5} = (60 - 40\log\frac{.5}{.1}, -180) = (32.04, -180)$

$C = (db, \phi)|_{1} = (60 - 40\log\frac{1}{.1}, -166.45) = (20, -166.45)$

$D = (db, \phi)|_{2} = (60 - 40\log\frac{2}{.1}, -166.45) = (7.96, -166.45)$

$E = (db, \phi)|_{\omega_{0db}} = (db, \phi)|_{3.16} = (0, \phi_m - 180) = (0, 4.61 - 180) = (0, -175.39)$

$F = (db, \phi)|_{\omega_{180}} = (db, \phi)|_{4} = (-6.11, -180) = (-4.08, -180)$

$G = (db, \phi)|_{5} = (-7.96, -184.36)$

$H = (db, \phi)|_{10} = (-13.98, -184.36 - 90\log(\frac{10}{5})) = (-13.98, -211.45)$

$I = (db, \phi)|_{20} = (-26.02, -184.36 - 90\log(\frac{20}{5})) = (-26.02, -238.55)$

$J = (db, \phi)|_{50} = (-49.90, -274.36)$

$K = (db, \phi)|_{100} = (-49.90 - 80\log\frac{100}{50}, -315) = (-73.98, -315)$

$L = (db, \phi)|_{200} = (-49.9 - 80\log\frac{200}{50}, -342.09) = (-98.06, -342.09)$

$M = (db, \phi)|_{500} = (-49.9 - 80\log\frac{500}{50}, -360) = (-129.9, -360)$

$N = (db, \phi)|_{1000} = (-153.98, -360)$

$\because$ Graph intersect 0db to the left of 180° ($\phi_m > 0$), Then system is stable

$\boxed{\dfrac{3d}{223}}$ $GH = \dfrac{K(s+2)(s+30)^2}{s^2(s+10)(s+15)(s+50)(s+1000)}$

$$\therefore GH(j\omega) = \frac{K(2+j\omega)(30+j\omega)^2}{-\omega^2(10+j\omega)(15+j\omega)(50+j\omega)(1000+j\omega)} =$$
$$= \frac{-3K/12500}{\omega^2} \cdot \frac{(1+j\omega/2)(1+j\omega/30)^2}{(1+j\omega/10)(1+j\omega/15)(1+j\omega/50)(1+j\omega/1000)}$$

Bode Plot: (Take 3K/12500 to be unity)

Six db corners at 2, 10, 15, 30, 50, 1000

+ Ten ϕ corners at .2, 1, 1.5, 3, 5, 20, 150, 300, 500, 10000

decade range is .1 → 10000 (5 decades)

⇒ db line as explained earlier:

$\omega = .1$	$\therefore GH = +40$ db	, slope is -40 db/dec till:
$\omega = 2$	$\therefore GH = -12.04$	after which slope is -20 db/dec till
$\omega = 10$	$\therefore GH = -26.02$	" " " " -40 db/dec "
$\omega = 15$	$\therefore GH = -33.06$	" " " " -60 " " "
$\omega = 30$	$\therefore GH = -51.13$	" " " " -20 " " "
$\omega = 50$	$\therefore GH = -55.56$	" " " " -40 " " "
$\omega = 1000$	$\therefore GH = -107.60$	" " " " -60 " " "
$\omega = 10000$	$\therefore GH = -167.60$	and continues with " " " " " ∞.

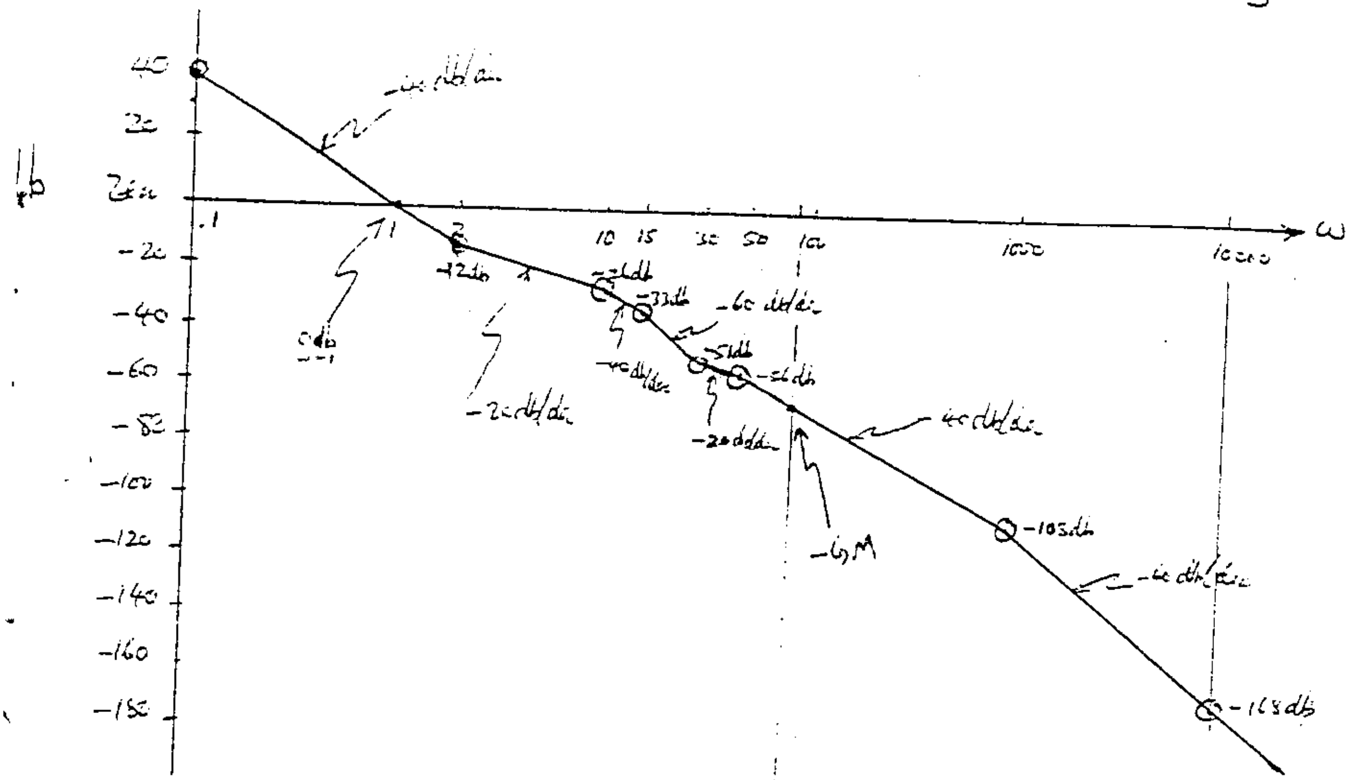
0 db occurs at $\omega \in (.1, 2)$ $\therefore \omega = 1$

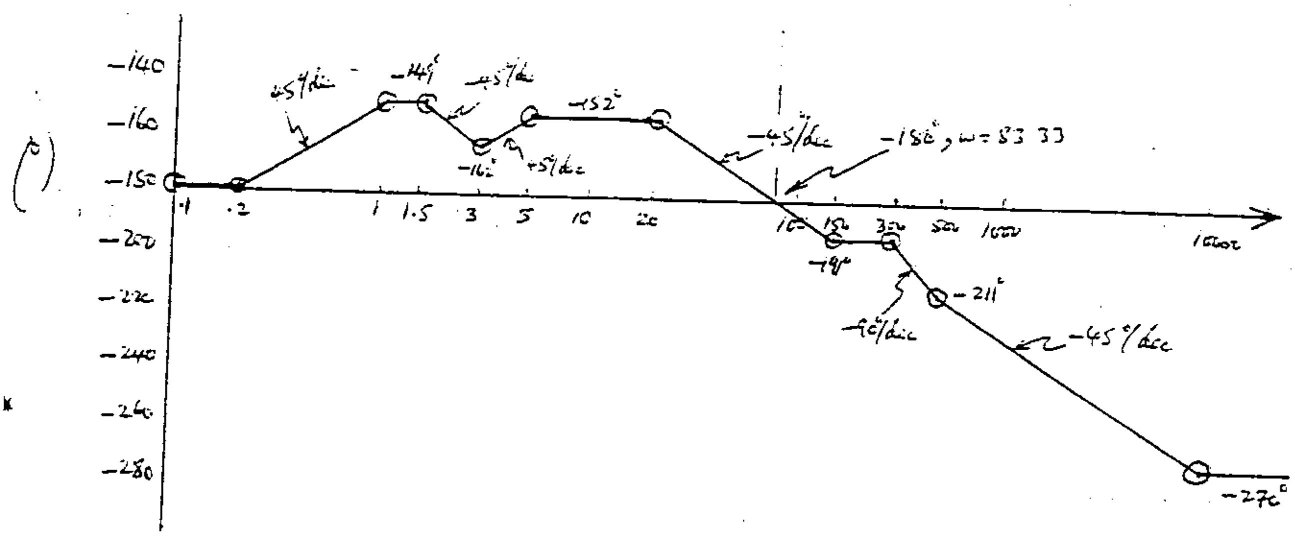
⇒ ϕ line:

$\omega = .1$	$\therefore \phi = -180°$	and continues at $-180°$ till
$\omega = .2$	$\therefore \phi = -180°$	after which slope is $+45°$/dec till:
$\omega = 1$	$\therefore \phi = -148.55$	" " " " $0°$/dec "
$\omega = 1.5$	$\therefore \phi = -148.55$	" " " " $-45°$/dec "
$\omega = 3$	$\therefore \phi = -162.09$	" " " " $+45°$/dec "
$\omega = 5$	$\therefore \phi = -152.11$	" " " " $0°$/dec "
$\omega = 20$	$\therefore \phi = -152.11$	" " " " $-45°$/dec "
$\omega = 150$	$\therefore \phi = -191.49$	" " " " $0°$/dec "
$\omega = 300$	$\therefore \phi = -191.49$	" " " " $-90°$/dec "
$\omega = 500$	$\therefore \phi = -211.45$	" " " " $-45°$/dec "
$\omega = 10000$	$\therefore \phi = -270.0$	" " " " $0°$/dec, so remains at -270 till ∞

$-180°$ phase occurs at $\omega \in (20, 150)$ $\therefore \omega = 83.33$

Range of plot gain $\in [40, -180]$, phase $\in [-140, -280]$





System is stable and has a GM of $-\left(55.56 - 40 \log \frac{83.33}{30}\right) = 64.4\,db$
$= 1666.0$

$\therefore$ Critical at $\frac{3K}{12500} = 1666.0 \Rightarrow K = 6.94 \times 10^6$ $(\omega_{0db} = \omega_{-180°})$

If K is smaller than 6.94×10^6, ω_{0db} goes left to $\omega_{-180} = 83.33$ and that corresponds to stable systems having phase margin of $\phi_m \in (0, 31°]$

$\therefore$ Stable for $K < 6.94 \times 10^6$

Nyquist Plot. (Take K to be unity)

$$\therefore GH(j\omega) = \frac{30^2+\omega^2}{\omega^2}\cdot\sqrt{\frac{(4+\omega^2)}{(100+\omega^2)(225+\omega^2)(2500+\omega^2)(10^6+\omega^2)}}\ \angle\frac{-\pi+\tan^{-1}\frac{\omega}{2}+2\tan^{-1}\ldots}{-\tan^{-1}\frac{\omega}{10}-\tan^{-1}\frac{\omega}{15}-\tan^{-1}\frac{\omega}{50}\ldots}$$

$$\therefore M = \frac{900+\omega^2}{\omega^2}\sqrt{\frac{(4+\omega^2)}{(100+\omega^2)(225+\omega^2)(2500+\omega^2)(10^6+\omega^2)}}$$

$$\& \ \phi = -\pi+\tan^{-1}\left(\frac{\omega}{2}\right)+2\tan^{-1}\left(\frac{\omega}{30}\right)-\tan^{-1}\left(\frac{\omega}{10}\right)-\tan^{-1}\left(\frac{\omega}{15}\right)-\tan^{-1}\left(\frac{\omega}{50}\right)-\tan^{-1}\left(\frac{\omega}{1000}\right)$$

$\therefore\ \omega$	M	$\phi\ (^\circ)$
0^+	∞	$(-180)^+$
.001	240	−179.98
.01	240	−179.78
.1	2.4E−2	−177.8
1	2.67E−4	−160.3
1.5	1.3E−4	−153.5
2	8.3E−5	−148.9
3	4.6E−5	−143.9
4	3.1E−5	−142.9
5	2.2E−5	−143.9
6	1.7E−5	−145.8
8	1.1E−5	−150.5
12	5.7E−5	−158.9
16	3.4E−6	−164.5
20	2.2E−6	−167.8
25	1.4E−6	−170.2
30	9.7E−7	−171.5
40	5.5E−7	−173.0
50	3.6E−7	−174.1
60	2.55E−7	−175.2
100	9.5E−8	−179.4
106	8.5E−8	−180.04
110	7.9E−8	−180.43
130	5.7E−8	[illegible]

