بسم الله الرحمن الرحيم

الحلول المختارة لطلاب الهندسة والعمارة

القياسات الكهربية والالكترونية

اعداد

سلمان محمد العاصي

الاستاذ المساعد بقسم الهندسة الكهربية والحاسبات

بجامعة ام القرى

بسم الله الرحمن الرحيم

الحمد لله والصلاة والسلام على سيدنا رسول الله وعلى آله وصحبه ومن والاه

وبعد، فهذه مجموعة من المسائل والحلول في مادة القياسات الكهربية

والالكترونية بدأ تدوينها د/ علي رشدي ود/ محمد الهنداوي ود/ عبدالحي

يوسف من أعضاء هيئة التدريس بقسم الهندسة الكهربية بجامعة

الملك عبدالعزيز إبان تعاقبهم على تدريسها بضع سنوات.

وقد قمت بمراجعتها وتبويبها والزيادة عليها بما يغطي الفروق المنهجية

بين ما يدرّس في القسم المذكور بعاليه وما يدرّس بقسم الهندسة

الكهربية والحاسبات بجامعة أم القرى حيث عُنِيَ الأول بقياس مختلف

الكميات الفيزيائية وعُنِيَ الآخر بقياس الكميات الكهربية فرعاً وحسب.

وقد أدرجت المسائل أولاً ثم أتبعتها بالحلول لبعض منها حسب

التبويب الذي سرت عليه إبان تدريسي لهذه المادة بقسم الهندسة

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والله أسأل أن يجعل هذا العمل مفيداً وأن يجزي من شارك

فيه خير الجزاء.

سلطان الشمري

١٤١٠/٧/٢٦

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PROBLEM SET One, Uncertainity.

1-1

A capacitive transducer of two parallel plates of overlapping area of 5 cm^2 ± 0.5% immersed in water. The capacitance, c, was found to be *950* PF. Calculate the Seperation, d between the two plates and the sensitivity $(\partial c/\partial d)$ of this transducer.

If the allowed uncertainty in d is 3%, what is the maximum permissible uncertainty in C ?

Phvsical Constants:

Magnetic Permeability for free space (μo) is $4\pi \times 10^{-7}$ H/m
Permittivity of Vaccum (ϵ_o) is *8.854* PF/m
Relative permittivity for water (ϵ_r) is *81*

1-2

The thermistor is a semiconductor resistance transducer used in the measurement of temperature. The output-input relation for that transducer is

$$R = R_0 \exp\left[\beta_0 (1/T - 1/T_0)\right]$$

where R_0 is the value of resistance at a reference temperature T_0. At the unknown temperature T, the resistance R is measured, then T is calculated as :

$$T = \frac{T_0}{1 + (T_0/B_0)\ ln(R/R_0)}$$

For T_0 = 300°K, β_0 = 3420°K R_0 = 1KΩ ± *30,* R = *2K0* ± *2%*

a) Calculate the nominal value of T.

b) Find the sensitivity $(\partial R/\partial T)$ of the transducer at the given operating point.

c) Show that : $(\Delta T/T)^2 = (T/\beta_0)^2 \left[(\Delta R_0/R_0)^2 + (\Delta R/R)^2\right]$

1-3

A voltage source of an internal resistance R_0 = 0 5 kΩ is measured 8 times with a moving coil meter The following readings were obtained.

6.73, 6.74, 6.69, 6.71, 6.70, 6.73, 6.72, 6.74 volts

The same source was also measured by a recently calibrated digital voltmeter whose input resistance is no less than 20 *MO.* The following 6 readings were obtained:

7.073, 7.069, 7.072, 7.074, 7.071, 7.073 volts.

a) Find the precision and accuracy of both sets of readings

b) How should the readings of the moving coil meter be recalibrated ?

1-4

A spring scale is a transducer that converts weight, W, to deflection, Y.

Its output-input relation is given by

$$Y = \frac{8 D^3 N}{E X^4} \cdot W$$

where D = mean coil diameter of the spring, X = its steel wire diameter, E = torsional elastic modulus for steel, and N = number of coil turns.

The deflection was measured to be Y = (10 ± 0.02) cm. for the following transducer specifications:

D = 2.5 cm. ± 2%, X = 2.5 mm. ± 1%,

E = 80 x 10^9 Pa ± 4%, and N = 50 ± 0.5

a) Calculate the nominal value of W.
b) Calculate the uncertainty in W. Rank the various variables according to their contribution to ΔW.
c) Find the nominal value of the transducer's static sensitivity. Plot this value as a function of the input W.

1-5

A satellite is observed so that its speed may be determined. On the first observation, it was found at a distance R from the observer = (30,000 ± 10) km. Five seconds later, this distance has increased by r = (125.0 ± 0.5) km and the change in angle was ϕ = (0.00750 ± 0.00002) radians. What is the speed of the satellite, assuming that it moves in a straight line and with constant speed in the given direction ?

(Hint = Use the cosine formula:

$$d^2 = (R + r)^2 + R^2 - 2R\,(R + r)\,\text{Cos}\,\phi$$

$$= r^2 + 2R\,(R + r)\,(1 - \text{Cos}\,\phi)$$

and note that $1 - \text{Cos}\,\phi = \phi^2/2 - \phi^4/24$ • ------).

1-6

A strain whose true value is conventionally known as 340 μm/m was measured by a certain guage. The following 12 readings were obtained:

346, 345, 347, 339, 342, 345, 347, 343, 340, 344, 351, 345 μm/m.

calculate:

a) the average value and the bias of this set of readings.
b) the precision and accuracy of the guage.

1-7 The output expression for a multiplier is

$$Z = y_1\, y_2$$

However, to include all sources of error, this expression is rewritten as:

$$Z = P \cdot y_1^{I_1} \cdot y_2^{I_2} + S$$

where $y_1 = y_{10} \pm W_{10}$, $y_2 = y_{20} \pm W_{20}$, $P = 1 \pm W_0$

$I_1 = 1 \pm W_{11}$, $I_2 = 1 \pm W_{21}$, $S = 0 \pm W_3$

Estimate an upper bound on the error in Z.

1-8 A metallic resistance thermometer has a linear variation of resistance with temperature $R = R_0[1 + \alpha(T - T_0)]$.

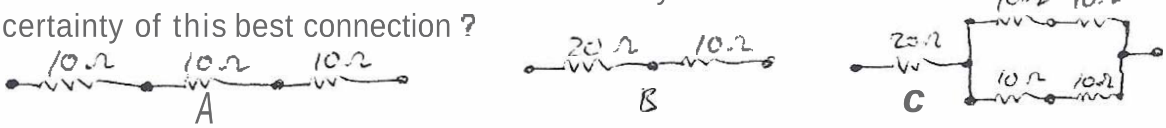
The resistance $R_0 = 20\ K\Omega \pm 0.1\%$, while at a temperature T the resistance R is found to be $R = 30\ K\Omega \pm 0.1\%$. The coefficient α is 0.00392 $^{o}K^{-1}$.

a) Write an explicit expression for T.
b) calculate the nominal value of T.
c) Find the static sensitivity $(\partial R/\partial T)$ of the thermometer.
d) Show that the uncertainty ΔT in T is given by

$$(\Delta T)^2 = (\Delta T_0)^2 + (1/\alpha^2)\,(R/R_0)^2\,[(\Delta R_0/R_0)^2 + (\Delta R/R)^2]$$

e) Find the value of uncertainity in T.

1-9 five resistors are available, one of 20 Ω and four of 10 Ω each. The uncertainty of the 20 Ω resistor is 5% and that of each 10 Ω resistor is 10%. Three possible connections using these resistors are shown below. Which connection would you use to obtain a resistance of 30 Ω with the least uncertainty ? What is the uncertainty of this best connection ?



1-10 The electronic counter can be used for measuring the time period of periodic signals. Show that the uncertainty in the measurement can be reduced by a factor of $1/\sqrt{N}$ if the average of N time periods is taken.

Hint: $T_{av} = 1/N\,(T_1 + T_2 + \ldots\ldots + T_N)$.

The T_i's are statistically independent, $T_i = T + \Delta T$)

1-11 The discharge coefficient C of an orifice can be found by collecting the water that flows through during a timed interval when it is under a constant head h. The formula is

$$C = \frac{W}{t \rho A \sqrt{2gh}}$$

Find C and its uncertainty if

$W = (865 \pm 0.5)\ lb_m$, $A = \dfrac{\pi d^2}{4}$, $d = (0.500 \pm 0.001)''$

$t = (600.0 \pm 2)\ S$, $g = 32.17\ ft/S^2 \pm 0.1\%$

$\rho = 62.36\ lb_m/ft^3 \pm 0.1\%$

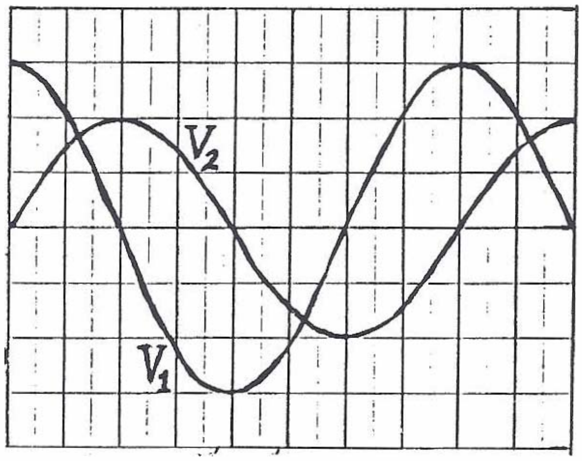
$h = (12.02 \pm 0.01)\ ft$

1-12 The dc current in a resistor $R = 10\ K\Omega \pm 5\%$ is measured to be $I = 10\ mA \pm 1\%$. Find the power it dissipates.

Problem Set Two, Scope.

2-1 Two sinusoidal signals V_1 and V_2 are applied to an oscilloscope in dual trace operation (V_1 applied to CH.I and V_2 applied to CH.2). The trigger source is CH.1 and the various sensitivities are S_{CH1} = 20 m V/cm, S_{CH2} = 1 V/cm, S_X = 20 ms/cm. For the dual trace shown, find

- the peak to peak values of V1 and V_2
- the ratio of V_2 to V_1 in decibels.
- the time period and the frequency of both signals.
- the trigger level and trigger slope.
- the phase shift between V, and V_2. Does V_1 lead or lag V_2 ?



2-2 The two voltage signals

$V_1(t) = 40 \sin(628 t + \pi/5)$ V

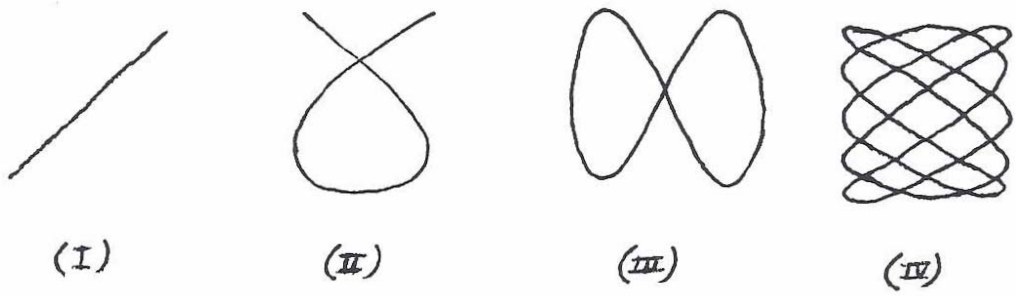
$V(t) = 60 \sin(628 t)$ V

are applied to channel 1 and *2* respectively of a dual-trace oscilloscope. Sketch the trace on the 8 cm x 10 cm CRT screen, given that:

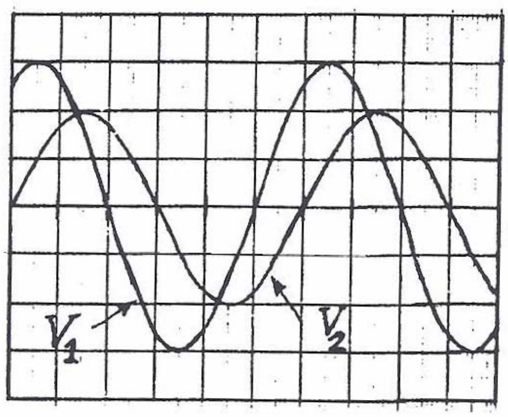
- The'vertical sensitivity of both channels = *20* V/cm.
- The time base sensitivity = 2 ms/cm
- The trigger source is channel 2, with a trigger level of 30 V and trigger slope = positive.

(b) If the oscilloscope is switched to XY operation, sketch the pattern that results on the screen if $V_1(t)$ is applied to the X-input and $V_2(t)$ is applied to the Y-input and both horizontal and vertical sensitivities are kept at 20 V/cm.

(c) For a horizontal frequency $f_H = 1$ KHz, the following Lissajous figures were obtained. Find the corresponding vertical frequency for each figure.



2-3 Two sinusoidal voltages $V_1(t) = V_1 \sin(wt + \emptyset_1)$ and $V_2(t) = V_2 \sin(wt + \emptyset_2)$ are applied to CH.1 and CH.2 respectively of an oscilloscope. The two vertical sensitivities are both equal to 50 mV/cm, and the time-base sensitivity is 20 ms/cm. For the shown trace determine the amplitudes V_1 and V_2, the circular frequency W, and the phase shift $\theta = \emptyset_2 - \emptyset_1$.



2-4

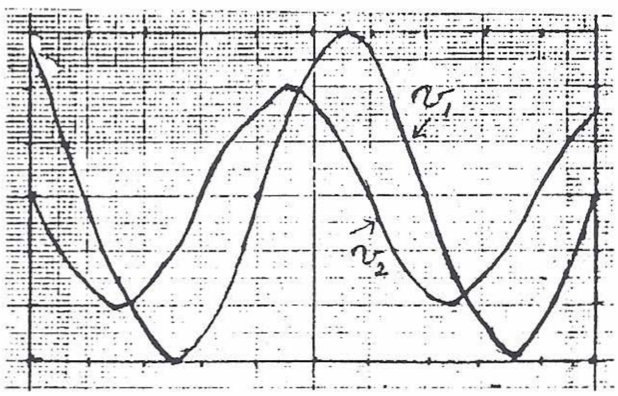
Two sinusoidal voltages V_1 and V_2 are applied to an oscilloscope, V_1 applied to channel I and V_2 applied to channel 2.

Given:

- Channel 1 Vertical Sensitivity = 0.5 V/cm.
- Channel 2 Vertical Sensitivity = 1.0 V/cm.
- Time Base Sensitivity = 10 m Sec/cm
- Trigger Source is Channel 1.
- See Fig. below

Determine:

- Peak to peak value of V_2
- Frequency of both signals.
- Phase-shift between V_2 and V_1. Which is leading ?
- Trigger level.
- Trigger slope, whether + ve or - ve.



2-5

An oscilloscope was used for the measurement of phase shift Ø between two signals V_1 and V_2 of the same frequency. The following results were obtained:

- for the ellipse method [Ø = $\sin^{-1}(y_0/y_m)$]

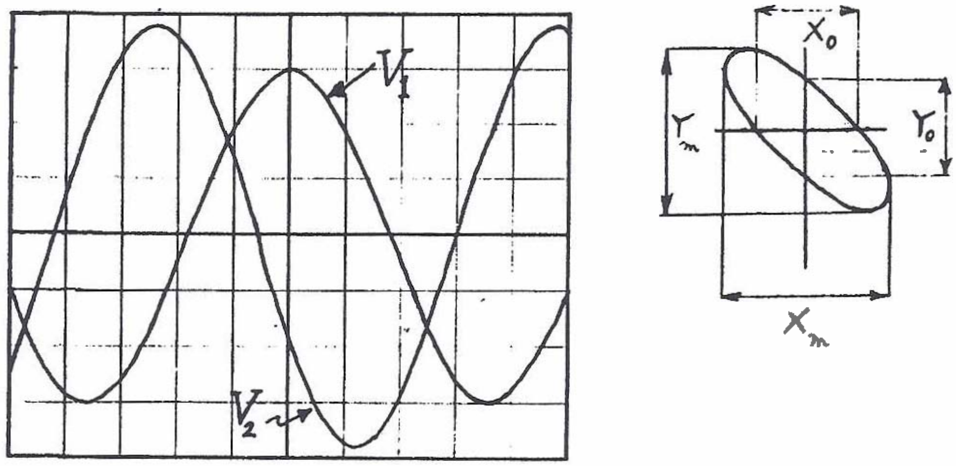
y_0 = (3.5 ± 0.05) cm, y_m = (5 ± 0.05) cm

- for the dual-trace method [$\varnothing = (d/D)\,2\pi$ rad]

  $d = (1 \pm 0.05)$ cm, $D = (8 \pm 0.05)$ cm.

  In both cases determine the phase shift $\varnothing$ and its uncertainty. Can any of the methods (a) or (b) be used to determine if V_1 leads or lags V_2?

2-6



Two sinusoidal voltages V_1 and V_2 are applied to an oscilloscope in dual trace operation (V_1 applied to channel 1 and V_2 applied to channel 2). The vertical sensitivities are S_{CH1} = 10 mV/cm and S_{CH2} = 0.05 V/cm. The time base has S_H = 10 mS/cm. The trigger source is CH.2. For the dual trace shown, find:

- The peak to peak values of V_1 and V_2.
- The time period and frequency of both signals.
- The trigger level and trigger slope.
- The phase shift between V_1 and V_2. Does V_1 lead or lag V_2?

