pesc '92
RECORD

23rd Annual
IEEE Power Electronics
Specialists Conference
Toledo, SPAIN

Formerly
POWER CONDITIONING SPECIALISTS CONFERENCE 1970-71
POWER PROCESSING AND ELECTRONIC SPECIALISTS CONFERENCE 1972

The 1992 PESC is sponsored by the
IEEE POWER ELECTRONICS SOCIETY
and is hosted by
UNIVERSIDAD POLITECNICA DE MADRID, SPAIN

IEEE

92CH3163-3

THE INDUCTANCE ANALYSIS OF THE TRAPIZOIDALLY RAILED LINEAR SYNCHRONOUS MOTOR AS COMPARED TO THE ZIG-ZAG VERSION FOR MAGLEV APPLICATIONS

Dr. Salman M. Al-Kasimi & Eng. A. O. BaQubas

Electrical and Computer Engineering Department,
Umm-Ul-Qura University, Saudi Arabia
Correspondance Address: P.O.Box: 6112, Makkah-Saudi Arabia

Abstract: *The trapizoidally railed linear synchronous motor provides a combined lift and thrust for maglev vehicles. Its magnet is U-shaped with split poles surrounded by inverter-fed coils distorting field among the four. Modified from Zig-Zag, its rail is trapizoidal and will propel to minimize reluctance. This paper obtains in theory the inductances of the new version compared to the old one for inverter design.*

INTRODUCTION

The Zig-Zag Linear Synchronous Motor, ZZLSM, providing a combined lift and thrust for maglev vehicles is shown in Fig1a. A meglev vehicle is a vehicle that is supported by magnetic attraction between a pair of iron tracks and controlled electromagnets fixed to its chassis. These are called boggies, one of which is shown in Fig1b. The main poles of ZZLSM are excited by field current through dc coils. Each of the main poles is split into two sub-poles that are surrounded by ac coils. McLean[1, 2] and West[1, 3] have chosen a rail shape shown in Fig.2a. Hence, with the ac coils around sub-poles excited such that the field is forced through sub-poles 1&3, the rail will move assuming constant air gap, z, to link sub-poles 1&3. When field is forced through sub-poles 2&4, the rail will move again to link sub-poles 2&4. As the switching of sub-pole coil-excitations synchronizes to rail position, a continuous motion is obtained. The ZZLSM is unable, due to symmetry, to start motion when the rail completely links sub-poles 1&3 or sub-poles 2&4 at a stop. This may be avoidable by another ZZLSM pair phased quarter of a cycle away.

Now for the same magnet, Al-Kasimi[4, 5] and BaQubas[4] noted that if a trapizoidally shaped rail like the one shown in Fig.2b is used, then the ac coils around sub-poles can be excited to force field through sub-poles 1&3. Hence, the rail will move assuming constant air gap, z, to link sub-poles 1&3. The field is then forced through sub-poles 2&3. Hence, pulling the rail to link sub-poles 2&3. Forced through sub-poles 2&4, the field then pulls the rail to link sub-poles 2&4. Finally, when the field is forced into sub-poles 1&4, the rail will move to link them both. This completes one cycle and as excitations of sub-pole ac coils are synchronized to rail position, a continuous motion is obtained.

This Trapizoidally Railed Linear Synchronous Motor, TRLSM, is noted to have no symmetry position throughout its cycle. Hence, no need for a starter exists. Another feature is that the switching frequency is half that of the ZZLSM for same speed, since the rail cycle length of the TRLHSM is twice that of the ZZLSM.

NOTATIONS

The symbols used in this paper are listed below:

- A sub-pole surface area
- A_i, A_i' overlap area of sub-pole i in TRLSM, ZZLSM respectively
- i_X, i_X' current flowing in phase X ac coils of TRLSM and ZZLSM respectively
- I_D, I_D' field current flowing in TRLSM, ZZLSM dc coils
- λ_n, λ_n' flux linkage of ac coil around sub-pole n in TRLSM, ZZLSM
- λ_p, λ_p' peak flux linkage per ac coil in TRLSM, ZZLSM
- λ_X, λ_X' flux linkage of phase X coils in TRLSM, ZZLSM
- L_X, L_X' self inductance of phase X coils in TRLSM, ZZLSM
- μ_0 permeability of air
- m, m' peak mmf excitation per phase per sub-pole in both TRLSM and ZZLSM respectively
- m_j, m_j' resultant mmf excitations around sub-pole j in both TRLSM and ZZLSM respectively
- m_X, m_X' ac mmf of phase X armature winding per sub-pole in TRLSM, ZZLSM
- M_{AD}' mutual inductance of phase A coils due to current I_D' in ZZLSM
- M_D, M_D' net dc mmf excitation of field winding per pole in TRLSM, ZZLSM
- M_{XD} mutual inductance of phase X coils due to current I_D in TRLSM
- N number of turns of ac coil around any sub-pole
- N_D number of turns of dc coil around any pole
- ω, ω' angular frequency of ac supply in TRLSM, ZZLSM
- ϕ_{ii} TRLSM flux due to m_i which leaves sub-pole i
- ϕ_{ij} TRLSM flux due to m_i which leaves sub-pole i to sub-pole j
- ϕ_k, ϕ_k' flux due to all sub-polar mmfs that leaves sub-pole k in TRLSM, ZZLSM
- p pole pitch
- θ, θ' phase angles of ac excitations in TRLSM, ZZLSM
- t time starting zero when rail completely links sub-poles 1&3
- T, T' time period to complete one cycle in TRLSM, ZZLSM
- u speed of rail relative to magnet
- z energized air gap

ASSUMPTIONS

For simplicity, the following assumptions were made in the analysis to come.

1. All sub-poles have the same surface area, A.
2. AC coils around sub-poles are identical.
3. Width of slot in the main poles is negligible.
4. Fringing and leakage are negligible.
5. Steel and copper losses are negligible.
6. Air gap, g, is homogeneous over the sub-poles.
7. The energized air-gap faces equal areas both at rail surface above it and at magnet pole surface below it throughout motion. This is justified in view of assumption 3 above.
8. The energized area per pole is composed of two portions. Each portion belongs to one of the two adjacent sub-poles within the main pole and is assumed to vary sinusoidally between zero and A.
9. Speed of motor relative to track, u, is constant.
10. Field mmf excitations are constants.
11. Flux linkage for any ac coil varies sinusoidally between a minimum of zero when its sub-pole is fully uncoupled to rail and a maximum when its sub-pole is fully coupled to rail.
12. AC coils are fed with sinusoidal ac current phase-locked to rail.
13. Windage, friction and other drag forces are ignored.

THEORITICAL ANALYSIS OF TRLSM INDUCTANCES

Coil Connections for TRLSM

When the rail of TRLSM moves in the direction indicated in Fig.2b, flux linkages of ac coils will assume linear segments and can be approximated to sinusoidal variation as shown in Fig.3a. Note that λ_1 & λ_2 vary in-phase with each another and that λ_3 & λ_4 also vary in-phase with each another but in quadrature lag with λ_1 & λ_2 variations. The period of these variations is given by:

$$T = 4p/u,$$

with angular frequency, ω, given by:

$$\omega = \pi u/2p. \qquad (2)$$

Hence, to obtain maximum variation in flux linkage per phase, the ac coils are connected as shown in Fig.4a. Note that phase A leads B by a quarter of a cycle, and that the dc coils are connected so as to build constructive flux components.

Magnetic Sub-polar Excitations for TRLSM

Each of the sub-poles, in Fig.4a could be considered excited with two components:

1- dc due to the field current I_D flowing in the dc coil of N_D turns, thus giving a dc excitation magnitude of:

$$M_D = N_D\, I_D, \qquad (3)$$

and:

2- ac due to the armature phase currents i_A or i_B flowing in the ac coil of N turns, thus giving an ac excitation magnitude of:

$$m_A(t) = N\, i_A(t) = m\cos(\omega t + \theta), \qquad (4)$$

or:

$$m_B(t) = N\, i_B(t) = m\sin(\omega t + \theta). \qquad (5)$$

Using Fig.4a and denoting m_1, m_2, m_3 and m_4 to be the resultant mmf excitations around sub-poles 1, 2, 3 and 4 respectively, with positive sense when forcing flux to leave the sub-pole, then:

$$m_1(t) = -M_D - m_A(t), \qquad (6)$$

$$m_2(t) = -M_D + m_A(t), \qquad (7)$$

$$m_3(t) = M_D + m_B(t)\ \& \qquad (8)$$

$$m_4(t) = M_D - m_B(t). \qquad (9)$$

Energized Sub-polar Areas for TRLSM

With A_1, A_2, A_3 and A_4 defined as above; than these areas vary trapizoidally as shown in Fig.3a during the rail motion indicated in Fig.2b. Note that the sum of any two adjacent areas is constant and equals A.