$\phi_{\omega=0} = (-180)^+ \therefore$ Asymptote // Real axis

& intercept at $\lim_{\omega\to 0} \operatorname{Im} GH(j\omega) = \lim_{\omega\to 0} M\sin\phi$

$$= \left(\lim_{\omega\to 0} M\right)\cdot\left(\lim_{\omega\to 0}\sin\phi\right) = \left(\lim_{\omega\to 0}\frac{900}{\omega^2}\sqrt{\frac{4}{100*225*2500\ldots}}\right)$$

$$\cdot\left(\lim_{\omega\to 0}\sin\left[-\pi+\left(\frac{\omega}{2}+\frac{2\omega}{30}-\frac{\omega}{10}-\frac{\omega}{15}-\frac{\omega}{50}-\frac{\omega}{1000}\right]\right)\right)$$

$$= \lim_{\omega\to 0}\frac{900*2}{10*15*50*10^3\,\omega^2}\cdot\left(-\sin\left(\frac{5685}{15000}\right)\omega\right) =$$

$$= \lim_{\omega\to 0}\frac{-3}{12500\,\omega^2}\cdot\frac{5685\,\omega}{15000} = \infty$$

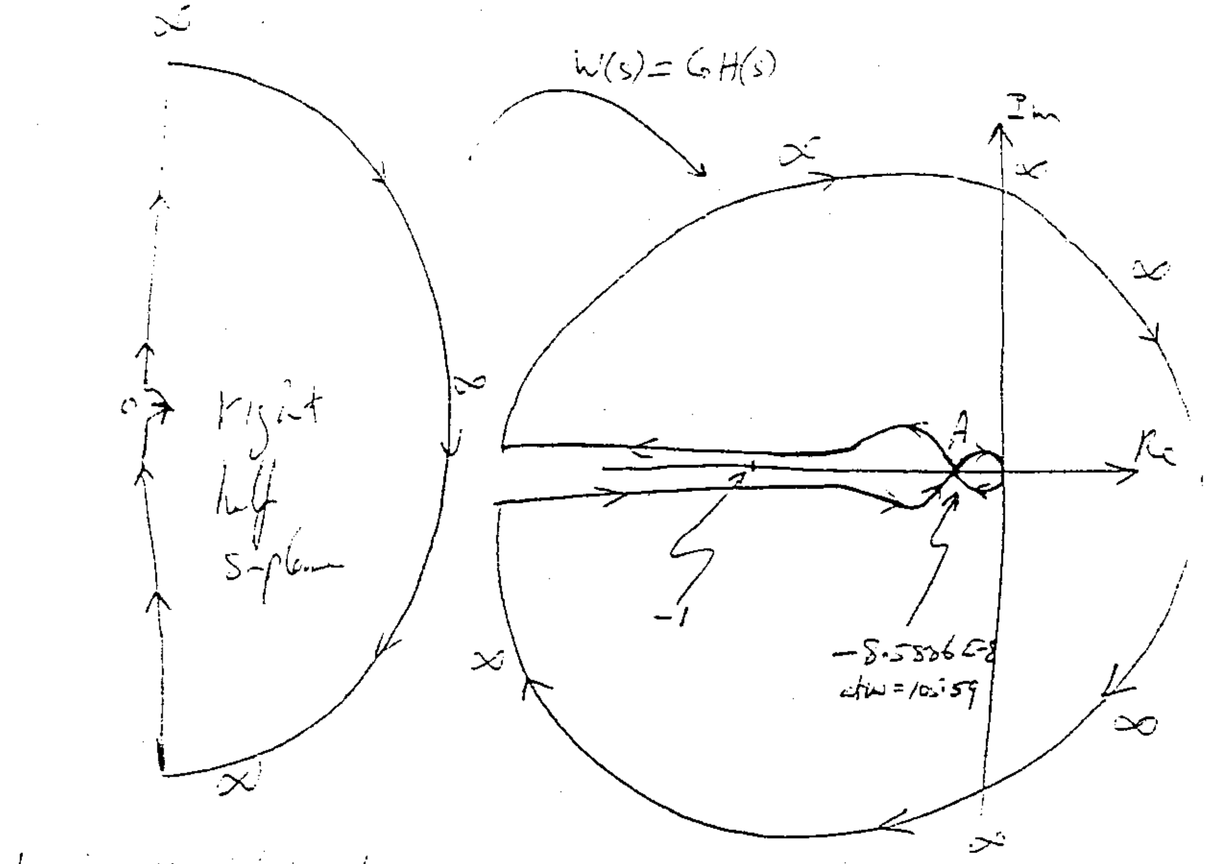
$\therefore$ It has no asymptote at $\omega = 0$

As for s near zero put $s=\epsilon\angle\theta$

$\therefore$ when $\theta\in[0, 90^\circ] \Rightarrow GH(s) = \frac{900*2\,K}{10*15*50*10^3\,s^2}$

$\therefore M=\infty,\ \phi\in[0, -180]$

Utilizing symmetry, the right half s-plane boundaries are mapped by $GH(s)$ into the contour shown.



boundaries of right half s-plane

Image.

$\therefore$ No -1 encirclement $\therefore n_{z|\text{right half s-plane}} = n_{p|} = 0$ $\therefore$ Stable

However, if K is increased, A will move left, and it will touch the -1 point when K is $\frac{1}{8.5856 \times 10^{-4}} = 1.164 \times 10^{7}$

If K is increased above 1.164×10^{7} the -1 will go into the loop and thus 2 encirclements will be obtained causing instability.

$\therefore$ Stable for $0 < K < 1.164 \times 10^{7}$

(Note: Bode gives $K_{critical} = 6.94 \times 10^{6}$ compared to true value of 1.164×10^{7} and this is due approximation to lines)

Root Locus: $(K \in [0, \infty))$

$\therefore F(s) = s^2(s+10)(s+15)(s+50)(s+1000) + K(s+2)(s+30)^2$

— the locus is symmetric with positive one. Consider the positive:

poles at $0, 0, -10, -15, -50, -1000$

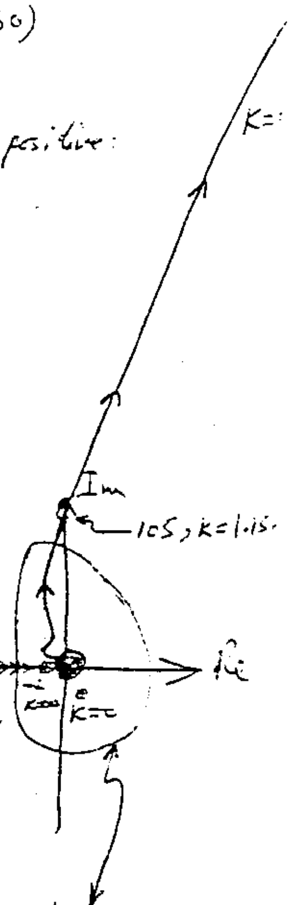
zeros at $-2, -30, -30$

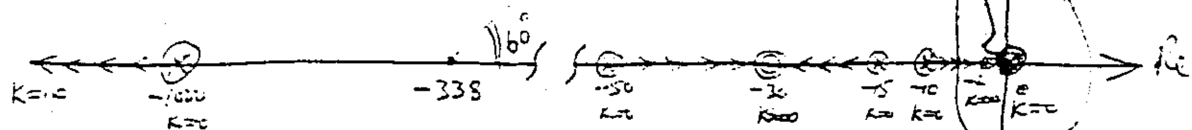
$\therefore$ Six branches of 3 zeros at ∞

Assymptotes:

Angle $= \pm 60^\circ, -180^\circ$

intercept $= \dfrac{-1075+62}{3} = -337.\overline{6}$





The poles at zero will go to $\pm 60^\circ$ assymptotes.

They will depart at $\pm 90^\circ$.

To see whether going to left or right:

$\therefore$ take $s = \epsilon e^{i\theta}$ where θ is $90^\circ + \delta^\circ$

$\therefore s = \epsilon\left(e^{i(90^\circ+\delta^\circ)}\right) = \epsilon\, e^{i90^\circ} \cdot e^{i\delta}$

$= \epsilon \cdot i \cdot (\cos\delta + i\sin\delta) = \epsilon i(1 + i\delta)$

$= \epsilon i - \epsilon\delta = -\epsilon\delta + i\epsilon$

checking the angle criteria and ignoring very far poles:

$\therefore -2\theta + \tan^{-1}\dfrac{\epsilon}{2-\epsilon\delta} = -180^\circ$

$\therefore -2(90^\circ+\delta) + \dfrac{\epsilon}{2-\delta\epsilon} = -180^\circ$

$\therefore -2\delta + \dfrac{\epsilon}{2} = 0 \quad \therefore \delta = \epsilon/4 > 0$

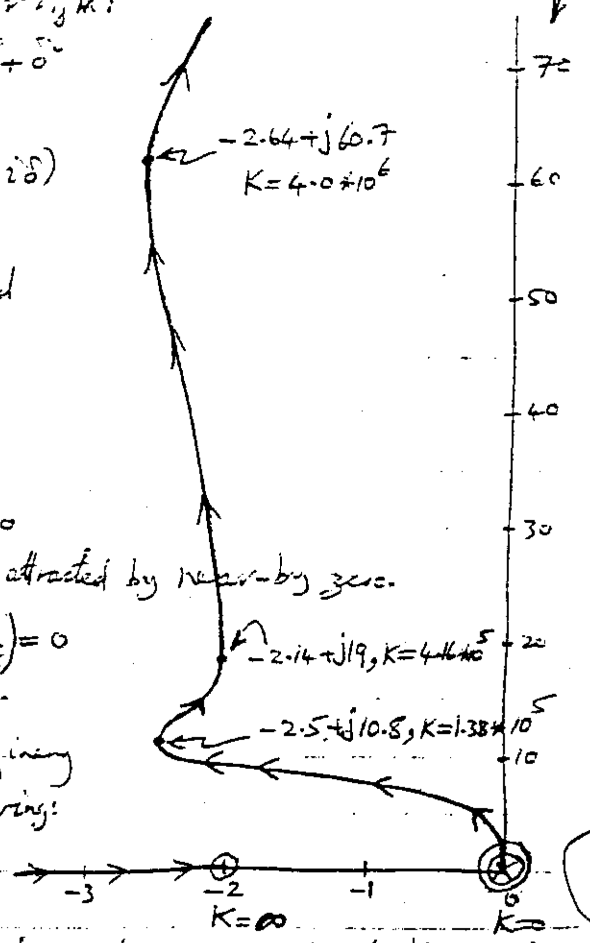
$\therefore$ After departure locus from pole is attracted by near-by zero.

Sides of locus at $f(s) = 0$ & $Re\left(\dfrac{ds}{dK}\right) = 0$

This gives the three points shown.

The Locus intersect the imaginary axis at w when $f(jw) = 0 + j0$ giving:

$w = 105.04$, $K = 1.15239 \times 10^7$



$\therefore$ Stable for $K \in [0, 1.15 \times 10^7)$

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$\boxed{\frac{5d}{224}}$ $H = 1$, $G = \frac{500(s+1)}{s(s+5)(s+10)}$ $\therefore GH = \frac{500(s+1)}{s(s+5)(s+10)}$

$\therefore GH(j\omega) = \frac{10}{j\omega} \cdot \frac{(1+j\omega)}{(1+j\omega/5)(1+j\omega/10)}$

$\therefore$ We have 3 corners for db line at 1, 5, 10

& " " 6 " " " ϕ " " .1, .5, 1, 10, 50, 100

We need at least 3 dec, take 4 decades .05 → 500

db line:

at $\omega = .05$ $\therefore GH = 46.02$ db, slope is -20 db/dec till

$\omega = 1$ $\therefore GH = 20.0$ db, after which slope is 0 db/dec till

$\omega = 5$ $\therefore GH = 20.0$ db, " " " " -20 db/dec "

$\omega = 10$ $\therefore GH = 13.98$ db, " " " " -40 db/dec "

$\omega = 500$ $\therefore GH = -53.98$ and keeps at -40 db/dec.