Now, the oscilloscope is switched to XY operation V_1 is connected, to the X input with S_X = 10 mV/cm, and V_2 is connected to the Y input with S_Y = 0.5 V/cm, so that the above ellipse results. Find

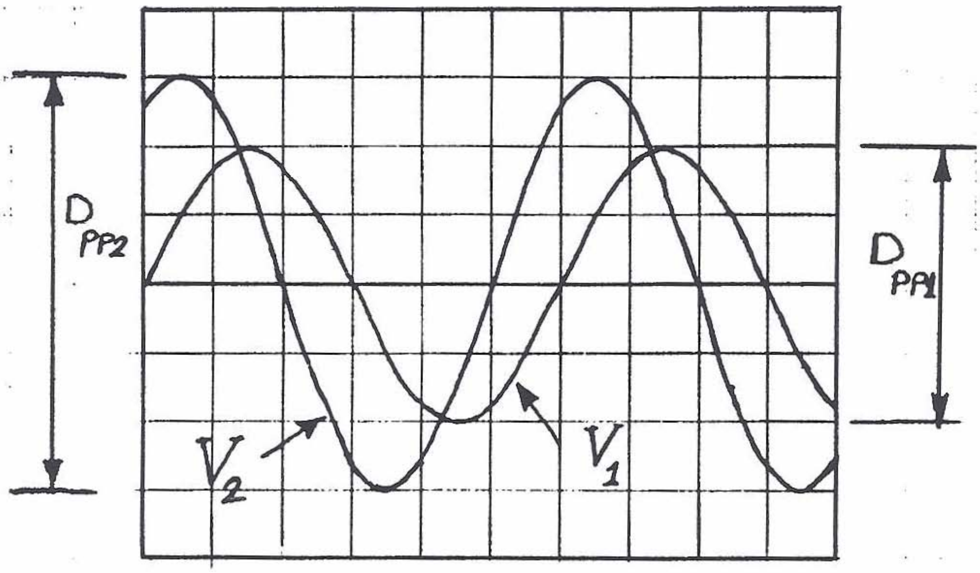
- The distances Y_m, Y_o, X_m, and X_o.

2-7

The input and output to an amplifier are the two sinusoidal voltages shown V_1 and V_2 respectively. These two voltages are applied to an oscilloscope in dual trace operation (V_1 applied to **CH.1** and V_2 applied to CH.2)

CH.1	vertical sensitivity :	S_1	=	20 mV/Cm
CH.2	vertical sensitivity :	S_2	=	0.5 V/Cm
Time base		SH	=	10 ms/Cm
Trigger source is		CH.2		

The trace obtained is **as** shown in the figure below. Assume an uncertainty of 0.5 mm. in all distances measured.



– if the gain G of the amplifier is defined in dB by:

$$G = 20\log_{10}\left(\frac{S_2 \cdot Dpp2}{S_1 \cdot Dpp1}\right)$$

Where S_1, S_2, Dpp1 and Dpp2 are defined as above. Show that the uncertainty in the gain *is* given by:

$$(W_G)^2 = (20\log_{10}e)^2\left[\left(\frac{W_{Dpp_2}}{D_{pp_2}}\right)^2 + \left(\frac{W}{D_{pp_1}}\right)^2\right]$$

- Calculate the value of G and W_G for the above case.
- Find the phase shift between V_1 and V_2 and its uncertainty. Does V, lead of lag V_2?

2-8

Two sinusoidal voltages V, and V_2 are applied to the oscilloscope; V, applied to **CH.1** and V_2 applied to **CH.2**

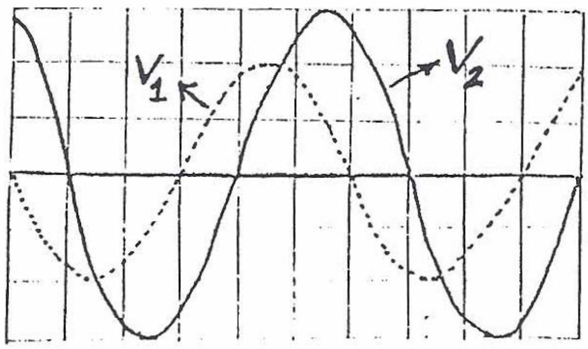
- CH.1 vertical sensitivity	=	50 mV/cm
- CH.2 vertical sensitivity	=	50 mV/cm
- Time base	=	20 mS/cm
- Trigger source	=	CH.1

For the shown trace determine:

- Peak to peak value of V_2
- Time period of both signals
- Phase shift

- Trigger level
- Trigger slope
- Assuming an uncertainty of ± 0.5 mm in all distances measured, what is the percentage uncertainties in the resusits of the first three parts

  If the 2 signals are applied to the oscilloscope in XY operation, sketch the resulting display.



2-9

Two voltages e_1 and e_2 are represented by

e_1 = 100 Sin (314 t) volt

e_2 = -150 Sin (628 t) volt

These are applied to channel 1 (Y_1) and channel 2 (Y_2) of a double beam oscilloscope respectively

a) Sketch the trace on the CRT screen, if

Vertical sensitivity of both channels	=	50 V/cm.
Time Rase	=	2 m Sec/cm
Width of the screen	=	10 cm
Height of the screen	=	8 cm
Trigger Source-Channel 1		
Trigger Level	=	0 (zero)
Trigger slope	=	positive

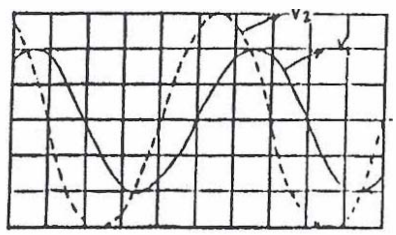
b) If the time base is switched off and XY operation is used with Y_2 input applied to Y-deflecting plates and Y_1 input applied to X deflecting plates, sketch graphically the pattern on the CRT screen.

2-10

Two voltages V_1 and V_2 were applied to dual channel oscilloscope. V_1 applied to Ch.1 and V_2 applied to Ch.2.

Ch.1 Vertical Sensitivity	=	50 mv/cm.
Ch.2 Vertical Sensitivity	=	0.5v/cm.
Time base	=	0.1 m sec./cm.
Trigger source		Ch.1

For the shown scope trace, determine.



- Peak to peak value of V_1
- Peak to peak value of V_2
- frequency of both signals
- Phase shift V_2 (leads) V_1 by ?
- Trigger level
- Slope (+ ve or -ve)

Sketch the scope waveform for a sine wave input 30 mv peak at a frequency 1000 Hz, given that

- Vertical sensitivity 10 mv/cm
- Time sensitivity 0.2 m sec/cm.
- Screen width 10 cm.
- Trigger level 0v
- Slope:- negative.

2-12 In problem 2 above sketch the wave form of another signal 20 mv peak at the same frequency and having a phase shift 50° leading, given that the second channel sensitivity is 10 mv/cm.

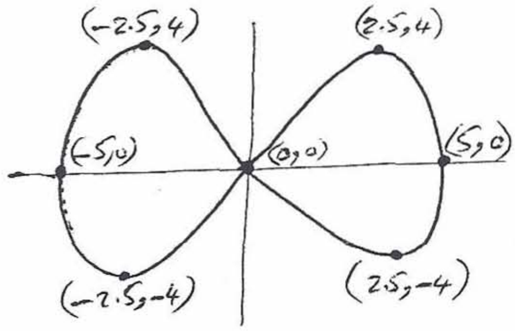
2-13 Sketch the resulting pattern on the Oscilioscope, vertical scale = 0.5 v/cm, horizontal scale 0.5 = v/cm. XY Mode

a) V_x = 1 sin wt	V_y = 45 sin (wt + 60°)
b) V_x = 2 sin wt	V_y = 1 sin (wt - 60°)
c) V_x = 0.5 sin wt	V_y = 1.5 sin (wt + 120°)
d) V_x = 1.5 sin wt	V_y = 2 sin (wt - 120°)
e) V_x = 2 sin wt	V_y = 1 cos wt
f) V_x = 2 sin wt	V_y = 2 cos wt
g) V_x = 2 sin (wt + ø)	V_y = 1.5 sin (wt + ø)
h) V_x = 2 sin (wt + ø)	V_y = 1.5 sin (wt - ø)

2-14 For the Lissajous figure shown:

X- Sensitivity = 5 V/Cm

$V_y(t) = 16 \sin(\omega t + \phi)$ Volts



Find:

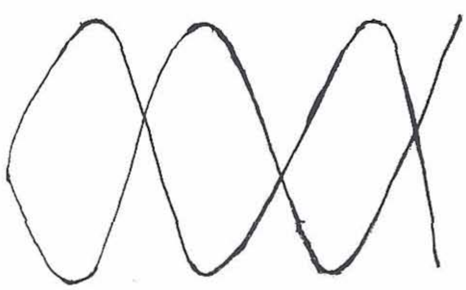
a) Y- Sensitivity

b) $V_x(t)$

c) Sketch trace on sweep mode given that $\omega = 40\pi$ rad/ms time base 5 μs/Cm, Trig. from ch X, level 2 Cm & slope positi

2-15 A scope is put into XY mode and the frequency of the X-Input was 50 Hz.

For the figure shown what is the Y-Input frequency?



$\boxed{2-16}$ If V_x was given by $V_x = 2 \sin 100\pi t$, Volts &

$V_y = 4 \cos 100\pi t$, Volts,

$S_x = 1V/Cm$ & $S_Y = 2V/Cm$

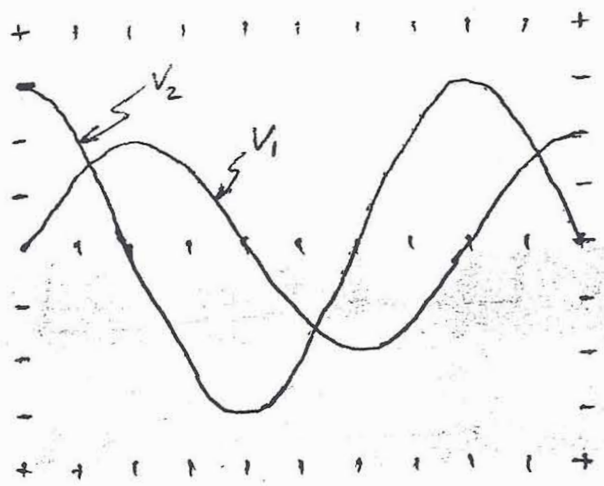
a) Plot screen on XY mode

b) Plot screen on time-sweep mode given that:

- V_x is applied to Ch 1 & V_y to ch 2, with same sensitivities,
- Trig. source is ch. 2, positive slope with level of 2 Volts
- Time-base is

I) 1 μsec/Cm & II) 2 msec/Cm

$\boxed{2-17}$ For the shown scope-Screen,



given:

- V_1 applied to ch1 at 1 V/Cm
- V_2 applied to ch2 at 20 mV/Cm
- Trig. source is ch1
- Time base at 20 msec/Cm
- Uncertainties of 1 mm

Find

a- $V_{1_{rms}}$, $V_{2_{rms}}$ with uncertainties

b- frequency of both signals with uncertainties

c- trig. level & slope

d- phase shift in degrees

e- screen at XY mode operation.

$\boxed{2-18}$ An xy mode screen shows the figure shown; what is the trace on sweep mode if $X = 2 \sin 60\pi t$ Cm, Trig. level 0 Cm, slope +ve, source ch x, Sweep rate = 10 msec/Cm.

2-19 Two signals V_1 & V_2 are applied to ch1 & ch2 respectively of a scope. They are given by:

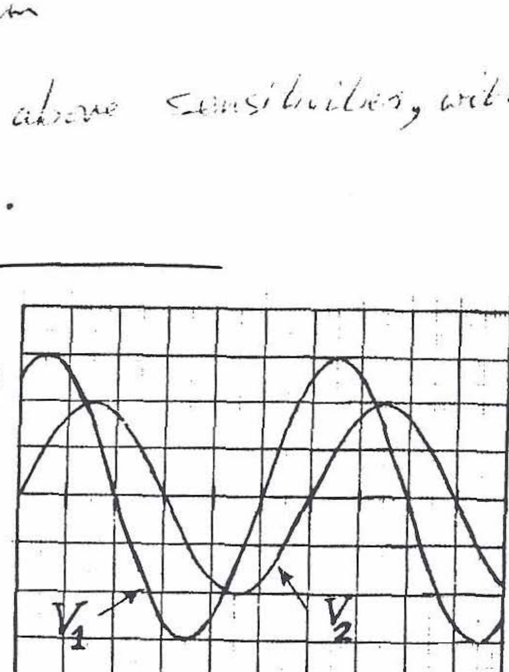
$V_1(t) = 15 \sin 50\pi t$ volts &

$V_2(t) = 20 \sin 100\pi t$ volts. Sketch the trace for:

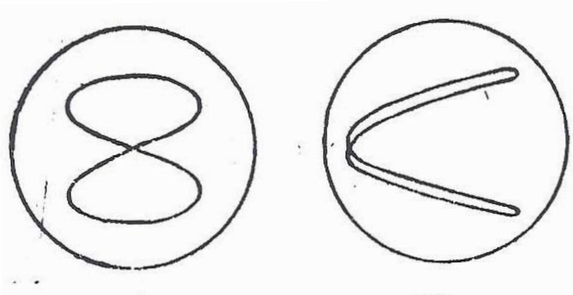
a) Trigger Mode, given: $S_1 = 5\,V/Cm$, $S_2 = 10\,V/Cm$, Trigger source ch1, slope +ve, level zero & Time base of 2 msec/Cm

b) XY-Mode, given the above sensitivities, with ch1 as x & ch2 as y.

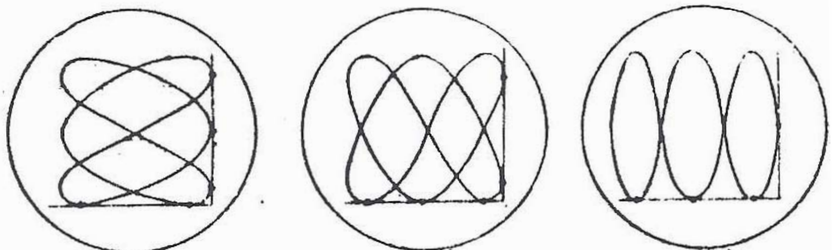
2-20 Two sinusoidal voltages $v_1(t) = V_1 \sin(\omega t + \phi_1)$ and $v_2(t) = V_2 \sin(\omega t + \phi_2)$ are applied to CH.1 and CH.2 respectively of an oscilloscope. The two vertical sensitivities are both equal to 50 mV/cm, and the time-base sensitivity is 20 ms/cm. For the shown trace determine the amplitudes V_1 and V_2, the circular frequency ω, and the phase shift $\phi = \phi_2 - \phi_1$.



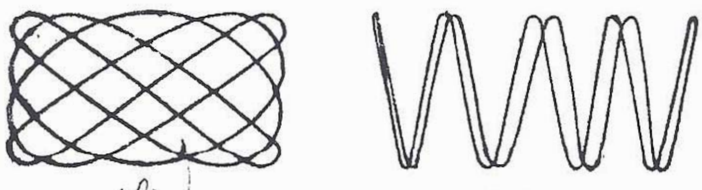
$\boxed{2\text{-}21}$ For the shown Lissajous figures $f_x = 240\,Hz$.
Find f_y.



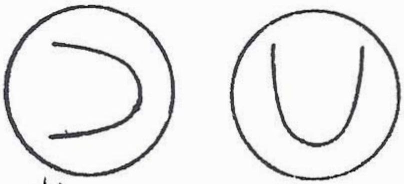
(a) (b)



(c) (d) (e)



(f) (g)



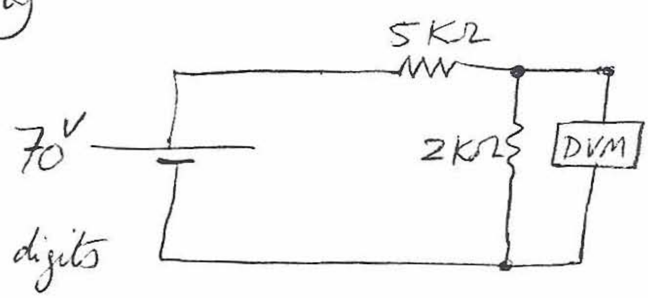
(h) (i)

Problem Set Three, Counters.

3-1 A DFM set to measure the period of a signal reads 7125.3925 μs. Find the interva[l] in which the correct period falls. Find also the frequency and its uncertainity.

3-2 The DFM was now set to measure the frequency of the above signal, what do you think the reading would be on the KHz scale range. Which method do you prefer to use for this signal? Why?

3-3 A DVM was used to determine the voltage of the following circuit. If the sensitivity of the DVM was 10KΩ/V of FSD and it has three ranges 1000 V, 100V & 10V, with 4½ digits



what range would you select? and what is the % error in this-range reading. Explain your comments.

3-4 A 4 digit DFM has the following ranges for unit count: 1ms, 10ms, 100ms, 1s, 10s, .1 Hz, 1 Hz, 10 Hz, 100 Hz & 1KHz.

a) If the reading on 100 Hz range was 0095, what is the likely frequency? Can you measure it better? How?

b) If the frequency was 2.745 KHz, what will the display be for each range?

3-5 A DFC screen can have full illumination of

1888888

with unit count range of:

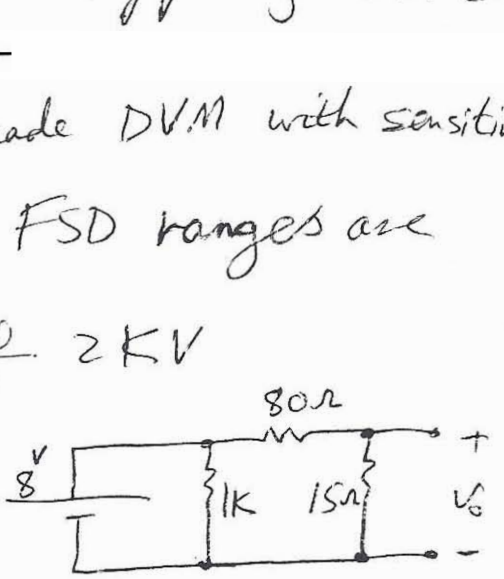
1μs, 10μs, 100μs, 1ms, 10ms, 10 Hz, 100 Hz, 1KHz, 10KHz & .1MHz

a) What is the no. of digits?
b) What is the best reading for $f = \pi$ Hz?
c) On what range is it obtained?
d) What is % uncertainity?
e) What is the range of measurement of f using this DFC?

3-6 The above DFC was made DVM with sensitivity of 1KΩ/V of FSD. The FSD ranges are

2mV, 0.2V, 20V & 2KV

Find: (a) best range,
(b) Reading &
(c) % error,
for measuring V_o shown in the figure.



$\boxed{3\text{-}7}$ A $5\frac{1}{2}$ Digits FC was used to measure 60 Hz signal.
If LSD can range as follows 10 ms, 1 s, 100 s, 100 Hz, 10 Hz
& 1 Hz, what is the best reading. Find its % e

A moving coil instrument has the following data:

No. of turns	=	100
Width of coil	=	2 cm
Depth of coil	=	3 cm
Flux density in the gap	=	0.1 Wb/m^2

Calculate the deflecting torque when carrying a current of 10 m A. Also calculate the deflection if the control spring constant is 20×10^{-7} N-m per degree.