For simplicity, these trapizoidal variations are assumed sinusoids between a maximum of A and a minimum of zero as shown by the dotted lines in Fig.3a. The equations describing these approximations are:

$$A_1(t) = (A/2)\,[1 + \cos(\omega t + \pi/4)], \qquad (10)$$

$$A_2(t) = (A/2)\,[1 - \cos(\omega t + \pi/4)], \qquad (11)$$

$$A_3(t) = (A/2)\,[1 + \cos(\omega t - \pi/4)]\ \& \qquad (12)$$

$$A_4(t) = (A/2)\,[1 - \cos(\omega t - \pi/4)]. \qquad (13)$$

Magnetic Sub-polar Fluxes for TRLSM

Let ϕ_{ij} be the flux due to m_i that leaves sub-pole i to sub-pole j. Hence, each of m_1, m_2, m_3 and m_4 mmfs is driving an associated magnetic circuit similar to that shown in Fig.5. Hence, using the equations above for A_i, then:

$$\phi_{ii}(t) = \mu_0\, A_i\, m_i\, (2A - A_i)/2A\,g\ \& \qquad (14)$$

$$\phi_{ij}(t) = \mu_0\, A_i\, m_i\, A_j/2A\,g. \qquad (15)$$

Hence, applying superposition theorm to find the flux $\phi_1(t)$ leaving sub-pole 1 due to all mmfs of m_1, m_2, m_3 and m_4; then:

$$\phi_1(t) = \phi_{11} - \phi_{21} - \phi_{31} - \phi_{41}.$$

Using (14) & (15) for ϕ_{11}, ϕ_{21}, ϕ_{31} & ϕ_{41} respectively and rearranging, then:

$$\phi_1(t) = \mu_0\, A_1\, [A_2(m_1 - m_2) + A_3(m_1 - m_3) + A_4(m_1 - m_4)]/2A\,g.$$

Using (6), (7), (8) & (9) for m_1, m_2, m_3 & m_4 and rearranging then:

$$\phi_1(t) = -\mu_0\, A_1\, [2M_D(A_3 + A_4) + m_A(2A_2 + A_3 + A_4) + m_B(A_3 - A_4)]/2A\,g.$$

Using (11), (12) & (13) for A_2, A_3 & A_4 and simplifying, then:

$$\phi_1(t) = -\mu_0\, A_1\, [2M_D + m_A\{2 - \cos(\omega t + \pi/4)\} + m_B\cos(\omega t - \pi/4)]/2g. \qquad (16)$$

Similariy: ϕ_2, ϕ_3 & ϕ_4 can be found to be:

$$\phi_2(t) = -\mu_0\, A_2\, [2M_D - m_A\{2 + \cos(\omega t + \pi/4)\} + m_B\cos(\omega t - \pi/4)]/2g, \qquad (17)$$

$$\phi_3(t) = \mu_0\, A_3\, [2M_D + m_B\{2 - \cos(\omega t - \pi/4)\} + m_A\cos(\omega t + \pi/4)]/2g\ \& \qquad (18)$$

$$\phi_4(t) = \mu_0\, A_4\, [2M_D - m_B\{2 + \cos(\omega t - \pi/4)\} + m_A\cos(\omega t + \pi/4)]/2g. \qquad (19)$$

Magnetic Flux Linkages for TRLSM

Now; the flux linkage λ_A of phase A can be seen from Fig.4a to be:

$$\lambda_A = N(\phi_2 - \phi_1).$$

Using (16) & (17) for ϕ_1 & ϕ_2 and simplifing, then:

$$\lambda_A(t) = -\mu_0 N [2(A_2 - A_1) M_D - m_A \{2(A_2 + A_1) + (A_2 - A_1) cos(\omega t + \pi/4)\} + (A_2 - A_1) m_B cos(\omega t - \pi/4)]/2z.$$