$\therefore$ cut is at $\omega \in (10, 500)$ $\therefore \omega = 22.36$

ϕ line:

at $\omega = .05$ $\therefore \phi = -90°$, slope at 0°/dec till

$\omega = .1$ $\therefore \phi = -90°$ after which slope is 45°/dec till

$\omega = .5$ $\therefore \phi = -58.55°$ " " " " 0°/dec "

$\omega = 1$ $\therefore \phi = -58.55°$ " " " " -45°/dec "

$\omega = 10$ $\therefore \phi = -103.5°$ " " " " -90°/dec "

$\omega = 50$ $\therefore \phi = -166.5°$ " " " " -45°/dec "

$\omega = 100$ $\therefore \phi = -180.0°$ " " " " 0°/dec "

$\omega = 500$ $\therefore \phi = -180.0°$ and keeps at 0°/dec.

$\therefore -180°$ phase is never crossed. It is approached assymptotically.

$\therefore$ System is always stable with gain margine between $(0°, 121.5°]$

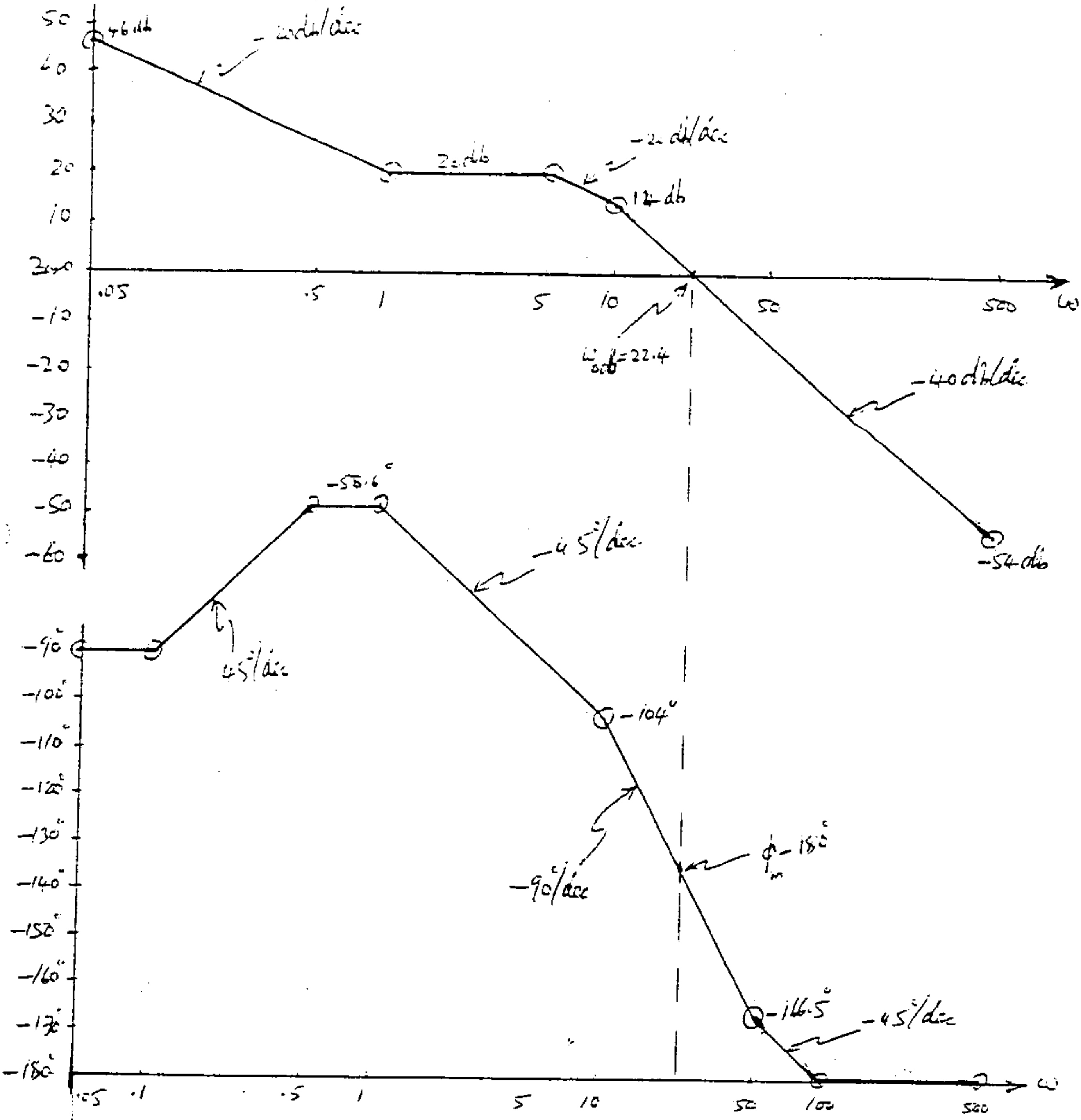
(check by Routh on $\frac{K(s+1)}{s(s+5)(s+10)} \Rightarrow \therefore s^3 + 15s^2 + (50+K)s + K = 0$

s^3	1	$50+K$
s^2	15	K
s	$750+14K$	0
1	K	0

$(*15)$ $\therefore 750 + 14K > 0$ $\therefore K > \frac{-750}{14} = -53.6$

$\therefore K > 0$

$\therefore$ For Stability $\therefore K > 0$ $\therefore K \in (0, \infty)$ i.e. stable for any $\underline{+ve}$ K = ok)



$$\therefore \quad \phi_m - 180 = \phi\Big|_{\omega_{0db}} = \phi\Big|_{22.4 \in (10,50)} = -103.5 - 90 \log \frac{22.4}{10} = -135.0^\circ$$

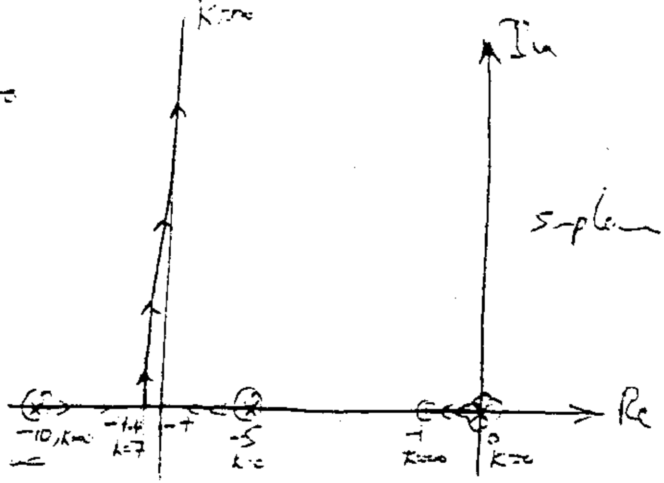
$\therefore$ ϕ_m the phase margine $= 45.0^\circ$

$\because$ No complex poles $\therefore$ No η & ω_n.

Root Locus:

$\therefore GH = \dfrac{K(s+1)}{s(s+5)(s+10)}$

$\therefore f(s) = s^3 + 15s^2 + 50s + K(s+1) = 0$



Symmetry is utilized to get the −ve part of the root locus.

Poles are at $0, -5, -10$

zeros " " : -1

$\therefore$ Three branches & 2 zeros at ∞

Angle of asymptotes $= \pm 90^\circ$

intercept $= \dfrac{-15+1}{2} = -7$

Breakaway at $x \in (-10, -5)$

$\therefore f(x) = f'(x) = 0$

$\therefore x = -7.435$ & $K = 7.216$

$\therefore$ Locus is confined in the left s-plane half $\therefore$ Stable for any $K > 0$

$\therefore$ OK

—

$\boxed{\frac{12}{226}}$ a) $\frac{e_o}{e_i} = \frac{.03}{s^2+2s+1} = \frac{.03}{(s+1)^2} = G(s)$

$\therefore G(j\omega) = \frac{.03}{(1+j\omega)^2}$

$\therefore$ One corner for db & 2 for ϕ

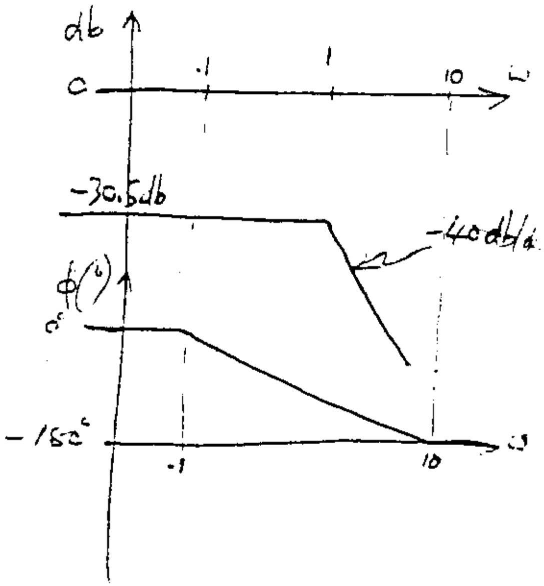
at $\omega \ll 1$ $\therefore dbG = -30.5$

$\therefore$ at $\omega = 1$ $db = -30.5$

and goes at -40 db/dec after that

at $\omega \le .1$ $\therefore \phi = 0$ then at $-90°$/dec till

$\therefore$ at $\omega = 10$ $\therefore \phi = -180°$ and stays so.



b) $f = 2$ cycles/sec

$\therefore \omega = 2\pi f = 4\pi$ rad/sec $= 12.6$ rad/sec

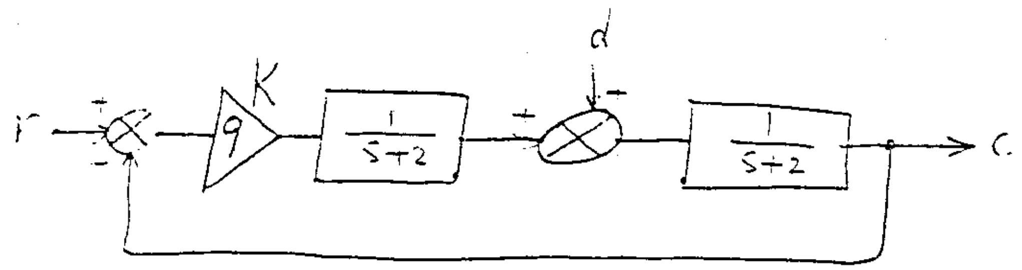
this corresponds to a db of

$-30.5 - 40 \log \frac{12.6}{1} = -74.4$ db

$= 1.9 \times 10^{-4}$ (very small)

$\therefore$ We don't use the accelerometer at $f > 2$ Hz because the output is very very small and reading will have high probability of error. This is because 2 Hz $\equiv$ 12.6 rad/sec is way off corner frequency or outside the bandwidth of the accelerometer.

Block diagram is as follows:



$$\therefore\; C(s) = \frac{c}{r}(s)\Big|_{d=0} \cdot r(s) + \frac{c}{d}(s)\Big|_{r=0} \cdot d(s)$$

$$= \frac{9}{(s+2)^2+9} \cdot r(s) + \frac{s+2}{(s+2)^2+9} \cdot d(s)$$

$$= \frac{9\,r(s) + (s+2)\,d(s)}{(s+2)^2+3^2}$$

$$\therefore\; c(t) = \frac{9}{(s+2)^2+9}\, r(t) + \frac{s+2}{(s+2)^2+9}\, d(t) + e^{-2t}\left(C_1 \sin 3t + C_2 \cos 3t\right)$$

$$\because\; r(t) = A + B\cdot t \quad \& \quad d(t) = D + E\cdot t$$

$$s^2 r(t) = 0, \quad s^2 d(t) = 0$$

$$\therefore\; c_{ss}(t) = \frac{9(A+Bt) + (s+2)(D+Et)}{s^2+4s+13} = \frac{9(A+Bt)+E+2D+2Et}{13\left(1+\frac{4s}{13}\right)}$$

$$= \frac{1}{13}\left(1 - \frac{4s}{13} + \left(\frac{4s}{13}\right)^2 + \cdots\right)\left(9A + E + 2D + t(9B+2E)\right) =$$

$$= \frac{1}{13}\left(1 - \frac{4s}{13}\right)\left(9A + E + 2D + t(9B+2E)\right) =$$

$$= \frac{1}{13}\left[9A + E + 2D + t(9B+2E) - \frac{4}{13}(9B+2E)\right] =$$

$$= \frac{1}{169}\left[117A - 36B + 26D + 5E + 13t(9B+2E)\right]$$

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$$\therefore\; c(t) = \frac{1}{169}\left[117A - 36B + 26D + 5E + 13t(9B+2E)\right] + e^{-2t}\left(C_1 \sin 3t + C_2 \cos 3t\right)$$

Assuming CIC

$$\therefore\ c(0) = 0 = \frac{1}{169}\left(117A - 36B - 26D + 5E\right) + C_2 \qquad \therefore\ C_2 = \frac{-1}{169}\left(117A - 36B + 26D + 5E\right)$$

$$\therefore\ \dot{c}(0) = 0 = \frac{1}{169}\left(13\left(9B + 2E\right)\right) + \left(3C_1\right) - 2\left(C_2\right)$$

$$\therefore\ C_1 = \left[2C_2 - \frac{1}{13}\left(9B + 2E\right)\right] / 3 = \frac{-234A + 72B - 52D - 10E - 117B - 26E}{507}$$

$$\therefore\ C_1 = \frac{-234A - 45B - 52D - 36E}{507}$$

$$\therefore\ c(t) = \frac{1}{507}\Big[\ A\left(351 - 234e^{-2t}\sin 3t - 351e^{-2t}\cos 3t\right)$$
$$+ B\left(-108 - 45e^{-2t}\sin 3t + 108e^{-2t}\cos 3t + 351t\right)$$
$$+ D\left(78 - 52e^{-2t}\sin 3t - 78e^{-2t}\cos 3t\right)$$
$$+ E\left(15 - 36e^{-2t}\sin 3t - 15e^{-2t}\cos 3t + 78t\right)\Big]$$

System type is zero for both r & d references.