The coil of a mc voltmeter is 4 cm long and 3 cm wide and has 100 turns on it. The control spring exerts a torque of 2.4×10^{-4} N-m when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air-gap is 0.1 Wb/m^2, estimate the resistance that must be put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected.

4-3 Design a multi-range d.c. ammeter using a basic movement with an internal resistance R_m = 50 Ω and a full scale deflection current I_m = 1mA. The ranges required are 0-10 mA, 0-50 Amp, 0-100 mA and 0-500 mA

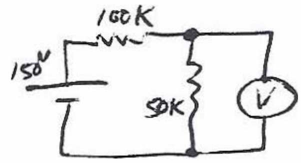
4-4 A mc instrument gives full scale deflection of 10 mA when the P.D. across its terminals is 100 mV. Calculate (a) the shunt resistance for a full scale deflection corresponding to 100 A and (b) the resistance for full scale reading with 1000 V. Calculate the power dissipated in each case.

4-5 A basic d' Arsonval meter movement with an internal resistance R_m = 100 Ω, a full scale current of I_m = 1mA, is to be converted into a multi-range d.c. voltmeter with ranges of 0-10V, 0-50V; 0-250V and 0-500V. Find the values of various resistance using the potential divider arrangement.

4-6 Calculate the multiplying power of a shunt of 200 Ω resistance used with a galvanometer of 1000 Ω resistance Determine the value of shunt resistance to give a multiplying power of 50.

4-7 It is desired to measure the voltage across a 50 K Ω resistor in the circuit shown in Fig. Two voltmeters are available for this purpose.

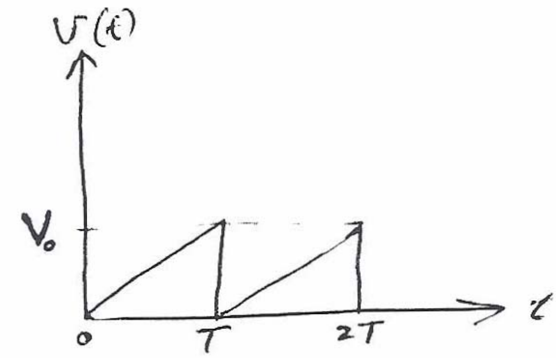
Voltmeter A with a sensitivity of 1000 Ω/V and
Voltmeter B with a sensitivity of 20,000 Ω/V.



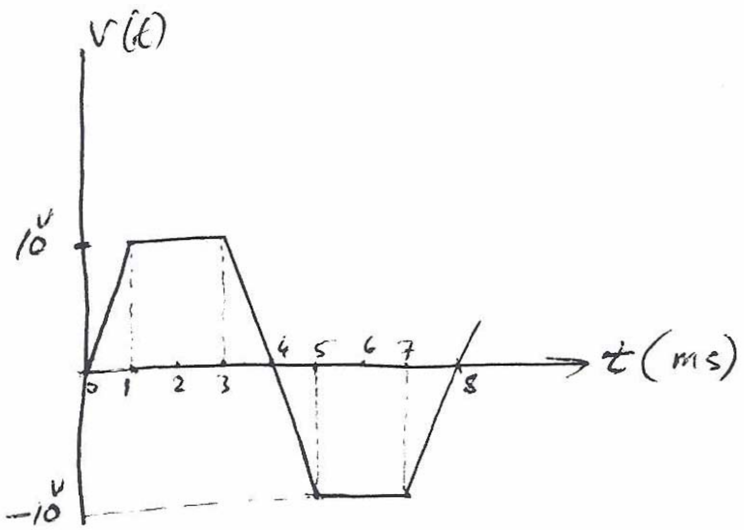
Both meters have 0-50V range. Calculate (a) the reading of each voltmeter and (b) the error in each reading expressed as a percentage of true V.

4-8 Two generators with 500 Ω internal resistance have a SAWTOOTH and Tropizoidal output voltages as shown The r.m.s. values of these outputs to be measured by a moving coil instrument whose internal resistance is 10 kΩ. The instrument has a full wave rectifier and is caliberated for sinusoidal wave forms. Calculate the errors due to the waveform and also the loading errors.

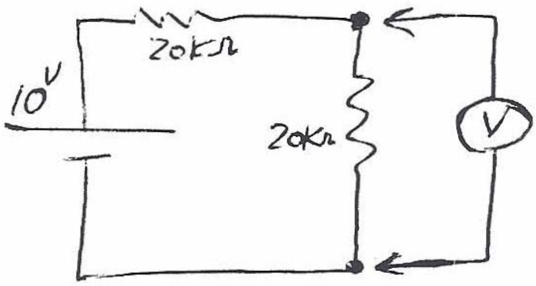
(a)



(b)



4-9 A voltmeter has a resistance of 20 KΩ/V; is used to measure the voltage on the shown circuit on a 10V range. Find the percentage loading error.



4-10 A voltage source of an internal resistance Ro = 0.4 KΩ is measured 5 times with a moving coil meter. The following readings were obtained

3.21 , 3.22 , 3.20 , 3.23 , 3.24 V

The same source was also measured by a recently calibrated digital voltmeter whose input resistance is 10 MΩ. The following 4 readings were obtained

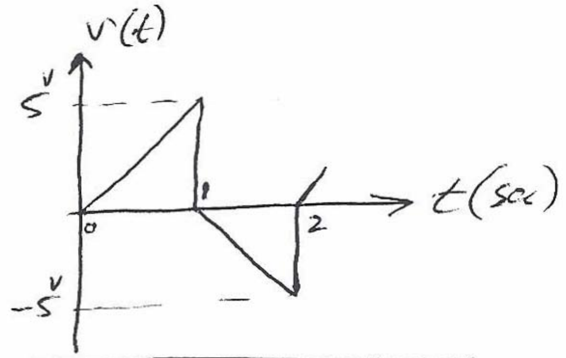
3.412 , 3.413 , 3.411 , 3.412 V

- Estimate the percentage loading error of the digital voltmeter.
- Find the precision and accuracy of both sets of readings.
- Does any of the readings need recalibration ? How ?
- If the bias in the moving coil meter is due solely to its loading error, estimate its input resistance.

4-11 A moving coil movement has 100 turns, 5 cm^2 coil area, and air-gap magnetic flux density of 0.1 t (Wm^{-2}). The control spring exerts a torque of 5 x 10^{-6} Nm at the full-scale deflection of 90^o. The potential difference accross the coil terminals at full-scale deflection is 100 mV. Using the above movement, design a multi-range DC ammeter with ranges [0, 50] mA and [0,1] A, and a multi-range DC voltmeter with ranges [0, 10] V and [0, 2001 V.

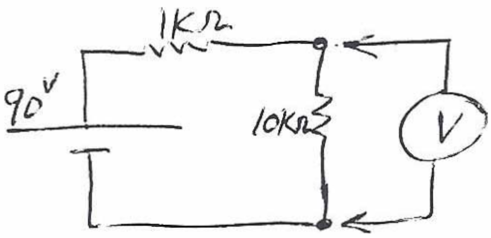
4-12 An average reading full-wave rectifier moving coil AC voltmeter is calibrated to read correctly the RMS value of applied sinusoidal voltages. The periodic waveform V (t) shown is applied to the meter

<u>Calculate</u> V_{RMS} for this waveform, Vindicated and the waveform error in it



4-13 A D'Arsonval movement gives full scale deflection of 1. mA when a voltage of **50** mV is applied across its terminals.

<u>Calculate</u> the resistance that should be added in series with this movement to convert it into a 0-100 V voltmeter. The above 0-100 V voltmeter is used to measure the voltage across the 10 KO resistor in the shown circuit. <u>Determine</u> the percentage loading error.



4-14 For a D'Arsonval movement, the reading is ø = 45° ± 0.5°. If the number of turns is N = **100** ± **0.3,** the coil area is A = 6 cm^2 ± 1%, the magnetic flux density is B = **0.2**t ± 1%, and the total spring constant is K = 5 x **10-8** Nm/o ± 2%,

<u>Find</u>

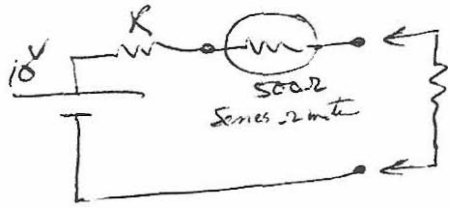
- the nominal value and percentage uncertainty of the current I.
- the nominal value of the movement sensitivity.

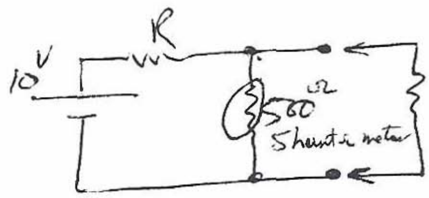
4-15 A D'Arsonval movement has a full-scale deflection current, I = 50 mA and a coil resistance R_m = 50 Ω. Use this movement to design

- an ammeter with range [0, 2] A.
- a voltmeter with range [0, 20] V.

4-16 What would you mark as readings in the Ω-scale for the shown Ω-meter circuits when the readings in the current scale is :

a) I_{FSD} = 100 μAmp
b) 50 μA
c) 30 μAmp
d) 10 μAmp



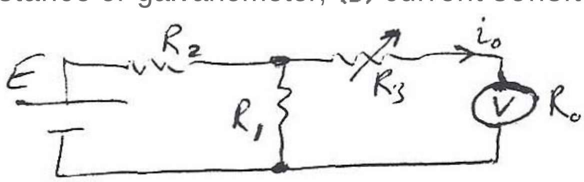


4-17 Find the steady state deflection of a moving coil galvanometer in degrees and in mm on a scale placed 1 m away when a current of 1 mA is passed through its coil. Also calculate the resistance required for critical damping if the galvanometer has a relative damping of 0.1 on open circuit. The data of the galvanometer is :

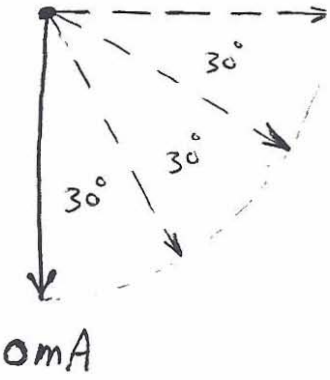
Dimensions of coil = 25 X 20 mm, number of turns = 100, flux density=0.1 Wb/m^2, inertia constant = 0.1 x 10^{-6} kg-m^2 and control constant = 25 x 10^{-6} Nm/rad.

4-18 The fig. shows a circuit for testing of a galvanometer where E=1.5 V, R_1 = 1.0 Ω, R_2 = 2500 Ω, R_3 is variable.

When R_3 = 450 Ω, the galvanometer deflection is 150.0 mm and with R_3 = 950 Ω, the feflection is reduced to 75 mm. Calculate (a) the resistance of galvanometer, (b) current sensitivity of galvanometer.



4-19 An MCV has a gravitational restoring force with zero position of needle as shown in the figure. The movement parameters are:



$N = 100$ Turn,

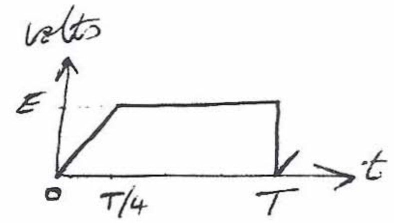
$B = 0.3$ Tesla,

$A = 4\ Cm^2$,

$m = 10$ grams &

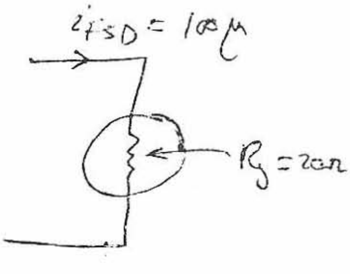
$C_g = 2$ Cm below pivot. Find the readings at 30° spaced positions off zero. What is i_{FSD}? Why?

4-20 An average half-wave rectifier AC MCV is calibrated to read correctly rms of sine signals.



Find %e in reading the ac of the signal shown above.

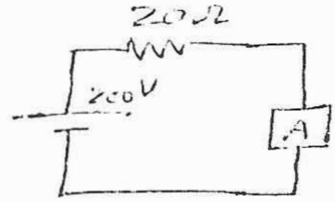
4-21 A MCV has $R_g = 20\,\Omega$, $i_{FSD} = 100\,\mu A$ was required to measure 10 A & 300 V ranges



a) Design to match these purposes.

b) What is the uncertainity of scale at maximum deflection if: $\Delta R_g = 2\%$, $\Delta i_{FSD} = 1\mu A$ & uncertainity in all used resistors is 10%.

4-22 Design a 10 Amp-range ammeter given a movement with $R_m = 100\,\Omega$ & $i_{FSD} = 100\,\mu Amp$.



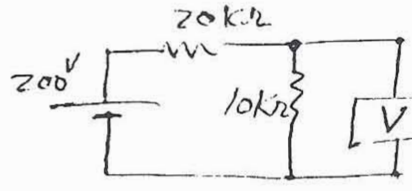
a) What is the reading using your ammeter if it was to measure the current through the circuit given aside?

b) What is the bias & % error? What causes this error?

c) Which range of 20A, 200A or 2000A ranges of $3\frac{1}{2}$ digits DAM with $50\,mV/Amp_{FSD}$ would you rather select? Why?

4-23 Design a 100-DCV range voltmeter given a movement with $R_m = 100\,\Omega$ & $I_{FSD} = 100\,\mu Amp$.

ⓐ What is the reading using your voltmeter if it was to measure the voltage on the circuit given aside?!



ⓑ What is the bias & % error?!

ⓒ Which range of 60^V, 70^V, 80^V ranges of 3 digit DVM with $1\,K\Omega/V_{FSD}$ would you select, why?.

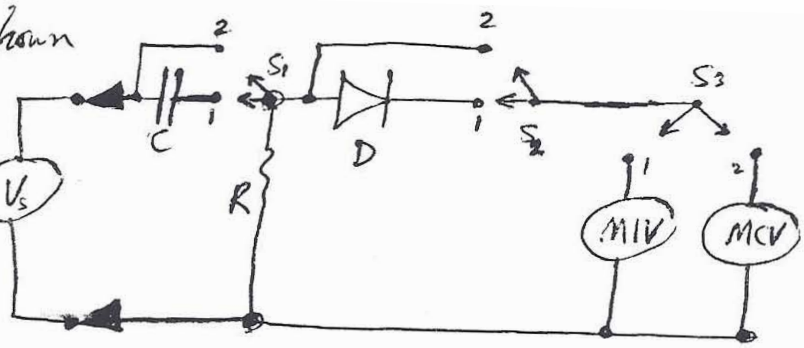
Problem Set Five, Moving Iron Meter.

5-1 A moving iron voltmeter is used to measure the rms of the following signal:

$$v(t) = 5 + 12\sin\omega t + 12\cos 2\omega t$$

What do you expect to give as reading.
Suppose now that the moving coil voltmeter were used what is the reading you expect to see? Which meter you prefer to use & why?

5-2 For the circuit shown in the fig, assuming ideal filter and diode, the switches S_1, S_2 & S_3 enable 4 readings for both MIV & MCV. Which reading is representative for the rms of V_s, if V_s is given by:

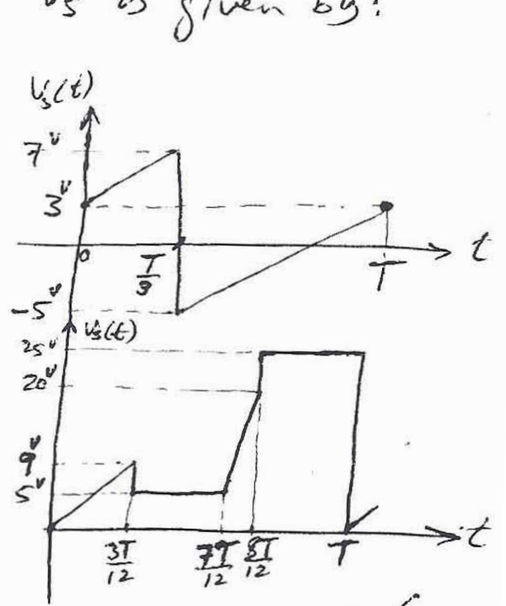


a) $V_s(t) = 8$ volts

b) $V_s(t) = 12 \sin\omega t$

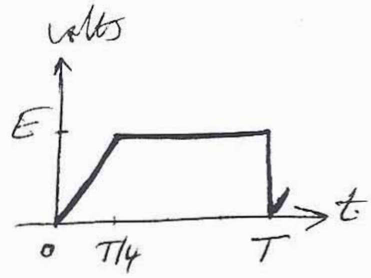
c) $V_s(t) = 8 + 12\sin\omega t$

d) $v_s(t)$ the waveform shown in figs.

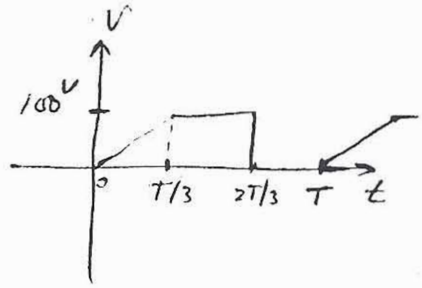


Calculate the errors and give your comments to every reading.

5-3 A source with 10 Ω internal resistance has the pattern shown in the figure. An MIV with $R_g = 1K\Omega$ was used to measure its rms value. Find the reading & %e. Comment.

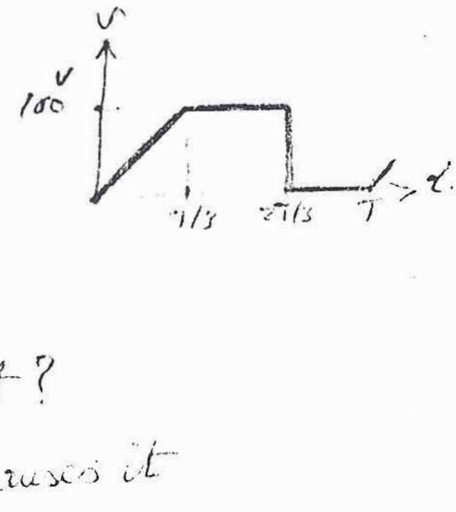


5-4 Two voltmeters with very high input resistance, MCV & MIVs are used to measure the rms of the voltage signal given here.