Using (10) & (11) for A_1 & A_2 and simplifying, then:

$$\lambda_A(t) = -\mu_0 A N [-2 M_D cos(\omega t + \pi/4) - m_A \{2 - cos^2(\omega t + \pi/4)\} - m_B cos(\omega t - \pi/4) cos(\omega t + \pi/4)]/2z.$$

This is simplified further to give:

$$\lambda_A(t) = \mu_0 A N [4 M_D cos(\omega t + \pi/4) + m_A \{3 + sin(2\omega t)\} + m_B cos(2\omega t)]/4z. \quad (20)$$

Similarly, the flux linkage λ_B of phase B can be found as:

$$\lambda_B(t) = \mu_0 A N [4 M_D cos(\omega t - \pi/4) + m_B \{3 - sin(2\omega t)\} + m_A cos(2\omega t)]/4z. \quad (21)$$

Self and Mutual Inductances for TRLSM

With inductance L_A, L_B, M_{AB}, M_{BA}, M_{AD} & M_{BD} as defined earlier and using (20), (21), (3), (4) & (5), then: L_A is the flux linkage of phase A coils due to unit current in phase A coils and is given as:

$$L_A = \mu_0 A N^2 \{3 + sin(2\omega t)\}/4z. \quad (22)$$

L_B is the flux linkage of phase B coils due to unit current in phase B coils and is given as:

$$L_B = \mu_0 A N^2 \{3 - sin(2\omega t)\}/4z. \quad (23)$$

M_{AB} is the flux linkage of phase A coils due to unit current in phase B coils and is given as:

$$M_{AB} = \mu_0 A N^2 cos(2\omega t)/4z. \quad (24)$$

M_{BA} is the flux linkage of phase B coils due to unit current in phase A coils and is given as:

$$M_{BA} = \mu_0 A N^2 cos(2\omega t)/4z = M_{AB}. \quad (25)$$

M_{AD} is the flux linkage of phase A coils due to unit field current and is given as:

$$M_{AD} = \mu_0 A N N_D cos(\omega t + \pi/4)/z \;\& \quad (26)$$

M_{BD} is the flux linkage of phase B coils due to unit field current and is given as:

$$M_{BD} = \mu_0 A N N_D cos(\omega t - \pi/4)/z. \quad (27)$$

REVIEW ANALYSIS OF ZZLSM INDUCTANCES

Coil Connections for ZZLSM

When the rail of ZZLSM moves in the direction indicated in Fig.2a, flux linkages of ac coils will assume linear segments and can be approximated to sinusoidal variation as shown in Fig.3b. The period of these variaions is given by:

$$T' = 2p/u, \quad (28)$$

with angular frequency, ω', given by:

$$\omega' = \pi u/p. \quad (29)$$

Hence, to obtain maximum variation in flux linkage per phase, the ac coils are connected as shown in Fig.4b.

Magnetic Sub-polar Excitations for ZZLSM

With treatment similar to that of TRLSM, then the sub-polar excitations in Fig.4b could be given as:

$$m'_1(t) = -m'_3(t) = -M'_D - m'_A(t), \quad (30)$$

$$m'_2(t) = -m'_4(t) = -M'_D + m'_A(t), \quad (31)$$

where M'_D & $m'_A(t)$ are given by:

$$M'_D = N_D I'_D \;\& \quad (32)$$

$$m'_A(t) = N i'_A(t) = m' cos(\omega' t + \theta'). \quad (33)$$

Energized Sub-polar Areas for ZZLSM

The ZZLSM sub-polar areas vary triangularly as shown in Fig.3b during the rail motion indicated in Fig.2a. These variations are assumed sinusoids with equations given by:

$$A'_1(t) = A'_3(t) = (A/2)[1 + cos(\omega' t)] \;\& \quad (34)$$

$$A'_2(t) = A'_4(t) = (A/2)[1 - cos(\omega' t)]. \quad (35)$$

Magnetic Sub-polar Fluxes for ZZLSM

With similar treatment to that of TRLSM, then these fluxes are given by:

$$\phi'_1(t) = -\phi'_3(t) = -\mu_0 A'_1 (M'_D + m'_A)/z \;\& \quad (36)$$

$$\phi'_2(t) = -\phi'_4(t) = -\mu_0 A'_2 (M'_D - m'_A)/z. \quad (37)$$

Magnetic Flux Linkages for ZZLSM

Now; the flux linkage λ'_A of phase A can be seen from Fig.4b to be:

$$\lambda'_A = N(\phi'_2 - \phi'_1 + \phi'_3 - \phi'_4).$$

Using (36) & (37) for ϕ'_1 & ϕ'_2 and simplifing, then:

$$\lambda'_A(t) = 2\mu_0 N [(A'_1 - A'_2) M'_D + (A'_1 + A'_2) m'_A]/z.$$

Using (34) & (35) for A'_1 & A'_2 and simplifying, then:

$$\lambda'_A(t) = 2\mu_0 A N [M_D cos(\omega t) + m_A]/z. \quad (38)$$

Self and Mutual Inductances for ZZLSM

With inductance L'_A, M'_{AD} as defined earlier and using (38), (32) & (33), then: L'_A is the flux linkage of phase A coils due to unit current in phase A coils and is given as:

$$L'_A = 2\mu_0 A N^2/z \;\& \quad (39)$$

M'_{AD} is the flux linkage of phase A coils due to unit field current and is given as:

$$M_{AD} = 2\mu_0 A N N_D cos(\omega t)/z. \quad (40)$$

CONCLUSION

The Zig-Zag Linear Synchronous Motor, ZZLSM, is a single phase linear machine that needs a starter; whereas the Trapizoidally Railed Linear Synchronous Motor, TRLSM, is a two-phase self-starting linear machine. Operating both machines with same linear speed. TRLSM is advantageous over ZZLSM in that it is operating at half the switching frequency.

The self inductance of ZZLSM is constant, whereas that of TRLSM per phase varies sinusoidally at twice the synchronous frequency between one half and one quarter of the value for ZZLSM self inductance.

The mutual inductances in both machines due to field vary sinusoidally at the synchronous frequency with ZZLSM peaking at twice the peak of TRLSM.

Finally, the mutual inductances in TRLSM due to other-phase current vary sinusoidally at twice the synchronous frequency, peaking at half the minimum value (or equally quarter the maximum value) of the self inductance.

ACKNOWLEDGMENTS

The authors would like to express their thanks to Dr. S. Abu-Shadi at The Electrical Engineering Department of King Abdul-Aziz University in Saudi Arabia for his valuable assistance and comments. They would also like to thank Eng. Mahir I. Rajab, a graduate of The Electrical Engineering Department at Umm-Ul-Qura University in Saudi Arabia, for his positive help and contributions.

REFERENCES

[1] G. W. McLean and A. N. West, "Combined Lift and Thrust for Maglev Vehicles Using the Zig-Zag Synchronus Motor", in Proceedings of The International Conference on Maglev Transport Now and for The Future, IMechE, 1984, pp. 87-97.

[2] G. W. McLean, "Review of Recent Progress in Linear Motors", IEE Proceedings, 1988, Vol. 35, Pt. B, No. 6, pp. 380-416.

[3] A. N. West, "The Control of a Linear Homopolar Synchronous Machine", PhD Thesis, Electrical Engineering Dept., Manchester University, 1982.

[4] S. M. Al-Kasimi and A. O. BaQubas, "The Acceleration Characteristics of The Trapizoidally Railed Linear Homopolar Synchronous Motor as Compared to The Zig-Zag Version for Maglev Applications", in Proceedings of Al-Azhar Engineering Second International Conference, AEC'91, Al-Azhar University, Egypt, Dec.1991, Vol. 6, pp. 392-409.

[5] S. M. Al-Kasimi, "A New Synchronous Motor for Mag-Lev Train With Integrated Lift and Thrust Using A Trapizoidal Rail", in Proceedings of The Second Middle East Power System Conference, MEPCON'92, Assiut University, Egypt, Jan.1992, pp, 67-69.