$\epsilon_{ss}(t) = r(t) - c_{ss}(t) = \frac{1}{507}\left[156A + B(108 + 156t) - 78D - E(15 + 78t)\right]$

Assuming that the gain was K rather than 9, then the problem can be analyzed for stability by Routh Criteria as follows:

$$C = \frac{C}{r}\Big|_{d=0} \cdot r + \frac{C}{d}\Big|_{r=0} \cdot d = \frac{K(s^2+3s+10)\,r + (s+2)(s^2+3s+10)\,d}{(s+2)^2(s^2+3s+10)+K}$$

$\therefore$ The characteristic equation is $f(s) = (s+2)^2(s^2+3s+10)+K=$

$\therefore f(s) = (s^2+4s+4)(s^2+3s+10)+K = s^4+7s^3+26s^2+52s+K+40$

The Routhian is:

s^4	1	26	$K+40$	
s^3	7	52	0	
s^2	130	$7K+280$	0	$(*\,7)$
s	$4500-49K$	0		$(*\,130)$ $\quad\therefore K < 97.96$
1	$7K+280$			$\therefore K > -40$

$\therefore K \in (-40, 97.96)$ for stable system.

\# It could also be analized by Root Locus:

$\therefore GH = \dfrac{K}{(s+2)^2(s^2+3s+10)}$, $f(s) = s^4+7s^3+26s^2+52s+K+40=$

Root Locus:

$-ve$ is obtained by symmetry from $+ve$ locus.

Poles at $-2, -2, -1.5 \pm j(\sqrt{31}/2) = -1.5 \pm j2.784$

Zeros: none

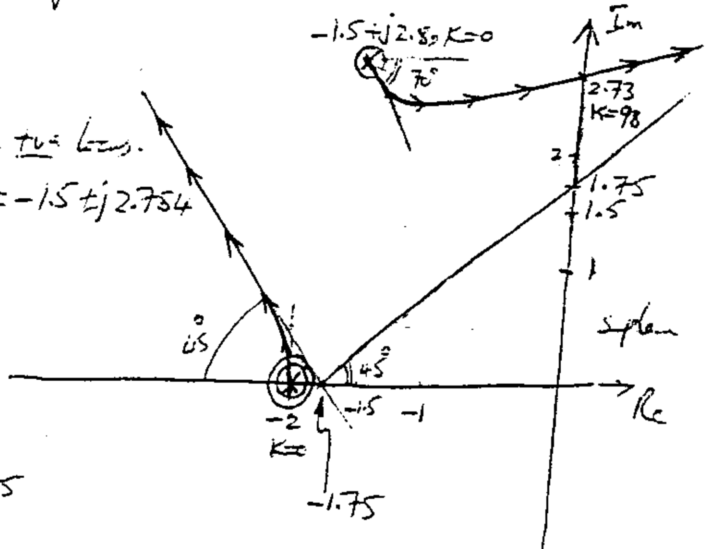
Branches: 4

4 zeros at ∞

ass. angle $= \pm 45^\circ, \pm 135^\circ$

ass. intercept $= \dfrac{-4-3}{4} = \dfrac{-7}{4} = -1.75$

no locus on real line



departure from $s=-1.5+j2.784$ at α

$\therefore -\alpha - 90^\circ - 2\tan^{-1}\dfrac{2.784}{.5} = -180^\circ \therefore \alpha = -69.64^\circ$, repulsion $\therefore$ it goes right

departure from $s=-2$ at β $\therefore -2\beta = \cancel{odd}\pi \therefore \beta = \mp 90^\circ$, repulsion $\therefore$ it goes left.

Im-axis intercept at ω $\therefore f(j\omega) = 0 + j0$ $\therefore \omega = \sqrt{\frac{52}{7}} = 2.726$, $K = 97.96$

$\therefore$ Stable for $K \in [0, 98)$

It could also be analyzed by Nyquist Method:

$$GH(j\omega) = (\text{assuming } K=1)\ \frac{1}{(2+j\omega)^2(10-\omega^2+3j\omega)} = M\angle\phi$$

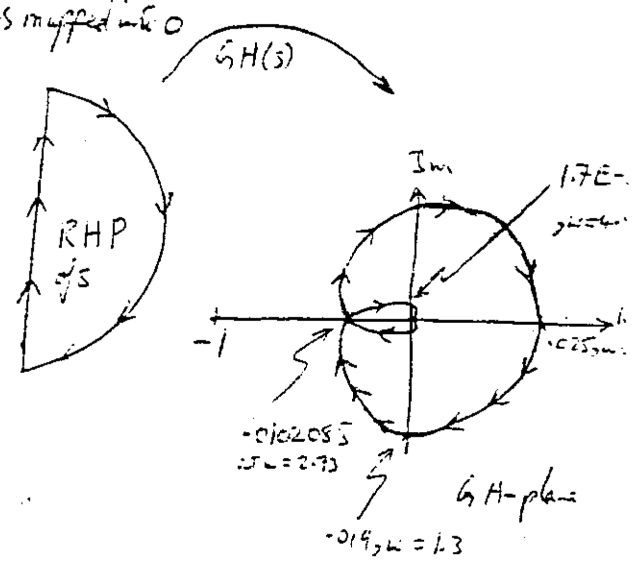
$$\therefore M = \frac{1}{(4+\omega^2)\sqrt{(10-\omega^2)^2+9\omega^2}}$$

$$\& \quad \phi = -2\tan^{-1}(\omega/2) - \tan^{-1}\left(\frac{3\omega}{10-\omega^2}\right)$$

ω	M	$\phi(°)$
0	.025	0
.1	.025	−7.44
.2	.025	−14.9
.3	.025	−22.3
.4	.024	−29.6
.6	.023	−44.0
.8	.022	−58.0
1	.021	−71.6
1.281	.019	−90.0
1.4	.018	−97.6
1.6	.017	−110
2.0	.015	−135
2.5	.012	−166
2.725564	.0102053	−180
3	8.5E−3	−196
3.5	5.7E−3	−223
4	3.7E−3	−243
4.936	1.7E−3	−270
5.5	1.1E−3	−280
6	7.9E−4	−288
7	4.3E−4	−300
10	1.0E−4	−319
15	2.0E−5	−333
20	6.3E−6	−340
100	1E−8	−356
∞	0	−360

Plot for $\omega<0$ is just symmetric &

plot for $s = Re^{j\theta}$, $R=\infty$, $\theta\in(90,-90°)$ is mapped into 0



$N = 0$

$\therefore n_z - n_p = 0$ but $n_p = 0$

$\therefore n_z = 0$

$\therefore$ Stable for $K = 1$.

If K is smaller the plot shrinks and hence still stable.

If K is made >1 the plot is magnified till it hits the −1 at

$$K_{critical} = \frac{1}{.0102053} = 97.96$$

after which $N = 2 = n_z - \cancel{n_p}^{0}$ $\therefore$ 2 RHP poles

$\therefore$ unstable

$\therefore$ Stable for $K \in [0, 97.96)$

≡ It could also be analyzed by Bode Plot:

$$\therefore GH(j\omega) = \frac{K}{4(1+j\omega/2)^2(1+.3j\omega-\omega^2/10)10} = \frac{K/40}{(1+j\omega/2)^2\left(1-\frac{\omega^2}{10}+.3j\omega\right)}$$

Take $K/40 = 1$ $\therefore K = 40$

$\therefore$ 2 db corners at 2 & $\sqrt{10}$

& 4 ϕ corners at $.2, \frac{1}{\sqrt{10}}, 20, 10\sqrt{10} (= 31.6)$

Take 3 decades $.1 \to 100$

$\therefore$ db line:

at $\omega = .1$ $\therefore GH = 0$ db and keeps at 0 db/dec till

$\omega = 2$ $\therefore GH = 0$ db, after which slope is -40 db/dec till

$\omega = \sqrt{10} = 3.16$ $\therefore GH = -7.96$ db after which slope is -80 db/dec till

$\omega = 100$ $\therefore GH = -127.96$ db and keeps at -80 db/dec &

ω_{0db} is at $\omega = 2$ (curve is asymptotic to 0db from below)

& ϕ line:

at $\omega = .1$ $\therefore \phi = 0°$ and keeps at $0°$/dec till:

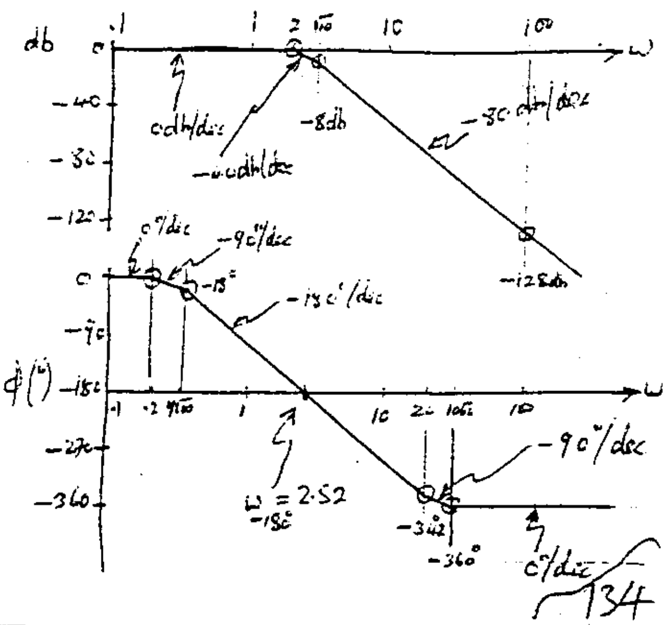
at $\omega = .2$ $\therefore \phi = 0°$ after which slope is $-90°$/dec till

$\omega = \frac{1}{\sqrt{10}} = .316$ $\therefore \phi = -17.91°$ " " " $-180°$/dec "

$\omega = 20$ $\therefore \phi = -342.1°$ " " " $-90°$/dec "

$\omega = 10\sqrt{10} = 31.62$ $\therefore \phi = -360°$ " " " $0°$/dec till ∞

$\omega_{-180°} \in \left(\frac{1}{\sqrt{10}}, 2\right)$ $\therefore \omega_{-180°} = 2.515$



$$GM = -GH\Big|_{\omega_{-180°}} = -GH\Big|_{2.52} =$$

$$= -(-3.98) = 3.98\text{ db} = 1.58$$

$\therefore \omega_{0db} = \omega_{-180°}$ when $K/40 = 1.58$

$\therefore K_{critical} = 63.25$

$\therefore$ Stable for $K \in [0, 63.25)$

(OK, within Bode approximation.)