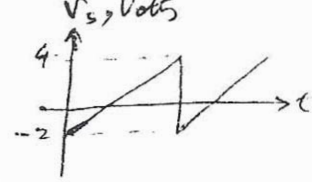
ⓐ Indicate the reading of both,
ⓑ which one has error, how much is it?
ⓒ what is the cause of it.

5-5 Two voltmeters with very high input resistances, MCV & MIV, were used to measure the dc of this signal.



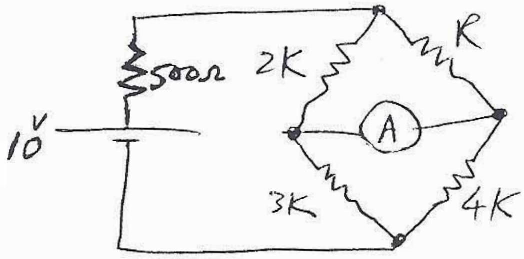
a) Indicate the reading of both.
b) How would you get your measurement?
c) What is the error in it & who causes it

5-6 Two voltmeters MCV & MIV were used to measure the rms of the signal shown find readings and errors



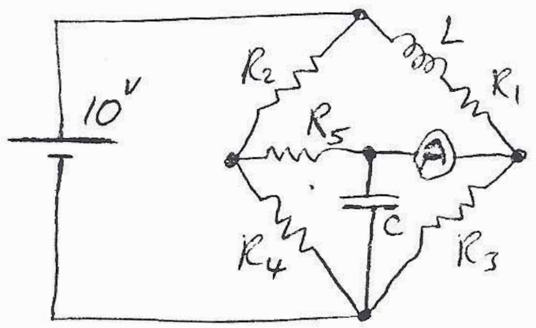
Problem Set Six, Bridges.

6-1 What is the unknown resistance, R in the shown Wheatstone Bridge.



6-2 If R is varied by + 2%, what would be the Ammeter reading. Take movement resistance of 100 Ω

6-3 In the Anderson Bridge shown the ac source was made a dc one. What are the conditions of balance?!



If $R_1 = R_3 = 10\,K\Omega$ &

$R_2 = R_4 = R_5 = 12\,K\Omega$ &

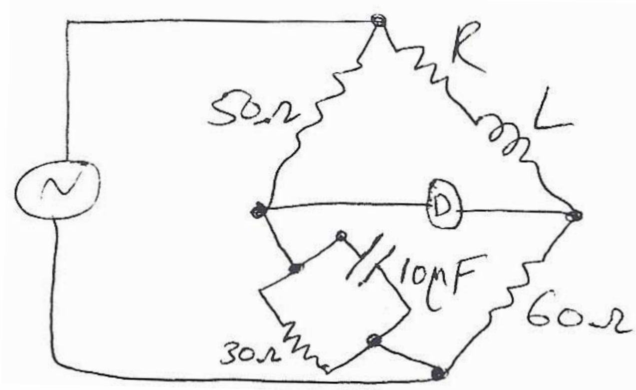
$C = 10\,\mu F$ & $L = 50\,mh$, then find the Ammeter reading if $R_m = 200\,\Omega$.

Repeat the calculations for +10% change in R_2.

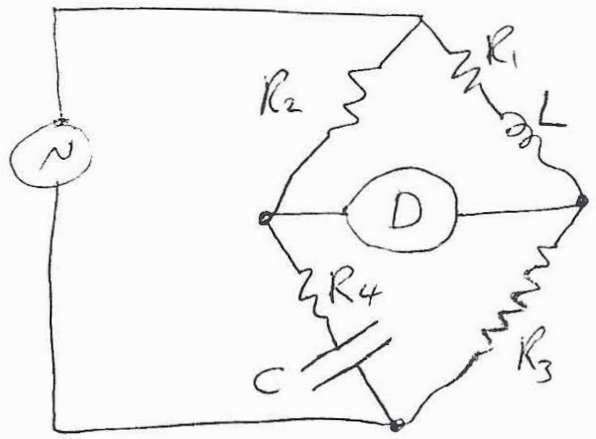
Find the voltages accross both L & C in both cases.

6-4 The above Anderson Bridge was fed from an ac source. What are the values of R_2 & C that are required for equilibrium?

6-5 Find the value of R & L for the shown Maxwell's Bridge.



6-6 Derive the balance conditions for Hay's Bridge. Does it change with frequency?

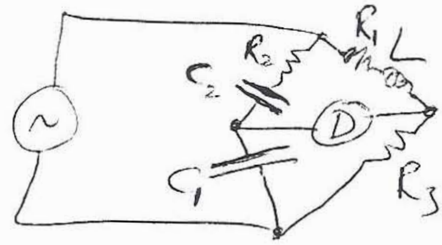


Find L & C if

$R_1 = 58\Omega$, $R_2 = 120\Omega$

$R_3 = 15\Omega$, $R_4 = 80\Omega$. Use mains frequency of 60 Hz.

6-7 Derive the balance conditions for Owen's Bridge. Is it affected by frequency?

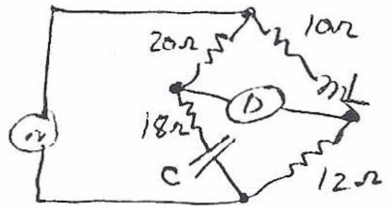


Find C_2 & R_2 if

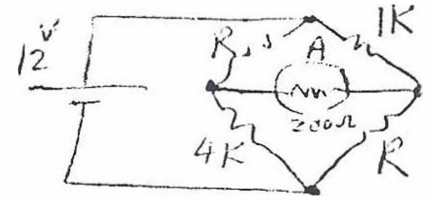
$R_1 = 1K\Omega$, $L = 50mh$, $R_3 = 500\Omega$ &

$C_1 = 40\mu F$ with mains frequency.

6-8 What are values of L & C for the given Hay's bridge at balance?! assume main frequency of 60 Hz.



6-9 Two identical thermometers with coefficient of .01/°C keeps the shown bridge balanced at an ambient of 25°C. What would be the temperature to cause 20μA deflection?



Problem Set Seven, Wattmeter.

7-1 A dynamometer wattmeter, having a current coil resistance of $0.5\,\Omega$ and a voltage coil resistance of $12.5\,K\Omega$ was used to measure the power in 250 DCV load at ⓐ 4 Amps & ⓑ 12 Amps.
Which connection would you use for each case? why?
Also find % errors.

7-2 If the reactance of the current coil of the above wattmeter was 1% of its resistance, repeat the answer for 250 ACV load at unity pf for
ⓐ 4 Amps & ⓑ 12 Amps

7-3 Repeat the above problem for pf of
ⓐ 0.8 , ⓑ 0.5 & ⓒ 0.1

7-4 Repeat exercises 2 & 3 if the reactance of the voltage coils is 0.1 % of its resistance.

$\boxed{7\text{-}5}$ How can we measure the power delivered to a 3-ϕ load connected (a) in star & (b) in Δ using: (I) one wattmeter (II) two wattmeters (III) three wattmeters. State also the conditions of measurement.

$\boxed{7\text{-}6}$ Can you measure the reactive power in 3-ϕ load using one wattmeter? If yes state how and conditions of measurement:

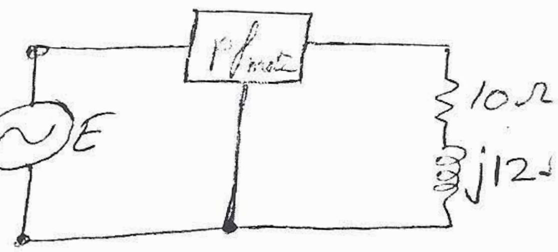
(a) If load is Y.

(b) If load is Δ.

Problem Set Eight, Power-factor Meter

8-1 A pf-meter has one voltage coil impedence of $(20{,}000 + j\cdot 1)\,\Omega$, find the voltage mat-coil impedence. If the applied input frequency was 10% higher than the designed one, how would you compensate the first coil.

8-2 The above pf meter was put accross the load in the given circuit, Find the reading and the percentage error in it for both possible connections if the current coil impedence was $(0.2 + j\,0.5)\,\Omega$. Which connection do you prefer?



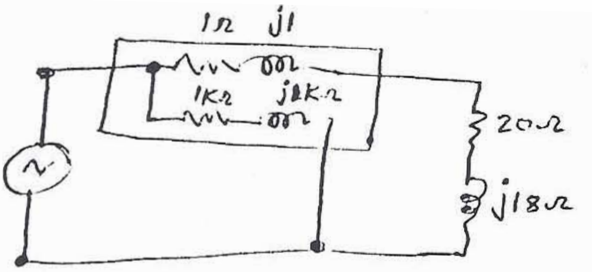
8-3 If, by mistake, the voltage & current coils were swapped over, find the two possible readings. Comment.

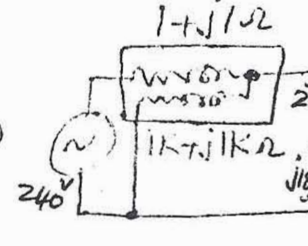
8-4 If, by ignorance the voltage & current coils were shorted together (a) accross the load & (b) in series with the load, find the readings and give your comments & explaination

8-5 Find the reading & % error for the 1-ϕ

(a) wattmeter shown.

(b) pf-meter shown





8-6 Find the reading & % error for the shown 1-ϕ (a) wattmeter & (b) pf meter

8-7 Suppose that the impedance of voltage coil-pairs of a pf meter were not equal and were $Z_R \angle \theta_R$ for the resistive dominated coil and $Z_L \angle \theta_L$ for the inductive dominated coil. Prove that the angle of deflection, ψ, is given by:

$$\psi = \tan^{-1}\left[\frac{Z_R \cos(\phi - \theta_L)}{Z_L \cos(\phi - \theta_R)}\right]$$

where ϕ is the actual power factor angle.

If $Z_R = R\angle 0^\circ$ &

$Z_L = R\angle 90^\circ$.

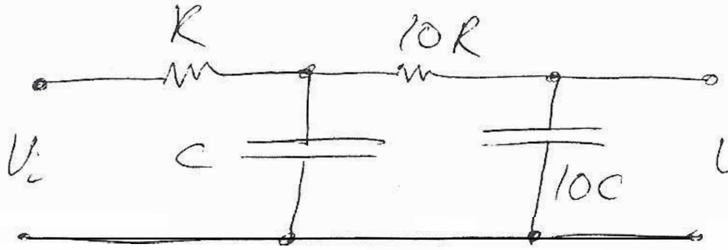
Show that the above formula becomes:

$$\psi = \phi$$

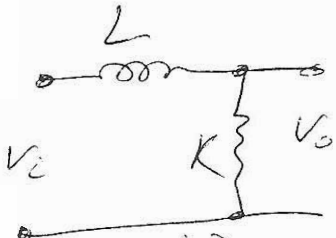
and hence the pf meter measures correctly.

Problem Set Nine, Filters.

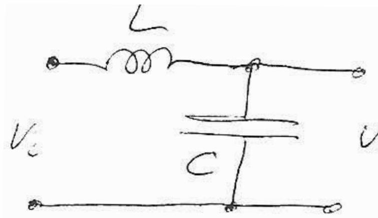
9-1 Find the Bandwidth of the following networks, and their dc gains



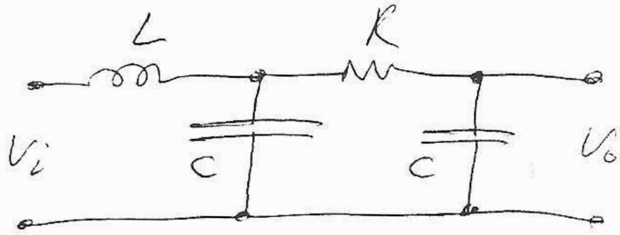
(a)



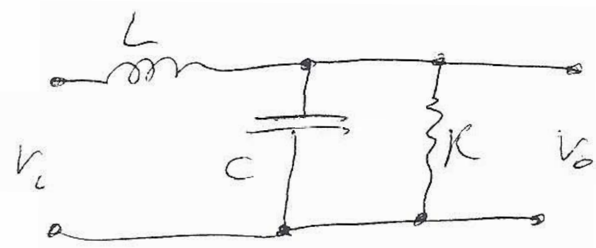
(b)



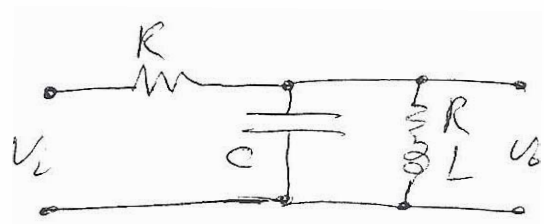
(c)



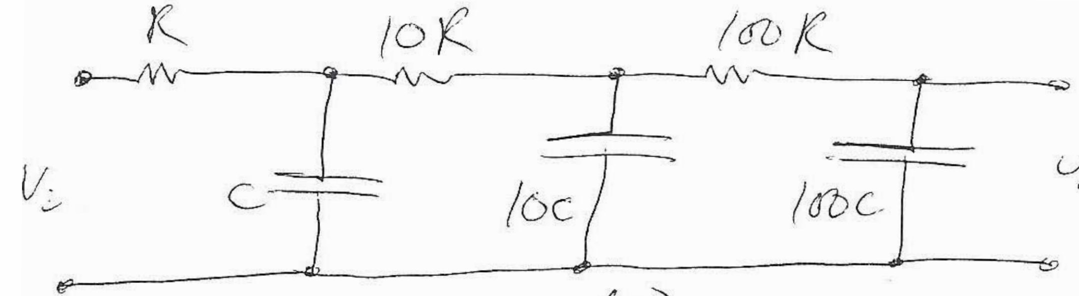
(d)



(e)

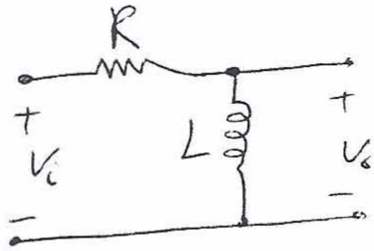


(f)

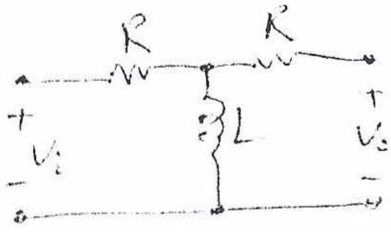


(g)

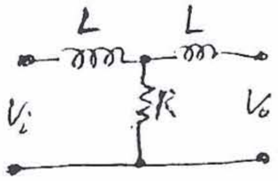
9-2 What type of filter is the one shown in the figure. Find f_L, f_h & gain at $f = 2 f_L$.



9-3 Classify the filter and find its f_L & f_H given $R = 10\,\Omega$, $L = 10\,mh$



9-4 Find the bandwidth of the shown network, given $L = 10\,mh$ & $R = 10\,\Omega$.



$\boxed{1\text{-}1}$ $\because C = \frac{\epsilon A}{d} = \frac{\epsilon_o \epsilon_r A}{d}$

$\because A = 5\ Cm^2 \pm .5\ \%$ $\therefore \frac{dA}{A} = .005$

$\because C = 950\ pF$

\# $\therefore d = \frac{\epsilon_o \epsilon_r A}{c} = \frac{8.854 * 10^{-12} * 81 * 5 * (10^{-2})^2}{950 * 10^{-12}}$

$= 3.775 * 10^{-4}\ m = 0.38\ mm$, the separation.

\# $\frac{\partial c}{\partial d} = \frac{-\epsilon A}{d^2} = \frac{-\epsilon_o \epsilon_r A}{d \cdot d} = \frac{-c}{d} =$

$= \frac{-950\ pF}{.38\ mm} = -2.52\ nF/mm$, the sensitivity.

\# $\left(\frac{dc}{c}\right)^2 = \left(\frac{dA}{A}\right)^2 + \left(\frac{dd}{d}\right)^2$ $\because \frac{dd}{d} = 3\% = .03$

$\therefore \frac{dc}{c} = \sqrt{(.005)^2 + (.03)^2} = .0304 = 3.04\ \%$

$\therefore$ The maximum uncertainity in c is $3.04\ \%$

1-2 a) $T = \dfrac{T_0}{1+(T_0/\beta)\ln(R/R_0)} = \dfrac{300}{1+(300/3420)\ln(2/1)} = 282.8\ {}^\circ K$

b) $S = \dfrac{\partial R}{\partial T} = R\beta_0\left(-\dfrac{1}{T^2}\right) = -\dfrac{\beta_0 R}{T^2} = -\dfrac{3420 * 2K}{(282.8)^2} = -85.5\ \Omega/K$

c) $\because R = R_0 \exp\left(\beta_0\left(\frac{1}{T} - \frac{1}{T_0}\right)\right)$

$\therefore dR = R_{R_0}\, dR_0 + R_T \cdot dT = (R/R_0)\, dR_0 + R\cdot\beta_0\left(-\frac{1}{T^2}\right) dT =$

$= R\cdot\left[\dfrac{dR_0}{R_0} - \dfrac{\beta_0}{T}\cdot\dfrac{dT}{T}\right]$

$\therefore \dfrac{dR}{R} = \dfrac{dR_0}{R_0} - \dfrac{\beta_0}{T}\cdot\dfrac{dT}{T} \Rightarrow \dfrac{\beta_0}{T}\cdot\dfrac{dT}{T} = \dfrac{dR_0}{R_0} - \dfrac{dR}{R}$

$\therefore \dfrac{dT}{T} = \dfrac{T}{\beta_0}\cdot\left(\dfrac{dR_0}{R_0} - \dfrac{dR}{R}\right)$ (vector equation.)

$\therefore \left(\dfrac{dT}{T}\right)^2 = \left(\dfrac{T}{\beta_0}\right)^2\left[\left(\dfrac{dR_0}{R_0}\right)^2 + \left(\dfrac{dR}{R}\right)^2\right]$ $\therefore$ OK & max $dT = 0.167\% = .473$

1-3 a) $MV_{mcm} = 6.72$ Volts

$MV_{DVM} = 7.072$ Volts $= TV$ since Loading is small & it is recently calibrated.

$\therefore$ $Prec._{mcm} = |6.72 - 6.69| = 0.03$ Volts.

& $Acc._{mcm} = |7.072 - 6.69| = 0.382$ Volts.