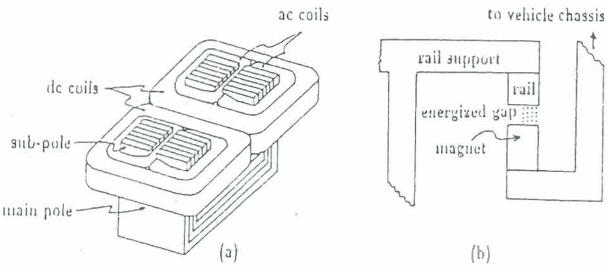

Fig1 (a) ZZLSM magnet and (b) vehicle boggie

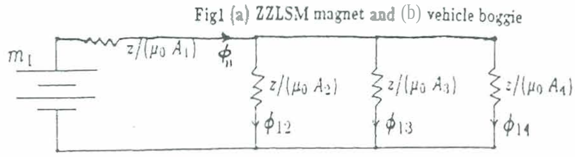

Fig5 The magnetic circuit driven by m_1 around sub-pole 1

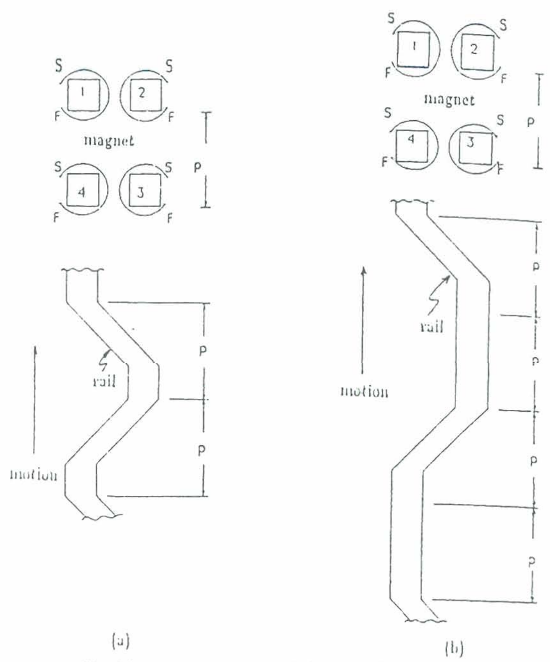

Fig2 Magnet and rail of (a) ZZLSM and (b) TRLSM

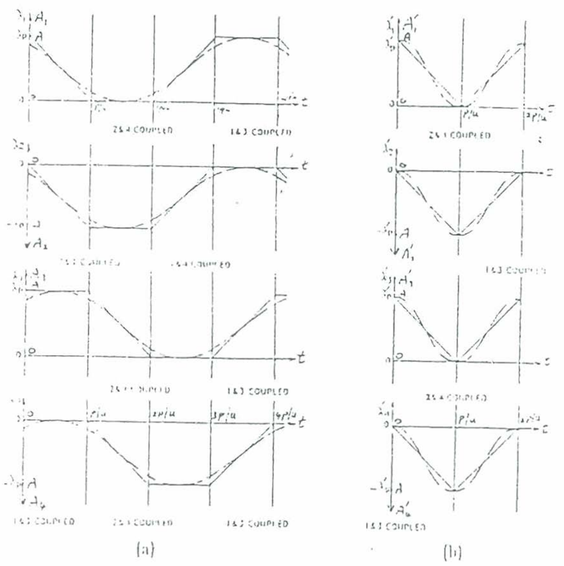

Fig3 Sub-polar flux-linkages and areas for (a) TRLSM and (b) ZZLSM

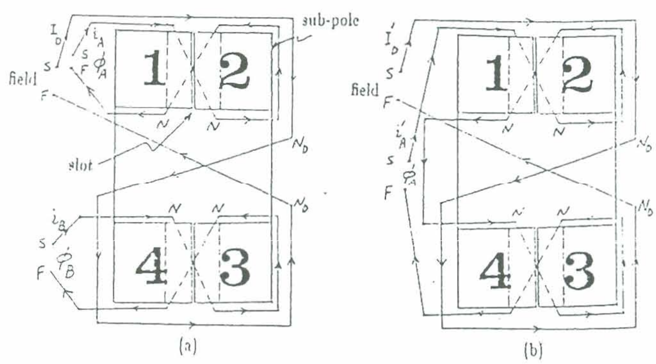

Fig4 Coil connections for (a) TRLSM and (b) ZZLSM