\# أولاً: إيجاد الخطأ في تقريب بود للدالة 1 + s

1.1. $$e_{db} = 20\log|1+j\omega| - \begin{cases} 0 & \omega<1 \\ 20\log\omega & \omega\geq 1 \end{cases}$$

$$= \begin{cases} 10\log(1+\omega^2) & 0\leq\omega\leq 1 \\ 10\log(1+\omega^2) - 20\log\omega = 10\log\left(\frac{1}{\omega^2}+1\right) & \omega\geq 1 \end{cases}$$

$$\therefore\ e_{db}\Big|_{max} = \left\{ \begin{matrix} 10\log 2 & \text{at } \omega=1^- \\ 10\log 2 & \text{at } \omega=1 \end{matrix} \right\} = 3\ db \text{ at } \omega=1$$

1.2. $$e_\phi = \tan^{-1}\omega - \begin{cases} 0 & \omega<.1 \\ 45^\circ * \log\frac{\omega}{.1} & \omega\in[.1,10] \\ 90^\circ & \omega>10 \end{cases}$$

$$= \begin{cases} \tan^{-1}\omega & \omega<.1 \\ \tan^{-1}\omega - 45^\circ\log 10\omega & \omega\in[.1,10] \\ \tan^{-1}\omega - 90^\circ & \omega>10 \end{cases}$$

$$\therefore\ e_\phi\Big|_{max} = \begin{cases} \tan^{-1}.1 & \text{at } \omega=.1^- \\ \because e'_\phi=0 & \omega\in[.1,10] \\ \tan^{-1}10 - 90^\circ & \text{at } \omega=10^+ \end{cases}$$

To find ω: $e'_\phi=0$, put e_ϕ in rad. form $\Rightarrow e_\phi = \tan^{-1}\omega - \frac{\pi}{4}\log 10\omega$

$\therefore\ e'_\phi = 0 \Rightarrow \frac{1}{1+\omega^2} - \frac{\pi}{4}\cdot\frac{1}{\ln 10}\cdot\frac{1}{\omega} = 0 \Rightarrow \omega^2 - \frac{4\ln 10}{\pi}\omega + 1 = 0$

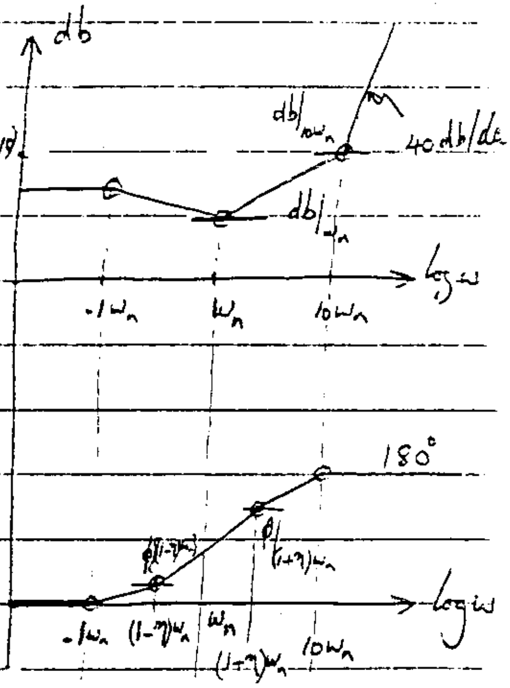
$\therefore\ \omega = \frac{4\ln 10 \pm \sqrt{(4\ln 10)^2 - 4\pi^2}}{2\pi} = .3941\ \&\ 2.5377$

$$\therefore e_\phi\Big|_{max} = \begin{cases} \tan^{-1}.1 = 5.71^\circ & \text{at } \omega = .1 \\ \tan^{-1}.3941 - 45^\circ \log 3.941 = -5.29^\circ & \text{at } \omega = .3941 \\ \tan^{-1}2.5377 - 45^\circ \log 25.377 = 5.29^\circ & \text{at } \omega = 2.5377 \\ \tan^{-1}10 - 90^\circ = -5.71^\circ & \text{at } \omega = 10^+ \end{cases}$$

$$\therefore e_\phi\Big|_{max} = 5.71^\circ \text{ at } \omega \in \{.1, 10\}$$

ثانياً: إيجاد أكبر خطأ في تقريب بود للدالة s^2+s+1

$$2.1.\quad e_{db} = 20\log\left|1-\omega^2+j\omega\right| - \begin{cases} 0, & \omega < .1 \\ \frac{0-0}{1}\log\omega, & \omega \in (.1,1) \\ \frac{20\log|-99+j10|}{1}\log\omega, & \omega \in (1,10) \\ 20\log|-99+j10| + 40\log\frac{\omega}{10}, & \omega > 10 \end{cases}$$



$$= 10\log(\omega^4-\omega^2+1) - \begin{cases} 0 & \omega < 1 \\ (10\log 9901)\log\omega, & \omega \in (1,10) \\ 10\log 9901 + 40\log .1\omega & \omega > 10 \end{cases}$$

$$= \begin{cases} 10\log(\omega^4-\omega^2+1), & \omega < 1 \\ 10\log(\omega^4-\omega^2+1) - 10\log 9901\cdot\log\omega, & \omega \in (1,10) \\ 10\log(\omega^4-\omega^2+1) - 10\log 9901 - 40\log\omega, & \omega > 10 \end{cases}$$

$\therefore$ a. max at $\omega < 1$ at $e'_{db} = 0 \Rightarrow \frac{4\omega^3-2\omega}{\omega^4-\omega^2+1} = 0 \Rightarrow \omega \in \{0, \frac{1}{\sqrt2}\}$

at $\omega = 0$, $e_{db} = 0$ (minimum) & at $\omega = \frac{1}{\sqrt2}$ $e_{db}\Big|_{max} = 10\log .75 = -1.25$ db

b. max. at $\omega \in (1,10)$ at $e'_{db} = 0 \Rightarrow$

$$\frac{4\omega^3 - 2\omega}{\omega^4 - \omega^2 + 1} - \log .9901 \cdot \frac{1}{\omega} = 0 \Rightarrow$$

$$4\omega^4 - 2\omega^2 - \log .9901 \cdot (\omega^4 - \omega^2 + 1) = 0 \Rightarrow$$

$$(4 - \log .9901)\,\omega^4 + (\log .9901 - 2)\,\omega^2 - \log .9901 = 0$$

$\therefore$ $\omega = 1.9936$ or -463.86, second rejected.

$\therefore$ at $\omega = 1.9936$, $e_{db}\big|_{max} = -.89313$ db

c. max. at $\omega > 10$ at $e'_{db} = 0 \Rightarrow$

$$\frac{4\omega^3 - 2\omega}{\omega^4 - \omega^2 + 1} - \frac{4}{\omega} = 0 \Rightarrow 4\omega^4 - 2\omega^2 - 4(\omega^4 - \omega^2 + 1) = 0$$

$\therefore$ $2\omega^2 = 4$ $\therefore \omega = \sqrt{2} \notin \omega > 10$

$\therefore$ max. occure at either sides of interval

$\omega = 10 \Rightarrow e_{db} = 0.0$

$$\omega \to \infty \Rightarrow e_{db}\big|_{max} = \lim_{\omega \to \infty} 10 \log\left(\frac{\omega^4 - \omega^2 + 1}{\omega^4}\right) - 10 \log .9901$$

$$= 0 - 10 \log .9901 = +.04321 \text{ db}$$

$$\therefore e_{db}\big|_{max} = \left\{\begin{array}{ll} -1.25 & \text{at } \omega = .7071 \\ -.893 & \text{at } \omega = 1.994 \\ +.0432 & \text{at } \omega \to \infty \end{array}\right\} = 1.25 \text{ db at } \omega = .707$$

$$2.2.\quad e_\phi = \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) - \begin{cases} 0 & , \omega < .1 \\ \frac{.5880}{\log 5}\cdot \log 10\omega & , \omega \in (.1, .5) \\ \frac{2.2655 - .5880}{\log 3}\cdot \log 2\omega + .5880 & ; \omega \in (.5, 1.5) \\ \frac{\pi - 2.2655}{\log\left(\frac{10}{1.5}\right)}\cdot \log\left(\frac{2\omega}{3}\right) + 2.2655 & , \omega \in (1.5, 10) \\ \pi & , \omega > 10 \end{cases}$$

(in rad)

$$= \begin{cases} \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) & , \omega < .1 \\ \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) - .84124 \log 10\omega & , \omega \in (.1, .5) \\ \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) - 3.5159 \log 2\omega - .5880 & , \omega \in (.5, 1) \\ \tan^{-1}\left(\frac{\omega}{\omega^2-1}\right) - 3.5159 \log 2\omega + 2.5536 & , \omega \in (1, 1.5) \\ \tan^{-1}\left(\frac{\omega}{\omega^2-1}\right) - 1.0633 \log\left(\frac{2\omega}{3}\right) + .87606 & , \omega \in (1.5, 10) \\ \tan^{-1}\left(\frac{\omega}{\omega^2-1}\right) & , \omega > 10 \end{cases}$$

$\therefore$ a. max in $\omega < .1$ at $e_\phi' = 0 \Rightarrow \dfrac{\frac{1-\omega^2+2\omega^2}{(1-\omega^2)^2}}{1+\left(\frac{\omega}{1-\omega^2}\right)^2} = 0 \Rightarrow$

$\dfrac{1+\omega^2}{1-\omega^2+\omega^4} = 0 \Rightarrow \omega^2 = -1$ $\therefore$ no solution $\therefore$ max occurs at

either ends of interval $\therefore$ $e_\phi\big|_{max} = \tan^{-1}\left(\frac{.1}{1-.1^2}\right) = 5.77^\circ$ at $\omega = .1$

b. max in $\omega \in (.1, .5)$ at $e_\phi' = 0 \Rightarrow \dfrac{1+\omega^2}{1-\omega^2+\omega^4} - \dfrac{.84124}{\ln 10}\dfrac{1}{\omega} = 0$

$\therefore \ \omega = 3.2703, \ .30579 \ \& \ -.4195 \pm j.90777,$

all rejected but $\omega = .30579$ at which $e\phi|_{max} = -4$

c. max in $\omega \in (.5, 1)$ at $e'_\phi = 0 \Rightarrow \dfrac{1+\omega^2}{1-\omega^2+\omega^4} - \dfrac{3.5159}{\omega \ln 10} = 0$

$\therefore \ \omega = .74142, \ 1.3488 \ \& \ -.7176 \pm j.6964,$

all rejected but $\omega = .74142$ at which $e\phi|_{max} = -.1646r = -9.43$

d. max. in $\omega \in (1, 1.5)$ at $e'_\phi = 0 \Rightarrow \dfrac{1+\omega^2}{1-\omega^2+\omega^4} + \dfrac{3.5159}{\omega \ln 10} = 0$

$\therefore \ \omega = -.74142, \ -1.3488 \ \& \ .7176 \pm j.6964$, all

rejected. Hence, max. is at either sides of interval.

$\therefore \ e\phi|_{max} = -\frac{\pi}{2} - 3.5159 \log 2 + 2.5536 = -.076 \text{ rad} = -4.33^\circ$ at $\omega = 1$

or: $e\phi|_{max} = -.8761 - 3.5159 \log 3 + 2.5536 = 0$ at $\omega = 1.5$, rejec

e. max. in $\omega \in (1.5, 10)$ at $e'_\phi = 0 \Rightarrow \dfrac{1+\omega^2}{1-\omega^2+\omega^4} + \dfrac{1.0633}{\omega \ln 10} = 0$

$\therefore \ \omega = -.3619, \ -2.7635 \ \& \ .4799 \pm j.8773$, all

rejected. Hence, max is at either sides of interval.