$Prec._{DVM} = Acc_{DVM} = |7.072 - 7.069| = 0.003$ Volts

b) The average of MCV should be increased by the bias $= 7.072 - 6.72 = 0.352$ Volts, to be true, and so is every reading of MCV.

1-4 a) $W = \dfrac{E X^4 Y}{8 D^3 N}$

$\therefore W = \dfrac{80 * 10^9 * (2.5 * 10^{-3})^4 (10 * 10^{-2})}{8 (2.5 * 10^{-2})^3 * 50} = 50 \ N$

b) $\dfrac{dW}{W} = \dfrac{dE}{E} + 4\dfrac{dX}{X} + \dfrac{dY}{Y} - 3\dfrac{dD}{D} - \dfrac{dN}{N}$ (Vectorially)

$\therefore \left|\dfrac{dW}{W}\right| = \sqrt{\left(\dfrac{dE}{E}\right)^2 + 16\left(\dfrac{dX}{X}\right)^2 + \left(\dfrac{dY}{Y}\right)^2 + 9\left(\dfrac{dD}{D}\right)^2 + \left(\dfrac{dN}{N}\right)^2} =$

$= \sqrt{16 + 16 + .04 + 36 + 1}\ \% = \sqrt{69.04}\ \% = 8.31\ \%$

$\therefore dW = 4.15 \ N$

& order of contribution as seen from the above figures is D, E & X, N,

c) $S = \dfrac{\partial Y}{\partial W} = \dfrac{8 D^3 N}{E X^4} = \dfrac{8(.025)^3(50)}{80 * 10^9 * (.0025)^4} = 0.002 \ m/N$

The sensitivity is independent upon W & hence the plot is constant line

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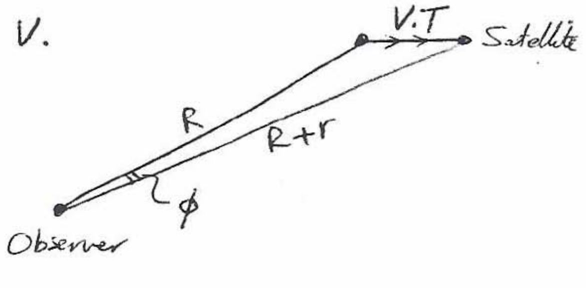
1-5 Assume the speed of satellite to be V.

& time between observations to be T



$\therefore (VT)^2 = R^2 + (R+r)^2 - 2R(R+r)\cos\phi$

$= 2R^2 + 2rR + r^2 - 2R^2\cos\phi - 2Rr\cos\phi$

$= r^2 + 2R(R+r)(1-\cos\phi)$

$\therefore V^2 = \dfrac{r^2 + 2R(R+r)(1-\cos\phi)}{T^2}$ $\qquad \therefore V = 51.56$ Km/sec

& $2VdV = \dfrac{1}{T^2}\cdot\left[(2r + 2R(1-\cos\phi))dr + (1-\cos\phi)(4R+2r)dR + 2R(R+r)\sin\phi . d\phi\right]$

$\therefore |dV| = \dfrac{1}{VT^2}\sqrt{\left[\{(r+R(1-\cos\phi))dr\}^2 + \{(1-\cos\phi)(2R+r)dR\}^2 + \{R(R+r)\sin\phi\, d\phi\}^2\right]}$

$= 0.1167$ Km/sec $= 116.7$ m/sec

$\therefore$ The satellite was moving at 51.56 Km/sec $\pm$ 116.7 m/sec

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$\boxed{1-6}$ True Value $= 340$ μm/m

Av. $= \dfrac{\Sigma \text{ all readings}}{12} = \dfrac{4134}{12} = 344.5$ μm/m

∴ Bias $= TV - AV = 340 - 344.5 = -4.5$ μm/m

Precision $= |AV - \text{Max. Deviated Reading}| =$

$= |344.5 - 351.0| = 6.5$ μm/m

Accuracy $= |TV - \text{Max. Deviated Reading}| =$

$= |340.0 - 351.0| = 11.0$ μm/m

$\boxed{1-7}$ $z = P \cdot y_1^{i_1} \cdot y_2^{i_2} + S$

$$\therefore \; z - S = P\, y_1^{i_1} \cdot y_2^{i_2}$$

$$\therefore \; \frac{d(z-S)}{(z-S)} = \frac{dP}{P} + \frac{i_1\, dy_1}{y_1} + \frac{i_2\, dy_2}{y_2} + (\ln y_1)\frac{di_1}{i_1} + (\ln y_2)\frac{di_2}{i_2}$$

$$\frac{dz - dS}{z - S} = \frac{w_0}{1} + 1 \cdot \frac{w_{10}}{y_{10}} + 1 \cdot \frac{w_{20}}{y_{20}} + (\ln y_{10})\frac{w_{11}}{1} + (\ln y_{20})\frac{w_{21}}{1}$$

$$\therefore \; \frac{dz - w_3}{(1 \cdot y_{10}^1 \cdot y_{20}^1 + 0) - 0} = w_0 + \frac{w_{10}}{y_{10}} + \frac{w_{20}}{y_{20}} + (\ln y_{10})\, w_{11} + (\ln y_{20})\, w_{21}$$

$$\therefore \; dz = w_3 + y_{10} \cdot y_{20}\left[w_0 + \left(\frac{1}{y_{10}}\right) w_{10} + \left(\frac{1}{y_{20}}\right) w_{20} + (\ln y_{10})\, w_{11} + (\ln y_{20})\, w_{21}\right]$$

$$\therefore \; |dz| = \sqrt{w_3^2 + y_{10}^2 \cdot y_{20}^2 \cdot \left(w_0^2 + \frac{w_{10}^2}{y_{10}^2} + \frac{w_{20}^2}{y_{20}^2} + w_{11}^2 \ln^2 y_{10} + w_{21}^2 \ln^2 y_{20}\right)}$$

$\boxed{1\text{-}8}$ $\quad R = R_0\left(1+\alpha(T-T_0)\right)$

$\# \therefore \quad T = \left[\left(\frac{R}{R_0}-1\right)/\alpha\right] + T_0$

$$= T_0 + \frac{R-R_0}{\alpha R_0}$$

> assume $T_0 = 25°C$
$= (273+25)°K$
$= 298°K$

$\# \quad T = 298 + \frac{30*10^3 - 20*10^3}{.00392 * 20*10^3} = 425.6\ °K$
$= 152.6\ °C$

$\# \quad \frac{\partial R}{\partial T} = R_0\alpha = 20*10^3 * .00392 = 78.4\ \Omega/°K$

$\# \quad dR = \{1+\alpha(T-T_0)\}\,dR_0 + R_0\alpha\, dT = \frac{R\,dR_0}{R_0} + \alpha R_0\, dT$

$\therefore \quad dT = \frac{dR}{\alpha R_0} - \frac{R\,dR_0}{R_0^2\alpha} = \frac{R}{\alpha R_0}\cdot\left[\frac{dR}{R} - \frac{dR_0}{R_0}\right]$

$\therefore \quad |dT| = \sqrt{(dR/R)^2 + (dR_0/R_0)^2}\cdot\frac{R}{\alpha R_0}$

$$= \sqrt{(.001)^2 + (.001)^2}\cdot\frac{30}{20*.00392} = .541\ °K$$

$\therefore$ Uncertainity in T assuming T_0 to be accurate is $0.541°K$

i.e. $0.127\ \%$

$\# \quad dT(T_0, R_0, R) = (1)\,dT_0 + \left(\frac{-R}{R_0^2\alpha}\right)dR_0 + \left(\frac{1}{\alpha R_0}\right)dR$

$\therefore \quad |dT|^2 = |dT_0|^2 + \left(\frac{R}{\alpha R_0}\right)^2\left(\left|\frac{dR_0}{R_0}\right|^2 + \left|\frac{dR}{R}\right|^2\right)$

$\boxed{1-9}$ Let $R_1 = 20\Omega \pm 5\%$ $\quad \therefore dR_1/R_1 = .05$

$\quad$ & $dR_1 = 1\,\Omega$

& let $R_2 = 10\,\Omega \pm 10\%$ $\quad \therefore dR_2/R_2 = .1$

$\quad$ & $dR_2 = 1\,\Omega$

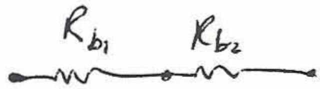
\# Connection A: $\quad \therefore R_A = R_{a_1} + R_{a_2} + R_{a_3}$

$$\therefore dR_A = dR_{a_1} + dR_{a_2} + dR_{a_3}$$

$$\therefore |dR_A|^2 = (dR_{a_1})^2 + (dR_{a_2})^2 + (dR_{a_3})^2 = 3\,(dR_2)^2 = 3 * 1^2$$

$$\therefore |dR_A| = \sqrt{3}\ \Omega$$

\# Connection B: $\quad \therefore R_B = R_{b_1} + R_{b_2}$

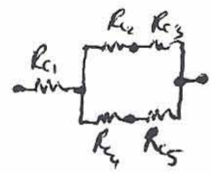


$$\therefore dR_B = dR_{b_1} + dR_{b_2}$$

$$\therefore |dR_B|^2 = |dR_{b_1}|^2 + |dR_{b_2}|^2 = |dR_1|^2 + |dR_2|^2 = 1^2 + 1^2 = 2$$

$$\therefore |dR_B| = \sqrt{2}\ \Omega$$

\# Connection C: $\quad \therefore R_C = R_{c_1} + \dfrac{(R_{c_2}+R_{c_3})(R_{c_4}+R_{c_5})}{R_{c_2}+R_{c_3}+R_{c_4}+R_{c_5}}$



$$\therefore dR_c = dR_{c_1} + \left[\frac{(R_{c_4}+R_{c_5})\Sigma - (R_{c_2}+R_{c_3})(R_{c_4}+R_{c_5})}{\underbrace{(R_{c_2}+R_{c_3}+R_{c_4}+R_{c_5})}_{\Sigma}{}^2}\right] dR_{c_2} + \text{similar terms for } dR_{c_3}, dR_{c_4} \,\&\, dR_{c_5}$$

$$\therefore |dR_c|^2 = |dR_{c_1}|^2 + \left(\frac{R_{c_4}+R_{c_5}}{\Sigma}\right)^4 \left[|dR_{c_2}|^2 + |dR_{c_3}|^2\right] + \left(\frac{R_{c_2}+R_{c_3}}{\Sigma}\right)^4 \left[|dR_{c_4}|^2 + |dR_{c_5}|^2\right]$$

$$= |dR_1|^2 + \left(\frac{R_2+R_2}{4R_2}\right)^4 \left[|dR_2|^2 + |dR_2|^2\right] + \left(\frac{R_2+R_2}{4R_2}\right)^4 \cdot \left[|dR_2|^2 + |dR_2|^2\right]$$

$$= |dR_1|^2 + \left(\tfrac{1}{2}\right)^4 \cdot 4\,|dR_2|^2 = |dR_1|^2 + \tfrac{1}{4}\,|dR_2|^2 = 1^2 + \tfrac{1^2}{4} = 1.25 = \tfrac{5}{4}$$

$$\therefore |dR_c| = \frac{\sqrt{5}}{2} = \sqrt{1.25} = 1.12\,\Omega$$

$\therefore \sqrt{3}, \sqrt{2}$ & $\sqrt{1.25}$, The best is connection C with $|dR_c| = 1.12\,\Omega$

$\boxed{1-10}$ $T_{av} = (T_1 + T_2 + \cdots\ T_N)/N$

$\therefore\ dT_{av} = (dT_1 + dT_2 + dT_3 + \cdots + dT_N)/N$

$\therefore\ |dT_{av}|^2 = \left[|dT_1|^2 + |dT_2|^2 + \cdots\ |dT_N|^2\right]/N^2$

$= N\ |dT|^2/N^2 = |dT|^2/N$

$\therefore\ |dT_{av}| = \dfrac{|dT|}{\sqrt{N}}$

$\therefore$ Uncertainity of measurement can be reduced by a factor of $\frac{1}{\sqrt{N}}$ if the average of N readings was taken.

$\boxed{2\text{-}2}$ a) $V_1 = 40 \sin(628t + \pi/5)$ V

$\therefore Y_1 = 2 \sin(2\pi * 100t + \pi/5)$ Cm

& $V_2 = 60 \sin 628t$ V

$\therefore Y_2 = 3 \sin(2\pi * 100 t)$ Cm

frequency of both signals = 100 Hz

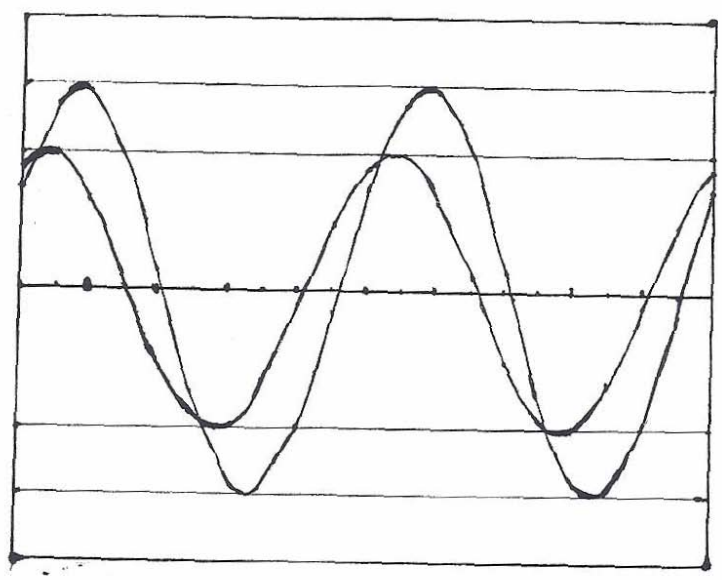
$\therefore$ Period = 10 m sec = 5 Cm

$\therefore$ Trig. level is 30 V of ch2 with +ve slope

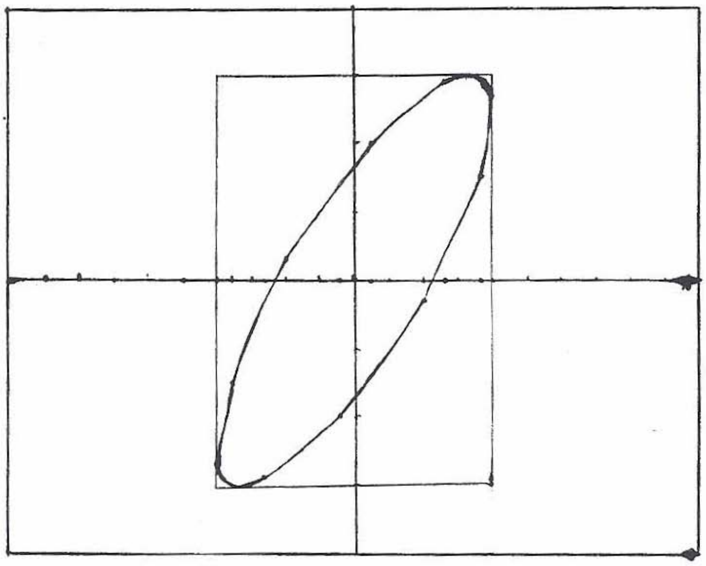
$\therefore$ Y_2 will start at left of screen at 1.5 Cm heading up.

$\therefore$ The trace will be as follows, for balanced display:

$\frac{\omega t}{2\pi}$	Y_1(Cm)	Y_2(Cm)
5/60	1.8	1.5
11/60	2.0	2.7
17/60	1.3	2.9
23/60	0.2	2.0
29/60	-1.0	0.3
35/60	-1.8	-1.5
41/60	-2.0	-2.7
47/60	-1.3	-2.9
53/60	-.2	-2.0
59/60	+1.0	-.3
65/60	1.8	1.5



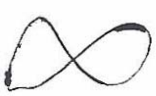
b) Now Y_1 stands for X & Y_2 for Y, hence with XY mode the pattern will look like this:



c) Lissajous figuers:

I)  $\therefore \frac{1}{2} f_h = \frac{1}{2} f_v \quad \therefore f_v = f_h = 1\ KHz$

II)  $\therefore 1 \cdot f_h = 1\frac{1}{2} f_v \quad \therefore f_v = \frac{2}{3} f_h = \frac{2}{3}\ KHz$

III)  $\therefore 2 f_h = 1 \cdot f_v \quad \therefore f_v = 2 f_h = 2\ KHz$

IV) $\therefore 3 f_h = 5 f_v \quad \therefore f_v = \frac{3}{5} f_h = 600\ Hz$

$\boxed{2\text{-}6}$ From trace $\therefore$ Period $= 7.2$ Cm $= 7.2 * 10ms = 72ms$

$\therefore f = \frac{1}{T} \simeq 14$ Hz $\therefore \omega = 2\pi f = 2\pi * 14 \simeq 88$ rad/sec

$\therefore Y_1 = 3$ Cm peak & $Y_2 = 3.8$ Cm peak

& start of Y_2 is -2.5 Cm at left of screen

$\therefore$ # peak to peak value of $V_1 = 3 * 2 * 10 = 60mV$

& # P-P value of $V_2 = 3.8 * 2 * .05 = 0.38V = 380m$

& # Period of both signals $= 72$ ms

& # frequency of both signals $= 14$ Hz

& # Trig. level $= -2.5 * .05 = -.125V = -125mV$

& # Trig. Slope = positive (because leading up)

& # phase shift $= \frac{2.4}{7.2} * 360 = 120°$, V_1 lags V_2

for xy mode, $X_{pp} = \frac{V_1}{10m} = \frac{60m}{10m} = 6$ Cm $= X_m$

& $Y_{pp} = \frac{V_2}{.5} = \frac{380m}{.5} = 0.76$ Cm $= Y_m$

& $X_o = X_m \sin\phi = 6 * \sin 120° = \frac{6\sqrt{3}}{2} = 5.2$ Cm

& $Y_o = Y_m \sin\phi = 0.76 * \sin 120° = 0.66$ Cm

2-14 $f_x n_x = f_y n_y$

$\therefore f_x * 2 = \frac{\omega}{2\pi} * 1 \Rightarrow f_x = \frac{\omega}{4\pi} \Rightarrow \omega_x = 2\pi f_x = \frac{\omega}{2}$

$\because$ Centre of screen is (0,0) and is a point in the figure.