$\therefore \ e\phi|_{max} = 0$ at $\omega = 1.5$ & $e\phi|_{max} = -.1007 \text{ rad} = -5.77^\circ$ at $\omega = 10$

ϕ. max. in $\omega > 10$ is as in (a) occurring at either sides of interval $\Rightarrow e_\phi\Big|_{max} = -5.77^\circ$ at $\omega = 10$

$$\therefore e_\phi\Big|_{max} = \left\{\begin{array}{l} 5.77^\circ \text{ at } \omega = .1000 \\ -4.76^\circ \text{ at } \omega = .3058 \\ -9.43^\circ \text{ at } \omega = .7414 \\ -4.33^\circ \text{ at } \omega = 1.000 \\ -5.77^\circ \text{ at } \omega = 10 \end{array}\right\} = 9.43^\circ \text{ at } \omega = .$$

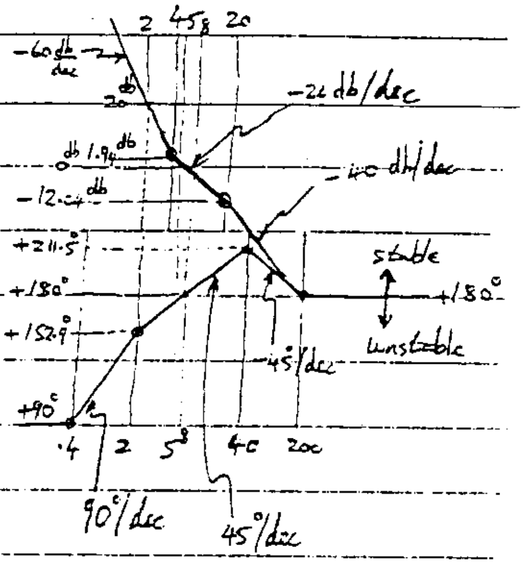
Summary:

	For $s+1$	For s^2+s+1
$\therefore$ max. e_{db}	3^{db} at $\omega = 1$	1.25^{db} at $\omega = .707$
ϕ max e_ϕ	5.71° at $\omega = .1$ or 10	9.43° at $\omega = .74$

$GH = \dfrac{K(s+4)^2}{s^3(s+20)}$.   انظر مسألة صفحة ٨٧ من الحلول

Lets plot Bode's for $K=100$

$\therefore\ GH(j\omega) = \dfrac{80\ \left(1+j\frac{\omega}{4}\right)^2}{(j\omega)^3\ \left(1+j\frac{\omega}{20}\right)}$



For db,

at $\omega = 2$ $\therefore$ db $= 20$, at -60 db/dec, till

at $\omega = 4$, $\therefore$ db $= 1.94$ after which at -20 db/dec

till $\omega = 20$, $\therefore$ db $= -12.04$ " " " -40 db/dec

0 db occurs at $\omega = 4 * 10^{(1.94/20)} = 5.00$

For ϕ,

at $\omega \leq .4$ flat at $90°$ after which it runs at $90°$/dec, till

at $\omega = 2$, $\therefore \phi = 152.9°$ " " " " " $45°$/dec, till

at $\omega = 40$, $\therefore \phi = 211.5°$ " " " " " $-45°$/dec, till

at $\omega = 200$, $\therefore \phi = 180°$ where it becomes flat to ∞

$180°$ occurs at $\omega = 2 * 10^{(180-152.9)/45} = 8.00$

$\therefore$ Phase margine $= 180 + \phi(\text{at } 0\text{db}) = 180 + \phi(\omega=5) = 180 + 152.9 + 45 * \log\frac{5}{2}$
$= 350.8 \equiv -9.2°$, unstable

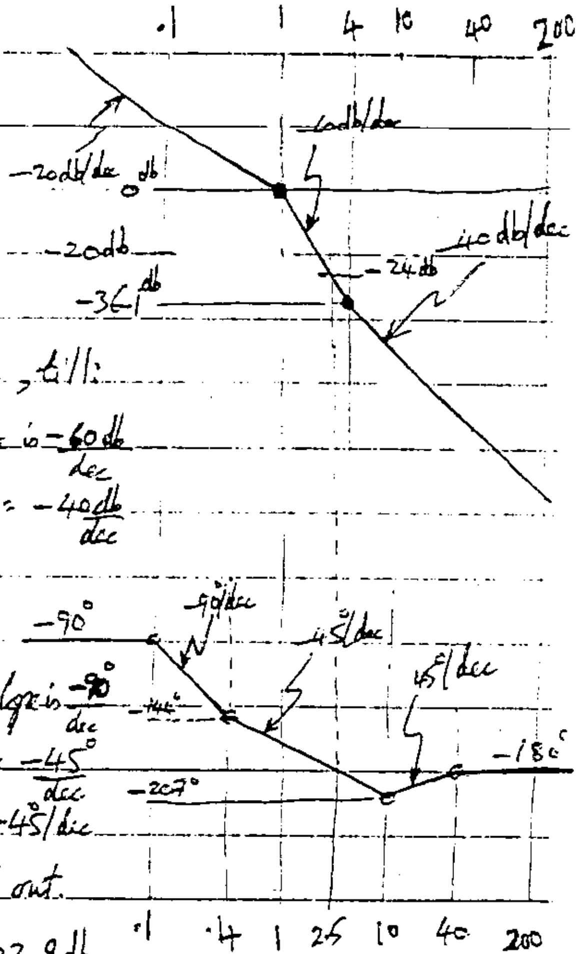
& Gain margine $= -\,\text{db}(\text{at } 180°) = -\,\text{db}(\omega = 8) = -\left[1.94 - 20\log\frac{8}{4}\right] = 4.08$ db, stable

Hence, at $K=100$ system is unstable. To make it stable, K must be additionally magnified by more than 4.08 db $\equiv 1.6$ times

$\therefore$ For stability, $K > 160$ [cf. $K > 67$, 140% error due to approx]

$GH = \frac{K(s+4)}{s(s+1)^2}$, limits for stability + GM & PM for K= Using Bode Plot.

$\therefore\ GH(j\omega) = \frac{1 + j\frac{\omega}{4}}{j\omega \cdot (1 + j\omega)^2}$



db plot:
Corners at 1, 4:
before $\omega = 1$, slope is -20 db/dec, till:
at $\omega = 1$, db $= 0$ after which slope is $-60\frac{db}{dec}$
till $\omega = 4$, db $= -36.1$ " " " $-40\frac{db}{dec}$
ϕ plot:
Corners at .1, .4, 10 & 40:
before $\omega = .1$, flat at $-90°$ after which slope is $-\frac{90°}{dec}$
till $\omega = .4$, $\therefore \phi = -144.2°$ after which slope is $-\frac{45°}{dec}$
till $\omega = 10$, $\therefore \phi = -207.1°$ " " " $+45°$/dec
till $\omega = 40$, $\therefore \phi = -180°$ & it flats out.
ϕ is $-180°$ at $\omega = 2.5$ where db $= -23.9$ db
db is zero at $\omega = 1$ where $\phi = -144.2 - 45\log 2.5 = -162.1°$

$\therefore$ Phase margin $= 180° - 162.1° = 17.9°$
& Gain margin $= -(-23.9\text{ db}) = 23.9$ db
$\therefore$ System is stable till K is magnified by more than 23.9 db $\equiv$ 15.6
$\therefore$ System is stable for $K < \frac{1}{4} \times 15.6 = 3.9$
$\therefore$ limits of K for stability is $K \in (0, 3.9)$ [cf $K \in (0, 1)$ ... $=402$]

Bode Plots for $GH = \dfrac{s^2+2s+10}{s^2(s-4)} = \dfrac{2.5}{\omega^2}\dfrac{1-\frac{\omega^2}{10}+j.2\omega}{(1-j\frac{\omega}{4})}$

I. db plot.

Corners for num. at $.1\sqrt{10}, \sqrt{10}, 10\sqrt{10}$
$(.316), (3.16), (31.6)$

" " denom. " 4

$\therefore \omega_n = \sqrt{10}$

& $\eta = \frac{\omega_n}{2} * .2$
$= .1\sqrt{10}$
$= .31623$

$\therefore$ Curve goes at -40 db/dec till:

$\omega = .1\sqrt{10} = .316$ & $db = 20\log\frac{2.5}{(.1\sqrt{10})^2} = 27.96$ db.

It then goes straight till:

$\omega = 3.16$ & $db = 20\log\left(\frac{2.5}{10} * .2\sqrt{10}\right) = -16.02$ db.

It then goes straight till:

$\omega = 4$ & $db = 20\log\left(\frac{2.5}{4^2} * \sqrt{\left(1-\frac{4^2}{10}\right)^2 + (.2*4)^2}\right) = -16.12$ db

It then goes straight till:

$\omega = 31.6$ & $db = 20\log\left[\frac{2.5\sqrt{(100)^2+40^2}}{100 \quad * 31.6/4}\right] = -29.98$ db after which it goes at -20 db/dec to infinity.

0 db occures at $\omega \in (.316, 3.16)$ $\therefore \omega = .316 * 10^{\frac{27.96}{27.96+16.02}} = 1.366$

II. ϕ plot

Corners for num. at $.1\sqrt{10}, (1-.1\sqrt{10})\sqrt{10}, (1+.1\sqrt{10})\sqrt{10}, 10\sqrt{10}$
$(.316), (2.1623), (4.1623), (31.6)$

Corners for denom. at $.4, 40$

$\therefore$ Curve goes flat at 0 due to neutralization of s^2 term with the negation sign; till $\omega = .316$ where it goes straight to $\omega = .4$ & $\phi = \tan^{-1} .2 * .4 = 4.65°$; then straight to $\omega = 2.1623$ & $\phi = \tan^{-1}\dfrac{.2 * 2.1623}{1-\frac{2.1623^2}{10}} + 45° * \log\dfrac{2.1623}{.4} = 72.06°$
$\quad 1 - .016$

straight to $\omega = 4.1623$ & $\phi = \tan^{-1}\dfrac{.2 * 4.1623}{\frac{4.1623^2}{10} - 1} + 180° + 45° \log\frac{4.16}{.4}$
$= 177.1°$

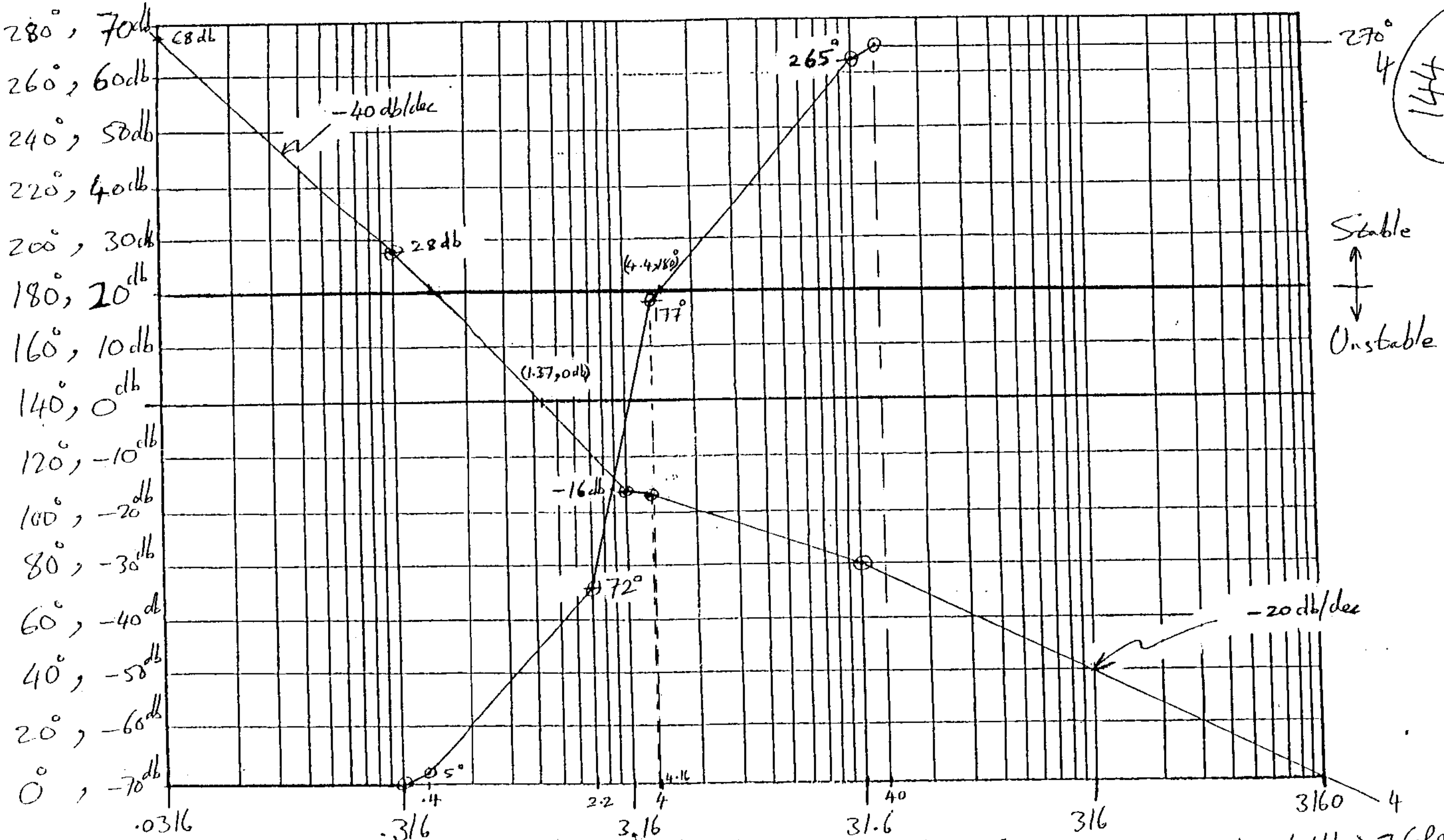
straight to $\omega = 31.6$ & $\phi = 180° + 45 \log\frac{31.6}{.4} = 265.4°$

straight at $+45°$/dec to $\omega = 40$ & $\phi = 265.4° + 45° \log\frac{40}{31.6} = 270°$

It then keeps flat at 270°. Plots are as shown.

180° occures at $\omega \in (4.1623, 31.6)$ $\therefore \omega = 4.1623 * 10^{\left[\frac{180-177.1}{265.4-177.1} * \log\frac{31.6}{4.16}\right]} = 4.449$

& db at $\omega = 4.449$ is $-16.12 + \left[\log\left(\frac{4.449}{4}\right)\right] * \dfrac{-29.98+16.12}{\log(31.6/4)} = -16.83$ db $\equiv \frac{1}{6.94}$



To make the system stable, K should be magnified by -(-16.83db) = 16.83db ≅ 6.94 or more ∴ K should be > 7 (cf. 9)

To solve for stability when, $GH = \dfrac{K(s^2+4s+13)}{s(s-2)(s-4)(s+10)}$:

a) by Routh

$\therefore f(s) = s(s-2)(s-4)(s+10) + K(s^2+4s+13) = 0$

or

$f(s) = s^4 + 4s^3 + (K-52)s^2 + 4(K+20)s + 13K = 0$

$\therefore$

s^4	1	$K-52$	$13K$
s^3	4	$4(K+20)$	
s^2	72	$13K$	
s	$85K+1440$		
1	$13K$		

$\therefore$ System is always unstable and has:

- one RHP root for $K < -\frac{1440}{85} = -16.94$
- one RHP root and two oscilatory roots for $K = -16.94$
- three RHP roots for $K \in (-16.94, 0)$
- two RHP roots for $K > 0$

b) by Root Locus:

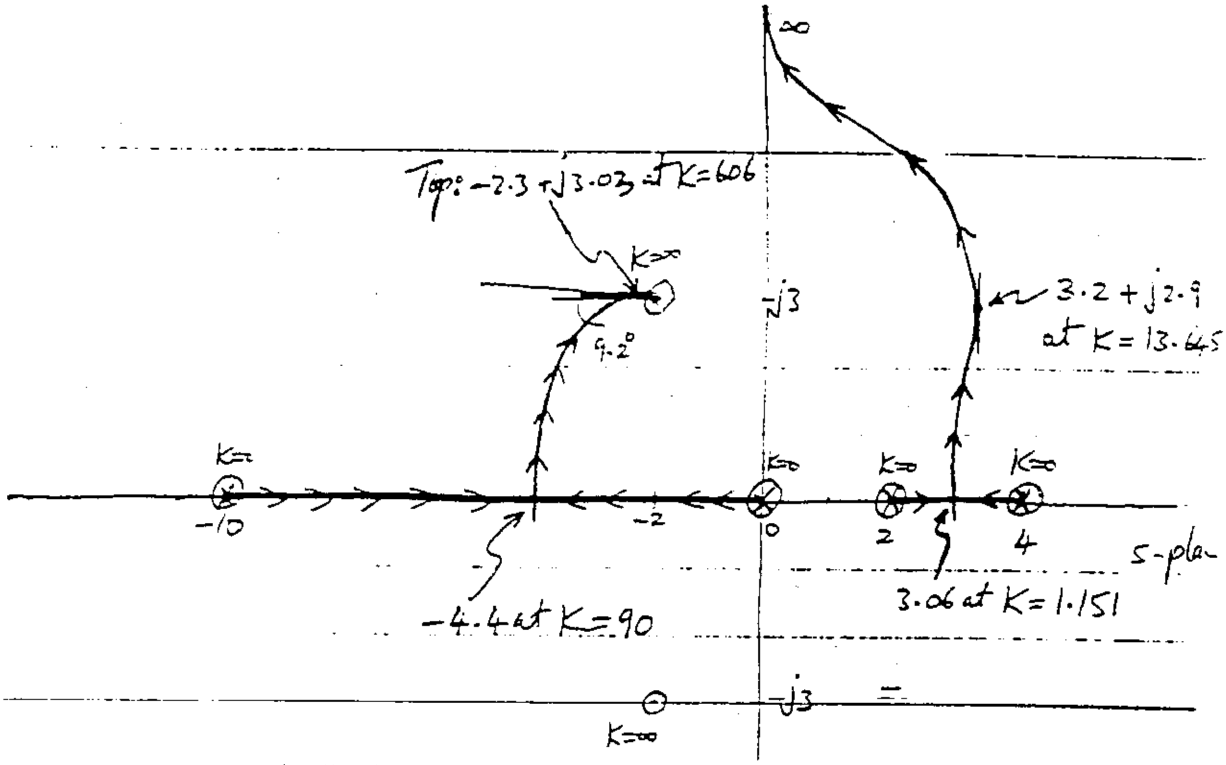
- poles: $0, 2, 4$ & -10 ; total 4
- zeros: $-2 \pm j3$; total 2

$\Big\}$ four branches

$\therefore$ 2 assymptotic zeros at $\pm\pi/2$ radiating from zero.

Hence, utilizing symmetry, Locus is as shown, where:

departure from $s = -2 + j3$ is at α :



$\tan^{-1}\frac{3}{8} + \left(\pi - \tan^{-1}\frac{3}{2}\right) + \left(\pi - \tan^{-1}\frac{3}{4}\right) + \left(\pi - \tan^{-1}\frac{3}{6}\right) - \left(\frac{\pi}{2} + \alpha\right) = \text{odd}\,\pi$

∴ $\alpha = 170.811^\circ$;

break away points at s : $f(s) = f'(s) = 0$

∴ in $(2, 4)$, $s = 3.0555$ at $K = 1.1508$

& in $(-10, 0)$, $s = -4.3921$ at $K = 89.746$

intercept with immaginary axis :

$f(j\omega) = 0$ ∴ $\omega^4 - (K-52)\omega^2 + 13K = 0$ ... (1) & $-4\omega^3 + 4(K+20)\omega = 0$ (2)

from (2) : $K = \omega^2 - 20$ ... (3) into (1) ∴ $\cancel{\omega^4} + 52\omega^2 + (20 - \cancel{\omega^2})\omega^2 + 13\omega^2 - 260 = 0$

∴ $85\omega^2 = 260$ ∴ $\omega^2 = \frac{260}{85}$ ∴ $K = \frac{260 - 1700}{85} = -\frac{1440}{85}$ (∴ 0 !)

∴ for $K > 0$ no intercept

∴ branches leaving $s = 3.0555$ go to assymptotic zeros, whereas those leaving $s = -4.3921$ go to $-2 \pm j3$ zeros.

Sides of loops : at $f(s) = 0 = \mathrm{Re}\,\frac{ds}{dK}$

∴ $s = 3.2003 + j\,2.9184$ at $K = 13.6448$

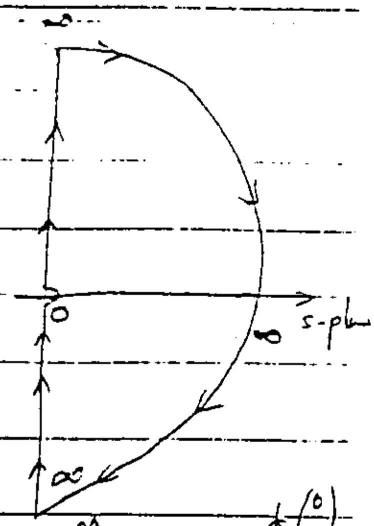
Top of loops at $f(s) = 0 = \mathrm{Im}\,\frac{ds}{dK}$ ⇒ $s = -2.3168 + j3.0260$ at $K = 605.80$

∴ System is always unstable for $K > 0$. It has two RH poles for any $K > 0$.

(∴ OK)

c) by Nyquist: (take $K=1$)

$\therefore GH(j\omega) = M\angle\phi$, where
$M = |GH|$ & $\phi = \angle GH$.
This will map the contour shown in the s-plane into that shown in the GH plane, where:



ω	M	$\phi(^\circ)$	ω	M	$\phi(^\circ)$
.0001	1625	-89.99	1.1	.1201	-31.62
.0005	325	-89.97	1.2	.1063	-26.63
.001	162.5	-89.95	1.3	.0946	-21.69
.005	32.5	-89.73	1.4	.0846	-16.79
.01	16.25	-89.45	1.5	.0760	-11.94
.05	3.246	-87.26	1.6	.0685	-7.12
.10	1.622	-84.51	1.7	.0619	-2.33
.15	1.078	-81.78	1.7489	$.0590\overline{27}$	0.000
.16169	1.00000	-81.14	1.8	.0562	+2.43
.20	.8063	-79.04	1.9	.0511	+7.17
.25	.6423	-76.31	2.0	.0467	+11.89
.30	.5325	-73.57	2.2	.0393	+21.29
.35	.4537	-70.87	2.5	.0310	+35.29
.40	.3942	-68.17	2.7	.0269	+44.52
.45	.3477	-65.47	3.0	.0224	+58.05
.50	.3102	-62.79	3.3	.0192	+70.96
.6	.2534	-57.45	3.6	.0170	+82.97
.7	.2123	-52.17	3.78944	.0159	+90.00
.8	.1810	-46.95	4.0	.0149	+97.25
.9	.1564	-41.78	5	.0121	+123.94
1.0	.1365	-36.67	10	.0062	+167.20

& as $\omega \to \infty$, $M \to 0$ & $\phi \to +180.00^\circ$. Resulting is the shown plot.

The $\pm 90^\circ$ asymptotes will have intercept of σ:

$$\sigma = \lim_{s\to j\epsilon\to 0} \mathrm{Re}\, GH = \lim_{\epsilon\to 0} \mathrm{Re}\left[\frac{13+4j\epsilon}{j\epsilon\,(j\epsilon-2)(j\epsilon-4)(j\epsilon+10)}\right] = \lim_{\epsilon\to 0} \mathrm{Re}\left[\frac{(13+4j\epsilon)}{80j\epsilon(1-j\frac{\epsilon}{2})(1-j\frac{\epsilon}{4})(1+j\frac{\epsilon}{10})}\right]$$

$$= \lim_{\epsilon\to 0} \mathrm{Re}\left[\frac{13}{80j\epsilon}\cdot\left(1+j\frac{4\epsilon}{13}\right)\cdot\left(1+j\frac{\epsilon}{2}\right)\left(1+j\frac{\epsilon}{4}\right)\left(1-j\frac{\epsilon}{10}\right)\right]$$

$$= \lim_{\epsilon\to 0} \mathrm{Re}\left[\frac{13}{80j\epsilon}\left(1+j\epsilon\left\{\frac{4}{13}+\frac{1}{2}+\frac{1}{4}-\frac{1}{10}\right\}\right)\right] = \frac{13}{80}\cdot\frac{80+130+65-26}{260}$$

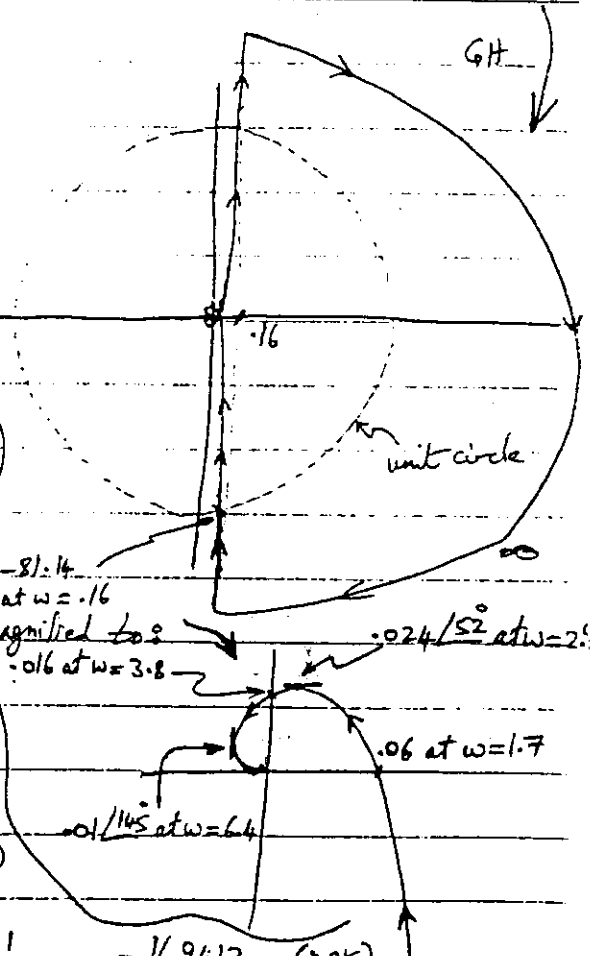
$$\therefore \sigma = \frac{249}{1600} = 0.155625 \simeq .16$$

top of loop at $\mathrm{Im}\frac{dGH}{d\omega} = 0$ or $\mathrm{Re}\frac{dGH}{ds} = 0$