$\therefore \omega t_0 + \phi = \frac{\omega t_0}{2} + \bar{\phi} = 0 \therefore \bar{\phi} = -\frac{\omega t_0}{2} = -\frac{1}{2}(-\phi) = \phi/2$

(OR $\omega t_0 + \phi = 0$ & $\frac{\omega t_0}{2} + \bar{\phi} = \pi \therefore \bar{\phi} = \pi - \frac{\omega t_0}{2} = \pi - \frac{1}{2}(-\phi) = \pi + \phi/2$)

$\therefore X(t) = 5 \sin\left(\frac{\omega t + \phi}{2}\right)$ Cm $\left[\text{or } X(t) = 5 \sin\left(\frac{\omega t + \phi + 2\pi}{2}\right)\right]$

& $Y(t) = 4 \sin(\omega t + \phi)$ Cm

a) $\therefore$ Y-sensitivity $= \frac{16V}{4C} = 4$ V/Cm

b) & $V_x(t) = 25 \sin\left(\frac{\omega t + \phi}{2}\right)$ Volts.

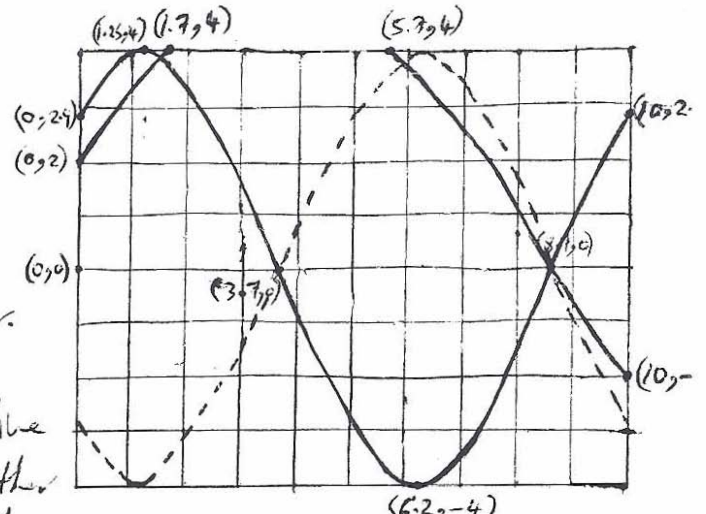
c) $\because \omega = 40\pi$ rad/ms

$\therefore f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi m} = 20$ KHz

$\therefore T = \frac{1}{20K} = 50\mu$sec

$\therefore$ Span of a cycle $= \frac{50\mu s}{5\mu s/C} = 10$ Cm

$\therefore$ One cycle of y would appear & half a cycle of x would appear.
The screen will start at X = 2 Cm & slope positive as shown here (note: the other posibility is also shown by the dashed line).



$\boxed{2\text{-}15}$ $n_x \cdot f_x = n_y \cdot f_y$

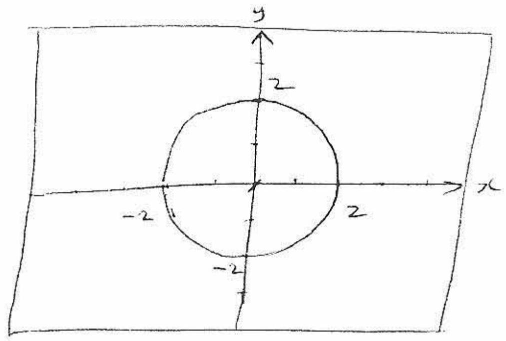
$\therefore (3\tfrac{1}{2}) * 50 = 1 * f_y$ $\qquad \therefore f_y = \frac{7}{2} * 50 = 175\ \text{Hz}$.

$\boxed{2\text{-}16}$ $X = \frac{V_x}{S_x} = \frac{2\sin 100\pi t}{1} = 2\sin 100\pi t$ Cm

$Y = \frac{V_y}{S_y} = \frac{4\cos 100\pi t}{2} = 2\cos 100\pi t$ Cm

$\therefore X^2 + Y^2 = 4$ $\qquad \therefore$ Circle radius 2 Cm

a) $\therefore$ The screen will look like this:



b) $\therefore Y_1 = 2\sin 100\pi t$

$Y_2 = 2\cos 100\pi t$

$\left.\right\}$ $f_1 = f_2 = \frac{100\pi}{2\pi} = 50\ \text{Hz}$

$\therefore T_1 = T_2 = \frac{1}{50} = 20$ msec

$\therefore$ Trig. level $= 2$ volts $= \frac{2}{2} = 1$ Cm

I) $T_1 = T_2 = 20$ m sec $= \frac{20\text{ m sec}}{1\,\mu\text{sec/Cm}} = \frac{20\text{ m}}{\mu}$ Cm $= 20{,}000$ Cm

$\therefore$ One period would be seen within 20,000 Cm.

$\therefore$ for 10 Cm wide scream the change is not visible

$\therefore Y_2\Big|_{\substack{\text{start}\\\text{of screen}}} = 1$ Cm, $100\pi t\Big|_{\substack{\text{start}\\\text{of screen}}} = \underbrace{\left[\cos^{-1}\left(\frac{1}{2}\right)\right]}_{\text{pos. in slope}} = -\pi/3$ & $Y_1\Big|_{\substack{\text{start}\\\text{of screen}}} = 2\sin(-\pi/3) = -\sqrt{3}$ Cm $= -1.732$ Cm ≈ -1.7 Cm

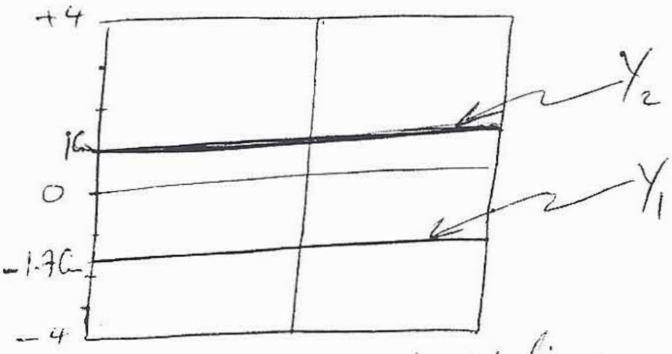
$$\left. Y_2 \right|_{\substack{\text{end}\\ \text{of screen}}} = 2\cos 100\pi\left(t_{\substack{\text{start}\\ \text{of screen}}} + 10\ \text{Cms}\right) = 2\cos 100\pi\left(t_{\substack{\text{start}\\ \text{of screen}}} + 10 * 1\mu s\right)$$

$$= 2\cos\left(100\pi\, t_{\substack{\text{start}\\ \text{of screen}}} + 100\pi * 10 * 10^{-6}\right) = 2\cos\left(-\frac{\pi}{3} + 10^{-3}\pi\right)$$

$$= 2\cos\left(-\frac{1}{3} + .001\right)\pi = 2 * .503 = 1.01\ \text{Cm}$$

$$\left. Y_1 \right|_{\substack{\text{end}\\ \text{of screen}}} = 2\sin\left(-\frac{1}{3} + .001\right)\pi = -1.729\ \text{Cm}$$

$\therefore$ The screen looks like this



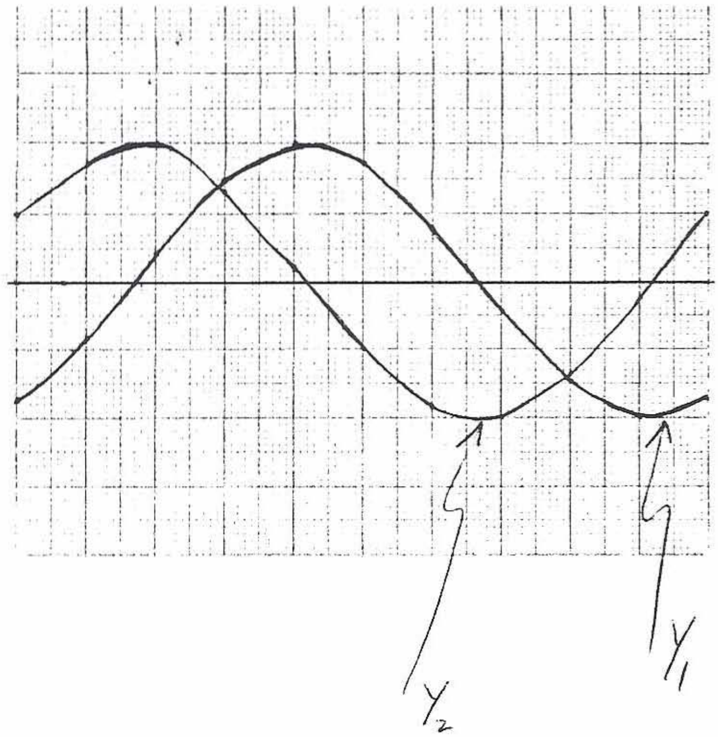
both are straight lines.

II) $T_1 = T_2 = \frac{20\ \text{msec}}{2\ \text{msec/C}} = 10\ \text{Cm}$

$\therefore$ One period will take the width of full screen

Y_2 (Cm)	T (Cm)	$100\pi t$	Y_1 (Cm)
1	0	$-\pi/3$ (start of screen)	-1.73
1.83	1	$-\frac{\pi}{3} + 100\pi * 2m = (-\frac{1}{3} + .2)\pi$	-0.81
1.96	2	$(-\frac{1}{3} + .4)\pi$	$+0.42$
1.34	3	$(-\frac{1}{3} + .6)\pi$	$+1.49$
0.21	4	$(-\frac{1}{3} + .8)\pi$	$+1.99$
-1	5	$(-\frac{1}{3} + 1)\pi$	$+1.73$
-1.83	6	$(-\frac{1}{3} + 1.2)\pi$	$+.81$
-1.96	7	$(-\frac{1}{3} + 1.4)\pi$	$-.42$
-1.34	8	$(-\frac{1}{3} + 1.6)\pi$	-1.49
$-.21$	9	$(-\frac{1}{3} + 1.8)\pi$	-1.99
1	10	$(-\frac{1}{3} + 2)\pi$	-1.73

Hence, screen looks as follows:



2-17 (a) $V_{1_{rms}} = S_1 * Y_1/\sqrt{2} = 1 * 2/\sqrt{2} = \sqrt{2}$ Volts $= 1.4142$ Volts

$dV_{1_{rms}} = S_1 \, dY_1/\sqrt{2} = 1 * 1*10^{-1}/\sqrt{2} = .070711$ Volts $= 70.711$ mV

$\therefore V_{1_{rms}} = 1.4142$ V ± 70.711 mV $= 1.4142$ V $\pm 5\%$

$V_{2_{rms}} = S_2 * Y_2/\sqrt{2} = 20m * 3/\sqrt{2} = 42.426$ mV

$dV_{2_{rms}} = S_2 \, dY_2/\sqrt{2} = 20m * 1*10^{-1}/\sqrt{2} = 1.4142$ mV

$\therefore V_{2_{rms}} = 42.426$ mV ± 1.4142 mV $= 42.426$ mV $\pm 3.33\%$

(b) $f_1 = f_2 = f$ & $T_1 = T_2 = T$ with $fT = 1$

$\therefore T = 8 * 20m = 160$ msec

$\therefore f = \frac{1}{160m} = 6.25\ Hz$

$\therefore f = \frac{1}{T} \qquad \therefore \frac{\delta f}{f} = -\frac{dT}{T}$

$\therefore$ % error in f = % error in $T = \frac{.1}{8} * 100 = 1.25\ \% = 7.8125*10$

$\therefore f = f_1 = f_2 = 6.25\ Hz \pm .078125\ Hz = 6.25\ Hz \pm 1.25\ \%$

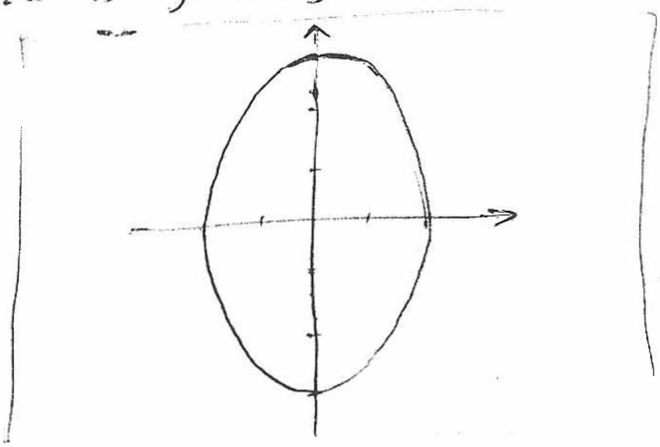
(c) Trig level = 0 Volts

slope positive.

(d) $\frac{\phi}{180} = \frac{2}{4} = \frac{1}{2} \qquad \therefore \phi = 90^\circ$

$\therefore V_2$ leads V_1 by 90° or V_1 laggs V_2 by 90°

(e) The screen at xy mode is given by



$Y = Y_2 = 3 \cos \omega t$

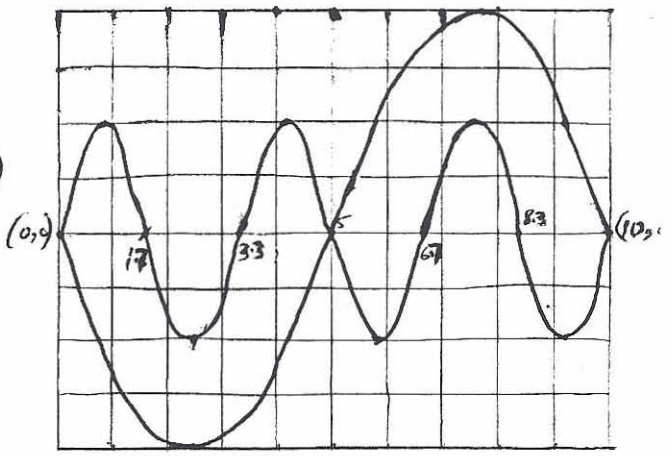
$X = Y_1 = 2 \sin \omega t$

$\therefore \left(\frac{Y}{3}\right)^2 + \left(\frac{X}{2}\right)^2 = 1$

Ellipse as shown

$\boxed{2-18}$ $n_x f_x = n_y f_y \Rightarrow \because n_x = \frac{1}{2},\ n_y = 1\frac{1}{2},\ f_x = 30\ Hz$

$\therefore f_y = \frac{1}{2} * 30 / 1\frac{1}{2} = 10\ Hz$



Since centre of screen (0,0) is one of the figure points and positive peaks form a corners,

$\therefore Y = -4 \sin 20\pi t$ Cm.

$T_y = \frac{1}{10}\ sec = \frac{100\ ms}{10\ ms/Cm} = 10\ Cm$

$\therefore$ A screen will display one period of y & 3 off x as shown above

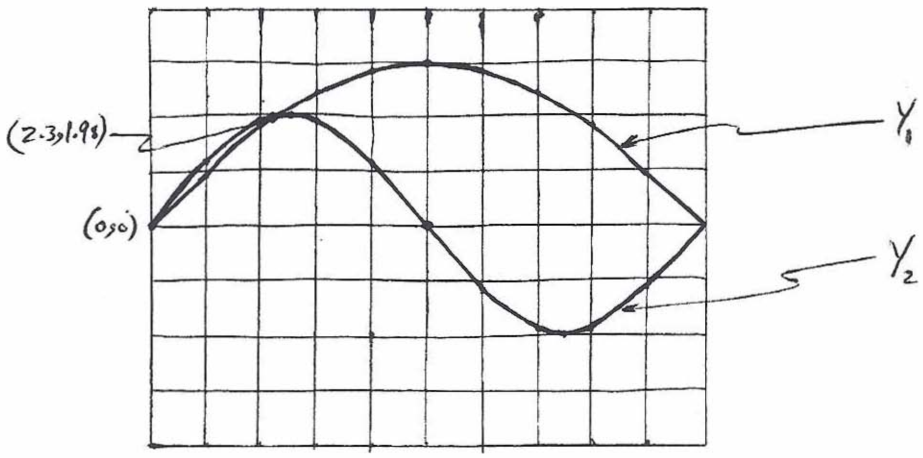
2-19 $Y_1(t) = \frac{15}{5} \sin 50\pi t = 3 \sin 50\pi t$ Cm

& $Y_2(t) = \frac{20}{10} \sin 100\pi t = 2 \sin 100\pi t$ Cm

a) $f_1 = \frac{50\pi}{2\pi} = 25$ Hz $\Rightarrow T_1 = 40ms = 20$ Cm

& $f_2 = \frac{100\pi}{2\pi} = 50$ Hz $\Rightarrow T_2 = 20ms = 10$ Cm

$\therefore$ Screen will show one cycle of Y_2 & half cycle of Y_1, as below

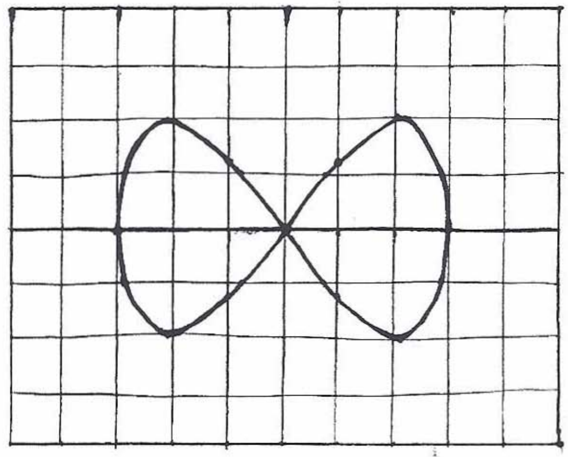


b) $X = Y_1 = 3 \sin 50\pi t$ & $Y = Y_2 = 2 \sin 100\pi t$

$\therefore Y = 2\sin(2 * 50\pi t) = 2\sin\left(2 * \sin^{-1}\frac{X}{3}\right) = 4 \cdot \sin\left(\sin^{-1}\frac{X}{3}\right)\cos\left(\sin^{-1}\frac{X}{3}\right)$

$= 4 \cdot \frac{X}{3}\sqrt{1-\left(\frac{X}{3}\right)^2} = \frac{4}{9}X\sqrt{9-X^2}$ symmetric about X & Y axis.