$\therefore$ top of loop is $0.02415\angle 52.31^\circ$ at $\omega = 2.8716$

& Side of loop at $\mathrm{Re}\frac{dGH}{d\omega} = 0$ or $\mathrm{Im}\frac{dGH}{ds} = 0$

$\therefore$ Side of loop is $.0099\angle 145.23^\circ$ at $\omega = 6.3721$



$\therefore$ For stability:

When $K > 0$: $n_z - n_p = n_{-1} \Rightarrow n_z = 2$

$\therefore$ always unstable with 2 RH roots ($\therefore$ OK)

When $K < 0$: $n_z - n_p = n_{+1} \Rightarrow n_z = 2 + n_{+1}$

$\therefore$ { $n_z = 2+1 = 3$ for $|K| < \frac{1}{.059027} = 16.9412$ ($\therefore$ OK)
$\therefore$ always unstable with 3 RH roots for $K \in (-16.94, 0)$
$n_z = 2-1 = 1$ for $|K| > 16.9412$
$\therefore$ unstable with one RH root for $K < -16.94$ ($\therefore$ OK)

Hence, for any value of $K \in (-\infty, \infty)$ system is unstable.

d) by Bode: (take $K = \frac{80}{13} = 6.1538$)

$\therefore\ GH(j\omega) = \dfrac{1 - \frac{\omega^2}{13} + j\frac{4\omega}{13}}{j\omega\left(1 - j\frac{\omega}{2}\right)\left(1 - j\frac{\omega}{4}\right)\left(1 + j\frac{\omega}{10}\right)}$

$\omega_n = \sqrt{13} = 3.6056$, $\eta = \dfrac{4/13}{2/\sqrt{13}} = \dfrac{2}{\sqrt{13}} = 0.5547$

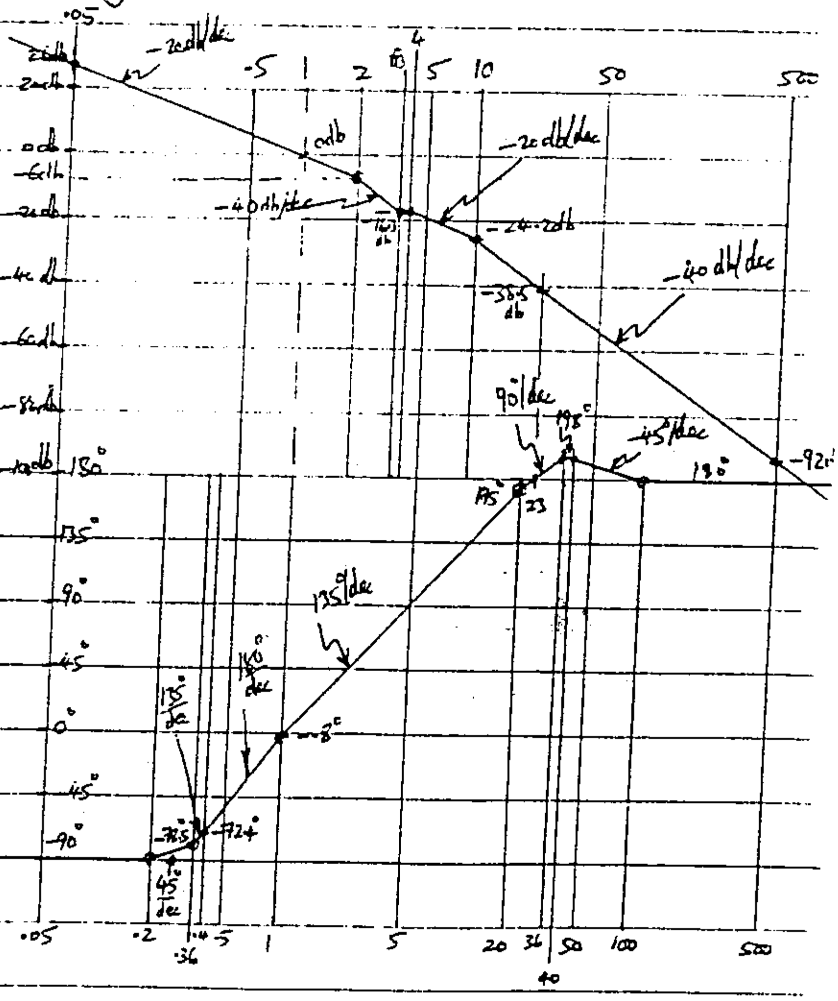
db corners: $2, \sqrt{13} = 3.6, 4, 10$

ϕ corners: $.2, .36, .4, 1, 20, 36, 40, 100$.

Chose 4 decades starting .05 & finishing 500

$\therefore$ db curve will come at -20 db/dec hitting .05 at $20\log 20 = 26$db till $(2, -6\text{db})$ after which at -40 db/dec till $(3.6, -16.3\text{db})$ level till $(4, -16.3\text{db})$ after which at -20 db/dec till $(10, -24.2\text{db})$ after which at -40 db/dec till $(500, -92.2\text{db})$

& ϕ curve will come at $-90°$ level till $(.2, 90°)$ then at $45°$/dec till $(.36, -78.5°)$ "
" $135°$/dec " $(.4, -72.4°)$ "
" $180°$/dec " $(1, -.77°)$ "
" $135°$/dec " $(20, 174.9°)$ "
" $90°$/dec " $(36, 197.9°)$ "
level till $(40, 197.9°)$ "
at $-45°$/dec " $(100, 180°)$ "
level at $180°$.



$\therefore$ System is unstable since 0 db occurs at $\omega = 1$ with $\phi_{0db} = -.8° \in$ 4th quad. This will also be the case for $0 < K < 6.15$. For $K > 6.15$; 0 db will correspond to $\phi \in$ 1 & 2 quad till $K \geq 520$ at $\omega \geq 22.8$ where $\phi_{0db} \in$ 3rd quad but $db_{\pi} \geq 0$. Hence, unstable for all $K > 0$. (30 OK) 5

To linearize: $X(\cos Y)e^{Z} + U^2 = 0$

∴ Let changes be indicated by small letters.

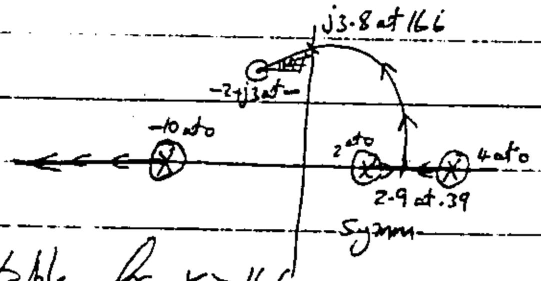
∴ $(e^{Z}\cos Y)x - (e^{Z}\sin Y \cdot X)y + (e^{Z} X \cos Y)z + (2U)u = 0$

a) dc gain of system $= \dfrac{G(0)}{1+G(0)} = \dfrac{13K/80}{1+13K/80} = \dfrac{13K}{13K+80}$.

& cut-off rate $= \dfrac{G(\infty)}{1+G(\infty)} = \dfrac{Ks^2/s^3}{1+Ks^2/s^3} = \dfrac{Ks^2}{s^3+Ks^2} = \dfrac{K}{s}$

∴ System will cut-off at -20 db/dec.

b) RL



stable for $K > 16.6$

c) Routh

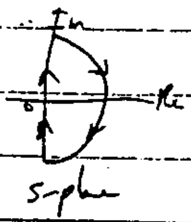
1	$4K-52$
$4+K$	$13K+80$
$4K^2-49K-288$	
$13K+80$	

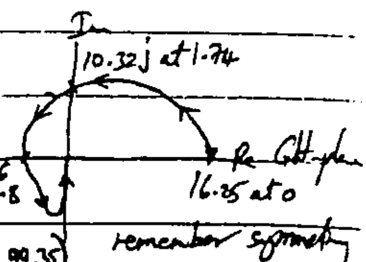
Stable for $K \in (16.6, \infty)$

d) type '0', $K_p = \dfrac{13K}{80}$, $K_v = K_a = 0$

e) $e_{ss}(t) = \dfrac{240t}{80+13K} - \dfrac{2988K}{(80+13K)^2}$ & $K > 16.6$; System can't cope.

f) Nyquist Plot is as shown





∴ $n_p = n_z = -2$ ∴ $n_p = 2-2 = 0$ ∴ stable

This is the case for $K > \frac{100}{6} = 16.\bar{6}$.

∴ $GM = \frac{1}{6} = -15.6$ db whereas $PM \cong 90^\circ$ (exact 89.98° at $\omega = 99.35$) remember symmetry

OR

j) Bode Plot: $GH(j\omega) = \frac{1300}{80} \cdot \frac{1-\frac{\omega^2}{13}+j\frac{4\omega}{13}}{(1+j\frac{\omega}{10})(1-j\frac{\omega}{4})(1-j\frac{\omega}{2})}$

db: corners at $2, \sqrt{13} \cong 3.6, 4, 10$

0 db/dec till (2, 24.22 db) after which at

−20 db/dec " ($\sqrt{13}$, 19.10 db) " "

+20 db/dec " (4, 20.00 db) " "

0 db/dec " (10, 20.00 db) " "

−20 db/dec till ∞

ϕ: corners at .2, .36, .4, 1, 20, 36, 40, 100

0°/dec till (.2, 0°) after which at

45°/dec " (.36, 11.52°) " "

135°/dec " (.4, 17.60°) " "

180°/dec " (1, 89.23°) " "

135°/dec " (20, 264.87°) " "

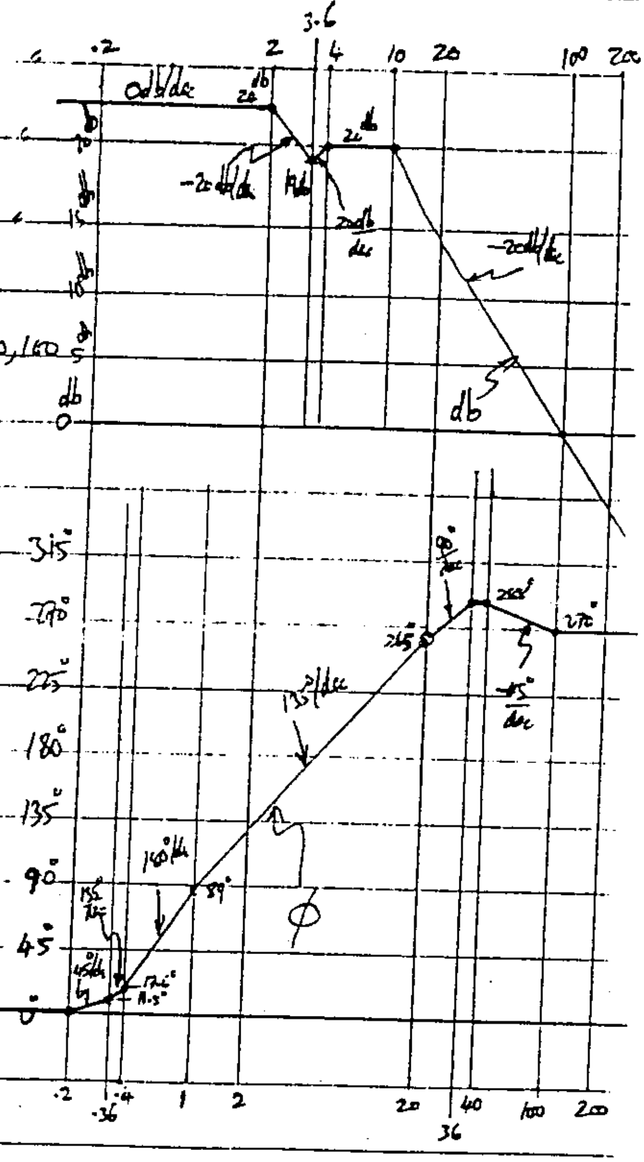
90°/dec " (36, 287.91°) " "

0°/dec " (40, 287.91°) " "

−45°/dec " (100, 270.00°) " "

0°/dec level at 270°

This is plotted as shown.



180° occurs at $\omega \in (1, 20)$

$\therefore \omega = 1 * 10 \wedge \left(\frac{180 - 89.23}{135}\right) = 4.703$

0 db occurs at $\omega > 10$ $\therefore \omega = 10 * 10 \wedge \left(\frac{0-20}{-20}\right) = 100$

$\therefore GM = -db(\omega = 4.703) = -\left(20 + 0 \log \frac{4.703}{4}\right) = -20.0$ db

& $PM = 180 + \phi(\omega = 100) = 180 + 270 = 450° \equiv 90°$