The shape look like ∞ and is plotted point by point as shown below



$\boxed{2\text{-}20}$

$V_1 = (3\ \text{cm})(50\ \text{mV/cm}) = 150\ \text{mV}$

$V_2 = (2\ \text{cm})(50\ \text{mV/cm}) = 100\ \text{mV}$

$T = (6\ \text{cm})(20\ \text{ms/cm}) = 120\ \text{ms} = 0.12\ \text{s}$

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.12} = 52.359878\ \text{rad/s} \simeq 52.36\ \text{rad/s}$

$\phi = \phi_2 - \phi_1 = -\frac{1}{6}\,360^\circ = \underline{-60^\circ}$

$\boxed{3\text{-}5}$ a) No. of digits are $6\frac{1}{2}$

b) $f = \pi\ Hz = 3.1415927\ Hz$

& $T = 0.31830989$ sec

The best reading is the one giving least gating error, i.e. max. possible no. of digits with high significancy.

I) If LSD is $1\mu s$ screen is 318309 with $\%e \leq \frac{1}{318309} * 100$

II) If LSD is $10\mu s$ " " 031830 " " $\leq \frac{1}{31830} * 100$

III) other period ranges and are seen to be all, but the first, inaccurate

IV) If LSD is 10 Hz screen is 000000 with $\%e = 100\%$

V) other frequency ranges are going also to give 000000 & $\%e = 100\%$

c) ∴ The best reading is obtained by pressing $1\mu s$ for LSD and is $318309\ \mu s$ i.e $f = 3.1416014\ Hz$

d) $\text{Uncertainity} = \frac{1}{318309} * 100 = 0.000314\ \%$ in both T & f

(note actual error $= 2.8 * 10^{-4}\ \% <$ uncertainity ∴ OK)

e) period ranges are 1999999 (10ms) & 000001 ($1\mu s$)
i.e. $T \in (2*10^6 * 10ms, 1\mu s]$ i.e $f \in (5*10^{-5}, 10^6]\ Hz$

& frequency ranges are 000001 (10 Hz) & 1999999 (100kHz)
i.e $f \in [10, 2*10^6 * 100 * 10^3)\ Hz = [10, 2*10^{11})\ Hz$

∴ The frequencies measurable by this DFC are $(5*10^{-5}, 10^6] \cup [10, 2*10^{11})\ Hz$
i.e $(5*10^{-5}, 2*10^{11})\ Hz$.

$\boxed{3\text{-}6}$ $V_o(\text{true}) = 8 * \frac{15}{80+15} = 1.2631579$ volts

I) 2mV range $\therefore R_m = \frac{1K\Omega}{V_{FSD}} * 2mV = 2\,\Omega$

$\therefore V_o = 8 * \frac{15||2}{15||2+80} = .17266187\overset{V}{0}5 = 172.6618705\,mV$ (too much loading)

$\therefore$ Reading is 1.999999 mV (overflow) & %e = 99.84%

II) .2V range $\therefore R_m = 1K * .2 = .2K = 200\,\Omega$

$\therefore V_o = 8 * \frac{15||200}{15||200+80} = 1.1881188188$ volts (little loading)

$\therefore$ Reading is .1999999 volts (overflow) & %e = 84.17%

III) 20V range $\therefore R_m = 1K * 20 = 20K\Omega = 20{,}000\,\Omega$

$\therefore V_o = 8 * \frac{15||20{,}000}{15||20{,}000+80} = 1.26236061434$ volts (less loading)

$\therefore$ Reading is 1.26236 volts & %e = 0.0632%

IV) 2000V range $\therefore R_m = 1K * 2000 = 2M\Omega = 2{,}000{,}000\,\Omega$

$\therefore V_o = 8 * \frac{15||2{,}000{,}000}{15||2{,}000{,}000+80} = 1.26314991694$ volts (least loading)

$\therefore$ Reading is 001.263 volts & %e = 0.0125%

$\therefore$ a) best range is the 2KV range, giving:

& b) The best reading of 001.263 Volts with:

& c) The least error of 0.0125%.

$\boxed{3-7}$ $f = 60$ Hz $\;\; + \;\; T = 16.\overline{66}$ ms

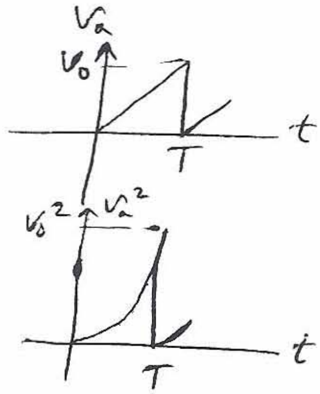
I) If LSD is	10 ms	screen is	00001	with %e $\leq \frac{1}{1} * 100$
II) If " "	1 sec	" "	00000	" " $= 100\%$
III) " " "	100 S	" "	"	" " " "
IV) " " "	100 Hz	" "	"	" " " "
V) " " "	10 Hz	" "	00006	" " $\leq \frac{1}{6} * 100$
VI) " " "	1 Hz	" "	00060	" " $\leq \frac{1}{60} * 100$

∴ The least uncertain range is the 1 Hz range for LSD

and best reading is 00060 Hz with actual error 0%
(note $0\% \leq \frac{100}{60}\% = 1.7\%$ ∴ OK)

4-8 # waveform (a)

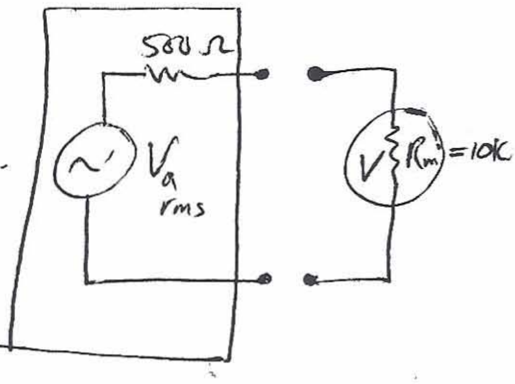
$$V_{a_{rms}} = \sqrt{\frac{1}{T} * \frac{1}{3} * V_0^2 * T} = \frac{V_0}{\sqrt{3}}$$



$\therefore$ Reading due to waveform (assuming no loading) =

$= 1.11 * \text{Av. of FWR signal} =$

$= 1.11 * \left(\frac{1}{T} * \frac{V_0 * T}{2}\right) = \frac{1.11}{2} V_0$

$\therefore$ 

$\therefore$ Waveform error $= \dfrac{\frac{V_0}{\sqrt{3}} - \frac{1.11}{2} V_0}{\frac{V_0}{\sqrt{3}}}$

$= \dfrac{2 - 1.11\sqrt{3}}{2} = .0387 = 3.87\%$

& Reading due to loading (assuming no waveform error) =

$= \dfrac{10K}{10K + 500} \cdot V_{a_{rms}} = 0.952\ V_{a_{rms}}$

$\therefore$ Loading error $= \dfrac{V_{a_{rms}} - .952\ V_{a_{rms}}}{V_{a_{rms}}} = .0476 = 4.76\%$

& Reading due to both waveform & loading =

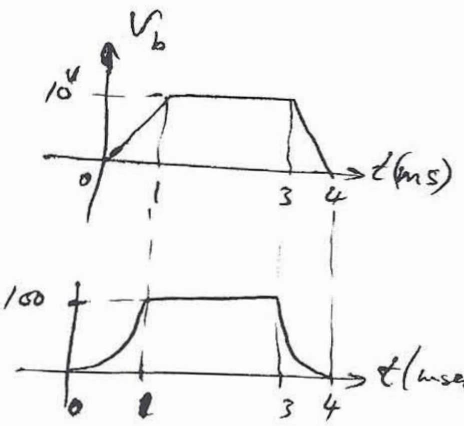
$= 1.11 * \text{Av. of FWR signal} = 1.11 * \left(\frac{1}{T} * \frac{10K\ V_0}{500 + 10K} * T/2\right) =$

$= \frac{1.11}{2} * .952\ V_0 = 0.528\ V_0$

$\therefore$ Total error $= \dfrac{V_0/\sqrt{3} - .528\ V_0}{V_0/\sqrt{3}} = .085 = 8.5\%$

(or waveform error + loading error = 3.87 + 4.76 = 8.6 %)

waveform (b)



$$V_{b_{rms}} = \sqrt{\frac{1}{4}\left(\frac{100*1}{3} + 100*2 + \frac{100*1}{3}\right)}$$

$$= \sqrt{\frac{1}{4}\left(\frac{200+600}{3}\right)} = \sqrt{\frac{800}{12}} = \frac{20}{\sqrt{6}}\ V$$

$\therefore$ Reading due to waveform (no loading) =

$$= 1.11 * \text{Av. of FWR signal} = 1.11 * \left(\frac{1}{4} * \left(\frac{10*1}{2} + 10*2 + \frac{10*1}{2}\right)\right)$$

$$= 1.11 * \frac{30}{4} = 8.33\ V$$

$\therefore$ Waveform error $= \left|1 - \frac{8.33}{20/\sqrt{6}}\right| = |-.0196| = 1.96\ \%$

& Reading due to loading is as before (assuming no waveform error) = 4.7

& Reading due to both waveform & loading =

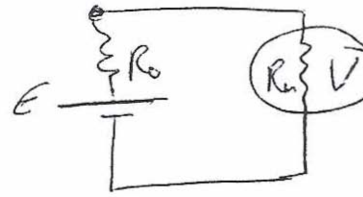
$$= 1.11 * \text{Av. of FWR signal} = 1.11 * \left(0.952 * \frac{30}{4}\right) =$$

$$= 7.93\ V$$

$\therefore$ Total error $= 1 - \frac{7.93}{20/\sqrt{6}} = .0293 = 2.93\ \%$

$\left(\approx \text{waveform error} + \text{loading error} = -1.96 + 4.76 = 2.8\ \%\right)$

$\boxed{4\text{-}10}$ # % loading error $= \left(1 - \dfrac{\text{Reading}}{\text{True Value}}\right) * 100$



$$= \left(1 - \frac{\frac{E \cdot R_m}{R_m + R_o}}{E}\right) * 100 = \frac{R_o}{R_m + R_o} * 100$$

$$= \frac{400}{400 + 10 * 10^6} * 100 = 0.004\ \%\quad \text{(almost nothing)}$$

$\therefore$ # $TV = AV_{DVM} = \dfrac{\sum \text{DVM readings}}{4} = 3.412$ Volts

$AV_{MCV} = \dfrac{\sum \text{MCV readings}}{5} = 3.22$ Volts

$\therefore$ Precision of MCV $= |3.22 - 3.24| = 0.02$ Volts

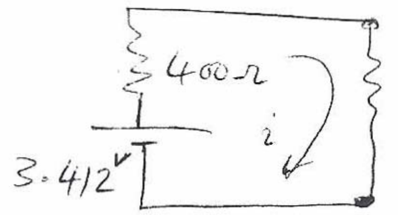
& Precision of DVM $= |3.412 - 3.413| = 0.001$ Volts = Accuracy of D

& Accuracy of MCV $= |3.412 - 3.20| = 0.212$ Volts

Only MCV readings need recalibration by adding

Bias $= 3.412 - 3.22 = 0.192$ Volts

Bias is due to drop at the 400 Ω battery resistance



$$i = \frac{0.192}{400} = .48 \text{ mA}$$

$$R_m' = \frac{3.412}{.48 * 10^{-3}} - 400 = 6.708 \text{ K}\Omega$$, I/p resistance of MCV

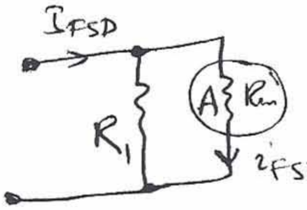
$\boxed{4\text{-}11}$ # $T_{FSD} = NBA\,i_{FSD} = 100 * .1 * 5*(10^{-2})^2 * \frac{V_{FSD}}{R_m} =$

$$= 5*10^{-3} * \frac{100*10^{-3}}{R_m} = \frac{5*10^{-4}}{R_m}$$

but $T_{FSD} = K\theta_{FSD} = 5*10^{-6}$

$\therefore \quad 5*10^{-6} = \frac{5*10^{-4}}{R_m} \qquad \therefore R_m = 100\,\Omega$

$\therefore i_{FSD} = \frac{100\,m}{100} = 1\,mAmp$

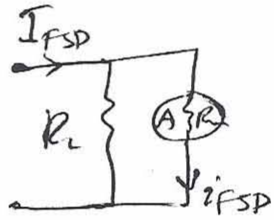


$I_{FSD} = 50\,mA$

$\therefore \frac{i_{FSD}}{I_{FSD}} = \frac{R_1}{R_1 + R_m}$

$\therefore \frac{50\,m}{1\,m} = 1 + \frac{R_m}{R_1} \Rightarrow 50 = 1 + \frac{R_m}{R_1} \quad \therefore \frac{R_m}{R_1} = 49$

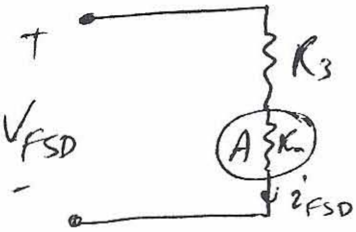
$\therefore$ Choose $R_1 = \frac{R_m}{49} = \frac{100}{49} = 2.041\,\Omega$ shunt.



$I_{FSD} = 1\,Amp$

$\therefore \frac{1}{1\,m} = 1 + \frac{R_m}{R_2} \quad \therefore \frac{R_m}{R_2} = 1000 - 1 = 999$

$\therefore$ choose R_2 to be $\frac{R_m}{999} = \frac{100}{999} = 0.1\,\Omega$ shunt



$V_{FSD} = 10\,V$

$\therefore R_3 = \frac{V_{FSD}}{i_{FSD}} - R_m = \frac{10}{1\,m} - 100$

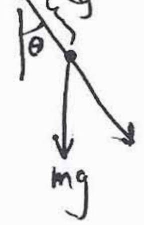
$= 10K - 100 = 9.9\,K\Omega$

$\therefore$ Choose R_3 to be $9.9\,K\Omega$ Series

$V_{FSD} = 200\,V$

$\therefore$ choose R_4 to be $\frac{200}{1\,m} - 100 = 200K - .1K = 199.9\,K\Omega$ Series

$\boxed{4\text{-}19}$ $T = BANi = mg \cdot C_g \cdot \sin\Theta$



$\therefore \; i = \dfrac{m \cdot g \cdot C_g \cdot \sin\Theta}{BAN} = \dfrac{10 * 10^{-3} * 9.81 * 2 * 10^{-2}}{0.3 * 4 * (10^{-2})^2 * 100} \cdot \sin\Theta$

$= 0.1635 \sin\Theta$ Amps

$\therefore$ for $\Theta = 30^\circ \Rightarrow i = 81.75$ mA

& for $\Theta = 60^\circ \Rightarrow i = 141.6$ mA

& for $\Theta = 90^\circ \Rightarrow i = i_{FSD} = 163.5$ mA

If i was increased beyond that no restoring torque can stop movement since the heaviest gravitational torque occurs $\Theta = 90$

$\boxed{4\text{-}20}$ $rms^2 = \frac{1}{T} \cdot \left[\frac{1}{3} * \frac{T}{4} * E^2 + E^2 * \frac{3T}{4}\right] = E^2 \cdot \left(\frac{1}{12} + \frac{3}{4}\right) = \frac{E^2}{12} * 10$

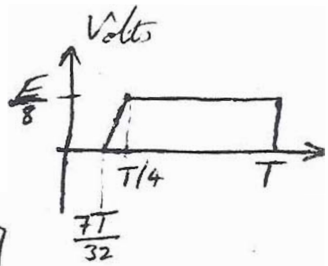
$\therefore \; rms = \sqrt{\frac{10}{12}}\, E$

$av. = \frac{1}{T} \cdot \left[\frac{1}{2} * \frac{T}{4} * E + E * \frac{3T}{4}\right] = E \cdot \left(\frac{1}{8} + \frac{3}{4}\right) = \frac{7E}{8} = dc$

$\therefore \; ac = \sqrt{rms^2 - dc^2} = \sqrt{\frac{10E^2}{12} - \frac{49E^2}{64}} = E\sqrt{\frac{160-147}{192}} = E\sqrt{\frac{13}{192}}$

Scale factor of HW-AC-MCV is $\frac{\pi}{\sqrt{2}}$

The signal input to the movement is as shown:



$\therefore$ Reading $= \frac{\pi}{\sqrt{2}} * $ Av. of it $= \frac{\pi}{\sqrt{2}} * \frac{1}{T} \cdot \left[\frac{E}{8} * \frac{T}{32} * \frac{1}{2} + \frac{E}{8} * \frac{3T}{4}\right]$

$= \frac{\pi}{\sqrt{2}} \cdot E \cdot \left(\frac{1+48}{512}\right) = \frac{49\pi}{512\sqrt{2}} \cdot E$

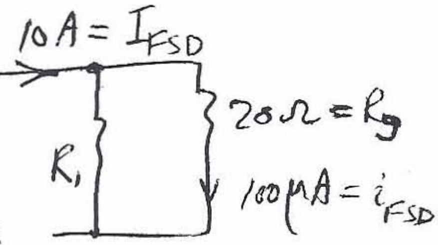
$\therefore \; \%e = \left| 1 - \frac{49\pi E / 512\sqrt{2}}{E\sqrt{13}/\sqrt{192}} \right| * 100 = 18.3\,\%$

4-21

a) $100\mu = 10 * \frac{R_1}{R_1+20}$

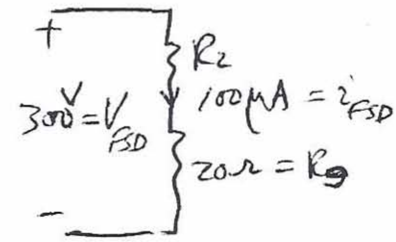
∴ $\frac{R_1+20}{R_1} = \frac{10}{100\mu} = 100K$

∴ $1+\frac{20}{R_1} = 100K \Rightarrow R_1 = 0.200002\,m\Omega$



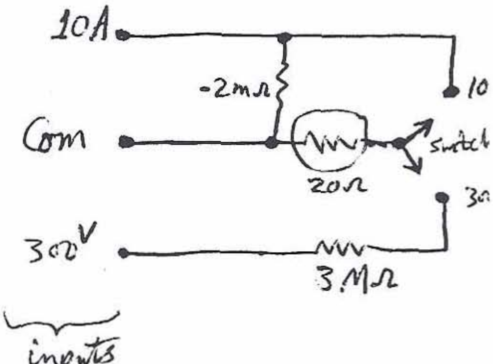
∴ We use a shunt of .200002 mΩ

& $\frac{300}{100\mu} = R_2 + 20 \Rightarrow R_2 = 2999980\,\Omega$



∴ We use a series of $\simeq 3\,M\Omega$

To arrange for both combination the following diagram is followed, where the current input is between Com & 10A terminals with switch on 10 Amp & the voltage input is between Com & 300V terminals with switch on 300V position.



b) $i_{FSD} = I_{FSD} * \frac{R_1}{R_1+R_g} \Rightarrow I_{FSD} = \left(\frac{R_1+R_g}{R_1}\right)\cdot i_{FSD} = \left(1+\frac{R_g}{R_1}\right) i_{FSD}$

∴ $\Delta I_{FSD} = \frac{i_{FSD}}{R_1}\Delta R_g + \left(1+\frac{R_g}{R_1}\right)\Delta i_{FSD} - \frac{i_{FSD}\cdot R_g}{R_1^2}\Delta R_1$

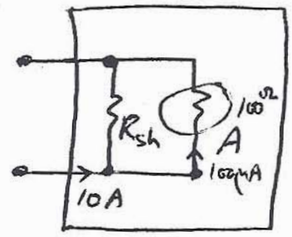
$= \frac{R_g}{R_1}\cdot i_{FSD}\left(\frac{\Delta R_g}{R_g} - \frac{\Delta R_1}{R_1}\right) + \left(1+\frac{R_g}{R_1}\right)\Delta i_{FSD}$

∴ $\Delta I_{FSD} = \sqrt{\left(\frac{20}{.2m}*100\mu\right)^2\cdot\left(.02^2+.1^2\right)+\left(1+\frac{20}{.2m}\right)^2(1\mu)^2} = 1.0247\ Amp = 10.25\%$

& $V_{FSD} = i_{FSD}*(R_g+R_2)$ ∴ $\Delta V_{FSD} = (R_g+R_2)\Delta i_{FSD} + i_{FSD}(\Delta R_g + \Delta R_2)$

∴ $\Delta V_{FSD} \simeq \sqrt{(3M)^2*(1\mu)^2+(100\mu)^2\cdot\left[(.02*20)^2+(.1*3M)^2\right]} = 30.15\ Volts = 10.05\%$

4-22 $R_{sh}\left(10 - 100 * 10^{-6}\right) = 100 * 100 * 10^{-6}$

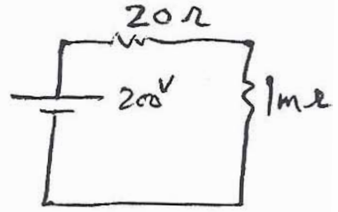


New Ammeter

∴ $R_{sh} = 1.00001\ m\Omega$

∴ We can obtain a 10 Amp range by shunting the movement with about 1mΩ resistor.

The new ammeter will now have 10 Amp FSD & an internal resistance $R_g = R_{sh} \| 100 = 1\,m\Omega$



a) The reading $= \dfrac{200}{20 + 1m} = 9.9995$ Amps

b) True value $= \dfrac{200}{20} = 10$ Amps

∴ Bias $= 10 - 9.9995 = .5$ mAmp

& $\%e = 0.005\%$

This error is caused by loading (little loading).

c) I) 20 A range ∴ $R_m = \dfrac{1}{50mV * 20} = 1\,\Omega$ ∴ $i = \dfrac{200}{20+1} = 9.5238$

∴ Reading is 9.52 Amp & $\%e = 4.8\%$ (too much loading little g

II) 200 A range ∴ $R_m = \dfrac{1}{50mV * 200} = 0.1\,\Omega$ ∴ $i = \dfrac{200}{20 + .1} = 9.95025$

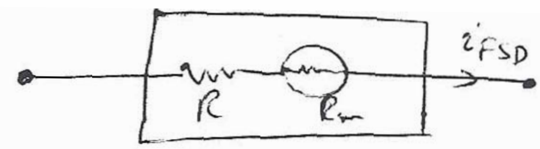
∴ Reading is 09.9 Amp & $\%e = 1\%$ (less loading high gating err

III) 2KA range ∴ $R_m = \dfrac{1}{50mV * 2K} = 10\,m\Omega$ ∴ $i = \dfrac{200}{20 + .01} = 9.99500$ A

∴ Reading is 009. Amp & $\%e = 10\%$ (least loading but highest gating err

∴ If DAM is used, the best range is the 200 Amp range since it gives least overall error of 1%.

4-23



$\therefore \frac{100}{i_{FSD}} = R + R_m \qquad \therefore R = \frac{100}{100\mu} - 100 = 1M - 100\,\Omega = 999.9\,K\Omega$

$\therefore$ We connect $999.9\,K\Omega$ series resistor to get 100 DCV voltmeter, hence V has $1M\Omega$ as R_m' with 100 DCV_{FSD}

ⓐ $\text{Reading} = 200 * \frac{(10K \,//\, 1M)}{20K + (10K \,//\, 1M)} = 66.2252\,\text{Volts}$

ⓑ $\text{bias} = \text{True Value} - AV = 200 * \frac{10}{10+20} - 66.2252 =$
$= 0.4415\,\text{Volts}$

$\%e = \frac{\text{bias}}{TV} = \frac{0.4415}{200/3} * 100 = 0.66\,\%$

ⓒ # 60V range $\quad \therefore R_{m_1} = 1K * 60 = 60\,K\Omega$

$\therefore \text{voltage}_1 = 200 * \frac{10 \,//\, 60}{20 + 10 \,//\, 60} = 59.9999\,\text{volts}$

$\therefore \text{Reading}_1 = 59.9\,\text{Volts} \quad \& \quad \%e = 10.0\,\%$

70V range $\quad \therefore R_{m_2} = 1K * 70 = 70K$

$\therefore \text{voltage}_2 = 200 * \frac{10 \,//\, 70}{20 + 10 \,//\, 70} = 60.8696\,\text{volts}$

$\therefore \text{Reading}_2 = 60.8\,\text{volts} \quad \& \quad \%e = 8.7\,\%$

80 V range $\quad \therefore R_{m_3} = 1K * 80 = 80K$

$\therefore \text{voltage}_3 = 200 * \frac{10 \,//\, 80}{20 + 10 \,//\, 80} = 61.5385\,\text{Volts}$

$\therefore \text{Reading}_3 = 61.5\,\text{Volts} \quad \& \quad \%e = 7.7\,\%$

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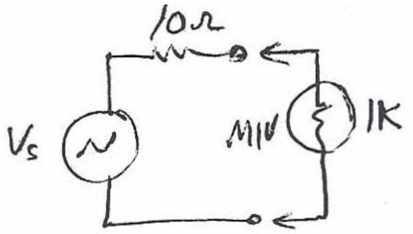
$\therefore$ we select the 80^V range since it gives least error of 7.7%

$\boxed{5\text{-}3}$ The rms as in $\boxed{4\text{-}20}$ is given by



$$rms = \sqrt{\frac{10}{12}} \cdot E = V_{s_{rms}}$$

$$\therefore \text{ reading of MIV} = \left(\frac{1K}{10+1K}\right) V_{s_{rms}}$$

$$= \frac{1000}{1010} * \sqrt{\frac{10}{12}} E = 0.9038\, E$$

$$\%e = \left| 1 - \frac{1000\sqrt{10}\, E / 1010\sqrt{12}}{\sqrt{10}\, E / \sqrt{12}} \right| * 100 = \left| 1 - \frac{1000}{1010} \right| * 100 = \frac{1000}{1010} \simeq 1\%$$

$\therefore$ Error is about 1% and it is due to loading of MIV.

$\boxed{5\text{-}4}$ MIV reads rms straight with no loading, hence it is correct

(a) $\therefore V_{rms} = MIV_{reading} = \sqrt{\left(100^2 * \frac{T}{3} * \frac{1}{3} + 100^2 * \frac{T}{3} + 0\right) / T} =$

$$= 100 \sqrt{\frac{1}{9} + \frac{1}{3}} = 100 \sqrt{\frac{1+3}{9}} = 100 * \frac{2}{3} = \frac{200}{3} = 66.\bar{6}$$

$\therefore$ MIV reading = RMS of waveform = $66.\bar{6}$ Volts

MCV reading = 1.11 * Av of FWR signal =

$$= 1.11 * \left[\left(100 * \frac{T}{3} * \frac{1}{2} + 100 * \frac{T}{3} + 0\right) / T\right]$$

$$= 1.11 * 100 \left(\frac{1}{6} + \frac{1}{3}\right) = 1.11 * 100 * \frac{1}{2} = 55.54$$

$\therefore$ MCV reading = 55.54 Volts

(b) MCV has an error of 11.13 Volts = 16.7%

(c) Error in MCV is due to waveform since it is made to measure sinusoidal voltage only not other waveforms

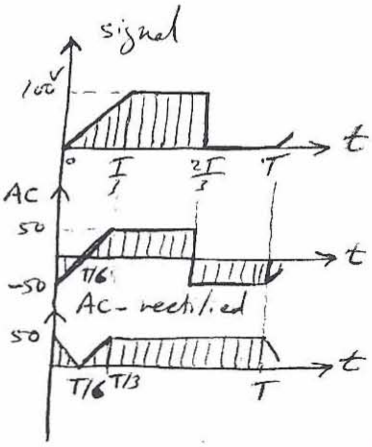
$\boxed{5-5}$ $RMS^2 = \frac{1}{T}\left[100^2 * \frac{1}{3} * \frac{T}{3} + 100^2 * \frac{T}{3} + 0\right] = \frac{100^2}{9}(1+3) = \frac{4}{9} * 100^2\, V^2$

$\therefore\ RMS = \frac{2*100}{3} = \frac{200}{3} = 66.\bar{6}$ volts

$DC = \frac{1}{T}\left[100 * \frac{1}{2} * \frac{T}{3} + 100 * \frac{T}{3} + 0\right] = \frac{100}{6}(1+2) = 50$ Volts

$\therefore\ AC = \sqrt{66.\bar{6}^2 - 50^2} = 44.09586$ volts

a) The MIV will read the RMS i.e $66.\bar{6}$ volts with 0%e.
The MCV will read good for AS sinusoidals.



$\therefore$ MCV reading $= \frac{\pi}{2\sqrt{2}} * (AC - rectified)_{av}$

$= \frac{\pi}{2\sqrt{2}} * \left[\frac{1}{T}\left(50*T - 50*\frac{T}{3}*\frac{1}{2}\right)\right]$

$= \frac{\pi}{2\sqrt{2}} * 50\left(1 - \frac{1}{6}\right) = \frac{25\pi}{\sqrt{2}} \cdot \frac{5}{6} = \frac{125\pi}{6\sqrt{2}}$

$\therefore$ MCV reading is 46.2800 Volts $\simeq$ AC

b) Since MIV gives RMS & MCV gives approximately AC

$\therefore\ DC = \sqrt{(RMS)^2 - (AC)^2} \cong \sqrt{(MIV)^2_{read} - (MCV)^2_{read}} = \sqrt{66.\bar{6}^2 - 46.28^2} = 47.985$

$\therefore$ My measurement will be the square-root of the squares-difference of both readings $= 47.985$ Volts

c) $\%e = \left|1 - \frac{47.985}{50}\right| * 100 = 4.03\%$

This error is caused by MCV and is due to waveform factor

$\boxed{5-6}$ $Av. = \frac{4-2}{2} = 1$ volt $=$ dc
$AC = \frac{4+2}{2\sqrt{3}} = \sqrt{3}$ volts
$RMS = \sqrt{AC^2 + dc^2} = \sqrt{3+1} = 2$ Volts

$\therefore$ MIV reads RMS of 2 Volts & $\%e = 0\%$
& MCV reads $1.11 * \frac{3}{2} = 1.666$ Volts & $\%e = 16.7\%$ (waveform error (frac+odc))

$\boxed{6-8}$ $(10+j\omega L)\left(18+\frac{1}{j\omega C}\right) = 20 * 12$

$\therefore (10+j\omega L)(1+j\omega C(18)) = 20*12\, j\omega C$

$\therefore 10 - \omega^2 LC(18) = 0 \implies \omega^2 LC = \frac{10}{18}$ (1)

& $180\,\omega C + \omega L = 240\,\omega C \implies \omega L = 60\,\omega C$ OR $L = 60C$ (2)

(2) into (1) $\therefore \omega^2 (60C) C = \frac{10}{18}$

$\therefore \omega^2 C^2 = \frac{10}{18*60} = \frac{1}{108}$ $\therefore C = \frac{1}{\omega\sqrt{108}} = \frac{1}{2\pi*60\sqrt{108}}$
$= 2.552*10^{-4}\,F$

$\therefore C = 255.2\,\mu F$

& $L = 60C = 15314.7\,\mu h = 15.315\,mh$

L is $15.3\,mh$ & C is $255.2\,\mu F$

$\boxed{6-9}$ At balance i.e at $25°C$ $R^2 = 4M$ $\therefore R = 2K$
Assuming little variation with temperature & taking equivalent at (A) ends:

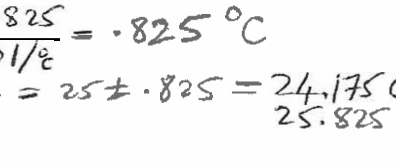
$\therefore R_{th} = (R \| 4K) + (R \| 1K) \approx 2\|4 + 2\|1 = \frac{8}{6} + \frac{2}{3} = 2K$

& $V_{th} = \left|12 * \left[\frac{4}{4+R+\Delta R} - \frac{R+\Delta R}{1+R+\Delta R}\right]\right| = \left|12 * \frac{4(3+\Delta R)-(2+\Delta R)(6+\Delta R)}{(6+\Delta R)(3+\Delta R)}\right|$

$= \left|\frac{12*(12+4\Delta R - 12 - 8\Delta R - (\Delta R)^2)}{(6+\Delta R)(3+\Delta R)}\right| \simeq \left|\frac{12*(-4\Delta R)}{18}\right| = \left|\frac{8\Delta R}{3}\right|$

$\therefore V_{th} = \frac{16}{3}\cdot\left|\frac{\Delta R}{R}\right|$ volts

$\therefore i = 20\,\mu A = \frac{V_{th}}{R_{th}+200} = \frac{\frac{16}{3}\left|\frac{\Delta R}{R}\right|}{2000+200}$



$\therefore \left|\frac{\Delta R}{R}\right| = .00825 = |\alpha\,\Delta T|$ $\therefore |\Delta T| = \frac{.00825}{.01/°C} = .825\,°C$
assumption is justified and new temperature $= 25 \pm .825 = 24.175\,C$
$25.825\,°C$

8-5 True power $= \frac{240^2}{20^2+18^2} * 20 = 1591.2\ W = 1.5912\ KW$

True pf $= \frac{20}{\sqrt{20^2+18^2}} = 0.74329$

(a) Wattmeter Reading $= 240 * \frac{240}{|21+j19|} * \frac{21}{\sqrt{21^2+19^2}} = 1508.2W = 1.5082\ KW$

$\therefore$ Reading $= 1.5082$ KW $\quad$ $f\%e = 5.212\%$

(b) pf reading $= \frac{21}{\sqrt{21^2+19^2}} = 0.74154$

$\therefore$ Reading $= 0.74154$ $\quad$ $f\%e = 0.2366\%$

8-6 a) Ideal power $= \left(\frac{240}{\sqrt{20^2+18^2}}\right)^2 * 20 = 1591.16$ Watts

Reading = power consumed by (load // voltage coil) =

(load // voltage coil) $= (20+j18) // (1+j1)K = 26.40553\angle 42.04^\circ$

$\therefore$ Reading $= \left|\frac{240}{1+j1+26.40553\angle 42.04}\right|^2 * 26.40553 \cos 42.04^\circ$

$= 1459.633$ watts

$\therefore \%e = 8.27\%$

b) Ideal pf $= \frac{20}{\sqrt{20^2+18^2}} = 0.743294$

Reading = pf of (load // voltage coil) $= \cos 42.04^\circ = 0.742637$

$\therefore \%e = 0.0884\% \approx .1\%$

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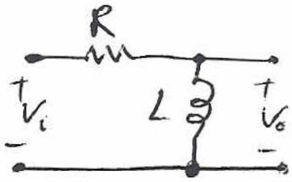
9-2 $\frac{V_o}{V_i} = \frac{sL}{R+sL} = \frac{1}{1+\left(\frac{R}{sL}\right)}$



at dc : $s=0$ $\therefore \frac{V_o}{V_i}=0$

at ac high frequency: $s=\infty$ & $\frac{V_o}{V_i}=1$

∴ This is a high-pass filter.

$\therefore f_h = \infty$ Hz

& $f_L = \frac{\omega_L}{2\pi}$ where ω_L is given by $\frac{R}{\omega_L L}=1 \Rightarrow \omega_L = \frac{R}{L}$

$\therefore f_L = \frac{R}{2\pi L}$

for $f = 2f_L = \frac{R}{\pi L}$ $\therefore \omega = 2\pi f = \frac{2R}{L}$ $\therefore s = \frac{j2R}{L}$ $\therefore sL = j$

$\therefore \frac{V_o}{V_i} = \frac{j2R}{R+j2R} = \frac{j2}{1+j2} = \frac{2}{\sqrt{5}}\angle 90-\tan^{-1}2 = \frac{2\sqrt{5}}{5}\angle 26.6^\circ = .894\angle 26.$

∴ Gain at $f=2f_L$ is $\underline{0.894}$

9-3 $\frac{V_o}{V_i} = \frac{sL}{R+sL} = \frac{j\omega L}{R+j\omega L}$ at $\omega=0$ $\therefore \frac{V_o}{V_i}=0$ at $\omega=\infty$ $\therefore \frac{V_o}{V_i}=1$

∴ It is a high pass filter & $\underline{f_H = \infty}$ Hz & $f_L = \frac{R/L}{2\pi} = \frac{10/10m}{2\pi} = 159.2$

9-4 $\frac{V_o}{V_i} = \frac{R}{R+sL} = \frac{1}{1+sL/R} = \frac{1}{1+j\omega\, 10m/10} =$

$= \frac{1}{1+j\omega m}$

$\therefore \omega_0 m = 1$ $\therefore \omega_0 = \frac{1}{m} = 1$ K rad/sec

$\therefore f_0 = \frac{\omega_0}{2\pi} = 159.2$ Hz

∴ The bandwidth is 159.2 Hz

69E